THEORY OF SOLAR OSCILLATIONS

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ABSTRACT

This paper reviews some of the most recent analyses of helioseismological data, and discusses briefly some theoretical issues they raise. It is intended not only to illustrate how currently available measurements are influencing our concept of the solar interior, but also to point the direction of some useful avenues of future enquiry. The properties of resonant modes of oscillation are briefly reviewed, for the purpose of subsequently understanding what aspects of the solar structure can be measured from their frequencies. A recent determination of the variation of sound speed through much of the interior of the sun is then described. The measurement reveals at least one error in a theoretical solar model, and together with some results described by Harvey in these proceedings suggests, albeit tentatively, the presence of macroscopic motion in the solar core. Future more sophisticated analyses of the data should provide us with important diagnostics of the outer layers of the sun, which hopefully will improve our understanding of the dynamics of the convection zone.

Keywords: Helioseismology, Solar interior, Solar rotation, Convection.

1. INTRODUCTION

Helioseismological data are now sufficiently extensive to enable us to impose quite detailed constraints on the internal structure of the sun. In the last year it has been possible to estimate the variation of the sound speed and the angular velocity throughout much of the solar interior. The former is in broad agreement with a conventional theoretical model of the sun, though there are systematic differences, which are of a form that had previously been suggested as a result of a direct comparison of the theoretical and observed frequencies. Now we are in a position to speculate what the cause of those differences might be. One possibility, which is discussed in this paper and which was motivated primarily by the measurements of angular velocity discussed by Harvey (Ref. 6), is that the central energy-generating core of the sun has recently started to overturn, possibly in an oscillatory fashion. The ramifications of this hypothesis are not discussed in detail here; the idea is raised to demonstrate that helioseismological diagnosis has now reached the point where it can seriously influence our thinking about the dynamics of the deep interior of the sun, with consequences that will be possible to test with more extensive data of a kind that we can reasonably expect to be able to acquire in the foreseeable future.

2. PROPERTIES OF OSCILLATION EIGENFREQUENCIES

Any dynamical scalar perturbation of a spherically symmetrical star can be decomposed into modes, each of which can be written as the product of a function of radius r, a sinusoidal function of time t with frequency ω, and a spherical harmonic of degree ℓ and order m. The sun appears to be almost spherically symmetrical, and therefore its oscillation eigenfunctions are very close to this form. I shall ignore the effects of symmetry breaking for the present. Then for each value of ℓ there are two discrete sequences of eigenfunctions in r, associated with well defined frequencies in nonoverlapping ranges, that are labelled with an integer n, called the order of the mode. Low-frequency modes are g modes; high-frequency modes are p modes. On the whole they may be regarded as standing internal gravity waves, and acoustic waves respectively. This description is unambiguous except when both n and ℓ are low; then there can be a mixture of character, with buoyancy dominating the restoring force in one part of the star and acoustic compression and rarefaction being more important elsewhere. When this is not the case, n measures the number of nodes in the vertical component of velocity. Counting starts at n = 1, and ω decreases or increases with n for g and p modes respectively. In addition there is an f mode, whose frequency lies between those of the g modes and the p modes, and except when ℓ is small it has no nodes; it is essentially a surface gravity wave.

I shall describe the radial variation of the eigenfunction by considering the function \( \Psi = \phi \xi \text{div} \Psi \), where \( \xi \) is the displacement of an oscillating fluid element from its equilibrium position, and \( \rho \) and \( c \) are the density and sound speed in the unperturbed state. I have chosen this variable because it approximately satisfies the simple equation

\[
\Psi'' + \kappa \Psi = 0, \tag{1}
\]

where a prime denotes differentiation with respect to \( r \). Here