Solar Differential rotation Maintained by Small- and Large-scale Convection

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Abstract. We investigate the solar differential rotation with special interest for the near surface shear layer (NSSL) in a high-resolution hydrodynamic numerical calculation. The sun is rotating differentially. Helioseismology has revealed the detailed structure of the solar differential rotation. One of the most important features is the NSSL. It is thought that the solar differential rotation is maintained by the turbulent thermal convection. In the NSSL convection time scales are short, leading to a regime with weak influence of rotation on convection. In order to reproduce the NSSL by the numerical calculations, we must use a large number of grids and integrate a large number of time steps for covering the broad spatial and temporal scales. This requirements for the NSSL is achieved using our recent efficient numerical method. In the calculation, the global scale and the 10 Mm-scale convection is established simultaneously. Then the solar like NSSL is partially reproduced. Around the NSSL, the convection transports the angular momentum radially inward and generates the poleward meridional flow. The small scale convection acts as the turbulent viscosity on the meridional flow. The turbulent viscous stress balances with the Coriolis force in the NSSL.

1. Introduction

The sun is rotating differentially. The solar internal structure of the differential rotation is inferred by using the helioseismology (e.g. Thompson et al. 2003; Howe et al. 2011). It is revealed that the solar differential rotation has three features that significantly deviate from the Taylor-Proudman theory (∂Ω/∂z = 0, where Ω and z are angular velocity and the direction of rotational axis, respectively). These are the conical profile of differential rotation in the middle of the convection zone, the tachocline, and the near surface shear layer (NSSL; see Fig. 1). The tachocline is the radial shear layer of the angular velocity at the base of the convection zone. For the conical profile of the differential rotation and the tachocline, there have been several studies which reproduce these in numerical calculations and suggest the maintenance mechanism (Rempel 2005; Miesch et al. 2006; Brun et al. 2011). These two features are essentially maintained by the latitudinal entropy gradient, which is ignored in the Taylor-Proudman theory. When the polar region is hotter than the equator region, the gradient suppresses the redistribution of the angular momentum by the meridional flow. The NSSL is, however, thought to be maintained by the different mechanism. There are two reasons: 1. The entropy gradient is thought to be generated in the boundary layer between the convection and radiation
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Figure 1. Inversion of the helioseismology from HMI data about the angular velocity ($\Omega/(2\pi)$) in the unit of nHz (Howe et al. 2011) describing the tachocline, the convection zone and the NSSL.

zones (Rempel 2005). The NSSL is farther from there than the middle of the convection zone, while the deviation from the Taylor-Proudman theory is larger. Since the NSSL is located in the near surface layer, it is possible to detect the temperature difference if it exists. The recent observation does not show such difference at the surface (Rast et al. 2008). Thus, the inertial term in the equation of motion, $-\nabla \cdot (\rho \mathbf{v} \mathbf{v})$, is thought to have a role in the NSSL, which is also ignored in the Taylor-Proudman theory, where $\rho$ and $\mathbf{v}$ are the density and the velocity, respectively (Miesch & Hindman 2011). In the NSSL the temperature is low and the corresponding density scale height is much smaller than that in the middle of the convection zone. This causes the small-scale convection there. In addition, the small density requires the large convection velocity in order to transport the solar luminosity as the enthalpy flux, which is $\rho c_p T_1 v_r$ for the ideal gas, where $c_p$, $T_1$, and $v_r$ are the heat capacity at constant pressure, the perturbed temperature, and the radial velocity. This small and fast convection has a short time scale which leads to a weak influence from the rotation. Since the inertial term (convective effect) is thought to be comparable to the Coriolis force in the NSSL, the NSSL requires a precise understanding of the inertial term. In terms of the numerical simulation, the NSSL requires huge computational resource, since we need to achieve small-scale convection with the scale of about 10 Mm in the large computational domain of the whole sun whose circumference is 4,400 Mm. In addition the time scale of small scale convection in the
NSSL is about 1 hour, while the angular momentum transport requires about 10 years. These requires huge numbers of grid and time step. This is achieved with using recently developed highly-optimized code using the reduced speed of sound technique.

2. Model

In this study, 3D hydrodynamic equations are solved with using the reduced speed of sound technique (RSST: Rempel 2005; Hotta et al. 2012). In the technique the perturbed density \( \rho_1 \) is increased \( \xi^2 \) times:

\[
\rho = \rho_0 + \rho_1 \rightarrow \rho = \rho_0 + \xi^2 \rho_1.
\]

Then the equation of continuity is transformed as

\[
\frac{\partial \rho_1}{\partial t} = -\frac{1}{\xi^2} \nabla \cdot (\rho \mathbf{v}),
\]

and the speed of sound is reduced \( \xi \) times. The advantages of this method compared with the anelastic approximation are: 1. Only the local communication is required. This leads a good scaling in the parallel computing. 2. The near surface layer where the anelastic approximation is not valid is included without loosing important physics with using inhomogeneous \( \xi \). The realistic equation of state considering the ionization of the Hydrogen and Helium is used (Hotta et al. 2014). The number of grid points is \( 384(N_r) \times 648(N_\theta) \times 1944(N_\phi) \times 2 \) in the Yin-Yang grid. In order to deal with the accelerated-pole problem, 18 times smaller luminosity than that in the Model S is used (e.g. Fan & Fang 2014, for the accelerated-pole problem in the sun).

3. Result

Figs. 2 shows the contour of the radial velocity at different depths. In the deep layer, the north-south aligned convection cell is prominent outside the tangential cylinder (Fig. 2c). This is called “banana cell”, the indication of the rotational influence. Since the Coriolis force is proportional to \( \mathbf{v} \times \mathbf{\Omega} \), the convective parcel rotates around the rotational axis when the rotational influence is strong. This is an analogy to the Larmor motion for the plasma particle with the Lorentz force of \( \mathbf{v} \times \mathbf{B} \). When we see the upper layer the banana cell like feature becomes insignificant (Fig. 2b). The almost isotropic 10 Mm-scale convective cells are seen around the top boundary (Fig. 2a). This shows that the calculation achieves two regimes with strong and weak rotational influence simultaneously.

The obtained differential rotation and the meridional flow are shown in Fig. 3. The equator acceleration is achieved thanks to the reduced luminosity and the reduced convective velocity. The NSSL like feature is reproduced especially in the high latitude. The differential rotation obeys the Taylor-Proudman theory in the middle of the convection zone, since we do not include the radiation zone, which is thought to have a role to generate the entropy gradient. The coherent poleward meridional flow is generated in the near-surface layer. This is caused by the inward angular momentum transport by the Reynolds stress. When the rotational influence is weak, the correlation \( \langle v'_r v'_\phi \rangle \) becomes negative. The angular momentum transports by the Reynolds stress and the poleward meridional flow are balanced.
We show the φ-component vorticity equation in order to understand the dynamics in the meridional plane.

\[
\frac{\partial (\omega_\phi)}{\partial t} = - \left( \nabla \times \left[ \frac{\nabla \cdot (\rho_0 vv)}{\rho_0} \right] \right)_{\phi} + r \sin \theta \frac{\partial (\Omega)^2}{\partial z} + \frac{g}{\rho_0 r} \left( \frac{\partial \rho}{\partial s} \right) \frac{\partial (s_1)}{\partial \theta}.
\]  

\(3\)

It is confirmed that the time derivative of the φ component vorticity \(\partial (\omega_\phi)/\partial t\) and the entropy gradient (the final term of eq. (3)) have small contributions in the NSSL. It is also confirmed that the contribution from the mean flow \(\langle v \rangle\) can be ignored for the first term of the right side in eq. (3). Figs. 4a and b show the value of \(-r \sin \theta \partial (\Omega)^2 / \partial z\), which is the negative value of the second term in the right hand side of eq. (3) derived from the meridional Coriolis force with showing deviation from the Taylor-Proudman theory and \(-[\nabla \times (\nabla \cdot (\rho_0 (v'v'))/\rho_0)]_{\phi}\), which is the contribution from the turbulent flow \(v'\) on the first term of the right side in eq. (3), respectively. It is clear that these two terms are balanced. The complete form for the value in the Fig. 4b is shown in Hotta et al. (2015). The turbulent effect comes from the correlation \(\langle v'_r v'_\theta \rangle\), the meridional Reynolds...
stress, which is generated by the shear of the poleward meridional flow. Fig. 4c shows the values $\partial (v_\theta)/\partial r$. The negative gradient is generated in the place where the NSSL is established. The turbulent effect acts as the turbulent viscosity on the meridional flow. This turbulent viscous stress balances with the Coriolis force generated due to the deviation of the Taylor-Proudman theory.

4. Summary

The high-resolution calculation with using the RSST partially achieves the NSSL. The analysis reveals the maintenance mechanism of the NSSL. The essential points are: 1. The inward angular momentum transport by the Reynolds stress. 2. Poleward meridional flow with increasing its amplitude along the radius. 3. Small-scale convection which acts as turbulent viscosity on the meridional flow.
In our calculation, the positive shear $\partial \langle v_\theta \rangle / \partial r$ is generated in the very near-surface layer where the tendency of the NSSL is opposite from the observation. Since it is currently not possible to reach the real solar surface in global calculation, we need an artificial thick cooling layer around the top boundary. This causes the sudden reduction of the radial velocity which suppresses the inward angular momentum transport. Since the real sun has a very thin cooling layer in the photosphere, we expect that the inward angular momentum transport extends closer to the surface than the calculation presented here. This is supported by the recent observation for the meridional flow with using the local helioseismology where the negative gradient of the meridional flow is established in the NSSL (Zhao et al. 2013). We need to explore the method of the open boundary condition for the top boundary in order to reproduce the NSSL properly.

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