Polarized Radiative Transfer: from Solar Applications to Laboratory Experiments

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Abstract. The theory of radiative transfer for polarized radiation, developed from Quantum Electrodynamics for the interpretation of solar observations, predicts the existence of a variety of physical phenomena that, in many cases, would deserve being directly tested through laboratory experiments, also in view of possible practical applications. In this report we will focus on the description of some of these atomic-physics phenomena that have been disregarded, or overlooked, in terrestrial laboratories.

1. Scattering experiments

Quite recently, some laboratory experiments have been performed with the aim of verifying, or contradicting, the main results derived from standard Quantum Electrodynamics relatively to the physical process of scattering polarization. Particular emphasis has been given to the polarization of the radiation scattered in the D 1-D 2 doublet of sodium or in the equivalent doublet of potassium. The main question that these experiments were supposed to answer was the presence of line-integrated linear polarization in the D 1 scattered radiation, a presence that is still debated even in solar observations. One of these experiments has been performed at ETH (Zürich, Switzerland). The experiment, which employs a tunable laser and a potassium cell, is described in the proceedings of the workshops Solar Polarization 4 and Solar Polarization 5 (Thalmann et al. 2006, 2009). A tentative interpretation of the results, based on non-standard quantum mechanics, has been presented in Stenflo (2009). In our opinion, the use of a tunable laser for producing the radiation to be scattered in the potassium cell makes the physics rather complicated and entails severe difficulties in the interpretation. Indeed, this potassium cell experiment has raised more questions than those that it has been capable of answering.

Another experiment has been carried on at HAO (Boulder, Colorado) by Steve Tomczyk (PI) and Roberto Casini. Some of the results obtained by the experiment have been presented by Steve Tomczyk at the workshop Solar Polarization 5. The
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HAO experiment is more “traditional”, in the sense that the radiation employed, to be scattered by the sodium cell, is produced by a “normal” lamp, not by a laser, a physical situation more similar to the one which is present in the solar atmosphere. The results have given an upper limit for the line-integrated linear polarization scattered in D1, an upper limit that the experimenters are trying to lower, as much as possible, before submitting a paper for publication. In any case, the results obtained for the polarization of the radiation scattered in D2 have been found to be very well reproduced by the “standard” theory (the one that is fully described in Landi Degl’Innocenti and Landolfi 2004, hereafter referred to as LL04) in a wide range of magnetic field values and geometries, and for a large number of polarimetric realizations of the experiment. The experiment has also shown that the buffer gas present in the cell has an important depolarizing effect, the depolarizing collisional parameter \( D^{(2)} \) being approximately 17 times larger than the Einstein coefficient for spontaneous de-excitation of the 3p levels of sodium. Qualitatively, the agreement between theory and experiment has indeed been found to be quite impressive, though it has to be still quantified in a precise way.

Apart from these scattering experiments, it is also interesting to report here on other possible laboratory experiments concerning radiative transfer for polarized radiation. This is a peculiar physical subject because it deals with objects, like stellar atmospheres, whose properties are rather difficult to mimic in terrestrial laboratories, where large optical thickness in the far wings of spectral lines can hardly be obtained. Therefore, it should not be very surprising that, on this subject, theory is capable of predicting phenomena that, as to the authors’ knowledge, have not yet been directly confirmed through laboratory experiments. In the following sections we describe two of such phenomena. The first concerns some properties of the transfer of polarized radiation, that are induced by optical pumping and that affect the wings of a spectral line in a non-magnetized medium. The second is a related effect of conversion of atomic polarization from alignment to orientation. In principle, such phenomena could be experimentally checked with a laboratory set-up similar to the one described in Courtade et al. (2002).

2. The atomic-polarization-induced Faraday pulsation effect

This effect comes into play when radiation propagates through a medium composed of atoms (or molecules) whose ground level is polarized due to the presence, e.g., of a directional pumping radiation covering, in its spectrum, an allowed transition of frequency \( \nu_0 \) originating from the same level and ending into an arbitrary upper level. It can be classified as an anomalous dispersion effect and is similar to the magneto-optical (or Macaluso-Corbino) effect, but it differs from this last one because there is no need of a magnetic field to produce it. A possible geometrical scenario is described in Fig. 1. Referring to this figure, the atoms are pumped by the unpolarized, unidirectional radiation coming from below. If the lower level has angular momentum \( J_z > 1/2 \), it can acquire atomic polarization, in particular alignment, so that its density matrix, defined in the reference system having its z-axis directed along the pumping radiation, and expressed in the representation of the statistical tensors, has a non-zero alignment term \( \rho_{00}(J_z) \). For instance, supposing \( J_z = 1 \), the sublevel \( M_z = 0 \) is overpopulated with respect to the sublevels \( M_z = \pm 1 \). In this situation, a “probe beam” of frequency \( \nu \approx \nu_0 \), propagating along the horizontal direction of Fig. 1, experiences dichroism and anom-
lous dispersion effects. At wavelengths sufficiently far from $\nu_0$, say in the far wings of the line, the dichroism terms go to zero and only the anomalous dispersion terms “survive”. In particular, assuming the $z$-axis as the reference direction for the definition of the Stokes parameters, only the anomalous dispersion coefficient $\rho_Q$ is different from zero, so that the Stokes parameters of the probe beam obey a transfer equation of the form:

$$\frac{dU}{ds} = -\rho_Q(\nu) V , \quad \frac{dV}{ds} = \rho_Q(\nu) U , \quad (1)$$

where $\rho_Q(\nu)$ is a quantity proportional to the amount of alignment present in the lower level and having a wavelength dependence typical of the anomalous dispersion effects (positive in a wing, negative in the other). Its explicit expression can be found by Eq. (7.16c) of LL04. Still referring to the geometry of Fig. 1, one has

$$\rho_Q(\nu) = -\frac{3}{2 \sqrt{2}} \frac{\hbar \nu}{4\pi} N_f B(J_f \rightarrow J_u) w_{J_u J_f}^{(2)} \sigma_0^2(J_f) \psi(\nu_0 - \nu) , \quad (2)$$

where $N_f$ is the number density of atoms in the ground level, $B(J_f \rightarrow J_u)$ is the Einstein coefficient for absorption, $w_{J_u J_f}^{(2)}$ is the symbol introduced in Eq. (10.11) of LL04, $\sigma_0^2(J_f)$ is defined by the ratio $\rho_0^2(J_f)/\rho_0^2(J_u)$, and finally $\psi(\nu_0 - \nu)$ is the anomalous dispersion profile which, in the wings, behaves as $[\pi(\nu_0 - \nu)]^{-1}$. According to Eq. (1), while the beam propagates along the medium, there is a continuous transformation of Stokes-$U$ into Stokes-$V$, and vice versa. In a different context, this phenomenon is known as “Faraday pulsation”, so that the phenomenon here described can be referred to as the “atomic-polarization-induced Faraday pulsation”.

This pulsation, however, has to be necessarily accompanied by another phenomenon, that can be considered as its complementary. When the probe beam propagates along the medium, it loses and acquires intrinsic angular momentum. We have to remember, in fact, that the intrinsic angular momentum of a radiation beam (its spin, or helicity), is directed along the beam itself and is proportional to the amount of circular polarization (Stokes parameter $V$). To this coming and going of angular momentum
of the radiation has to correspond an opposite coming and going of the angular momentum of the atoms of the medium. Obviously, the total angular momentum (atom + radiation) has to be conserved since the interaction Hamiltonian between the atom and the radiation beam is invariant under arbitrary rotations. But the angular momentum of the atom is connected with the amount of atomic orientation, so that the probe beam has to produce a sort of pulsation between alignment and orientation.

3. The alignment-to-orientation pulsation effect

The existence of this phenomenon has been put forward several years ago (see Landi Degl’Innocenti 1982) but it has shown, through detailed calculations, of not being very relevant for astrophysical applications (for instance in prominences) because it works only when the radiation interacting with the atom (the probe beam in our example) has a sharp wavelength dependence across the line profile. Indeed, it disappears if the radiation is “spectrally flat”. An amount of non-resonant Stokes-\(U\) in the probe beam affects the statistical tensor \(\rho^Q_1\) (defined in the reference system with the quantization axis directed along the same beam) in such a way that the total angular momentum (atom + radiation) is conserved.

To be more quantitative, let us consider the time variation of the intrinsic angular momentum carried by the probe beam in a particular point along the medium. Denoting by \(\vec{\Omega}\) the unit vector directed along the propagation, from Eq. (A3.7) of LL04 one gets, for a single mode of the radiation field characterized by the frequency \(\nu\) and direction \(\vec{\Omega}\)

\[
\left[ \vec{J}_{\text{rad}} \right]_{\text{single mode}} = -\frac{\epsilon^2}{2\pi\nu^3} V(\nu) \vec{\Omega} \ ,
\]

from which, supposing that the probe beam is spectrally concentrated in a small frequency interval \(\Delta\nu\) centered around \(\nu\) and in a small solid angle \(\Delta\Omega\) centered around the direction \(\vec{\Omega}\), one obtains the intrinsic angular momentum per unit volume carried by the probe beam as

\[
\vec{J}_{\text{rad}} = \frac{\nu^2}{c^3} \Delta\nu \Delta\Omega \left[ \vec{J}_{\text{rad}} \right]_{\text{single mode}} = -\frac{1}{2\pi c} \frac{\Delta\nu \Delta\Omega}{\nu} V(\nu) \vec{\Omega} \ .
\]

From Eq. (1), one thus gets

\[
\frac{d\vec{J}_{\text{rad}}}{dt} = c \frac{d\vec{J}_{\text{rad}}}{ds} = -\frac{1}{2\pi} \frac{\Delta\nu \Delta\Omega}{\nu} \rho_Q U(\nu) \vec{\Omega} \ ,
\]

or, substituting for \(\rho_Q\) its expression given by Eq. (2)

\[
\frac{d\vec{J}_{\text{rad}}}{dt} = \frac{3}{2} \sqrt{2} \hbar N_f B(J_{\ell} \rightarrow J_{\mu}) w^{(2)}_{J_{\ell},J_{\mu}} \frac{\Delta\nu}{\Delta\Omega} \frac{\Delta\Omega}{\pi^2} \frac{U(\nu)}{\sigma_0^2(J_{\ell})} \vec{\Omega} \ .
\]

\[1\] Note that this phenomenon is not mentioned in LL04 because it does not show up under the “flat spectrum approximation”, on which most of the results presented in the monograph are based.
On the other hand, the $\hat{Q}$-component of the angular momentum per unit volume due to the atoms located at the same point of the medium is given by (see Eqs. (3.108) and (3.109) of LL04)

$$J'_{at} = \hbar N t \sqrt{\frac{J_t(J_t+1)}{3} \frac{[\rho_0^1(J_t)]'}{[\rho_0^0(J_t)]'} \Omega} ,$$

where $[\rho_0^i(J_t)]'$ is the statistical tensor in the reference system having its $z$-axis directed along $\Omega$. The time evolution of $J'_{at}$ is thus controlled by the time evolution of this statistical tensor, which has been derived in Landi Degl’Innocenti (1982). Particularizing the equations of that paper (deduced for the more general case of a two-term atom$^2$ to the case of a two-level atom, one finds that the statistical tensor $[\rho_0^K(J_t)]'$ relaxes, due to the non-resonant radiation of the probe beam, through the equation

$$\frac{d}{dt} [\rho_0^K(J_t)]' = -(2J_t + 1)B(J_t \rightarrow J_o) \sum_{K'QK_0} \sqrt{\frac{3}{2}} \sqrt{K + 1)(2K' + 1)(2K_t + 1)} (-1)^{J_t - J_t + K + Q'}$$

$$\times \left[ \begin{array}{cccc}
K & K' & K & 1 \\
J_t & J_t & J_t & 1 \\
K & K' & K & 1 \\
Q & Q & Q & 1
\end{array} \right] \zeta \cdot i F^K_{Q_0} [\rho_0^K(J_t)]' , \quad (3)$$

where

$$\zeta = \frac{1}{2} \left[ 1 - (-1)^{K + K'} \right] ,$$

and where the non-resonant tensor of the radiation field, $F^K_{Q_0}$ is given, for our probe beam having Stokes parameters $S_i(\nu, \hat{Q})$, by the expression

$$F^K_{Q_0} = \sum_{i=0}^{3} T^K_{Q_0}(i, \hat{Q}) S_i(\nu, \hat{Q}) \frac{\Delta \Omega}{4\pi} \frac{\Delta \nu}{\pi(\nu_0 - \nu)} ,$$

with $T^K_{Q}$ the polarization tensor introduced in LL04. We now particularize Eq. (3) to the case $K = 1$, $Q = 0$, and take into account that the only non-vanishing elements of the statistical tensors $[\rho_0^K(J_t)]'$, that can be related by a simple rotation to the statistical tensors defined in the “vertical” reference system, $\rho_0^0(J_t)$ and $\rho_0^2(J_t)$, are $[\rho_0^1(J_t)]'$, $[\rho_0^2(J_t)]'$, and $[\rho_0^2(J_t)]'$. The transformations, performed through Eq. (3.98) of LL04, are the following

$$[\rho_0^0(J_t)]' = \rho_0^0(J_t) , \quad [\rho_0^2(J_t)]' = -\frac{1}{2} \rho_0^0(J_t) \quad [\rho_0^2(J_t)]' = \sqrt{\frac{3}{8}} \rho_0^0(J_t) .$$

We also take into account that, being $K = 1$, the sum over $K_t$ brings a non-vanishing contribution only for $K_t = 2$ and $Q_t = \pm 2$, so that, being

$$F^2_{\pm 2} = \frac{\sqrt{3}}{2} \left[ -Q(\nu) \mp iU(\nu) \right] \frac{\Delta \Omega}{4\pi} \frac{\Delta \nu}{\pi(\nu_0 - \nu)} ,$$

$^2$The relevant equation is the one for $\frac{2}{\pi} F^2_{\pm 2}$, leading to upper levels at page 305.
and also being

\[
\{ J_\ell \begin{pmatrix} 2 \\ J_\ell \end{pmatrix} \} = (-1)^{2J_\ell+1} \sqrt{\frac{3}{10J_\ell(J_\ell + 1)(2J_\ell + 1)}} ,
\]

\[
\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & -2 \end{pmatrix} = - \begin{pmatrix} 1 & 2 & 2 \\ 0 & -2 & 2 \end{pmatrix} = \frac{2}{\sqrt{15}} ,
\]

one finally obtains, by means of simple substitutions

\[
\frac{d\vec{\mathbf{J}}_{at}}{dt} = - \frac{d\vec{\mathbf{J}}_{rad}}{dt} ,
\]

an equation showing that the total angular momentum of the system (atom+radiation) is conserved.

4. Practical applications

Though the phenomena that we have described are of marginal importance in the astrophysical context (the alignment-to-orientation pulsation effect indeed disappears in the case of a “spectrally flat” probe beam), they can be conveniently used, at least in principle, for practical applications in atomic physics laboratories. For instance:

- The pulsation of the “probe beam” polarization from \( U \) to \( V \), and vice versa, depends on the amount of atomic alignment present in the lower level. This effect can be used, at least in principle, for giving an accurate measurement of the alignment.

- Ground level atomic alignment can be partially converted into atomic orientation by means of a non-resonant beam, thus avoiding altering the overall populations of the atomic levels. The amount of orientation can be conveniently gauged by simply shifting the frequency of the non-resonant probe beam. In particular, one can change the sign of the resulting atomic orientation either by changing the polarization of the probe beam from positive \( U \) to negative \( U \), or, alternatively, by changing the frequency of the probe beam from the red wing to the blue wing of the line.

References