Shock Wave Structure in the Radiation Spectrum During Bose Condensation of Photons

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The spectrum of photons interacting with electrons via the induced Compton effect is considered. Assuming weak energy transfer per collision, we previously showed that a discontinuity may arise in the dependence of intensity on frequency ("shock wave" in phase space). In the present paper it is shown that if the finite temperature of the electrons is taken into account, the slope of the spectrum increases only up to a certain limit, after which an oscillatory dependence of intensity on frequency arises on the shortwave side of the spectrum. The structure of the shock wave is thus found to be more complex than previously assumed. It is similar to a collisionless wave in a plasma and not to a viscous wave in a neutral gas.

1. INTRODUCTION

The discovery of powerful compact sources of low-frequency radiation in astronomical objects has attracted attention to the interaction between intense electromagnetic waves having a brightness temperature \( kT_b \gg m_e c^2 \) and free electrons.

Let us consider a spatially-homogeneous and isotropic situation, wherein the radiation spectrum is specified by the spectral energy density \( \sigma_\nu \) of the radiation or by the occupation number \( n(\nu) = (c^2/8\pi\hbar\nu^4)\sigma_\nu \).

The space is filled with electrons with density \( N_e \) and temperature \( T_e \), which we assume constant and, although the results remain unchanged for a variable, and particularly self-consistent temperature (which the electrons acquire in the given radiation field).

The scattering of the radiation by the free electrons leads to a redistribution of the energy and of the intensity over the spectrum; thus, the object of the investigation is the function \( \sigma_\nu(t, \nu) \), where \( t \) is the time. Instead of using \( \sigma_\nu \), we can characterize the radiation by the brightness temperature \( T_b \), which is connected with \( \sigma_\nu \) (we are considering long-wave unpolarized radiation) by the Rayleigh-Jeans relation

\[ \sigma_\nu = 8\pi kT_b \nu^3, \quad \sigma_\nu = 8\pi kT_c \nu^3 / c^2. \]

Since \( \sigma_\nu \) is in the general case not in equilibrium, \( T_b(\nu, t) \) is also a function of the frequency. We are investigating the case of long-wave radiation of high-intensity, so that \( T_b \gg T_e \) in a wide frequency range.

As is well known\(^{1,2}\), the interaction of the radiation having the higher brightness temperature with the colder electrons is then accompanied by drawing of energy from the radiation\(^{1,2}\), as a result of which the radiation spectrum is altered in the low-frequency region\(^{3,2}\). If, furthermore, \( kT_b \gg m_e c^2 \gg kT_e \), then (i) the induced scattering is stronger than the spontaneous scattering and (ii) the integral equation for the realignment of the spectrum can be transformed into a differential equation at a spectrum width \( \Delta \nu \gg \Delta \nu_D = \nu' / 2kT_e / m_0 c^2 \). Here \( \Delta \nu_D \) is the Doppler width of the spectrum and corre-

\(^1\)To realize this case it is necessary that the electrons lose energy in some manner that does not depend on the considered interaction with the low frequency radiation.

\(^2\)This effect was apparently observed in experiment\(^{3,2}\).

\[ \frac{\partial \sigma}{\partial t} = \frac{\sigma_\nu N_e}{m_e} \frac{\partial}{\partial \nu} \left( \nu^3 + n + \frac{kT_e}{h} \frac{\partial \sigma}{\partial \nu} \right). \]

As applied to our problem, this equation can be written in simpler form

\[ \frac{\partial g}{\partial t} = \frac{\partial g}{\partial \nu}, \]

where

\[ g = \nu^3 n = \frac{c^2}{8\pi k} \sigma_\nu, \quad dt = \frac{2N_e}{m_e} \frac{\partial \sigma}{\partial \nu}; \quad \sigma_\tau = \frac{8\pi}{3} \left( \frac{c^2}{m_0 c^2} \right)^3. \]

This nonlinear equation was studied by Levich and one of the authors\(^{3,2}\); its characteristics are the lines

\[ \frac{\partial g}{\partial \nu} = -g, \]

corresponding to a decrease of frequency at a rate proportional to the spectral density of the radiation energy (more accurately, proportional to the quantity \( g \), which is connected with this density).

Under definite initial conditions, the spectrum evolution in accordance with Eq. (2) leads in the course of time to the formation of an infinite derivative \( \partial g / \partial \nu \). For this purpose it is necessary and sufficient that there exist a point of inflection on the low-frequency side of the \( g(\nu, 0) \) curve, i.e., the derivative \( \partial g(\nu, 0) / \partial \nu \) should have a maximum at definite values \( \nu = \nu_0 \) and \( g = g_0 \):

\[ \frac{\partial g}{\partial \nu}(\nu_0, 0) > 0, \quad \frac{\partial^2 g}{\partial \nu^2}(\nu_0, 0) = 0, \quad \frac{\partial g}{\partial \nu^2}(\nu_0, 0) < 0. \]

The situation is mathematically similar to nonlinear propagation of an acoustic wave in a gas, wherein the dependence of the wave velocity on the amplitude gives rise first to an infinite derivative \( \partial p / \partial x \rightarrow \infty \) and then to a shock wave. In analogy, the formation of a "shock wave" was predicted in\(^{2,3}\) also for the spectrum of the electromagnetic radiation under the conditions described above.

It should be noted that such situations were considered earlier as applied to plasma oscillations. A nonlinear equation similar to (2) was derived in\(^{4,5}\) for longitudinal plasma waves and it was noted that the evolution leads to a narrowing of the wave front. In its idea, this reference (see also\(^{3,2}\)) anticipates the results of\(^{4,5}\).