Magnetic fields in the first galaxies: Dynamo amplification and limits from reionization

Dominik R.G. Schleicher\textsuperscript{1}, Jennifer Schober\textsuperscript{2}, Christoph Federrath\textsuperscript{2,3}, Francesco Miniati\textsuperscript{4}, Robi Banerjee\textsuperscript{5}, Ralf S. Klessen\textsuperscript{2}

\textsuperscript{1}Institut für Astrophysik, Georg-August-Universität Göttingen, Friedrich-Hund-Platz 1, 37077 Göttingen, Germany,
\textsuperscript{2}Zentrum für Astronomie der Universität Heidelberg, Institut für Astrophysik, Albert-Ueberle-Str. 2, 69120 Heidelberg, Germany,
\textsuperscript{3}Ecole Normale Supérieure de Lyon, F-69364 Lyon, France,
\textsuperscript{4}Physics Department, Wolfgang-Pauli-Strasse 27, ETH-Zürich, CH-8093 Zürich, Switzerland,
\textsuperscript{5}Hamburger Sternwarte, D-21029 Hamburg, Germany,

Abstract

We discuss the amplification of magnetic fields by the small-scale dynamo, a process that could efficiently produce strong magnetic fields in the first galaxies. In addition, we derive constraints on the primordial field strength from the epoch of reionization.

1 Introduction

In the local Universe, magnetic fields are observed on virtually all scales (e.g. Beck 1996), and observations confirm the presence of magnetic fields also at high redshift. Such evidence has been found through the Faraday Rotation imprint of line-of-sight galaxies on distant QSOs (Bernet et al. 2008, Kronberg et al. 2008), as well as through the far-infrared - radio correlation (Murphy 2009). They further exist in the intergalactic medium (IGM). For instance, galaxy clusters exhibit $\mu$G fields (Clarke et al. 2001) that require non-negligible initial seeds (Banerjee & Jedamzik 2003, Ryu et al. 2008, Miniati & Martin 2011).

Even in cosmic voids, weak magnetic fields seem to exist, as suggested by recent gamma-ray experiments (Neronov & Vovk 2010, Tavecchio et al. 2010, Taylor et al. 2011), but see Broderick et al. (2011). Various astrophysical mechanisms have been proposed for the origin of intergalactic and cosmological magnetic fields (Miniati & Bell 2011, Bertone et al. 2006, Gnedin et al. 2000, Ando et al. 2010). In particular, the model in Miniati & Bell (2011) makes consistent predictions for the magnetic fields observed in the cosmic voids, providing a relevant seed for subsequent dynamo amplification.

Primordial models of magnetogenesis (e.g. Grasso & Rubinstein 2001) provide an alternative scenario. Magnetic fields from the early universe could be potentially strong, and need to be constrained observationally. This concerns in particular inflationary scenarios (Turner & Widrow 1988), the electroweak phase transition (Baym et al. 1996), or the QCD phase transition (Quashnock et al. 1989, Cheng & Olinto 1994, Sigl et al. 1997).

In this contribution, we discuss the amplification of magnetic fields via the small-scale dynamo, which may provide a strong tangled magnetic field already in the first galaxies (e.g. Schleicher et al. 2010, Sur et al. 2010, Federrath et al. 2011. We further describe upper
## The small-scale dynamo for different turbulence models

The small-scale dynamo provides an efficient amplification mechanism of magnetic fields in the presence of turbulence. It was originally proposed by Kazantsev (1968), and subsequently explored by various authors using analytical models and numerical simulations (e.g., Subramanian 1997, 1999, Schekochihin et al. 2002, 2004, Haugen et al. 2004a, 2004b). To calculate the growth rate of the magnetic field, the induction equation, given as

\[
\frac{\partial B}{\partial t} = \nabla \times \mathbf{v} \times B - \eta \nabla \times \nabla \times B,
\]

can be rewritten in terms of the so-called Kazantsev equation

\[
-\kappa_{\text{diff}}(r) \frac{d^2 \psi(r)}{d^2 r} + U \psi(r) = -\Gamma \psi(r),
\]

where \( \psi \) is related to the spatial dependence of the magnetic field correlation function, \( \kappa_{\text{diff}} \) denotes the turbulent diffusion coefficient and \( U \) denotes a function depending on the properties of turbulence. In a recent study by Schober et al. (2011), we solved this equation and derived the dependence of the growth rate on the turbulence model, the magnetic Prandtl number and the Reynolds number. For this purpose, we considered the turbulence models given in Table 1, where we also provide the critical magnetic Reynolds number for magnetic field amplification, as well as the growth rate, which is normalized in terms of the eddy timescale. In Fig. 1, we show the dependence of the growth rate on the turbulence model, the Prandtl and the Reynolds number. The interested reader is referred to Schober et al. (2011) for the derivation of these results.

### Table 1: The critical magnetic Reynolds number \( R_{m,\text{crit}} \) and the normalised growth rate of the small-scale dynamo \( \bar{\Gamma} \) in the limit of infinite magnetic Prandtl numbers. We show our results for different types of turbulence, which are characterised by the exponent \( \vartheta \) of the slope of the turbulent velocity spectrum, \( v(\ell) \propto \ell^\vartheta \). The extreme values of \( \vartheta \) are 1/3 for Kolmogorov turbulence and 1/2 for Burgers turbulence.

<table>
<thead>
<tr>
<th>Model/Reference</th>
<th>( \vartheta )</th>
<th>( R_{m,\text{crit}} )</th>
<th>( \bar{\Gamma} ) (( Pm \rightarrow \infty ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolmogorov (1941) 1/3</td>
<td>( \sim 107 )</td>
<td>( \frac{36}{35} Re^{1/2} )</td>
<td>0.94 ( Re^{0.48} )</td>
</tr>
<tr>
<td>intermittency of Kolmogorov turbulence (She &amp; Leveque 1994)</td>
<td>0.35</td>
<td>( \sim 118 )</td>
<td>0.84 ( Re^{0.46} )</td>
</tr>
<tr>
<td>driven supersonic MHD-turbulence (Boldyrev et al. 2002)</td>
<td>0.37</td>
<td>( \sim 137 )</td>
<td>0.79 ( Re^{0.45} )</td>
</tr>
<tr>
<td>observation in molecular clouds (Larson 1981)</td>
<td>0.38</td>
<td>( \sim 149 )</td>
<td>0.54 ( Re^{0.40} )</td>
</tr>
<tr>
<td>solenoidal forcing of the turbulence (Federrath et al. 2010)</td>
<td>0.43</td>
<td>( \sim 227 )</td>
<td>0.34 ( Re^{0.36} )</td>
</tr>
<tr>
<td>compressive forcing of the turbulence (Federrath et al. 2010)</td>
<td>0.47</td>
<td>( \sim 697 )</td>
<td>0.34 ( Re^{0.36} )</td>
</tr>
<tr>
<td>observations in molecular clouds (Ossenkopf &amp; Mac Low 2002)</td>
<td>1/2</td>
<td>( \sim 2718 )</td>
<td>( \frac{11}{36} Re^{1/3} )</td>
</tr>
<tr>
<td>Burgers (1948)</td>
<td>1/2</td>
<td>( \sim 2718 )</td>
<td>( \frac{11}{36} Re^{1/3} )</td>
</tr>
</tbody>
</table>

Limits on the primordial field strength from recent reionization data (Schleicher & Miniati 2011).
Magnetic fields in the first galaxies

3 The Mach number dependence of magnetic field amplification

Another quantity with a significant impact on the growth rate and saturation level of the magnetic field is the Mach number of the turbulence. One of the first studies exploring this quantity considered a range of Mach numbers from 0.65 up to 1.14 (Haugen et al. 2004).

We considerably extended the range of Mach numbers in a recent study, exploring the range from Mach 0.02 up to Mach 20, using both solenoidal and compressive driving schemes (Federrath et al. 2011). Our results for the growth rate, the saturation level and the amount of solenoidal turbulent energy are shown in Fig. 2. We derived analytic fits using the fit function

\[ f(x) = \left( p_0 + \frac{p_1}{x} + p_2 + p_3 + p_4 + p_5 \right) x^{p_6}, \]

with the fit parameters given in Table 2. In the regime of low Mach numbers, all quantities depend considerably on the driving scheme, with solenoidal driving leading to more efficient amplification. Also at high Mach numbers, solenoidal driving is more efficient, but the difference is less pronounced. For the growth rate of the magnetic field, a transition seems to occur at about Mach 1, where the growth rate drops considerably in the presence of shocks. Towards even larger Mach numbers, we observe an approximate scaling as \( M^{1/3} \) for both types of driving. Overall, our results show that the small-scale dynamo works for a large range of Mach numbers, as well as for compressively driven and solenoidally driven turbulence.

4 Upper limits on the primordial field strength

If strong primordial fields have been created in the early Universe, they will subsequently affect structure formation via the magnetic Jeans mass, which is given as (Subramanian & Barrow 1998, Schleicher et al. 2009)

\[ M_J^B = 10^{10} M_\odot \left( \frac{B_0}{3 \, nG} \right)^3, \]

Figure 1: Left: Normalized growth rate for different turbulence models as a function of the Reynolds number, in the limit of an infinite magnetic Prandtl number. Right: Normalized growth rate as a function of the magnetic Prandtl number in the case of incompressible (Kolmogorov) turbulence, for different Reynolds numbers.
Figure 2: Growth rate (top), saturation level (middle), and solenoidal ratio (bottom) as a function of Mach number, for all runs with solenoidal (crosses) and compressive forcing (diamonds). The solid lines show empirical fits given by Federrath et al. (2011).

with $B_0$ the co-moving field strength. The latter is related to the physical field strength $B$ via $B_0 = B/(1+z)^2$. To explore the implications for the epoch of reionization, we follow the time evolution of the ionized volume fraction $Q_{HII}$ as Madau et al. (1999), yielding

$$\frac{dQ_{HII}}{dt} = -\frac{Q_{HII}}{t_{\text{rec}}} + \frac{SFR(z)f_{\text{esc}}10^{53.2}}{n_H(0)},$$  \hspace{1cm} (5)$$

with $t_{\text{rec}}$ the recombination timescale, $f_{\text{esc}}$ the escape fraction, $n_H(0)$ the co-moving number density. The star formation rate is calculated from the observed Schechter function provided by Bouwens et al. (2011). The further details of this approach are given by Schleicher & Miniati (2011). From the ionized volume fraction as well as the ionized fraction in the neutral component, we calculate the effective ionization degree $x_{\text{eff}}$ and the reionization optical depth

$$\tau_e = \frac{n_H(0)c}{H_0} \int_{z=0}^{z=s} x_{\text{eff}}(z)\sigma_T \frac{(1+z)^2}{\Omega_\Lambda + \Omega_m(1+z)^3} dz,$$  \hspace{1cm} (6)$$

with $c$ the speed of light, $H_0$ the Hubble constant, $\sigma_T$ the Thompson scattering optical depth.
Magnetic fields in the first galaxies

\[ \Gamma \left[ \frac{t}{t_{\text{ed}}} \right] (\text{sol}) \quad \frac{(E_m/E_k)_{\text{sat}}}{(E_{\text{sol}}/E_{\text{tot}})} (\text{sat}) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sol</th>
<th>Comp</th>
<th>Sol</th>
<th>Comp</th>
<th>Sol</th>
<th>Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 )</td>
<td>-18.71</td>
<td>2.251</td>
<td>0.020</td>
<td>0.037</td>
<td>0.808</td>
<td>0.423</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>0.051</td>
<td>0.119</td>
<td>2.340</td>
<td>1.982</td>
<td>2.850</td>
<td>1.970</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>-1.059</td>
<td>-0.802</td>
<td>23.33</td>
<td>-0.027</td>
<td>1.238</td>
<td>0</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>2.921</td>
<td>25.53</td>
<td>2.340</td>
<td>3.601</td>
<td>2.850</td>
<td>1.970</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>1.350</td>
<td>1.686</td>
<td>1</td>
<td>0.395</td>
<td>1</td>
<td>0.535</td>
</tr>
<tr>
<td>( p_5 )</td>
<td>0.313</td>
<td>0.139</td>
<td>0</td>
<td>0.008</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( p_6 )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

Table 2: Parameters in Eq. (3) for the fits in Fig. 2.

Figure 3: Left: The expected reionization optical depth as a function of the co-moving field strength for different cosmological parameters. Right: The expected ionization degree at different redshifts as a function of the co-moving field strength.

and the cosmological density parameters \( \Omega_\Lambda \) and \( \Omega_m \). We explore the uncertainties due to the cosmological parameters, the reionization parameters as well as the uncertainty in the observed Schechter function on the reionization optical depth, and the ionization degree at different redshifts. Some of our results are given in Fig. 3, and additional details are provided by Schleicher & Miniati (2011). Overall, they lead to a 2\( \sigma \) constraint of \( \sim 2 \) nG on the co-moving field strength.

Acknowledgements. C.F. acknowledges funding from a Discovery Projects Fellowship of the Australian Research Council (grant DP110102191) and from the European Research Council (FP7/2007-2013 Grant Agreement no. 247060). C.F., R.B., and R.S.K. acknowledge subsidies from the Baden-Württemberg-Stiftung under research contract P-LS-SPII/18 and from the German Bundesministerium für Bildung und Forschung via the ASTRONET project STAR FORMAT (grant 05A09VHA). R.S.K also thanks the Deutsche Forschungsgemeinschaft (DFG) for financial support via grants KL1358/10 and KL1358/11, as well as via the SFB 881 *The Milky Way System*. R.B. acknowledges funding by the Emmy-Noether grant (DFG) BA 3706.
References
Burgers J., 1948, Advances in Applied Mechanics, 1, 171
Cheng B., Olinto A. V., 1994, PRD, 50, 2421
Kazantsev A. P., 1968, Sov. Phys. JETP, 26, 1031
Kolmogorov A., 1941, Akademiia Nauk SSSR Doklady, 30, 301
Schekochihin A. A., Cowley S. C., Hammett G. W., Maron J. L., McWilliams J. C., 2002, New Journal of Physics, 4, 84
Shi Z.-S., Leveque E., 1994, Physical Review Letters, 72, 336
Subramanian K., Barrow J. D., 1998, PRD, 58, 083502
Turner M. S., Widrow L. M., 1988, PRD, 37, 2743