SOLAR DIFFERENTIAL ROTATION DETERMINED BY TRACING LOW AND HIGH BRIGHTNESS TEMPERATURE REGIONS AT 8 MM

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Abstract. At the wavelength of 8 mm absorption features (Low brightness Temperature Regions, LTRs) and emission features (High brightness Temperature Regions, HTRs) can be traced for determination of solar rotation. From earlier studies it is known that about two thirds of LTRs are associated with Hα filaments. The goal of the present analysis is to determine the heights of these solar structures and their rotational velocities. We used the method for the simultaneous determination of the solar synodic rotation velocity and the height of tracers. The rotation velocities were determined by the linear least-square fit of their central meridian distances as a function of time. The mean value of the low brightness temperature regions’ heights is about 45 600 km. The results of solar rotation determined by tracing LTRs and HTRs are mutually compared and also compared with the results using other tracers and methods. The method for the simultaneous determination of the solar synodic rotation velocity and the height of the tracers could be applied properly only on LTRs, since a wide distribution over latitudes and central meridian distances of a large data set is necessary, which was not available for HTRs. Observational findings that HTRs rotate systematically faster than LTRs and the possibility that they can be observed at and outside the solar limb are consistent with relatively high altitudes of HTRs. It was concluded that the radiation mechanism of HTRs is thermal bremsstrahlung, probably associated with flaring active regions.

Key words: Sun - rotation
1. Introduction

The brightness temperature $T_b$ is often used in radio astronomy for measuring radiation intensity. Structures with higher brightness temperature (HTR) and lower brightness temperature (LTR) than the quiet Sun temperature level (qsl) are distinguished on full-disc solar images taken at cm and mm wavelengths. These structures are often used as tracers for solar rotation. But, projection effects in the determination of the exact positions of HTRs and LTRs represent a problem in the analysis of the solar rotation because objects detected at mm and cm wavelengths are located in the solar chromosphere and corona. For determining the height of tracers a special method is used (Roša et al., 1998). That method enables to correct the differential rotation profile and to determine the average heights of the used tracers. So, with this method we can simultaneously determine the height and the solar synodic rotation velocity of tracers. But, to apply this method properly it is necessary that the tracers used for determination of the solar rotation are distributed widely over central meridian distances and latitudes.

In this work a series of daily full-disc solar maps recorded at the wavelength of 8 mm in the years 1979–1982 and 1987–1991 at the Metsähovi Radio Observatory are used. For the purpose of the rotational analysis we used only objects whose brightness temperatures are considerably different from the quiet Sun level. In earlier studies (Brajša et al., 1997) rotational velocities are determined approximately by a graphic method while in this work all velocities are calculated numerically.

2. Measurements and Observations

The data set was obtained at the Metsähovi Radio Observatory, Helsinki University of Technology. Observations were performed using the 14 m dish radio-telescope. The receivers used for solar observations at the radio-telescope are capable of detecting frequencies in the range from 10 to 100 GHz. In this analysis we used daily full-disc solar maps taken at the frequency of 37 GHz ($\lambda = 8$ mm). The beam width of the telescope amounts to 2.4 arc min at this wavelength and the quiet Sun level is estimated at a brightness temperature $T_{\text{qsl}} = 7800$ K (see e.g., Urpo, Pohjolainen and Teräsrranta, 1994). The sensitivity of the receivers enables for a 0.1 s.f.u.
resolution. In the temperature scale this corresponds to a resolution of better than 100 K and it is limited by short term changes in the atmospheric attenuation (Brajša et al., 1997).

More than five hundred full-disc solar contour maps recorded in the years 1979-1982 and 1987-1991 were used for this analysis. Two examples are given in Figures 1 and 2. As tracers for the determination of the solar rotation the brightness temperature maxima (minima) inside the contours of HTRs (LTRs) were used. Their positions on the solar disc were determined by their central meridian distance (CMD) and heliographic latitude.

Objects on the maps should be traced at least for 2 to 4 days, depending on position on the solar disc, as the estimated uncertainty of the heliographic coordinates at medium latitudes is 3–5 deg. If objects are traced for shorter time, unreliable results might be obtained. The accuracy of the determina-
tion of solar coordinates was 1 deg, better than the resolution of the radio telescope. Overall, 155 HTRs and 620 LTRs were traced. HTRs were followed in 2–13 consecutive days and LTRs in 2–10 consecutive days. Figure 3 graphically represents the number of identified LTRs and HTRs according to the number of days when they were traced. From this Figure it can be seen that in the analysis of LTRs the most common tracing interval was between 2 to 4 days. Figure 4 represents the number of LTRs and HTRs over latitudes and Figure 5 their number over CMD.

3. Data Reduction Methods

To determine the rotational velocities of HTRs and LTRs we used the linear least-square fit of their CMD as a function of time, $t$. The transfor-
Figure 3: The number of identified LTRs and HTRs traced during $d$ days.

Figure 4: The number of LTRs and HTRs distributed over latitudes.

Information from the synodic to the sidereal rotational velocity was not calculated as a seasonal dependent function but as a constant factor ($\Delta\omega = 360 \, \text{deg}/365 \, \text{days}$) because of the given precision of the position determination.

The solar differential rotation is represented by the equation:

$$\omega = A + B \sin^2 \psi,$$

(1)

where $\omega$ is the sidereal rotation velocity in deg/day, $A$, $B$, are the solar
differential rotation parameters and $\psi$ is the solar latitude in degrees.

A method for the simultaneous determination of the solar rotation velocity corrected for the height of tracers and projection is here briefly described according to Roša et al. (1998). The main assumption of the method is that the height of tracers does not significantly change during the tracing period, i.e., the height is constant. Then, unusually high rotation velocities that are measured at high latitudes and large CMD are a result of projection effects as the tracers are positioned in the solar chromosphere and corona. Finally, proper motions of the tracers are supposed to be smaller than the spatial resolution of the radio telescope.

To distinguish measured quantities from the corrected ones, the measured quantities are denoted with an asterisk (*). The parameter $\beta$ connects the height parameter, $\epsilon = h/R$, where $h$ is the height above the surface of the Sun and $R$ is the solar radius, with the observed, $\psi^*$, and corrected, true latitude of the tracer, $\psi$

$$\beta = (1 + \epsilon) \frac{\cos \psi}{\cos \psi^*} . \tag{2}$$

This equation can be approximated by

$$\beta = \frac{\sqrt{(1 + \epsilon)^2 - \sin^2 \psi^*}}{\cos \psi^*} = \text{constant} . \tag{3}$$
SOLAR DIFFERENTIAL ROTATION

if $B_0 \approx 0$, where $B_0$ is the heliographic latitude of the solar disc centre. Even in the most inconvenient cases when the tracer is close to the limb and $B_0$ reaches the maximum value, the relative deviation of the parameter $\beta$ from a constant value, due to the change in the projected heliographic value, is less than 2% (Roša et al., 1998).

Then, the parameter $\beta$ is connected with the mean observed CMD, $\lambda^*$, observed $\omega^*$, and corrected rotation velocity of the measured tracer $\omega$

$$\omega_i^* = \frac{\sqrt{\beta^2 - \sin^2 \lambda_i^*}}{\cos \lambda_i^*} \omega.$$  \hspace{1cm} (4)

Then, the following abbreviations are used:

$$a = \sum_{i=1}^{N} \omega_i^{*2},$$ \hspace{1cm} (5)

$$b = \sum_{i=1}^{N} \frac{1}{\cos^4 \lambda_i^*},$$ \hspace{1cm} (6)

$$c = \sum_{i=1}^{N} \frac{\omega_i^{*2}}{\cos^2 \lambda_i^*},$$ \hspace{1cm} (7)

$$d = \sum_{i=1}^{N} \frac{1}{\cos^2 \lambda_i^*},$$ \hspace{1cm} (8)

$$e = \sum_{i=1}^{N} \frac{N}{\cos^4 \lambda_i^*},$$ \hspace{1cm} (9)

where the summation refers to 10 deg latitude bins and $N$ is the number of measured pairs of velocity, $\omega_i^*$, and mean CMD, $\lambda_i^*$, in each bin. Now we can calculate the corrected rotation velocity $\omega$ and the parameter $\beta$ by:

$$\omega^2 = \frac{ab - cd}{e - d^2},$$ \hspace{1cm} (10)

$$\beta^2 = \frac{b - d + c/\omega^2}{b}.\hspace{1cm} (11)$$

The true height of a tracer can now be calculated by:


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\[ h = R \left( \sqrt{\beta^2 \cos^2 \psi^* + \sin^2 \psi^*} - 1 \right), \]  
\[ \psi^* = \frac{1}{N} \sum_{i=1}^{N} \psi_i^*. \]

where \( \psi^* \) represents the mean latitude value for each latitude bin

\[ \omega^* = \frac{1}{N} \sum_{i=1}^{N} \omega_i^*, \]

for each latitude bin. Then, the corrected latitude \( \psi \) is calculated by

\[ \cos \psi = \frac{\beta \cos \psi^*}{\sqrt{\beta^2 \cos^2 \psi^* + \sin^2 \psi^*}}. \]

As mentioned earlier, the method relies on a homogenous distribution of the measured rotation velocities in latitude and CMD. For this reason, the HTR data set is too small for a reliable and consistent application of the height correction method. For example, there were only 7 HTRs identified in the latitude bin 30-40 deg and none of them at latitudes higher than 40 deg (Figure 4). So, the height correction was applied only to the LTR data set.

Tracing times of the LTRs were up to 10 days, and after a few days the CMD span was too large for observed rotation velocities to be properly connected with the corresponding mean CMD. Because of this, rotational velocities for the whole LTR data set were determined with the method of the daily shift, i.e., by dividing the observed change of CMD by the elapsed time in pairs of consecutive images obtained on different days, which is still the lower limit for reliable heliographic coordinate determination. Using this method, after filtering out 124 unreasonably small or large rotational velocities, 1518 rotation values of LTRs were obtained together with their corresponding latitudes and CMDs. In that data set of 1518 LTRs, both the CMDs and latitudes up to 50 deg are fairly well covered (Figures 4 and 5), justifying the application of the method of the height correction for LTRs. Then all sidereal velocities less than 11.0 deg/day were discarded and 1365 values remained for further analysis. Because the extremely high velocity
SOLAR DIFFERENTIAL ROTATION

values are considered as a consequence of projection effects, no upper limit for the rotation velocity was applied.

The procedure to calculate the height correction is as follows. All measured rotation velocities (after an application of the low filter, as mentioned earlier) with the corresponding mean values of the CMD are sorted in 10-deg latitude bins. Data from both solar hemispheres were taken together. Then, for each latitude bin the mean value of the corrected rotation velocity $\omega$ and the parameter $\beta$ are calculated according to Equations (10) and (11), respectively. The measured rotational velocities $\omega^*_i$ are now fitted as a function of the measured CMD $\lambda^*_i$ using Equation (4) and taking into account the corrected rotation velocity $\omega$ and the value of the parameter $\beta$ for each latitude bin. Then, the height of the object above the Sun’s surface (expressed in units of the solar radius $R$) is calculated according to Equation (12) for each latitude bin taking into account the mean latitude of all measurements in each bin using Equation (13). Finally, the corrected mean latitude for each bin is calculated by Equation (15).

Now we have for each latitude bin the mean observed values of latitude and rotation velocity, $\psi^*$ and $\omega^*$, calculated using Equations (13) and (14) and the corresponding corrected values, $\psi$ and $\omega$, calculated by Equations (10) and (15). With these values the corrections specific for each latitude bin are calculated as $\Delta \omega = \omega - \omega^*$ and $\Delta \psi = \psi - \psi^*$ and used to correct all individual measured pairs of the latitude and rotation velocity, for 620 rotation velocities of identified LTRs traced in 2 to 10 consecutive images. The corrected data set of LTRs velocities was established with this procedure.

4. Results

The results on solar differential rotation are presented for three cases: LTRs with and without height correction and HTRs without the height correction (Tables I-III). The running-average velocity filter over latitudes was applied to reject the statistically unreasonable velocity values due to imprecise position determination and possible false tracer identifications (as used in the rotational studies of coronal bright points, Brajša et al., 2002a). Two filters were used to exclude extreme rotation velocity values. In the first filter, all sidereal rotation velocity values lower than 8 deg/day and higher than 18 deg/day were excluded regardless of the tracer’s latitude. The rotation velocity parameters from Equation (1) were then found for all remaining
Figure 6: The mean rotation profiles of HTRs and LTRs. For the LTRs, results with and without the height correction are presented, as indicated in the legend. The differential rotation profile obtained tracing sunspot groups from the Greenwich measurements (Balthasar, Vázquez and Wöhl, 1986) is also given for comparison.

Table I: Differential rotation parameters $A$ and $B$, in deg/day for HTRs, without the height correction. The sidereal parameters and their standard errors ($M$) are expressed in deg/day for two different filters. Both solar hemispheres are treated together and the number of tracers $n$ is also given in the Table. Filters are explained in the text.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\pm M_A$</th>
<th>$-B$</th>
<th>$\pm M_B$</th>
<th>$n$</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.94</td>
<td>$\pm 0.15$</td>
<td>3.18</td>
<td>$\pm 1.23$</td>
<td>149</td>
<td>1st filter</td>
</tr>
<tr>
<td>14.91</td>
<td>$\pm 0.10$</td>
<td>2.56</td>
<td>$\pm 0.84$</td>
<td>141</td>
<td>2nd filter</td>
</tr>
</tbody>
</table>

data points. With the second filter we excluded all velocity values differing by $\delta = 2.0$ deg/day or more from the mean curve and finally new parameters were calculated. The solar differential rotation parameters are given in Tables I-III after applying the first and the second filter. The differential rotation profiles, obtained after using both filters, are presented in Figure 6.
**SOLAR DIFFERENTIAL ROTATION**

**Table II:** Similar to Table I for LTRs without the height correction.

<table>
<thead>
<tr>
<th>A</th>
<th>±M_A</th>
<th>-B</th>
<th>±M_B</th>
<th>n</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.33</td>
<td>± 0.09</td>
<td>1.89</td>
<td>± 0.39</td>
<td>595</td>
<td>1st filter</td>
</tr>
<tr>
<td>14.44</td>
<td>± 0.06</td>
<td>1.91</td>
<td>± 0.26</td>
<td>494</td>
<td>2nd filter</td>
</tr>
</tbody>
</table>

**Table III:** Similar to Table II for LTRs with the height correction.

<table>
<thead>
<tr>
<th>A</th>
<th>±M_A</th>
<th>-B</th>
<th>±M_B</th>
<th>n</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.54</td>
<td>± 0.13</td>
<td>2.09</td>
<td>± 0.74</td>
<td>561</td>
<td>1st filter</td>
</tr>
<tr>
<td>13.92</td>
<td>± 0.06</td>
<td>4.09</td>
<td>± 0.34</td>
<td>462</td>
<td>2nd filter</td>
</tr>
</tbody>
</table>

Using Equation (12), the effective heights of LTRs are calculated for each 10-deg latitude bin and the results are presented in Table 6 in the paper by Brajša et al. (2009). The first six latitude bands, i.e., latitudes up to 60 deg are taken into account according to the latitudinal distribution of LTRs. The calculated heights in that Table refer to the effective solar radius \( R = R_0 + \Delta R = 696\,260\,\text{km} + 500\,\text{km} = 696\,760\,\text{km} \) used in the coordinate transformation. Following heights of LTRs relative to the mentioned effective solar radius were obtained: \( \bar{h} = (45\,580 \pm 12\,808)\,\text{km} \) for the latitudes in the range 0-50 deg and \( \bar{h} = (35\,406 \pm 15\,283)\,\text{km} \) for the latitudes 0-60 deg. These results were obtained by averaging the mean values for 5 and 6 latitude bins, respectively. The standard errors are also given, expressed as \( \sigma/\sqrt{N} \), where \( \sigma \) is the standard deviation and \( N \) the number of bins under consideration. The height calculated in the last latitude band (50-60 deg) is excluded from the further analysis because this result is unreliable due to the small number of data points at latitudes higher than 50 deg. Using the method of daily shifts only 27 LTRs were traced in this latitude band (50-60 deg, Figure 4). This is a relatively small number of tracers compared with the other 10-deg latitude ranges where it varied between 118 and 554. A negative value of the height obtained for this latitude band would indicate that a tracer's height is below the effective solar radius, which obviously does not make sense. It is a consequence of the monotonically decreasing function \( \omega(\text{CMD}) \), as was pointed out by Vršnak et al. (1999). In our case...
that function can not be properly determined because the number of data points is too small in the latitude bin (50-60 deg) under consideration.

5. Discussion

In this work a simultaneous determination of the solar synodic rotation velocity and the height of tracers is presented. This method can be properly applied if there are reasonably large data sets of rotation velocity measurements provided by observations, which are well distributed over CMDs and solar latitudes. This was the case only with LTRs in our data set. As a consequence of the height correction, the solar differential rotation curve was shifted to the systematically lower values and the profile became more differential. The mean value of the LTRs' height determined by this method was found to be about 45 600 km. The above described method could not be applied in the case of the HTRs because the data set was not large enough.

Thermal bremsstrahlung can explain both the LTRs associated and not associated with Hα filaments (Brajša et al., 2009). A decrease in the brightness temperature of 2–14 % was found for prominence models (LTRs associated with Hα filaments) and coronal condensation models (LTRs not associated with Hα filaments), in agreement with observations. For interpreting the HTR phenomena we propose following possibilities, taking into account the observed rotational properties of HTRs:

A) The observed systematically higher values of rotation velocity of HTRs (compared to LTRs, Figure 6) are consistent with projection effects, and are due to the uncorrected heights of HTRs. This might be interpreted by larger average HTRs altitudes than LTRs altitudes ($h_{LTR} \approx 45 \text{ 600 km}$). In that case, for an interpretation of the radiation there are two possibilities:

A1) Thermal bremsstrahlung.

We have modelled the solar atmosphere above active regions in the range of heights 10000–80000 km (Brajša et al., 2009). Both temperature and density are increased in these models and the calculated brightness temperature of HTRs is enhanced by 5–20 % in agreement with observations. One consequence of this interpretation is that it would imply the appearance of HTRs at and outside the solar limb and this is in fact the prevailing case occurring in about two thirds of all analysed solar maps. One example is given in Figure 2.

A2) Thermal gyromagnetic radiation.
As an alternative, we consider the case of thermal gyromagnetic radiation for HTRs (Brajša et al., 2009). The emission is caused by nonrelativistic electrons spiralling in the magnetic field and is also known as cyclotron radiation. The observing frequency of 37 GHz would require a magnetic field of 6600 Gauss for emission at the second harmonic of the electron gyrofrequency. Such a field has never been reported for the Sun and thus we can exclude the second harmonic. The third and fourth harmonics could contribute significantly and this would require a magnetic field of 4400 Gauss or 3300 Gauss, respectively. These values are very unlikely at the heights where HTRs are assumed to be located according to this interpretation ($h_{HTR} \approx 50000–80000$ km).

B) HTRs are mostly related to ordinary active regions. In Figure 6 the differential rotation profile of sunspot groups is overplotted for comparison with the differential rotation of HTRs and LTRs. It is noticeable that the slope of the differential rotation curve of HTRs is very similar to that of sunspot groups, although the HTR curve is systematically shifted to higher values. The observed systematically higher rotation velocity of HTRs, which are almost always placed above ordinary active regions, may be interpreted in terms of the rotational properties of sunspot groups. A pronounced asymmetry of positive and negative rotation velocity deviations from the average differential rotation profile was reported (Godoli, Mazzucconi, and Piergianni, 1998; Brajša et al., 2002b); the positive deviations being larger than the negative ones. Moreover, during the evolution the following part of a sunspot group disperses and dissolves, shifting the centre of gravity towards the leading sunspot. So, the subregion of maximal brightness temperature inside a HTR might be shifted in the direction of solar rotation during the tracing, leading to a faster measured rotation velocity. At the same time the mean position of the sunspot group may not be affected, because the following sunspot is still visible, leaving the sunspot group rotation measured by the geometric method unchanged. Therefore, the observed systematically higher rotation velocity of HTRs than of LTRs could be interpreted not as a consequence of the height difference between them, but rather as a consequence of the derived rotation of HTRs.

In this case thermal gyromagnetic radiation from the chromosphere might be the radiation mechanism responsible for the observed HTR emission, since the required magnetic field of the order of several thousand Gauss is a reasonable value for sunspots. In that case the radiation is the slowly-
varying or S-component (e.g., Tapping and Harvey, 1994) and the HTRs should emit circularly polarized emission. The radiation is indeed circularly polarized at the mm wavelengths, as reported by Urpo and Pohjolainen (1987) for observations performed with the Metsähovi radio telescope and by Feix (1969) and Kundu and McCullough (1972) for measurements using other radio telescopes. Also, HTR emission should be directional, i.e., significantly weaker near and at solar limbs, which was not often the case in our data set. Future observations near the limb with high spatial resolution may help to resolve this question.

6. Conclusion

Finally, we conclude that the observational fact, found in the present analysis, that HTRs can be seen at and outside the solar limb strongly supports our possibility A), i.e., that HTRs occur at surprisingly high altitudes. Then the most probable interpretation of radiation is thermal bremsstrahlung, as discussed in A1). So, the HTRs are not the short wavelength extension of the slowly varying component (gyromagnetic radiation from the chromosphere). Urpo et al. (1986) find that in most 22 and 37 GHz sources beyond the solar limb (12 out of 16 events) a flare of C5 class or larger occurred within 4 hours before the mm map was recorded. For the other events there was strong evidence for flares behind the limb. Therefore, a possible interpretation of present results concerning HTRs is persistent flare activity enhancing the density at high coronal altitudes above active regions. Concerning the circular polarization, which was reported for mm wavelengths in many studies, we note that optically thin emission is proportional to the emissivity and inversely proportional to the square of refractive index. The latter is different for the two modes of radiation (e.g., Benz, 2002, Equations 11.1.10 and 11.1.2). Thus thermal emission may also be circularly polarized. A continuation of the flare - HTR relationship and an analysis of data obtained in the solar activity minimum are planned for a subsequent study.

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References


