Intrinsically Polarized Blend Lines

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Abstract. The Second Solar Spectrum formed by coherent scattering processes in the Sun, is highly structured. It is characterized by numerous blend lines, both intrinsically polarizing and depolarizing, superposed on the background continuum. These blend lines play an important role in the interpretation of the Second Solar Spectrum. Since blend lines affect the shapes of the neighboring spectral lines they have to be treated in a sophisticated manner in order to efficiently model a given spectral line of interest. The depolarizing blend lines – mostly considered to be formed under LTE conditions – depolarize the background continuum and thereby affect the absolute scale of the polarization measurement. An understanding of the influence of the blend lines leads to a proper determination of the zero-point of the polarization scale. With this motivation we extend a previously developed framework to include many blend lines formed under NLTE conditions, in the radiative transfer equation. The results are shown for the particular case of two blend lines situated on either side of the main spectral line.

1. Introduction


In the Second Solar Spectrum, the spectral lines superpose on the background continuum. To correctly estimate the line polarization, a knowledge of the continuum polarization is therefore essential. Blend lines serve as a tool in fixing the zero point of the polarization scale (Stenflo 2005). It is not possible to directly determine the absolute polarization scale with a precision that comes close to the polarimetric precision of the relative polarization scale. The depolarizing blend lines can be used to obtain a good estimate of the zero point of the polarization scale. Due to this fact the depolarizing blend lines are of profound importance for all observational works on the Second Solar Spectrum. The problem of blend lines is of considerable interest due to its relevance to the measurement and interpretation of the polarized solar spectrum.

The theoretical modeling of the line polarization in the Second Solar Spectrum involves incorporating the blend lines, as they invariably affect the shapes of the polarized main lines. Depolarizing blend lines are usually treated by assuming that they are formed under LTE conditions (see Fluri & Stenflo 1999, 2001). This is a good approximation when blend lines are formed in high density layers of the atmosphere. On the other hand, if the blend lines are formed in low density layers, it becomes necessary to treat them in NLTE. Attempts have been made to treat the blends in NLTE, but by
assuming the scattering to be isotropic which results in zero polarization (see Fluri & Stenflo 2003). Recently, Sowmya et al. (2012) developed a framework to include in the polarized line transfer equation an intrinsically polarized blend line formed under NLTE conditions. In the present paper we extend that formalism to treat many blend lines with intrinsic polarization, that occur in the wings of the main lines. They interact with the main line through radiative transfer effects.

The number of strongly polarizing lines is modest in the visible part of the Second Solar Spectrum. However, as we go down in the UV, the Second Solar Spectrum gets increasingly more crowded with strongly polarizing lines. There we can find several good examples of polarizing blend lines (see the UV atlas of Gandorfer 2005). Therefore, the theoretical studies presented in this paper become relevant in the analysis of the scattering polarization of the lines in the UV region of the Second Solar Spectrum.

In Section 2 we formulate the relevant transfer equation. Section 3 deals with the numerical methods used to solve the transfer equation. In Section 4 we present the results of this so called ‘multiline transfer’ with resonance scattering. Concluding remarks are presented in Section 5.

2. The Transfer Equation

The main and the blend lines are considered to be formed in a one-dimensional, plane-parallel, static, isothermal atmosphere. Throughout this paper the symbols ‘l’ and ‘ba’ stand for the ‘main’ line and the ‘blend’ lines respectively, where the subscript ‘a’ takes values 1, 2, ..., n, with n being the number of blend lines considered. In the absence of a magnetic field, the total source vector in the Stokes vector basis may be written as

\[
S(\tau, \lambda, \mu) = \frac{1}{k_{tot}(\lambda)} \left( \beta_c \phi_l(\lambda) S_l(\tau, \lambda, \mu) + \sum_{a=1}^{n} \beta_{ba} \phi_{ba}(\lambda) S_{ba}(\tau, \lambda, \mu) + \beta_{sc} S_{sc}(\tau, \lambda, \mu) + B(\lambda) U \right),
\]

where \( \mu = \cos \theta \), with \( \theta \) being the co-latitude, and

\[
k_{tot}(\lambda) = \beta_c + \sum_{a=1}^{n} \beta_{ba} + \beta_{sc} + 1,
\]

with

\[
\beta_c = \frac{k_l}{k_c}; \quad \beta_{ba} = \frac{k_{ba}}{k_c}; \quad \beta_{sc} = \frac{\sigma_{sc}}{k_c}.
\]

Here \( k_l \) and \( k_{ba} \) are the frequency integrated main and blend line absorption coefficients respectively. \( \sigma_{sc} \) and \( k_c \) are the continuum scattering and absorption coefficients. \( \phi_l \) and \( \phi_{ba} \) denote the absorption profiles for the main and the blend lines. The total optical depth scale \( \tau \) is defined by \( d\tau = -k_{tot}(\lambda) k_c dz \). In Eq. (1), \( U = (1, 0)^T \). The source vectors for the main and the blend lines \( S_l \) and \( S_{ba} \), and the continuum scattering source vector \( S_{sc} \) are given respectively by

\[
S_l(\tau, \lambda, \mu) = \epsilon_l B(\lambda) U + (1 - \epsilon_l) \int_{-1}^{+1} \frac{d\mu'}{2} \int_0^{\infty} d\lambda' R_l(\lambda, \lambda', \mu, \mu') \frac{\phi_l(\lambda)}{\phi_l(\lambda)} I(\tau, \lambda', \mu'),
\]

\[
S_{ba}(\tau, \lambda, \mu) = \epsilon_{ba} B(\lambda) U + (1 - \epsilon_{ba}) \int_{-1}^{+1} \frac{d\mu'}{2} \int_0^{\infty} d\lambda' R_{ba}(\lambda, \lambda', \mu, \mu') \frac{\phi_{ba}(\lambda)}{\phi_{ba}(\lambda)} I(\tau, \lambda', \mu'),
\]

\[
S_{sc}(\tau, \lambda, \mu) = \epsilon_{sc} B(\lambda) U + (1 - \epsilon_{sc}) \int_{-1}^{+1} \frac{d\mu'}{2} \int_0^{\infty} d\lambda' R_{sc}(\lambda, \lambda', \mu, \mu') \frac{\phi_{sc}(\lambda)}{\phi_{sc}(\lambda)} I(\tau, \lambda', \mu'),
\]

where \( \epsilon_l, \epsilon_{ba}, \text{ and } \epsilon_{sc} \) are the linear polarization efficiencies.
where $\epsilon_l$ and $\epsilon_{b_0}$ are the thermalization parameters for the main and blend lines respectively, and $B(\lambda)$ is the Planck function. $B(\lambda)$ is taken as the same for both the main and the blend lines. The continuum is assumed to scatter coherently through Rayleigh and Thomson scattering. $P(\mu, \mu')$ is the Rayleigh phase matrix (see e.g., Chandrasekhar 1950). The redistribution matrix $R(\lambda, \lambda', \mu, \mu')$ contains the physics of scattering. For simplicity, here we consider the factorized form of the redistribution matrix, given by

$$R(\lambda, \lambda', \mu, \mu') = R(\lambda, \lambda') P(\mu, \mu'),$$

where $R(\lambda, \lambda')$ is the angle-averaged redistribution function of Hummer (1962) which contains only the frequency correlations between the incident and scattered photons.

The angular dependency of the source vectors are eliminated by working in the irreducible basis (see Frisch 2007). In this basis, the total and the line source vectors have the form:

$$S(\tau, \lambda) = \frac{k_L(\lambda)S_L(\tau, \lambda) + B(\lambda)U}{k_{tot}(\lambda)},$$

and

$$S_L(\tau, \lambda) = \frac{1}{k_L(\lambda)} \left[ \beta_c \phi_l(\lambda) \epsilon_l + \sum_{a=1}^{n} \beta_{b_a} \phi_{b_a}(\lambda) \epsilon_{b_a} \right] B(\lambda)U + \frac{1}{k_L(\lambda)} \int_{-1}^{1} \frac{d\mu'}{2}$$

$$\times \int_0^\infty d\lambda' \left[ \beta_c (1 - \epsilon_l) R^c(\lambda, \lambda') W^l + \sum_{a=1}^{n} \beta_{b_a} (1 - \epsilon_{b_a}) R^{b_a}(\lambda, \lambda') W^{b_a} \right.$$

$$\left. + \beta_{sc} \delta(\lambda - \lambda') \Psi(\mu') I(\tau, \lambda', \mu') \right],$$

where $k_L(\lambda) = \beta_c \phi_l(\lambda) + \sum_{a=1}^{n} \beta_{b_a} \phi_{b_a}(\lambda) + \beta_{sc}$. Here $U = (1, 0)^T$, $E$ denotes the $2 \times 2$ unity matrix, and $R^{b_a}(\lambda, \lambda')$ is either given by the type-II redistribution function ($R_{II}^{b_a}(\lambda, \lambda')$) of Hummer (1962) or CRD. $\Psi(\mu')$ is the $2 \times 2$ Rayleigh phase matrix in the irreducible basis. The matrices $W$ are given by

$$W = \begin{pmatrix} 1 & 0 \\ 0 & W_2 \end{pmatrix},$$

where $W_2$ are polarizability factors. They depend on the angular momentum quantum numbers of the upper and lower levels. For a normal Zeeman triplet transition ($J = 0 \rightarrow 1 \rightarrow 0$), this factor is unity.

The one-dimensional polarized line transfer equation in the irreducible basis is then given by

$$\mu \frac{\partial I(\tau, \lambda, \mu)}{\partial \tau} = I(\tau, \lambda, \mu) - S(\tau, \lambda),$$

where $I$ is the irreducible intensity vector. To solve this transfer equation we use the scattering expansion method (SEM) proposed by Frisch et al. (2009).
3. Numerical Method of Solution

SEM is based on the Neumann series expansion of the polarized component of the source vector. Single scattered solution is computed at first, and this solution is then used for calculating the higher order scattering terms.

In the absence of a magnetic field, the source vector components in the irreducible basis can be written as

\[ S_{0,L}^K(\tau, \lambda) = \frac{1}{k_L(\lambda)} \left[ \beta_c \phi_c(\lambda)e_i + \sum_{a=1}^{n} \beta_{ba} \phi_{ba}(\lambda)e_{ba} \right] B(\lambda) \delta_{K0} + \frac{1}{k_L(\lambda)} \int_{-1}^{+1} \frac{d\mu'}{2} \times \int_{0}^{\infty} d\lambda' \left[ \beta_c(1 - \epsilon_i)R^c(\lambda, \lambda')W_{2}^l + \sum_{a=1}^{n} \beta_{ba}(1 - \epsilon_{ba})R^b_{ba}(\lambda, \lambda')W_{2}^{ba} + \beta_{sc} \delta(\lambda - \lambda') \right] \Psi^{K'0}(\mu', \mu') \right] \]

where \( \Psi^{K'0}(\mu', \mu') \) are the elements of the \( \Psi \) matrix with \( K, K' = 0, 2 \). For the calculation of Stokes I, one can neglect the contribution from the linear polarization \( Q \) to a good approximation because the linear polarization resulting from Rayleigh scattering in the solar atmosphere is small. Therefore the dominant contribution to Stokes I comes from the component \( I_{00}^0 \). The corresponding source vector component, neglecting the \( K \neq 0 \) terms, is given by

\[ S_{0}^0 \approx \frac{1}{k_L(\lambda)} \left[ \beta_c \phi_c(\lambda)e_i + \sum_{a=1}^{n} \beta_{ba} \phi_{ba}(\lambda)e_{ba} \right] B(\lambda) + \frac{1}{k_L(\lambda)} \int_{-1}^{+1} \frac{d\mu'}{2} \int_{0}^{\infty} d\lambda' \left[ \beta_c(1 - \epsilon_i)R^c(\lambda, \lambda') + \sum_{a=1}^{n} \beta_{ba}(1 - \epsilon_{ba})R^b_{ba}(\lambda, \lambda' + \beta_{sc} \delta(\lambda - \lambda') \right] I_{00}^0(\tau, \lambda', \mu'). \]

Here \( \tilde{S}_{0}^0 \) stands for approximate value of \( S_{0}^0 \). We calculate it using the approximate lambda iteration method of solution with the frequency-by-frequency technique (see Paletou & Auer 1995).

Retaining only the contribution from \( I_{00}^0 \) on the RHS of \( K = 2 \) component of \( S_{0,L}^K \) in Eq. (12), we obtain the single scattering approximation as

\[ \left[ \tilde{S}_{0,L}^2(\tau, \lambda) \right]^{(1)} \approx \frac{1}{k_L(\lambda)} \int_{-1}^{+1} \frac{d\mu'}{2} \int_{0}^{\infty} d\lambda' \left[ \beta_c(1 - \epsilon_i)R^c(\lambda, \lambda')W_{2}^l + \sum_{a=1}^{n} \beta_{ba}(1 - \epsilon_{ba})R^{ba}_{2}(\lambda, \lambda')W_{2}^{ba} + \beta_{sc} \delta(\lambda - \lambda') \right] \Psi_{00}^{20}(\mu', \mu') \tilde{R}_{00}^0(\tau, \lambda', \mu'). \]

The superscript \( (1) \) stands for single scattering. The single scattered polarized radiation field \( \tilde{I}_{00}^1 \) is calculated using a formal solver. This solution is used as a starting point to calculate the higher order scattering terms. Thus the iterative sequence at order \( n \) is

\[ \left[ \tilde{S}_{0,L}^2(\tau, \lambda) \right]^{(n)} \approx \left[ \tilde{S}_{0,L}^2(\tau, \lambda) \right]^{(1)} + \frac{1}{k_L(\lambda)} \int_{-1}^{+1} \frac{d\mu'}{2} \int_{0}^{\infty} d\lambda' \left[ \beta_c(1 - \epsilon_i)R^c(\lambda, \lambda')W_{2}^l + \sum_{a=1}^{n} \beta_{ba}(1 - \epsilon_{ba})R^{ba}_{2}(\lambda, \lambda')W_{2}^{ba} + \beta_{sc} \delta(\lambda - \lambda') \right] \Psi_{00}^{22}(\mu', \mu') \left[ \tilde{I}_{00}^0(\tau, \lambda', \mu') \right]^{(n-1)}. \]
The iteration is continued until the maximum relative change in the surface polarization becomes less than the convergence criteria of $10^{-8}$.

4. Results

When modeling the specific lines of the Second Solar Spectrum, the blend lines are generally treated in LTE. In that case, the blend usually depolarizes the main line polarization. In this section, we present the effects of polarizing blends on the main line of interest. For illustration we consider two blends located to the left (denoted $b_1$) and to the right (denoted $b_2$) of the main line. A detailed parametric study involving only one polarizing blend line is presented in Sowmya et al. (2012).

We consider a self-emitting isothermal atmospheric slab characterized by $(T, r, B) = (10^8, 10^{-5}, 1)$. Here $r = 1/\beta_c$ is the ratio of continuum to the main line opacity. The grids used in the computations have the resolution $(N_d, N_x, N_\mu) = (5, 401, 5)$. $N_d$ is the number of depth points in a decade of the logarithmically spaced $\tau$-grid with the first point being $10^{-2}$. $N_x$ is the total number of points in the equally spaced wavelength grid with a separation of 5 mÅ. $N_\mu$ is the colatitude grid represented by a 5-point Gaussian quadrature formula. The main line strength $\beta_c = 10^5$ and the continuum scattering coefficient $\beta_{sc} = 0$. The photon destruction probabilities are given by $\epsilon_l = 10^{-4}$ and $\epsilon_{bl} = 5 \times 10^{-2}$. The damping parameters for the three lines are $a_l = 2 \times 10^{-3}$ and $a_{bl} = 10^{-4}$. The main line is centered at 5000 Å. The two blends are placed 20 mÅ away on either sides of the main line. The Doppler widths of all the lines are 25 mÅ unless specified otherwise.

The Stokes profiles obtained from this multiline transfer are presented in Fig. 1. The positive $Q$ corresponds to the vibrations of the electric vector perpendicular to the limb. In panels (a), (b), and (c), dotted lines represent $(I, Q/I)$ profiles computed without blends, while dashed lines represent those computed with blends. In panels (a) and (b) we show the effects of polarizing blends ($W_{b_2} = 1$) on the intrinsically unpolarized main line ($W_{l_2} = 0$). Blends of unequal strength are considered in panel (a), while in panel (b) they are of equal strength. In panel (a), the blend at 4999.98 Å has a strength of $\beta_{b_1} = 5 \times 10^2$ and the one at 5000.02 Å has a strength of $\beta_{b_2} = 5 \times 10^3$. Blends make the main line core in $I$ relatively narrow and less deep (compare dotted and dashed lines in Fig. 1a). In $Q/I$ the main line does not produce any signal in the absence of blends as expected (see dotted line). The blends show triple peak structure (not all peaks are seen here) due to the PRD scattering mechanism. The wing peaks of the blends differ in amplitude because of the difference in their relative strengths. The main line polarization which should have been zero according to the expectations ($W_{l_2} = 0$) is now clearly non-zero (see dashed line in Fig. 1a). This is an effect of the proximity of the polarizing blends. The central and the wing PRD peaks of the two blends combine so as to give a distorted signal in $Q/I$. It appear as though the $Q/I$ signal that we are seeing is due to the main line. One has to be therefore careful in identifying these effects in the process of modeling.

The parameters used to compute the Stokes profiles in panel (b) are similar to panel (a) but for blend lines of equal strengths ($\beta_{b_1} = 5 \times 10^4$). We see, as before, a shallow and narrow absorption line in Stokes $I$ when blends are included (see dashed line) compared to the case when they are neglected (see dotted line). Since both the blends are of equal strength, the $Q/I$ profile is symmetric and shows a double peak structure with a dip at
Figure 1. Emergent Stokes profiles are shown for a line-of-sight $\mu = 0.047$. See Section 4 for details on the cases presented and parameters used for the panels (a), (b), (c), and (d).

the main line center (5000 Å). The increase in the percentage of polarization compared to panel (a) is attributed to the increase in the blend line strengths.

The case in which all the three lines are strongly polarizing ($W_2^I = W_2^b = 1$) is presented in panel (c). The blend lines have equal strengths ($\beta_b = 5 \times 10^4$). The dotted line corresponds to the main line case without blends. In this case the main line appears as a pure absorption line in intensity and shows a strong polarization of about 7% at main line center. When the blends are included, they bring down the line center polarization of the main line to nearly 4% (see dashed line). The short peaks on either side of the main line core correspond to blend line peaks. The signs of the $Q/I$ at the main line center and at the blend lines centers are opposite. This is indicative of a switch-over from limb brightening to the limb darkening of the radiation field.

The effect of variation of the Doppler width is shown in panel (d). Here the dotted and dashed lines represent the cases where the Doppler widths for the blends are 15 mÅ and 40 mÅ respectively. The Doppler width of the main line is 25 mÅ in both the cases. The strength of the blends continues to be $5 \times 10^4$. All the three lines have an intrinsic polarizability factor $W_2 = 1$. The intensity $I$ profile becomes broader in the main line core, with an increase in the the Doppler width of the blends. Magnitude of $Q/I$ in the main line core decreases with an increase in the blend line width.
5. Conclusions

In this paper we have shown how theoretically the total source vector can be generalized to include many blend lines. Blend lines are generally present in the wings of the main spectral line. For the cases considered in this paper, blend lines lie in the core of the main line. In practice, it is possible to have such situations where the blend lines are placed very close to the main line, thereby showing a considerable effect. We have seen in our studies that the blends affect the main line polarization significantly when they are closer to the main line. Including these effects helps to achieve a better modeling of a given spectral line. These studies are essential in a fine analysis of the Second Solar Spectrum, and help in our understanding of the solar atmosphere.

References

Chandrasekhar, S. 1950, Radiative Transfer (New York: Dover Publications)