SOLAR CONVECTION DYNAMICS DERIVED FROM LONG TIME SERIES OBSERVATIONS

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Abstract. Long time series of solar granulation are extremely difficult to be obtained from ground based observations because of the unstable Earth’s atmosphere. The Hinode-SOT instrument provided long term stable time series of solar granulation at different wavelengths in the visible. After appropriate calibration, these data can be used for studies of long time series of several hours. In this study we concentrate on the question of whether granulation can be considered as an ergodic phenomenon. The answer to such question is very important when comparing observational results with theoretical models since these models are limited either in time or in the extension of the spatial grid. We have analysed a series of 8 h of Hinode-SOT blue continuum images, the average separation between the successive images was at maximum about 50 sec. The images were aligned in order to minimize tracking problems. A quiet region located near the solar disc centre at the beginning of the observations was selected. The analysis shows that it seems to be that solar granulation is at least near the behaviour of ergodicity. That means, that the behaviour along the time axis and along a spatial coordinate become similar on a long interval. From sufficient spatial sampling the time behaviour could be derived.

Key words: Hinode-SOT - photosphere - granulation evolution

1. Introduction

Hinode is a Japanese mission developed and launched by ISAS/JAXA, collaborating NAOJ, NASA and STFC (UK). The operation of the Hinode mission is conducted by the Hinode science team organized at ISAS/JAXA. Support for the post-launch operation is provided by JAXA and NAOJ (Japan), STFC (U.K.), NASA (U.S.A.), ESA, and NSC (Norway). It was
launched successfully on Sept. 22, 2006. We are interested in solar granulation observation of high resolution. Therefore, Solar Optical Telescope, SOT (50 cm diameter, diffraction limited images (0.2-0.3") in 388-668 nm range), data in the blue continuum band will be used here. The image stabilization system consists of a piezo-driven tip-tilt mirror in the optical tube assembly in a closed-loop servo using a displacement error estimated from correlation tracking of solar granulation (correlation tracker, CT).

In this paper we study solar surface convection with Hinode long time series. These series have the advantage that they are completely free of any seeing effects induced by turbulence in the Earth’s atmosphere. Therefore turbulence observed in the data is completely of solar origin and no further artificial corrections of any atmospheric seeing effects are required.

Solar convection is important since it provides a (i) mixing (ii) shocks which contribute to heating of upper atmosphere iii) is important for the Alpha-Omega Dynamo process and (iv) without convection no solar activity would be observed.

Magnetic fields on the Sun are generated by dynamo action. The motion of an electrically charged plasma produces magnetic fields. These fields are destroyed by Ohmic diffusion processes. In principle the phenomena occurring on the Sun can be explained by two types of dynamos:

1. Small scale dynamo theory (also called fluctuation dynamo theory): the field is generated on scales smaller than those of the turbulent eddies. This explains phenomena like the magnetic carpet. The magnetic carpet could be also explained by interaction of large-scale magnetic flux due to turbulent motions.

2. Large scale dynamo theory: field is generated at scales larger than that of turbulence. This explains the solar cycle, sunspots, active regions etc.

For more details see the review given by Tobias (2008). A correlation between magnetic activity and solar luminosity changes was studied by Spiegel and Weiss (1980). They claimed that the cause of luminosity variations over spot cycles should be sought in more deep-seated global features and not in directly observable features like sunspots or active regions. Strong magnetic fields at the base of the convective zone can alter the local convection. The resulting changes in thermal energy are large enough to produce variations
SOLAR CONVECTION DYNAMICS

of order 0.1% in the solar luminosity over the 11-yr sunspot cycle. Thus convective energy transport is closely related to solar luminosity variations.

Considering a selected field on the solar surface, convective patterns changes due to evolution of granular field and oscillations but also due to the influence of small scale magnetic fields. In this paper the time series analysed comprises practically quiet Sun. The magnetic field is negligible. We want to investigate whether the behaviour of the data can be called ergodic or not. Ergodicity means that using a data set in spatial extension and extracting the information leads to similar conclusions than using a time series of such a data set. Methods of non linear dynamics are applied and the results are compared.

2. Data

2.1. Data analysis

From Hinode time series of images, the evolution of the solar granulation structure can be studied by different methods. Classical methods involve autocorrelation function, power spectrum analysis of intensity variations. These data give information about the size and size distribution of granular elements. By comparing subsequent images, information about the horizontal velocity field can be derived when using data near the solar disc centre. In this paper another approach was made for studying the solar granulation, namely methods of non linear dynamics.

2.2. Methods of nonlinear dynamics

One of the basic questions is to establish a measure for the embedding dimension of a given system. The idea to derive such a quantity is as follows. This quantity is required for an estimation of the time delay which is used by the method of delays. The method of delays is the most important phase space reconstruction technique. Dynamical systems describe the time evolution of a system in a phase space. Let us consider a series of measurements \( s_n \). Such a series does not represent the multidimensional phase space of the dynamical system. One has to design some method to unfold the multidimensional structure of the data. This is done in the method of delays. From time delayed values of the scalar measurements vectors in the new space,
called the embedding space are constructed:

\[ s_n = (s_{n-(m-1)\tau}, s_{n-(m-2)\tau}, \ldots, s_n) \]  \hspace{1cm} (1)

The number \( m \) is called the embedding dimension and \( \tau \) is called the delay. The theorems of Takens (1981) and Sauer et al. (1991) state that if the sequence \( s_n \) consists of scalar measurements of the state of a dynamical system, then the time delay embedding provides a one-to-one image of the original set \( x_n \) provided if \( m \) is large enough.

If there exist \( N \) scalar measurements, then the number of embedding vectors is \( N_{(m-1)\tau} \).

Therefore, the first step in applying methods of non linear dynamics will be to estimate \( m \), the embedding dimension. One method that was developed is found in Kennel et al. (1992) and is called false nearest neighbours. The principle is simple. Suppose \( m_0 \) is the minimal embedding dimension for a time series \( s_i \). In a \( m_0 \)-dimensional delay space, the reconstructed attractor is a one-to-one image of the attractor in the original phase space. The neighbours of a given point are mapped onto neighbours in the delay space. The Lyapunov exponents determine the evolution of a dynamical system. Thus, shape and neighbourhood of points will be changed according to the Lyapunov exponents. However neighbourhoods will be mapped into neighbourhoods again. The topological structure will be preserved. This is not the case, when \( m < m_0 \) is taken for the embedding. Here, the topological structure is no longer preserved, points are projected into neighbourhoods of other points to which they would not belong in higher dimensions. Such points are called false neighbours. A description of the false nearest neighbour methods, FNNs, was given by Rhodes and Morari (1997).

Therefore, in the subsequent analysis we first calculate the embedding dimension by the method of false neighbours and then investigate the method of delays. Also the mutual information (see e.g. Kantz and Schreiber, 1997) is derived which is a generalization of the well known autocorrelation function. In information theory, mutual information is defined by:

\[ I(X,Y) = \int_X \int_Y f_{XY}(x,y) \log_a \frac{f_{XY}(x,y)}{f_X(x)f_Y(y)} \, dx \, dy \]  \hspace{1cm} (2)

\( X, Y \) denote two variables, \( f_{XY}(x,y) \) is the joint probability density function of \( X \) and \( Y \), and \( f_X(x), f_Y(y) \) are the probability density functions of \( X \) and \( Y \) respectively. The units of information \( I(X,Y) \) depend on the base
a of the logarithm (usually \( a = 2 \)). Assuming a partition of the domain of \( X \) and \( Y \) the double integral becomes a sum over the cells. For a time series \( \{ X_t \}_{t=1}^{n} \) sampled at fixed times \( \tau \), the mutual information is defined as a function of the delay \( \tau \) assuming the two variables \( X = X_t \) and \( Y = X_{t-\tau} \), i.e. \( I(\tau) = I(X_t, X_{t-\tau}) \). More information can be found in the above cited paper.

The ergodic behaviour of a system can be tested by the following consideration: the mean value of a function (in our case time series) is calculated from the integral:

\[
< F > = \lim_{T \to \infty} \frac{1}{T} \int_0^T F(x) dt
\]  

(3)

The evaluation of equation 3 must be independent from the choice of the initial spatial position.

In astrophysics several applications of ergodic theory were made. Easton (1981) discussed applications of ergodic theory and area preserving mappings mainly for orbits. Zimbardo, Veltri and Mangeney (1990) studied the correlate propagation of two nearby photons in phase space in the solar corona and derived a general diffusion equation for a two-photon probability density. The effects of large scale, average refraction are included, and they found that an exponential separation of the ray paths occurs in a medium with fluctuating refractive index, giving rise to chaotic behaviour and to fractal structures. Plyukhin (2008) investigated the generalized Fokker-Planck equation, Brownian motion, and ergodicity.

2.3. Data sets

As we have mentioned already, there exists a continuous 48h time series of solar granulation observations made by the Hinode satellite in Aug. 2007. A description of the Hinode mission in general was given by Kosugi et al. (2007), and more detailed information about the SOT instrument can be found in the paper of Tsuneta et al. (2008). From this time series, we used the first half of data. In order to study ergodicity, we selected: (a) a series of 1000 measurements (covering about 8 hours) of intensity variations due to granular/intergranular structures in the blue continuum, (b) we selected a random spatial position and followed the evolution at this position along the time. The reason why we have not selected the whole time series was, that by selecting only 1000 subsequent images gives the same number of
Figure 1: Mutual information and false nearest neighbour calculations for the two cases, (i) fixed time, (ii) fixed spatial position. In this case position ‘840’ was selected from the data. The designation false nearest neighbours on the y-axis means the fraction of FNNs.

data points than by selecting one row of 1000×1000 pixel image. The data were calibrated first using the routines given by solarsoft. In order to ensure that the satellite always pointed to the same physical location on the solar surface, the data were aligned. Again it has to be stressed that such satellite data are absolutely free of any further random influences induced e.g. by seeing effects in the Earth’s atmosphere.

3. Results

3.1. Embedding dimension

The embedding dimension was calculated as it was described in the previous section. We calculated the embedding dimension for two cases in order to test the data for ergodicity: (i) for a fixed time of the images; a vector
containing the continuum intensity variations was extracted, (ii) for a fixed spatial position; a vector containing the continuum intensity variations as a function of time was extracted. The results for the mutual information calculation and for the false nearest neighbour analysis are shown in Figure 1.

Both curves (for fixed time and fixed space vectors) are very similar for the mutual information. This suggests an embedding dimension about 5. In the case for the false nearest neighbours, the two curves (for fixed time and fixed space vectors) are similar again, however they differ from those of Figure 1 showing a smaller slope. Small values below 0.1 are reached for $m \geq 15$. This suggests a higher embedding dimension.

Taking these estimates for the embedding dimension into account, we applied the method of delay coordinates for the two datasets. The results of two delays, $d$ are shown here: (a) delay=5, (b) delay =5 (a higher value is suggested by the result of the false nearest neighbours).

In Figure 2 we have applied the method of delay coordinates for two

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**Figure 2:** Delay coordinates, $d = 5$ and $d = 10$ for (i) fixed time, (ii) fixed spatial position. In this case position '840' was selected from the data.
Figure 3: Mutual information and false nearest neighbour calculations for the two cases, (i) fixed time, (ii) fixed spatial position. In this case position '140' was selected from the data. The designation false nearest neighbours on the y-axis means the fraction of FNNs.

cases (i) delay =5, (ii) delay =10, again for the two types of vectors that have been defined above.

In the case of \( d = 5 \), the results are similar for the time fixed and space fixed vectors. A larger delay, \( d = 10 \) clearly shows differences. There appear no closed curves any more. Only in the case of time fixed vector, some indication of a structure appears.

In order to investigate whether these results depend on the position or time step selected, we present in Figures 3 and 5 the results for an arbitrarily selected second position. A similar structure is obtained.

Finally, for comparison in Figure 6 the data are shown for a completely arbitrary series that was calculated by a random number generator.

In this case, the mutual information steeply decreases to zero values, the delay coordinates show no structures.

In Figure 4 we give the test for ergodicity by evaluating the integral
Figure 4: Test for ergodicity by evaluating the long time average (see Equation 3) at 50 different spatial positions.

(Equation 3) at 50 different spatial positions. In the upper panel the results are shown for the Hinode data. It is clearly seen that the values are different, however the variation around the mean is less than 2%.

In the lower panel the results are shown for completely random data. Here the fluctuations are considerably stronger.

4. Discussion

Ergodic behaviour means that the time average of a function along the trajectories exists almost everywhere and is related to the space average. In mathematics, for an Ergodic Random Process we can exchange Ensemble Averages for Time Averages.

Ergodicity of granulation continuum intensity data was tested in this paper by applying methods of non-linear dynamics to two data vectors $I(x_i)$ where $x_i$ denotes spatial positions at a fixed time $t_0$ and $I(t_i)$, where
Figure 5: Delay coordinates, $d = 5$ and $d = 10$ for (i) fixed time, (ii) fixed spatial position. In this case position '140' was selected from the data.

$t_i$ denotes the time evolution at a specified spatial point $x_0$. For completely random data (Figure 6) the mutual information steeply declines to zero. This has to be expected since the data are completely random, there exists no mutual information i.e. the joint probability for finding a value in the $i$-th interval and after a delay $\tau$ in the $j$-th interval goes to zero. However, when applying this method to our granulation intensity measurements, this probability must be $\geq 0$ because granular patterns exist, that extend both in space and time. The question however, if the spatial and time extension of a data set contains qualitatively the same information can only be answered by applying tools of non linear dynamics.

The calculation of the mutual information and the false nearest neighbours showed that there is qualitative and even close quantitative similarity in the results for the vectors $I(x_i)$ and $I(t_i)$. This similarity can be seen by comparing Figures 1 and 2. The behaviour of the mutual information and false nearest neighbours is practically identical for both series. The same is
valid when taking a different initial point. Thus the results do not depend
on the starting point.

Similar methods (mutual information and false nearest neighbour calculations) were applied in other fields of physics such as e.g. non linear
dynamic rupture in sapphire (Sherman and Bery, 1997).

The structure found in the delay coordinates is different. This may be a
result of the relatively bad resolution in time (more than 30 s).

Our results favour the idea that granulation is an ergodic phenomenon.
It must be stressed that this data set is from the observational point of view
the best data set available for such investigation. The evaluation of the long
time average (integral in Equation 3) shows small fluctuations below 5%
around the mean which is considerably smaller than the fluctuations for
random data (compare upper and lower part in Figure 4). Further investiga-
tions including other parameters than intensity and filtering the data
for oscillations will be made. At this stage it is highly probable that the
dynamics of it can be deduced both from time series as well as from a static picture taken into account that the spatial extension of the data is long enough.

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