Axisymmetric MHD Instabilities in Solar/Stellar Tachoclines

M. Dikpati,1 P. S. Cally,1,2 P. A. Gilman,1 and M. S. Miesch1
1High Altitude Observatory, National Center for Atmospheric Research, Boulder, CO, USA
2Center for Stellar and Planetary Astrophysics, School of Mathematical Sciences, Monash University Clayton, Victoria, Australia

Abstract. We show that banded toroidal fields in the tachoclines of the Sun and other stars should be unstable to 3-D axisymmetric overturning modes if the peak toroidal field is $\sim 100$ kG or more. This instability should fragment and limit the amplitude of toroidal fields in tachoclines.

1 Introduction and Motivation

Any star that rotates and has a convection zone is virtually certain to have a “tachocline” at the interface between the convection zone and radiative interior, where the differential rotation of the convection zone interfaces with the rotation of the radiative zone. Tachoclines are also likely to contain toroidal fields generated by dynamo action in the convection zone, thus playing an important role in solar and stellar dynamos. It is therefore important to understand the global dynamics and MHD that takes place in tachoclines. For example, under what conditions would differential rotation and toroidal fields there be unstable to global perturbations, and how is the instability manifested? We report on instability for disturbances that are axisymmetric about the axis of rotation.

Previous investigations of global MHD instabilities in tachoclines (Gilman, Dikpati & Miesch 2007; Arlt, Sule and Rüdiger 2007) have focused on non-axisymmetric modes of instability. These studies have shown that instability, particularly to modes with longitudinal wavenumber $m = 1$, occurs for a wide range of profiles and toroidal field amplitudes for solar differential rotation. Unstable modes double in amplitude in about a year. Toroidal fields “tip” away from a longitudinal orientation.

There is a long history of study of axisymmetric modes of instability in more general stellar interiors (Fricke 1969; Tayler 1973). Such studies commonly focus on instabilities whose growth rates and disturbance structures are strongly influenced by one or more forms of diffusion. Here we focus on instabilities on dynamical time scales, generally much shorter than the period of a typical solar or stellar magnetic cycle, and omit all forms of diffusion. Instability to axisymmetric modes does not exist in 2D (longitude-latitude); in quasi-3-D “shallow water” models (Dikpati, Gilman & Rempel 2003), it exists only for concentrated toroidal bands with extremely high peak fields, $\sim 2$ MG. We show here that in 3-D, instability is excited for toroidal fields of $\sim 100$ kG.
2 Equations and Integral Theorem

We developed a hydrostatic $m = 0$ model of the tachocline based on formulations in Gilman, Dikpati & Miesch (2007) and a nonhydrostatic but Boussinesq model starting from the formulation in Cally (2003). For both systems, the equations for instability can be reduced to a single second-order equation, from which an integral theorem is derived. From this theorem some qualitative features of the instability can be inferred. Details on equations, integral theorems and results from them are given in Dikpati et al. (2008) and Cally, Dikpati & Gilman (2008).

3 Qualitative and Quantitative Results

In the Boussinesq limit we can state some qualitative results common to the hydrostatic and nonhydrostatic systems: (i) There are exponentially growing and decaying modes, and neutral oscillations. (ii) A necessary condition for instability is that the quantity $\mu(1 - \mu^2)\frac{d}{d\mu} (\omega_0^2 - \alpha_0^2) - 4\omega_0^2 \mu^2 > 0$ somewhere in the domain. Here $\mu$ is sine latitude, $\omega_0$ is the differential rotation, and $\alpha_0 = 2B/r\omega_0\sqrt{\rho}$ is the toroidal magnetic field scaled to units of angular velocity. It follows that the angular momentum per unit mass, $(1 - \mu^2)\omega_0$, decreasing monotonically toward the poles, such as in the case of solar differential rotation, is stabilizing. But in stars that have a polar acceleration, the differential rotation may be destabilizing. (iii) Broad toroidal-field profiles, such as $\alpha_0 = a\mu$ (peaks of opposite signs at 45° N and S latitude), are also stabilizing for solar-type differential rotation. For example, the combination $\alpha_0 = a\mu$ and $\omega_0 = (1 - s\mu^2)$ ($0 \leq \mu \leq 1$) is stable in both systems, no matter what the peak field strength is. But banded toroidal fields of sufficiently large field strength can be unstable, since $\mu\frac{d}{d\mu} \alpha_0^2 < 0$ on the poleward side of such bands. This feature is the source of unstable modes, we show in later sections. This effect was also noted in Pitts & Tayler (1985).

To get quantitative results we have focused on parameter choices that are plausible for the solar tachocline. We specify a differential rotation of 0.18 relative to the reference frame. We choose values of the effective gravity parameter, $G$, appropriate for the tachocline, between 0.1 for the overshoot part and 10 for the radiative part. We specify toroidal band widths of 10° (FWHM) placed at latitudes between 15° and 60°. Toroidal field strengths in the tachocline are unknown from observations, so we consider a wide range of peak values, from well below 100 kG, for which $m = 0$ modes are always stable, up to 1 MG, probably well above what can occur in the Sun.

Figure 1a,b shows that the two different approximations of the equations give similar disturbance growth rates as functions of toroidal field and stratification parameters $G$ or $N$ (the Brunt-Väisälä frequency, $N$, in the nonhydrostatic system). In both cases, instability occurs for toroidal fields above threshold values (straight diagonal lines) deduced from the equations and the integral theorem. The presence of rotation and differential rotation leads to a diagonal corridor of stability (shown in black) immediately to the right of the white line. Further to the right (higher toroidal fields), the growth rates increase linearly with $a$. A growth rate of 1 corresponds to an e-folding time of 3.6 days.
(1 year/100); the instability is much more powerful than for nonaxisymmetric modes. $a = 1$ corresponds to a toroidal field peak of 100 kG, so the instability first sets in for much higher fields than for nonaxisymmetric modes. The largest differences in growth rate for hydrostatic and nonhydrostatic systems occur for MG toroidal fields and low subadiabatic stratification (very small $G$ and $N$; the lower right of each figure). In this part of parameter space the hydrostatic approximation starts to break down, so the nonhydrostatic equations are probably more accurate. But it takes rather extreme conditions to produce a significant difference.

![Figure 1](image1.png)

**Figure 1.** Growth rate contours in the $G$-$a$ plane for (a) a hydrostatic, non-Boussinesq system and (b) a nonhydrostatic-Boussinesq system for a 10\degree toroidal band at 30\degree latitude, where $a$ is $\alpha_0/\mu$.

Figures 2a,b show velocity and magnetic patterns for an unstable mode, radial index $n = 5$, that has grown from an initial distribution of toroidal magnetic flux distributed quasi-uniformly from the bottom of the tachocline to the top between latitudes 25\degree and 30\degree (17\% amplitude relative to the toroidal peak). Fig 2a is for the overshoot tachocline ($G = 0.1$); Fig 2b is for the radiative tachocline ($G = 10.0$). In both cases, the disturbances modify the toroidal field profile in radius, leading to possible formation of toroidal rings stacked in radius. We see also that most of the deformation occurs on the poleward side of the peak in the undisturbed band; this is consistent with the integral-theorem result that showed that instability can occur where the toroidal field amplitude decreases rapidly toward the poles. The velocity patterns extend much further in latitude away from the toroidal field peak than do the magnetic patterns. They also extend much further in latitude in the radiative tachocline than in the overshoot tachocline. This latter effect is caused by the more subadiabatic stratification in the radiative case, which tends to suppress radial motions, forcing the disturbance circulation to be much flatter.

4 Implications for the Sun and Stars

Previous studies have shown that bands of toroidal fields in solar/stellar tachoclines are unstable to nonaxisymmetric disturbances. We have shown here that such bands are also unstable to axisymmetric perturbations. Axisymmetric
Figure 2. Frames a and b show meridional cuts of flow (arrow vectors) and magnetic (color shades) fields rendering early development of an unstable mode (radial order n = 5) from magnetic flux initially distributed quasi-uniformly in a 10°-wide sector in differential latitude centered at 30° for (a) $G = 0.1$ (overshoot) and (b) $G = 10$ (radiative) tachoclines. The plot domains span 25°–40° in latitude and from the bottom (lower left) to the top (upper right) of the tachocline in radius.

perturbations set in at much higher field strengths but with much higher growth rates. A probable consequence of axisymmetric instabilities is toroidal band fragmentation in depth, implying a mechanism for producing repeated pulses of flux eruption.

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References