Implementation of Data Assimilation Methods for Dynamo Models to Predict Solar Activity

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**Abstract.** Cyclic variations of solar activity are a result of a complicated dynamo process in the convection zone. Despite the regular cyclic variations of solar activity, the chaotic variations of sunspot number from cycle to cycle are difficult to predict. The main reasons are the imperfect dynamo models and deficiency of the necessary observational data. Data assimilation methods iterate observational data and models for possible efficient and accurate estimations of physical properties, which cannot be observed directly. We apply the Ensemble Kalman Filter method for assimilation of the sunspot data into a non-linear mean-field dynamo model, which takes into variations of magnetic helicity and parameters of the solar convection zone from helioseismology. We present the results of application of this data assimilation method for representation of the solar cycles and prediction of variations of the sunspot number, and discuss potentials of data assimilation methods for solar dynamo modeling.

1 Introduction

Investigation of any natural phenomenon consists of three basic parts: observation, construction of a model and prediction of future state of the system. Predictions based on a model determine correctness of our understanding of the physical processes, simplifications and assumptions, and accuracy of the model. Taking into account that observational data include errors and that a model constructed on their basis is characterized by some approximations, a prediction of the next set of observations will deviate from the real data. Nevertheless, an estimate of possible uncertainties of the model and observations allows us to correct the model solution according to the information obtained from measurements. Thus, the updated observational data and a consistent correction of the model solution allow us to improve the simulation results, more accurately describe the system behavior and forecast its future states.

We consider an application of a data assimilation approach to a problem of solar activity in the context of sunspot number variations. There is no doubt that the 11-year cyclic variations of the sunspot number are connected to the dynamo process inside the Sun. Therefore it is natural to search for a solution by investigating nonlinear dynamo models, and use them for predicting the solar cycles. The current predictions of the next solar cycle, number 24, show
a wide range of the expected sunspot number (reviewed by Kane 2007; Pesnell 2008). In this paper we discuss initial results of application of the Ensemble Kalman Filter method (EnKF) to a simple nonlinear dynamo model for analysis of the solar activity cycles. For modeling of the solar cycle properties we use a nonlinear MHD dynamo model of Kleeorin and Ruzmaikin (1982), which takes into account dynamics of turbulent magnetic helicity.

2 Basic Formulation of Data Assimilation

The main goal of any model is an accurate description of properties of a system in the past and present times, and the prediction of its future behavior. However, a model is usually constructed with some approximations and assumptions, and has errors. Therefore, it cannot describe the true condition of a system. On the other hand, observational data, \( d \), also include errors, \( \epsilon \), which are often difficult to estimate. The data assimilation methods such as the Kalman Filter (Kalman 1960) allow us, with the help of an already constructed model and observational data, to determine the initial state of the model, which will be in agreement with a set of observations, and obtain a forecast of future observations and estimate its errors (Evensen 2007; Kitiashvili 2008). For instance, in our case we know from observations the sunspot number (with some errors) and want to estimate the state of the solar magnetic fields, described by a dynamo model.

In general, if the state, \( \psi \), of a system can be described by a dynamical model \( d\psi/dt = g(\psi, t) + q \), with initial conditions \( \psi_0 = \Psi_0 + p \), where \( g(\psi, t) \) is a nonlinear vector-function, \( q \) and \( p \) are the errors of the model and in the initial conditions, then the system forecast is \( \psi^f = \psi^t + \phi \), where \( \psi^t \) is the true system state, and \( \phi \) is the forecast error. The relationship between the true state and the observational data is given by a relation \( d = M[\psi] + \epsilon \), where \( d \) is a vector of measurements, \( M[\psi] \) is a measurement functional, which relates the model state, \( \psi \), to the observations, \( d \).

For a realization of the data assimilation procedure in the case of nonlinear dynamics it is convenient to use the Ensemble Kalman Filter (EnKF) method (Evensen 2007). The main difference of the EnKF from the standard Kalman Filter is in using for the analysis an ensemble of possible states of a system, which can be generated by Monte Carlo simulations. If we have an ensemble of measurements \( d_j = d + \epsilon_j \) with errors \( \epsilon_j \) (where \( j = 1, ..., N \)) then we can define the covariance matrix of the measurement errors \( C_{\epsilon\epsilon} = \bar{\epsilon}\bar{\epsilon}^T \), where the over bar means the ensemble-averaged value, and superscript \( T \) indicates transposition. Using a model we always can describe future states of a system, \( \psi^f \). However, errors in the model, initial conditions and measurements do not allow the model result to be in total agreement with observations. To take into account this deviation, we consider a covariance matrix of first guess estimates (our forecast related only to model calculations): \( (C^{\psi^f}_\psi)^T = (\psi^f - \bar{\psi}^f)(\psi^f - \bar{\psi}^f)^T \). Note that the covariance error matrix is calculated for every ensemble element. Then the estimate of the system state is given by:

\[
\psi^a = \psi^f + K \left( d - M\psi^f \right),
\]
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where \( K = (C_e^\psi)^f M^T \left( M(C_e^\psi)^f M^T + C_e^\psi \right)^{-1} \), is the so-called Kalman gain (Kalman 1960; Evensen 2007). The covariance error matrix of the best estimate is calculated as: \( (C_e^\psi)^a = (\psi^a - \overline{\psi}^a)(\psi^a - \overline{\psi}^a)^T = (I - K_eM) (C_e^\psi)^f \). We can use the last best estimates obtained with the available observational data as the initial conditions and make the next forecast step. At the forecast step, we calculate a reference solution of the model, according to the new initial conditions, then simulate measurements by adding errors to the model and to the initial conditions. Finally we obtain a new best estimate of the system state, which is our forecast. A new set of observations allows us to redefine the previous model state and make a correction to the predicted state.

In order to implement EnKF for prediction of sunspot cycles it is necessary to define a dynamo model that describes the evolution of the system parameters in time.

3 Formulation of the Dynamo Model with Magnetic Helicity

Currently, there is no generally accepted model of the solar dynamo. However, most of the models are based on the Parker’s oscillatory \( \alpha \Omega \)-dynamo mechanism (Parker 1955), which includes turbulent helicity and magnetic field stretching by the differential rotation. Recent observational and theoretical investigations (e.g. Sokoloff 2007; Brandenburg & Subramanian 2005) revealed an important role of magnetic helicity (Pouquet et al. 1976). Thus, for this investigation we added to the original Parker’s model an equation describing the evolution of the magnetic helicity, \( \alpha_m \). This equation was derived by Kleeorin and Ruzmaikin (1982) from the conservation of the total magnetic helicity. The dynamo model can then be written as (Kitiashvili & Kosovichev 2008a)

\[
\begin{align*}
\frac{\partial A}{\partial t} &= \alpha B + \eta \nabla^2 A, \quad \frac{\partial B}{\partial t} = G \frac{\partial A}{\partial x} + \eta \nabla^2 B, \\
\frac{\partial \alpha_m}{\partial t} &= \frac{Q}{2\pi \rho} \left[ \langle B \rangle (\nabla \times \langle B \rangle) - \frac{\alpha}{\eta} \langle B \rangle^2 \right] - \frac{\alpha_m}{T},
\end{align*}
\]

where \( B \) is the toroidal component of magnetic field, \( A \) is the vector potential of the poloidal component of the mean magnetic field, \( \langle B \rangle = \langle \mathcal{B} \rangle = \mathcal{B}_P + \mathcal{B}_T \) (\( \mathcal{B}_P = \text{curl}(0, 0, A) \), \( \mathcal{B}_T = (0, 0, B) \) in spherical coordinates), \( \eta \) is the total magnetic diffusivity; \( G = \partial \langle v_x \rangle / \partial y \) is the rotational shear, coordinates \( x \) and \( y \) are in the azimuthal and latitudinal directions respectively, parameter \( \alpha \) is helicity represented in the form \( \alpha = \alpha_h/(1 + \xi B^2) + \alpha_m \), \( \alpha_h \) and \( \alpha_m \) are the kinetic and magnetic parts; \( \xi \) is a quenching parameter, \( \rho \) is density, \( T \) is a characteristic time of dissipation magnetic helicity and, \( Q \sim 0.1 \). Following the approach of Weiss et al. (1984) we average the system of equations (2) in a vertical layer to eliminate \( z \)-dependence of \( A \) and \( B \) and consider a single Fourier mode propagating in the \( x \)-direction, assuming \( A = A(t)e^{ikx} \), \( B = B(t)e^{ikx} \); then we get the following
Figure 1. Variations of the magnetic field for the middle convective zone and different initial conditions. Panel a) shows toroidal component of magnetic field variation; b) vector-potential, $A$, of the poloidal magnetic field, c) magnetic helicity variations and d) evolution of the model sunspot number.

The system of equations

$$\frac{dA}{dt} = \alpha B - \eta k^2 A, \quad \frac{dB}{dt} = ikGA - \eta k^2 B, \quad \frac{d\alpha_m}{dt} = -\frac{\alpha_m}{T} - \frac{Q}{2\pi \rho} \left[-ABk^2 + \frac{\alpha}{\eta} (B^2 - k^2 A^2)\right].$$

For the interpretation of the solutions of the dynamical system in terms of the sunspot number properties we use the imaginary part of the toroidal component $B(t)$ because it gives the amplitude of the antisymmetric harmonics, and approximate the sunspot number, $W$, as $(\text{Im}B)^{3/2}$, following Bracewell’s suggestion (Bracewell 1953, 1988). This dynamo model has been investigated in detail by Kitiashvili & Kosovichev (2008a).

Figure 1 shows typical nonlinear periodic solutions and the corresponding model sunspot number, which reproduce typical observed solar cycle profiles with fast growth and slow decay. The profile of the toroidal field variations becomes nearly sinusoidal for small amplitude.

3.1 Implementation of the Data Assimilation Method

For assimilation of the sunspot data into the dynamo model we selected a class of periodic solutions (Figure 1) that corresponds to parameters of the middle convective zone and describes the typical behavior of the sunspot number variations. The implementation of the EnKF method consists of 3 steps: preparation of the observational data for analysis, correction of the model solution according to observations, and prediction.

**Step 1: Preparation of the observational data.** Following Bracewell (1953, 1988), we transform the annual smoothed values of the sunspot number for the period of 1856 - 2007 in the toroidal field values using the relationship $B \sim W^{2/3}$ and alternating the sign of $B$. We also select the initial conditions of the model so that the reference solution coincides with the beginning of the first cycle of our series, cycle 10, which started in 1856. In this paper we do not consider the previous solar cycles because of the uncertainties in the early sunspot number measurements (Svalgaard et al. 2007). Then we normalize the toroidal field in the model in such a way that the model amplitude of $B$ is equal to the mean "observed" toroidal field. In addition, we normalize the model time scale assuming that the period of the model sunspot variations corresponds to the typical solar cycle duration of 11 years.
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Figure 2. Predictions for the solar cycles 22-24. The black dashed curves show the model reference solution. The grey solid curves show the best estimate of the sunspot number using the observational data (empty circles) and the model, for the previous cycles. The black solid curves show the prediction results. In panel c) the model solution is shown for 3 different estimates of the sunspot number for 2008: 3 (grey dashed curve), 5 (dashes) and 10 (dots).

Step 2: Assimilation for the past system state. Unfortunately we do not have observations of the magnetic helicity, toroidal and poloidal components of magnetic field. Therefore, in the first approximation, we generate observational data as random values around the reference solution with a standard deviation of $\sim 12\%$, which was chosen to roughly reproduce the observed variations of the sunspot number. Similar random errors are also added to the model equations as described in Sec. 2. Then we calculate the covariance error matrixes of the observations, $C_{\epsilon\epsilon}$, and the forecast, $(C_{\psi\psi})^f$. After combining the observational and model error covariances in the form of Kalman gain, $K$, we obtain the best estimate of evolution of the system, $\psi^a$ from Equation 1.

Step 3: Prediction. To obtain a prediction of the next solar cycle we determine the initial conditions from the best estimated solution for the previous cycle in terms of the amplitude and phase to continue the model calculations. Then after incorporating the reference solution with the new initial conditions we simulate future observational data by adding random noise and repeat the analysis. This provides the best EnKF estimate of the future state of the system (forecast).

The analysis described has been tested by calculating predictions of the previous cycles. Figure 2 (a,b) shows examples of the EnKF method implementation for forecasting the sunspot number of cycles 22-23. For these forecasts, we first obtain the best estimated solutions (grey curves) using the observational data prior to these cycles (open circles). After this, we obtain the exact solution (black curves) according to the initial conditions of the time of the last measurement and simulate a new set observation (black dots) by adding random noise. Then, we obtain the EnKF estimates using the simulated observations that give us the prediction (Figure 2, black curves). These experiments show that this approach can provide a reasonable forecast of the strength of the next solar cycles. However, there are significant discrepancies.

The main uncertainties are caused by inaccuracies in determining the time of the end of the previous cycle from the sunspot number data, and, of course, by the incompleteness of the model and insufficiency the sunspot number data. In particular, we found the forecast is inaccurate when the sunspot number changes...
significantly from the value of the previous cycle. Also, our forecast experiments showed a strong dependence on the phase relation between the reference model solution and the observations. The same analysis scheme is applied to prediction of the next solar cycle, cycle 24 (Figure 2c). According to this result, solar cycle 24 will be weaker than the current cycle by approximately 30%. To check the stability of the prediction we used two other sets of initial conditions in 2008 and obtained close results (Figure 2c).

4 Results and Discussion

The results of assimilation of the annual sunspot number data into the solar dynamo model and the prediction of the previous solar cycles demonstrate a new method of forecasting the solar activity cycles. Using the EnKF method and a simple dynamo model we obtained reasonable predictions usually for the first half of the sunspot cycles with the error $\sim 8 - 12\%$, and in some cases also for the declining phase of the cycles. This method predicts a weak solar cycle 24 (Figure 2c) with the smoothed annual sunspot number at the maximum of approximately 80. It is interesting that the simulations show that the previous cycle does not finish in 2007 as was expected, but it still continues. According to the estimates the maximum of the next cycle will be approximately in 2013.

The application of the data assimilation method, EnKF, for modeling and predicting the solar cycles shows the power of this approach and encourages further development. It also reveals significant uncertainties in the model and the data. Among these are the uncertainties in the determination of the start of a solar cycle from the sunspot number series (in particular, when the cycles overlap), leading to the uncertainty in the phase relation between the model solution and the data. Also, there are significant uncertainties in the relationship between the sunspot number data and the physical properties of the solar magnetic field, in the absence of magnetic field and helicity data, and, of course, in the dynamo model. Our conclusion is that for robust and accurate predictions of the solar cycles the information contained in the sunspot number data is insufficient. Our future plan is to develop a data assimilation procedure for synoptic magnetic field data available for the past 3 cycles, using a 2D dynamic solar dynamo model such as the model of Brandenburg et al. (1992).

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