Comparing Properties of Convection in the Surface Layers of Stars With Different Masses and Evolutionary States

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Abstract. By comparing convective properties computed from simulations derived from a series of G and K type stellar models, we find marked differences between simulations and mixing length theory. These differences increase with the strength of the turbulence (as measured by the size of the turbulent pressure). Two of the most obvious differences are in the run of the superadiabaticity versus depth and in the size of the convective eddies. Provided the turbulence is weak (i.e. the ratio of turbulent pressure to gas pressure is a few percent) the estimate of eddy size used in a stellar model eddy (i.e. $\alpha H_p$) is quite close to the mean size of the simulated eddies. However, as the strength of the turbulence is increased (as happens in more evolved or hotter or more massive stars) the mixing length estimate becomes a very poor approximation to the actual eddy size. We also show results of a comparison study between different simulation codes conducted by an independent collaborator. Despite significant differences in input physics, numerical methods and treatments of radiation, the resolved convective quantities differ by less than 10%.

1 Introduction

The convective transport of energy in stars plays an important role in accurately determining stellar structure, particularly in the outer layers. Commonly used treatments of convection in stellar modeling such as mixing length theory (MLT) (Böhm-Vitense 1958), are a large source of uncertainty, especially when applied to transition regions between radiative and convective energy transport. MLT works very well in deep layers, however, near the surface where radiation carries a significant fraction of the total energy flux, the MLT approximation breaks down.

Due to recent advances in computing power it is now possible to perform realistic 3 dimensional simulations of stellar convection that include a range of scales spanning 2-3 orders of magnitude. Though the actual range of scales in stellar convection is many orders of magnitude greater than this, provided the smallest modeled scale (approximately the grid size) is 2-3 orders of magnitude less than the largest resolved scale (e.g. granules), then a subgrid model can be applied and the simulation should be able to realistically represent coherent structures in the surface layers of the star. This is the idea behind large eddy simulations. Here we will use the term ‘simulation’ to describe 3D LES models that resolve a range of convective eddies and the term ‘model’ to describe the stellar model in which convection is parametrized.

Parameterizing and including the effects of turbulence in stellar models will allow for the computation of more realistic stellar structure and a superadiabatic
layer (SAL) that matches that from a simulation. In addition, turbulence is an important factor in determining realistic surface boundary conditions for stellar models with convection zones.

2 Modeling Realistic Stellar Surface Convection

A 3D simulation is characterized by its surface gravity, effective temperature and chemical composition. We get the surface gravity and stellar flux (which is close to $\sigma T_{\text{eff}}^4$, where $\sigma$ is the Stefan-Boltzmann constant) of each star from the 1D stellar evolution model, and then put the values in the 3D code by hand.

Due to the huge range of scales in the solar convection zone the computational domain is typically a tiny box (depth less than 0.01% of the stellar radius) located at the top of the convection zone. For such a small domain, curvature and radial variation in gravity are minimal and can be ignored. The box itself has a width equal to that of a few granules and a depth of 9 or more pressure scale heights (PSH), with the top of the box located about 2 PSH above the photosphere. The size of the domain is important as boundary effects can be reduced (though never eliminated) by increasing the vertical extent of the box until quantities within 1 PSH of the upper and lower boundaries remain unchanged if further increases are made. The vertical walls are periodic and the horizontal walls free slip and impenetrable (closed box).

The code is a finite-volume code on a staggered grid and simultaneously solves the mass-, momentum- (Navier-Stokes) and energy conservation equations consistently coupled with the radiative transfer equations. The subgrid-scale model employed is the one from Smagorinsky (1963). The code is numerically stable for surface gravities resembling the Sun and stars with higher surface gravity, and no additional artificial viscosity is needed to stabilize the code. The radiation flux is computed using the 3D Eddington approximation (Unno & Spiegel 1966). The same authors thoroughly prove that the 3D Eddington approximation does hold in the optically thin regions. Retardation effects are neglected, but matter and radiation are treated without any simplifying assumptions about thermal equilibrium among each other. Furthermore, the radiation flux is consistently coupled to the energy equation. Opacities and equation of state are taken from the OPAL project (Alexander & Ferguson (1994), Rogers et al. (1996) and Iglesias & Rogers (1996)).

Including turbulence in stellar models alters the structure of the outer layers. Because the MLT prescription for convection lacks dynamical effects, 3D convection simulations produce turbulent pressure that can be in excess of 10% of the gas pressure. In particular, the superadiabaticity is larger and occurs closer to the surface in turbulent simulations than in static 1D models. Figure 1 compares the SAL from a 3D turbulent simulation to the SAL computed with MLT. The difference in the SAL is greater in the higher mass model, where the turbulent pressure is larger.

2.1 Comparison of Stellar Convection Codes

A typical 3D simulation solves 100$^3$ equations every time step and requires a million or so time steps to adjust from the initial state of thermal and hydrostatic equilibrium to radiative hydrodynamical equilibrium (thermal relaxation) and
then another million or so time integration steps to compute reliable statistical quantities (statistical convergence). After such a long and complex computation how can one be sure what is being computed is a realistic representation of stellar granulation? One answer is to compare to simulated and observed quantities, another is to compare results with those from other simulation codes used to model the Sun.

One observational test is the computation of $p$-mode frequencies and comparing with values derived using helioseismology. Li et al. (2002) showed that when parametrized turbulence taken from a 3D simulation of the Sun is included in the computation of the $p$ modes then the match between observed and modeled $p$-mode frequencies is improved significantly. The same method has been applied to $\eta$-Bootis (Straka et al. 2006) and $\alpha$-Centauri (Straka et al. 2007) with similar success.

As far as simulations are concerned, two other groups that have done extensive 3D modeling of solar granulation are Nordlund and Stein (1990), see also Stein & Nordlund (1989, 1998) and the CO5BOLD group (Freytag et al. 2002). F. Kupka has kindly provided us with results of a comparison of dynamical and thermal quantities from all three groups.

Figures 2 and 3 compare the mean temperature structure and the root mean square of the vertical velocity found in different numerical simulations. The domains of the simulations begin in the convection zone and end in the photosphere. CKS refers to our code (Robinson et al. 2003). The steep temperature gradient near the top of the domain is almost identical in all simulations, producing similar structure in the peak of the SAL ($\nabla - \nabla_{ad}$). The differences in $\langle u^2 \rangle^{1/2}$ are primarily a result of the different boundary conditions. A closed
box produces a more gentle velocity gradient in the deeper layers and forces an abrupt drop near the lower boundary.

These figures show that despite significant differences in input physics (ray integration versus 3D non-local Eddington approximation), opacity binning versus gray atmosphere and differences in simulation boundary conditions (open versus closed top and bottom surfaces) away from the boundaries the differences in the resolved convection (large eddies) is less than 10%.

![Figure 2](image.png)

Figure 2. Comparison of average temperature as a function of height above surface in solar simulations computed with different codes. Courtesy of F. Kupka.

3 Results

3.1 Testing Mixing Length Theory in the Surface Layers

In a stellar model, the radius is set by the entropy jump in the outer layers, which is determined by the surface boundary conditions. In conventional 1D stellar models, the entropy jump is set by the mixing length parameter $\alpha$, where $\alpha$ is set to a value that reproduces the observed solar radius. Using the solar value of $\alpha$ does not work for all stellar models and can vary with depth. Using 3D simulations, we show that eddy sizes cannot be represented simply by $\alpha H_p$, where $H_p$ is the pressure scale height. While a single value of $\alpha$ works reasonably well for the Sun, it is not constant for stellar models elsewhere in the $\log(g)$-$\log(T_{\text{eff}})$ plane.

One estimate of the mixing length in the simulations is the full width at half maximum of the auto-correlation of either the vertical velocity or entropy or temperature. This metric tells us approximately how far a convective element
rises before mixing with the surrounding fluid. Figure 4 compares the FWHM of vertical velocity to MLT at various depths for several stellar models with different mass and turbulent pressure. The mixing length is defined by $\alpha H_p$ where $\alpha$ is the solar calibrated value. In these plots, the granule sizes are weighted by $\alpha H_p$, so that MLT is represented by a constant value at unity. In the SAL, the simulation granule sizes deviate from MLT in the top few PSH, but in deep layers, MLT and the simulations agree. The bending of the curve within one pressure scale height of the bottom is a result of the lower boundary, which suppresses convective motion. The simulation of the evolved subgiant is not deep enough for the convective motions to approach MLT. Models with higher turbulent pressure show greater discrepancies between MLT and simulation.

4 Conclusion

It is standard practice to calibrate the convective mixing length for 1D stellar models to the solar value. We have used 3D radiative hydrodynamic simulations to demonstrate that the mixing length is different for stars with different mass and evolutionary state. Further, we show that the effective mixing length is variable with depth in the superadiabatic layer. The deviation from the solar calibrated value is greater in simulations with higher turbulent pressure.

We also present comparisons of numerical methods for simulations of solar granulation. The comparisons show that similar results are achieved using codes that implement the physics in different ways.
Figure 4. Estimates of the mixing length for several simulations. The simulation mixing length is estimated by the FWHM of the auto-correlation of vertical velocity.

Acknowledgments. We would like to thank F. Kupka of the Max Planck Institute for Astrophysics for providing the independent comparison of simulation codes. This research was supported in part by the NASA EOS/IDS Program (FJR) and by NASA grant NAG5-13299 (PD). SB is partially supported by NSF grants ATM 0348837 and ATM 0737770.

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