Wavelength-diverse Polarization Modulators for Stokes Polarimetry

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Abstract. An increasing number of astronomical applications depend on the measurement of polarized light. For example, our knowledge of solar magnetism relies heavily on our ability to measure and interpret polarization signatures introduced by magnetic field. Many new instruments have consequently focused considerable attention on polarimetry. For solar applications, spectro-polarimeters in particular are often designed to observe the solar atmosphere in multiple spectral lines simultaneously, thus requiring that the polarization modulator employed is efficient at all wavelengths of interest. We present designs of polarization modulators that exhibit near-optimal modulation characteristics over broad spectral ranges. Our design process employs a computer code to optimize the efficiency of the modulator at specified wavelengths. We will present several examples of modulator designs based on rotating stacks of Quartz waveplates and ferroelectric liquid crystals (FLCs). An FLC-based modulator of this design was recently deployed for the ProMag instrument at the Evans Solar Facility of NSO/SP.

1. Introduction

Our knowledge of solar magnetism relies heavily on our ability to detect and interpret the polarization signatures of magnetic fields in solar spectral lines. There is now a strong drive toward instruments that observe over broad spectral ranges. In the context of understanding magnetism in the solar atmosphere, observations of spectral lines formed at different heights are needed to constrain the field geometry in three dimensions, and simultaneous observation of multiple spectral lines greatly enhances the diagnostic potential of Zeeman-effect observations through application of a line-ratio technique (Stenflo 1973). Improving technologies (e.g., IR detector arrays) have made it possible to take advantage of the increased sensitivity of the Zeeman effect with wavelength through the observation of infrared spectral lines (Stenflo et al. 1987; Rüedi et al. 1995).

The next generation of polarimeters will have the capability to observe in a variety of spectral lines over a wide wavelength range, coupled with the ability to observe several lines simultaneously. This is reflected in the design of the recently completed Spectro-Polarimeter for InfraRed and Optical Regions (SPINOR, Socas-Navarro et al. 2006), installed at the Dunn Solar Telescope (DST) of the National Solar Observatory on Sacramento Peak (NSO/SP, Sunspot, NM), that can observe between 430 and 1600 nm. The instruments planned for the Advanced Technology Solar Telescope and
the European Solar Telescope will span from the UV to the near-IR (Keil et al. 2003; Collados 2008).

From the need for wavelength diversity follows the requirement that the polarization modulation scheme is **efficient** at all wavelengths of interest. Typically, one attempts to achieve this goal by achromatizing the polarimetric response of a modulator. Tomczyk et al. (2010) introduced a new paradigm for the design of efficient polarization modulators that are not achromatic in the above sense, since they have polarimetric properties that vary with wavelength. However, they do so in such a way that they can be operated over a wide wavelength range with near optimal polarimetric efficiency. We refer to these modulators as **polychromatic**.

### 2. Traditional Polarimeters

Most polarimeters operate by employing retarders and polarizers in a configuration that encodes polarization information into a modulated intensity signal. This intensity modulation is then measured with a detector and analyzed to infer the input Stokes vector. A thorough treatment of the theoretical operation of Stokes polarimeters has been given by Collados (1999) and del Toro Iniesta & Collados (2000). In the following, we will assume that the reader is familiar with the concepts introduced in those references. We urge the reader to review Sect. 2 of Tomczyk et al. (2010), as it contains a concise discussion on the subject.

There are several well-known configurations for optimally efficient polarization modulators at a single wavelength. In Table 1, we present two such examples of balanced modulators. These employ a pair of liquid crystal variable retarders (LCVRs) with variable retardation and fixed orientation of the fast axis (e.g., Tomczyk et al. 2008), and a pair of ferroelectric liquid crystals (FLCs) with fixed retardance but variable orientation of the fast axis. The particular solution presented here for the FLC modulator is implemented in the Diffraction-Limited Spectro-Polarimeter (DLSP, Sankarasubramanian et al. 2006), installed at the NSO/SP DST.

To create polarimeters that operate over a large wavelength range, instrument developers have typically tried to select optical materials in order to make the polarimeter modulation matrix as independent of wavelength as possible, i.e., achromatic. As an example, the modulator of NSO/SPINOR is shown in Fig. 1. This method would, in principle, allow the possibility of applying a single demodulation scheme to infer the Stokes vector at any wavelength. However, in practice, careful calibration of the polarimeter is still necessary because of unavoidable residual wavelength dependence of the Mueller matrix of the modulator across the spectral range of interest.

### 3. The Polychromatic Modulator

The preservation of a given form of the polarimeter’s Mueller matrix with wavelength is a very limiting, and arguably unnecessary, constraint. The fundamental driver in the design of a polarization modulator for multi-line applications is the achievement of near-optimal modulation efficiencies in all Stokes parameters at all wavelengths of interest. The Mueller matrix of such a modulator can be completely arbitrary, and even
Table 1. Examples of optimal monochromatic modulation schemes, based on typical retarding devices. The orientation of the fast axes (third column) are positive counterclockwise, when looking at the light source. The modulation vector (last column) for a given \(i\)-th state corresponds to the \(i\)-th row of the modulation matrix. The last example is the modulator solution adopted for the NSO/DLSP.

<table>
<thead>
<tr>
<th>Modulator Type</th>
<th>Retardance</th>
<th>Orientation</th>
<th>Modulation Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCVRs (#1,#2)</td>
<td>(180°, 360°)</td>
<td>(0°, 45°)</td>
<td>(1, +1, 0, 0)</td>
</tr>
<tr>
<td></td>
<td>(180°, 180°)</td>
<td>&quot;</td>
<td>(1, −1, 0, 0)</td>
</tr>
<tr>
<td></td>
<td>(90°, 90°)</td>
<td>&quot;</td>
<td>(1, 0, +1, 0)</td>
</tr>
<tr>
<td></td>
<td>(90°, 270°)</td>
<td>&quot;</td>
<td>(1, 0, −1, 0)</td>
</tr>
<tr>
<td></td>
<td>(180°, 90°)</td>
<td>&quot;</td>
<td>(1, 0, 0, +1)</td>
</tr>
<tr>
<td></td>
<td>(180°, 270°)</td>
<td>&quot;</td>
<td>(1, 0, 0, −1)</td>
</tr>
<tr>
<td>FLCs (#1,#2)</td>
<td>(180°, 102.2°)</td>
<td>(0°, −18.1°)</td>
<td>(1, +1/√3, +1/√3, −1/√3)</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>(0°, +18.1°)</td>
<td>(1, +1/√3, −1/√3, +1/√3)</td>
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<tr>
<td></td>
<td>&quot;</td>
<td>(45°, +18.1°)</td>
<td>(1, −1/√3, +1/√3, +1/√3)</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>(45°, −18.1°)</td>
<td>(1, −1/√3, −1/√3, −1/√3)</td>
</tr>
</tbody>
</table>

Figure 1. Modulation efficiency curves of the bi-crystalline achromatic rotating retarder modulator used with the SPINOR instrument as a function of wavelength. It is used from 450 to 1600 nm, indicated by the arrows on the top axes. Solid curves: optimal demodulation. Dashed curves: using the simple direct addition/subtraction demodulation scheme currently employed. Dotted and dashed-dotted curves in the top left panel: RSS of the efficiencies in \(Q\), \(U\), and \(V\) for the optimal and current demodulation schemes, respectively. The horizontal solid lines in the four panels indicate the maximum theoretical modulation efficiencies that can be achieved simultaneously for the four Stokes parameters (1 for \(I\), and \(1/√3\) for \(Q\), \(U\), and \(V\)).

strongly dependent on wavelength. Since calibration with wavelength is required in any case, such wavelength dependence will not impact the precision of the polarimetric
measurements. Real-time approximations of the Stokes vector can be computed using a demodulation matrix calculated theoretically from the modulator design.

Tomczyk et al. (2010) propose a new paradigm where full-Stokes polarization modulators are designed to satisfy the constraint of having optimal and balanced polarimetric efficiency at all wavelengths of interest. This generally results in polarimeters that have modulation and demodulation matrices that are strong functions of wavelength. We refer to these modulators as polychromatic. A computer code is used to optimize polarization modulators consisting of combinations of fixed and variable retarders. Varying the retardance and orientation of the optical components that comprise a modulator, and maximizing the polarimetric efficiency for all wavelengths of interest, results in realizable configurations with a high degree of wavelength diversity. We illustrate this concept with polychromatic modulators for polarimetric applications using FLCs and Quartz retarders. We have also designed polychromatic modulators using LCVRs, but for the sake of brevity we omit examples here.

Figure 2. As Fig. 1, but now for an FLC-based modulator. Continuous curve: optimal and balanced solution at 630 nm (indicated by the central arrow) corresponding to the DLSP modulator (Sankarasubramanian et al. 2006). Dotted curve: polychromatic solution corresponding to a simple modification of the DLSP modulator, where the first FLC is rotated by an additional angle of 67.5°. Dashed curve: polychromatic solution obtained from the former solution through the addition of a fixed quartz retarder between the two FLCs. This solution was optimized between 500 and 900 nm (indicated by the two outmost arrows). The horizontal solid lines in the four panels indicate the maximum theoretical modulation efficiencies that can be achieved simultaneously for the four Stokes parameters (1 for $I$, and $1/\sqrt{3}$ for $Q$, $U$, and $V$).

3.1. DLSP Tweak

As a first example, we start with the FLC modulator used by the Diffraction Limited SpectroPolarimeter (DLSP) of NSO/SP. The DLSP was designed to operate at 630 nm only. We define the region of acceptable performance of a polarimeter as the region over which the efficiency is greater than the optimal efficiency divided by $\sqrt{2}$ in all Stokes parameters. Figure 2 (continuous curve) shows that the DLSP, as configured,
has optimal and balanced efficiency at 630 nm with a spectral range of acceptable performance of 128 nm. We then optimized the efficiency of the DLSP modulator between 500 and 900 nm, by allowing the orientation angles of the FLCs to vary. The resulting solution (dotted curve) is a simple modification of the DLSP configuration, where the first FLC is rotated such that the new switching angles are 67.5° and 112.5°. With this new configuration, the usable range is twice as large as before (259 nm). Next, we added a fixed retarder between the two FLCs, and allowed the orientations of the FLCs and the retardance and orientation of the fixed retarder to vary. The resulting solution (dashed curve) yields a polarimeter with a usable range of 553 nm. It must be noted that one can obtain even larger usable ranges with this type of modulator, when the optimization is extended to all the retarding devices. A clear example of this (even if still subject to some external constraints) is the ProMag modulator illustrated in Sect. 3.3.

The design of the polarimeter for the Yunnan Solar Telescope involved a similar optimization of the efficiency over a broad wavelength range by adjusting the orientation of the fast axes of the retarding elements (Xu et al. 2006). Their modulator, in our opinion, is polychromatic in nature. However, they did not optimize the retardances of the elements in their polarimeter, nor did they adopt an optimal demodulation scheme. They also argue for the importance of minimizing crosstalk, which is not necessary to achieve an efficient polarimeter.

### 3.2. Compound Quartz Retarder

The second type of modulator is based on a stack of retarders glued in a fixed set of relative orientations, that is then rotated as a whole to the required set of positions to perform the measurement of the Stokes vector. For this type of rotating modulator, we consider the usual scheme with 8 measurements at positions $kn/8$, with $k = 0, \ldots, 7$, i.e., stepping rather than continuously rotating. The example in Fig. 3 shows the modulation efficiency of a stack of three Quartz retarders.

![Figure 3](image-url)

**Figure 3.** As Fig. 1, but now for a modulator consisting of a stack of three quartz retarders rotated over 8 discrete steps of 22.5 degrees. The modulator was optimized between 380 and 1600 nm (indicated by the two arrows).
3.3. The ProMag Modulator

Figure 4. Theoretical efficiency curves for the ProMag modulator, consisting of two FLCs followed by a quartz retarder. The modulator was optimized at 587.6, 656.3, 769.9, and 1083.0 nm (indicated by arrows). The crosses indicate the measured efficiencies at 587.6 and 1083.0 nm after deployment of the instrument.

Figure 5. Theoretical modulation matrix of ProMag. The crosses indicate the measured modulation amplitudes at 587.6 and 1083.0 nm. The first column of the matrix is identically equal to 1 and is not shown.

The polarimeter of the HAO Prominence Magnetometer (ProMag, Elmore et al. 2008) was re-designed using the methods described by Tomczyk et al. (2010). This instrument
was conceived to observe solar prominences and filaments in the spectral lines of He I at 587.6 and 1083.0 nm, and also in Hα (656.3 nm). The re-design was prompted by a series of failed attempts at fabricating the ProMag modulator following the original design, which was based on a stack of six FLCs (Elmore et al. 2008). The new design utilizes two FLCs followed by a fixed retarder, constrained to use one of the FLCs from the original design. The second FLC and the fixed quartz retarder were obtained according to the optimized solution. The retardances of all devices were measured in the laboratory. Since the measured retardances differed somewhat from the specified ones, a second optimization was performed using their measured retardances and varying only the orientations of the devices. Figure 4 shows the predicted efficiencies for the Stokes parameters resulting from the optimization.

The modulator was then constructed according to the design resulting from the second optimization. Actual efficiencies at 587.6 and 1083.0 nm were measured after deployment of the instrument at the Evans Solar Facility of NSO/SP, and they are in good agreement with the theory (see crosses in Fig. 4). The discrepancy observed at 1083.0 nm is likely caused by a less precise determination of the retardances of the modulator optics at that wavelength, possibly due to an undetected leak in the interference filter used during the measurement in the laboratory, combined with the different spectral responses of the ProMag NIR camera and the photo-diode used in the lab measurement.

The theoretical modulation matrix is shown as a function of wavelength in Fig. 5. The first column is unity and therefore is omitted. Measured modulation amplitudes at 587.6 and 1083.0 nm are also shown to be in close agreement with the theory.

### 3.4. A Reconfigurable Modulator

![Figure 6](image-url)  
**Figure 6.** As Fig. 2, but now for a modulator consisting of two stacks of two quartz retarders, rotated over 8 discrete steps of 45° and 67.5°, respectively. The modulator was optimized between 500 and 700 nm (indicated by the two arrows). By changing the phase of the two rotation stages, the modulator may be configured for balanced operation (continuous curve), as a longitudinal magnetograph (dotted curve), or with emphasis on linear polarization (dashed curve).
Specific science applications of polarimeters may require giving a higher priority to selected spectral lines and/or Stokes polarization parameters. For this reason, in our optimization code both the optimizing wavelengths and the four Stokes parameters can be attributed different weights in the search of a high-efficiency modulator configuration. We can use this feature to search for modulators that can be reconfigured to favor some Stokes parameters over others. Figure 6 shows such a modulator, consisting of two stacks of two quartz retarders each. The stacks are independently rotated in 8 discrete steps over 360° and 540°, respectively. By changing the phase of the rotation of the second stack, the modulator may be tuned for efficient modulation in Stokes V only, in Stokes Q and U, or in all Stokes parameters. Though the optimization was carried out between 500 and 700 nm, the balanced scheme remains highly efficient beyond 900 nm.

4. Summary

We have reviewed the paradigm of the polychromatic polarization modulator as introduced by Tomczyk et al. (2010). They argued that the effort of achromatizing the response matrix of a polarimeter is both a very limiting and unnecessary constraint, and that the figure of merit is the polarimetric efficiency of the modulator at all wavelengths of interest. We have developed a search technique in order to explore the parameter space, and find near-optimally efficient solutions compatible with the particular type of modulator. We illustrated the power of this method by improving the modulation efficiency of existing polarimetric instruments, as well as by designing new modulators with near optimal and balanced efficiency over very large spectral ranges. Finally we have demonstrated the applicability of this new paradigm by showing the measured performance of a prototype modulator, which was designed and optimized through the method presented in this paper.

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References