STRUCTURE ANALYSIS OF A MODEL SOLAR PHOTOSPHERE

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Abstract. The structure of the solar photosphere has been studied by means of correlation analysis. The data analysis is based on a 3D radiation hydrodynamics-code modelling solar surface convection with high resolution in both, space and time. The variation of thermodynamic quantities with depth have been evaluated as well as the dependencies among those quantities as a function of depth. This gives an insight into the structure of the convective-radiative transition layer. We determined height levels for regions of thermal convection, convective overshoot, and for the near-surface layer up from where radiation takes over the role of the outward energy transport.

Key words: solar photosphere - RHD near-surface solar convection model - convection

1. Introduction

The successful implementation of numerical simulations of the solar granulation can be dated back to the early eighties. It is essential to mention the models of Cloutman (1979), Nordlund (1982), Gigas and Steffen (1984) in this context. Due to the enormous enhancement of computational performance numerical modelling of near-surface solar convection has become an increasingly important part in the physics of stellar granulation ever since.

In this paper, we present analyses of data obtained from a numerical granulation model. We analysed the height dependencies and correlation between various physical quantities in the solar photosphere in order to study the structure and physical processes in the photosphere. Since correlation analysis provides a simple approach for the determination of height levels in the photosphere, this method has been widely used before, see. e.g. Pravdjuk (1982), Durrant and Nesis (1982), Kushnir (1982) and Gadun et
al. (2000). Furthermore, we used so-called two-point correlations that reveal linear dependencies between different quantities and the brightness at the visible surface layer.

2. Model and Data

The ANTARES\textsuperscript{1} software package, developed by Muthsam \textit{et al.} (2007) is a radiation hydrodynamic (RHD) code used here to model near-surface solar convection. The code is not limited to modelling solar granulation but also handles other astrophysical flows successfully.

The ANTARES model works with essentially non-oscillatory (ENO) high-resolution schemes. A second- or third-order Runge-Kutta scheme, preserving the properties of the ENO schemes has been used for the temporal integration of the equations of radiation hydrodynamics. At an optical depth of $\tau \sim 10^4$, the model is smoothly switched from full 3D radiative transfer to the diffusion approximation. For the current model magnetic fields have not been considered so far. Thus, this RHD-case accords to the quiet sun. A detailed description of the code-development itself, the numerical procedures, constraints and applicabilities can be found in Obertscheider (2007). With a 2D modelling of solar granulation based on the ANTARES-code a very high resolution of $1.78 \text{ km} \times 2.84 \text{ km}$ has been achieved. This surpasses typical 3D model resolutions considerably, while two dimensions imply quite a strong restriction for studying the 3D granular topology. Such a 2D analysis of plumes, pulses, and the photospheric turbulence that occurs therein is discussed in Muthsam \textit{et al.} (2007).

The computational domain of the 3D model used here is a cartesian cuboid of $N_x \times N_y \times N_z = 190 \times 151 \times 151$ grid points that spreads over $6 \text{ Mm} \times 6 \text{ Mm}$ in horizontal and $3.6 \text{ Mm}$ in vertical direction, respectively. Thus, the spatial resolution of the data set is $\Delta y = \Delta z \approx 40 \text{ km}$ in horizontal, and $\Delta x \approx 20 \text{ km}$ in vertical direction. The time step is $30 \text{ s}$. While the resolution of the 2D case mentioned above is significantly higher, the accuracy of the 3D model used here is of the same order of magnitude as observational data. E.g. the high-resolution Swedish Vacuum Solar Telescope (SVST) resolves structures on the solar disc of 0.1 arcsec that correspond to approx. 70 km.

\textsuperscript{1}A Numerical Tool for Astrophysical RESearch
3. Methods

In order to divide the photospheric medium, different techniques of correlations have been used. One-point correlations serve as a measure for local linear dependencies among the thermodynamic quantities. This method uses the linear correlation coefficient between horizontal cuts of two quantities. The two-point correlations correspond to the correlation coefficient two horizontal cuts of two quantities, one of them being fixed at the bottom photosphere, where \( \langle \tau \rangle = 1 \), the other varied over the vertical direction. Finally, a two-component representation gives a separate insight into both, the run of the thermodynamic quantities of up- and downflows of the photospheric matter. By comparison of these correlation functions we are able to study the model photospheric structure with height, and to determine different levels of relevance in the convective-radiative transition regime. Note, however, that these levels have been evaluated globally, and result from an averaging over space and time. Of course, locally, they vary strongly with the underlying granular pattern as all quantities do, they are based on.

4. Results

The visible solar surface has been calculated via quadrature of the equation of optical depth \( d\tau(x)_{jk} = \kappa(x)_{jk} \rho(x)_{jk} \, dx \) column-by-column by using the data values of the Rosseland mean opacity \( \kappa \), and the density \( \rho \). The result has been averaged over all columns with indices \( \{(j,k) \mid j,k \in \mathbb{N}_0, 0 \leq j, k \leq 150\} \) and time and weighted subsequently in order not to overvalue \( \tau \) unity above granules that covers a larger area than the intergranular region. With \( \langle \tau \rangle = 1 \) as a level of reference, we always refer to the plane of the averaged geometrical depth \( \langle x \mid \tau = 1 \rangle_{j,k} \approx 420 \text{ km} \).

4.1. Two-component representation

A first insight into the vertical structure is given by the two-component representation of relative fluctuations \( (Q - \langle Q \rangle)/\langle Q \rangle \) of a thermodynamic quantity \( Q \), such as temperature \( T \), pressure \( P \), and density \( \rho \). For this purpose, the cartesian cuboid \( \Omega = \{(i,j,k) \mid i,j,k \in \mathbb{N}_0; 0 \leq i \leq 189; 0 \leq j, k \leq 150\} \) has been divided into the two subsets \( U = \{(i,j,k) \mid v_{x,ijk} \leq 0 \} \) and \( D = \{(i,j,k) \mid v_{x,ijk} > 0 \} \) that correspond to upflows and downflows of
matter, respectively, where $v_x$ denotes the cartesian, vertical flow velocity vector component, $\mathbf{e}_x \cdot \mathbf{v}$. The fluctuations have been averaged over each vertical level and over a model time period of approximately 15 minutes. Averages over up- and downflows at level $i$ have been calculated via $\langle Q \rangle_{\text{up,}i} = \sum_{(i,j,k) \in U_i} Q_{ijk}/|U_i|$ and $\langle Q \rangle_{\text{down,}i} = \sum_{(i,j,k) \in D_i} Q_{ijk}/|D_i|$, respectively. Thus, what is called $\langle T \rangle$ in Figure 1 represents such averages.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Temperature fluctuations $(T - \langle T \rangle)/\langle T \rangle$ as a function of (geometrical) depth from the top of the computational box downwards, i.e. for $x \leq 3.6$ Mm (left panel) and zoomed to the photosphere (right panel) where $\langle \tau \rangle = 1$ marks the origin, respectively. Solid lines denote $T$-fluctuations in upflows, dashed lines denote $T$-fluctuations in downflows. Note that the second graph, by contrast, is plotted as a function of photospheric height as is a usual convention in solar astrophysics.}
\end{figure}

Figure 1 shows the relative temperature fluctuations in up- and downflows. The separation starts at a depth of $x \approx 3$ Mm and is most pronounced just below the photosphere, indicating the fully convective behaviour of that region. Hot parcels showing a positive temperature excess are elevated while cooled parcels coming from the photosphere have lost their thermal energy due to radiation and merge to fast downdrafts in the intergranular regions. From the graph on the right hand side of Figure 1, which is zoomed to the photosphere and is plotted versus height above $\tau = 1$, we see that this behaviour inverts above the bottom of the photosphere. This buoyancy breaking effect is well known and can be reduced to the fact that as the upflowing matter overshoots into the convectively stable region, it starts to lose energy by radiation due to the strong decrease in opacity. On the other hand, downflows become compressed and heated as they enter into denser regions. The temperature fluctuations become equal in a height of
$h \approx 120\text{ km}$, while the inversion is most pronounced at $h \approx 230\text{ km}$. However, in the upper photosphere these processes invert again, since there is a compression and heating of oscillating upflows that are progressing into the chromosphere, while cooled matter from above sinks again due to its gravitational pull. The second intersection of the graphs occurs at $h \approx 330\text{ km}$. These values agree well with the steady-state model of solar granulation of Nelson and Musman (1976).

The variation of pressure and density fluctuations as a function of depth shows a negative correlation between these two quantities in subphotospheric layers that are convectively unstable and belong to the outer region of the convection zone. As we progress into the photosphere, we enter the regime of overshooting convection and the above mentioned correlation becomes positive. Besides, the absolute value of pressure fluctuations $|P - \langle P \rangle| / \langle P \rangle$ exceeds the absolute value of density fluctuations $|\rho - \langle \rho \rangle| / \langle \rho \rangle$, while $(T - \langle T \rangle) / \langle T \rangle$ remains positive in upflows and negative in downflows. The same behaviour has been detected for 2D granulation model data by Gadun et al. (2000). This zone extends to a height of less than 100 km in the photosphere. Somewhat higher up, at $120\text{ km} \leq h \leq 330\text{ km}$, temperature fluctuations in downflows exceed those in upflows. Only here, the intergranular downflows are hotter than the granular upflows. The modulus of pressure and density fluctuations in up- and downflows becomes nearly equal here with a relative difference of no more than 0.02, while these functions intersect at the same heights as the temperature fluctuations do. Here, the situation is reversed, $|\rho - \langle \rho \rangle| / \langle \rho \rangle > |P - \langle P \rangle| / \langle P \rangle$.

4.2. One-point correlations

The one-point correlations $\varrho(\delta v_x, \delta Q)$ dealt with in this section serve as a measure for local linear dependencies among spatial fluctuations $\delta v_x$, $\delta Q$ of the vertical velocity $v_x$ and different thermodynamic quantities $Q$. The correlation coefficient is calculated at each height level separately so that the correlation procedure is symmetric in its arguments, $\varrho(\delta V_i, \delta Q_i) = \varrho(\delta Q_i, \delta V_i)$. An asymmetric correlation procedure will be discussed in the forthcoming section. Again, the correlations have been calculated as a function of height for all columns $(j, k)$ and averaged over the whole horizontal domain and over time.

In Figure 2 we show that there is a negative correlation between the
vertical velocity fluctuations $\delta v_x$ and the gas density fluctuations $\delta \rho$ up to a level of $h \approx 40$ km. This anti-correlation is an indicator for convective instability, see e.g. Gadun et al. (2000) and we may conclude that thermal convection is present to that level of height.

$\varrho(\delta v_x, \delta T)$, plotted in the same part in Figure 2, demonstrates high correlation in the low photosphere. As the upflows start to propagate into optically thinner layers, temperature and velocity fields become increasingly inverted. The interval of negative correlation corresponds to the inversion of temperature fluctuations, dealt with in section 4.1. For these two graphs error bars have been plotted that represent the variation of the correlation graphs with time. They are intended to show the oscillatory behaviour of the quantities and their correlation as time elapses. They are nearly constant with time below the photosphere whereas the temporal variation increases continuously from $h = 0$ upwards\textsuperscript{2}. This corresponds to an increase of oscillations with photospheric height that can be attributed to the acoustic $p$-modes.

Figure 2 (lower panel) shows the same qualitative run for the correlation functions $\varrho(\delta v_x, \delta P)$ and $\varrho(\delta \rho, \delta P)$. Yet, $\varrho(\delta v_x, \delta P)$ is too low to indicate linear dependency, while $\delta \rho$ and $\delta P$ start to correlate increasingly from the low photosphere upwards. It shows a very high correlation of $\varrho \approx 0.9$. This corresponds to the upper end of the thermal convection zone.

4.3. TWO-POINT CORRELATIONS

In this section, two point correlations between spatial variations of the temperature $\delta T$ at the fixed level $\tau = 1$ and spatial variations of thermodynamic quantities increasing with height, $\delta Q_i$, are investigated. Of course, in this case the correlation procedure is no longer symmetric in its arguments, but all quantities are “compared” to the same profile of $\delta T$, fixed at the level of reference. These correlations can be compared directly with observational intensity correlations. This correspondence is due to the fact that the 2D brightness field reflects the horizontal structure of the temperature and temperature fluctuations can be considered as brightness inhomogeneities, see e.g. Karpinsky (1990), Pravdjuk (1974). The two-point correlations as a

\textsuperscript{2}We note that this result has to be regarded cautiously at the very upper end of the photosphere, $h > 400$ km, and should at least in part be attributed to the fixed boundary conditions.
Figure 2: One-point correlation of thermodynamic quantities $g(\delta v_x, \delta \rho)$, $g(\delta v_x, \delta T)$ (at the top) and $g(\delta v_x, \delta P)$, $g(\delta \rho, \delta P)$ (at the bottom). The error bars of the correlation functions involving $v_x$ indicate the temporal variation and thus the amplitude of oscillations that increases significantly with photospheric height.
function of height give an insight into the vertical structure of the photosphere.

![Diagram showing correlation functions](image)

*Figure 3:* Two-point correlations, based on the reference point \( \delta T \big|_{\tau=1} \): \( \rho(\delta T, \delta \rho_i) \), \( \rho(\delta T, \delta P_i) \), \( \rho(\delta T, \delta T_i) \), and \( \rho(\delta T, \delta v_{x,i}) \)

Figure 3 shows the results of several two-point correlations. The correlation between temperature fluctuations \( \delta T \) and density fluctuations \( \delta \rho \) is negative to a level of about 40 km. This behaviour is what should be expected, since less dense matter is hotter and also brighter in the subphotospheric layer. As the correlation graph changes the sign, buoyancy breaking occurs. Up from here, in the optically thin regime, the opposite is true. The correlation function \( \rho(\delta T, \delta P_i) \) on the other hand is positive in the whole photosphere with the strongest correlation occurring near the photospheric bottom, at a height of \( h \approx 60 \text{ km} \).

Besides we show how spatial variations of the temperature at \( \langle \tau \rangle = 1 \) are correlated with spatial temperature variations at greater heights in the photosphere. This is illustrated by the dashed curve in Figure 3. Of course, the temperatures are auto-correlated at \( \langle \tau \rangle = 1 \) and peak at \( (x, y) = (\langle x \big|_{\tau=1} \rangle, 1) \). In subphotospheric layers and the lower photosphere, this graph shows a strong correlation between the temperature profile at the photospheric bottom and temperature profiles beneath and beyond that.
level. As mentioned before, the reversal of temperature fluctuations leads to the decrease of correlation above $h \approx 120$ km. Again, the anticorrelation is most pronounced at the middle photosphere, at $h \approx 270$ km. Here, temperature fluctuations represent a mirror image of the granular brightness field, see e.g. Gadun et al. (2000). Due to the second reversal of temperature fluctuations the anticorrelation decreases but $\rho$ does not become positive again since oscillations are hindering the quasi-columnar structure here.

Contrary to $\rho(\delta T, \delta T_i)$, there is no reversal field of vertical velocities and thus $\rho(\delta T, \delta v_{x,i})$ is strongly correlated from subphotospheric regions to the upper photosphere. The correlation peaks at the bottom of the photosphere, where temperature and vertical velocity almost accord to each other. Compared to the one-point correlation $\rho(\delta v_x, \delta T)$ this conformance is even more distinctive below the photosphere.

5. Summary and Conclusions

We studied the structure of the solar photosphere from a RHD-model by means of correlation analysis. We find that in the low photosphere, up to a height of about 40 km, the thermal convection zone comes out of the visible surface layer at $\tau = 1$, which we conclude from the one-point correlation between the vertical velocity and the mass density $\rho(\delta V, \delta \rho)$ that remains negative below that level. Adjacent, there is a zone of convective overshoot to a height of approx. 120 km. This region is characterized by pressure fluctuations slightly exceeding the density fluctuations, while temperature fluctuations remain positive in upflows and negative in downflows. From that level upwards to approx. 330 km we enter the very region where temperature fluctuation inversion occurs. This region can be seen as a transition layer where the convective columnar structure still exists. It has been shown that this inversion is most pronounced at a height of approx. 230 km. Above 330 km almost all correlation coefficients decrease below $\rho = 0.3$ to become insignificant. This marks the breakdown of the columnar photospheric structure. Above this layer the photospheric medium is driven by the characteristic solar oscillations.
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