Dependence of Velocity Distributions of Small-Scale Magnetic Fields Derived from Hinode/SOT G-band Filtergrams on the Temporal Resolution of the Used Data Sets

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Abstract. The dynamics of isolated small-scale fields in terms of velocities of magnetic bright points (MBPs) is addressed in this contribution. The empirically determined linear relation between the observed width parameter for the Rayleigh velocity distribution of MBPs versus the temporal cadence of the acquired data is studied by simulations and a simple analytical model. The results of the model and the simulation agree with the found relation for the observations. The conclusion we draw from the model is that there may be no characteristic velocity for MBPs at all.

1. Introduction

The dynamics of the atmosphere of the Sun are dominated by magnetic fields. These fields span from very extended and strong fields down to the yet known smallest field concentrations which are composed of single flux tubes. These small-scale magnetic fields in the solar photosphere can be identified in high-resolution magnetograms or in high resolution G-band filtergrams as magnetic bright points (MBPs).

Utz et al. (2009) derived the lifetime and velocity distributions for isolated MBPs using SOT/Hinode data (Tsuneta et al. 2008). They found an exponential decrease of the number of MBPs for increasing lifetimes. For different temporal resolutions of the data acquisition the linear relation between the decaying parameter of the distribution and the temporal resolution of the used data set was determined. From this relation they obtained the true physical decaying parameter of the exponential law related to the mean lifetime found to be 2.50 ± 0.05 min. The velocity distributions derived were of Gaussian form for the x- and y- component and of Rayleigh shape for the effective velocity. These distributions were found to depend on the used temporal resolution as well. A decrease in temporal resolution of the data leads to a decreasing broadness of the velocity distributions. For more details we refer to Utz et al. (2009).
In this paper, we compare these observational findings to results of a simulation and an analytical model in order to verify these findings.

2. Simulation Results

In this section we verify the observational relation found in Utz et al. (2009) between temporal resolution ($\Delta t$) and the measured fit parameter ($\sigma$) of the Rayleigh distribution of MBP velocities.

For this purpose we simulate a random walk with gaussian distributed $x$- and $y$-velocity components. In our used approach we generate gaussian distributed random numbers for the two components. A summing up of these values gives a random walk (path of MBPs on the solar surface), i.e. that the position of the MBP at time $t$ is given by the sum of $t$ generated random numbers. The next position will be given by adding one more random number to this sum and so on. An example of such a random walk is shown in Fig. 1. We recalculate then the velocity distributions ($x/y$/effective) for this random walk with different temporal samplings (resolutions). Interestingly, we find the same behaviour as for the observations, i.e. increasing probability of high velocities with increasing time cadence (compare the left to the right plot of Fig. 2). Figure 3 shows the relation between the obtained $\sigma$-parameter versus the time cadence for the measured and the simulated distributions. We note that the linear fit for the simulated data goes nearly through the origin (we think that in theory it should pass perfectly through it), whereas for the observed data the $\sigma$-parameter has a non-zero value. The implication of the zero passing for the simulation data is that there is no characteristic velocity for the MBPs, which implies that the velocity distribution can get arbitrary high values for high time cadence (continuous observations). This is an interesting and important finding for coronal AC-heating models as for such models one of the most important parameters is the velocity distribution of the initial disturbance. We conclude that the interplay with the temporal resolution and initial displacements have to be considered in more detail in AC-heating models.

3. Analytical Model

In the last section we showed the agreement between observations and simulations. In this section we want to prove these findings also from the viewpoint of an easy analytical model. In our analytical approach we assume a gaussian velocity distribution with equal $\sigma$ and zero $\mu$ parameters (no intrinsical drift) for both velocity components ($x$ and $y$; The calculation below is only given for $x$ but holds in the same way also for $y$):

$$f(x, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right). \quad (1)$$

We now calculate the probability distribution for velocities if we skip one position measurement, i.e. if we reduce the temporal resolution by a factor of 2. Therefore we have to find the probability of a certain displacement $x_2 = \text{Pos}_2 - \text{Pos}_0$ after two time steps. This displacement can be reached by moving a distance $x_1 = \text{Pos}_1 - \text{Pos}_0$ and then $x_2 = \text{Pos}_2 - \text{Pos}_1$. All available combinations have to be considered, i.e. all different intermediate positions 1 from which the end position 2 can be reached. This gives us
The following equation for the probability $P$ to find an MBP on a certain position:

$$P(\text{Pos}_0 \rightarrow \text{Pos}_2) = \int_{\text{Pos}_0 = -\infty}^{\text{Pos}_0 = +\infty} P(\text{Pos}_2 \rightarrow \text{Pos}_1) * P(\text{Pos}_0 \rightarrow \text{Pos}_1) d\text{Pos}_1$$  \hspace{1cm} (2)

With the assumption of gaussians for the single displacements and the transition of positions to distances ($x$) we can solve the integral in the following way:

$$P(x_2) = \frac{1}{2\pi \sigma^2} \int_{x_1 = -\infty}^{x_1 = +\infty} \exp\left(-\frac{1}{2} \left(\frac{x_1}{\sigma}\right)^2\right) \cdot \exp\left(-\frac{1}{2} \left(\frac{x_2 - x_1}{\sigma}\right)^2\right) dx_1$$

$$= \frac{1}{2 \sqrt{\pi} \sigma^2} \exp\left(-\frac{x_2^2}{4\sigma^2}\right)$$

We see that the $\sigma$ value has increased by a factor of $\sqrt{2}$ compared to the original gaussian distribution (which means that equal probable distances have increased in length by $\sqrt{2}$) whereas the time between the measurements has doubled. Therefore the equal probable velocities have decreased by a factor of $\sqrt{2}$. As we started with an arbitrary
Figure 3. The measured Rayleigh $\sigma$-fit parameters are plotted versus the square root of the temporal sampling rate $\Delta t$ together with a first order fit. The dashed lines represent the 1-$\sigma$ bandwidth of the resulting fit. The dashed-dotted lines give the 3-$\sigma$ bandwidth. Both observation (left) and simulation (right) reveals a linear increase of $1/\sigma$ with the square root of the temporal resolution.

reference time, and this behaviour is valid for other factors too, we conclude that it is a general behaviour of a gaussian random walk that the broadness of the measured velocity distribution (described by $\sigma$) gets narrower with the square root of the temporal cadence. QED.

4. Conclusions

We proved the observed relation between estimated Rayleigh fit parameters for the effective velocity distribution of MBPs versus the time cadence of the measurements from the viewpoint of simulation. Furthermore an analytical solution for an easy model velocity distribution agrees with this observed relation. From the found relation we conclude that MBPs may have no characteristic velocity at all.

Acknowledgments. Hinode is a Japanese mission developed and launched by ISAS/JAXA, with NAOJ as domestic partner and NASA and STFC (UK) as international partners. It is operated by these agencies in co-operation with ESA and NSC (Norway). This work was supported by FWF Fonds zur Förderung wissenschaftlicher Forschung grant P20762. D. U. and A. H. are grateful to the ÖAD Österreichischer Austauschdienst for financing research visits at the Pic du Midi Observatory. M.R. is grateful to the Ministé des Affaires Etrangé és et Européennes, for financing a research visit at the University of Graz. This work was partly supported by the Slovak Research and Development Agency SRDA project APVV-0066-06 (JR).

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