Results from Fitting Long and Very-Long MDI
Time-Series at Low and Intermediate Degrees

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Abstract. I present results from fitting long and very-long MDI time series of spherical harmonics coefficients, at low and intermediate degrees. The fitting methodology used, initially developed for very-long time series, incorporates several key aspects not present in the “production” Michelson Doppler Imager (MDI) (or Global Oscillation Network Group (GONG)) fitting methodologies. The fitting has since been extended to higher degrees and applied to shorter time series, resulting in fitting 2088-day long, as well as, 728, 364 and 182-day long time series, covering nearly 11 years of observations. The 2088-day long time series has been fitting up to $\ell = 125$. Nine overlapping 728-day long time series have been fitted up to $\ell = 95$, while nineteen and thirty nine overlapping 364 and 182-day long time series, respectively, have been fitted up to $\ell = 47$. I present and discuss some of the characteristics of the observed temporal changes. Presentation and discussion of remaining “issues” with the fitting, as well as the inferred rotation inversion are shown in separate contribution to these proceeding.

1 Introduction

The fitting methodology used here was initially developed for fitting 2088-day long time-series (Korzennik 2005). This method incorporates the following features: fitting individual modes for each $m$, using an asymmetric profile, and fitting all $m$ for a given $(\ell, n)$ simultaneously. The FWHM and the asymmetry are independent of $m$. It uses an optimized multitaper as power spectrum estimator, and the complete leakage matrix (horizontal and vertical components). The method iterates to include mode contamination, i.e. non-fitted $(\ell', n')$ modes present in fitting window, and performs a “sanity check”: it fits only the modes whose amplitude is above a given threshold, i.e.: $A_{n,\ell,m} > x_{\text{rej}} \times \text{RMS(residuals)}$, where $x_{\text{rej}} = 3$. The method was at first applied to Global Oscillation Network Group (GONG) and Michelson Doppler Imager (MDI) data, and limited to $1 \leq \ell \leq 25$.

Since there is nothing intrinsic to the methods that limits it to low $\ell$ or very-long time series, I have since extended the method to higher degree and applied it to shorter time series: up to $\ell = 125$ for 2088-day long time-series; up to $\ell \leq 95$ for 9 overlapping 728-day long time series; and up to $\ell \leq 47$ for 364- and 182-day long time series (19 & 39 overlapping segments respectively). The fitting covers some 3,600 days (1996/04/30 to 2006/04/19), and has so far been carried out using MDI time-series. Figure 1 compares the results of the 2088-day long fit to a similar one carried out by the MDI team with their pipeline (Schou 1992).
This comparison shows the improved coverage in $\ell - \nu$, and a discrepancy in frequencies that peaks up to $\approx 100\mu$Hz (around 1–3 mHz), but then increases again at higher $\nu$. This effect is more than just due to including (or not) the asymmetry. The mode line-width, $\Gamma$, resulting from my fitting appears more consistent (collapses into a single curve) but not the mode power. The opposite is observed for the MDI results, which remains puzzling. Estimates of error bars, $\sigma_\nu$, are self-consistent, but show a systematic discrepancy with MDI’s estimate by a factor 4. This and other remaining issues are presented and discussed elsewhere in these proceedings (Korzennik 2009).

2 Changes with Activity

Figure 2 shows changes in frequencies, after binning, as a function of frequency, resulting from fitting the 728-day long time series. That figure also shows the frequency differences multiplied by the normalized mode mass, frequency differences divided by the effective line-width, and frequency differences divided by the fitting uncertainty. While the amplitude of the frequency changes increases with frequency, they do not simply scale with the mode mass. In fact, at high frequencies, the magnitude of the changes increase faster than the inverse mode mass.

That figure also shows changes in frequencies, after binning, as a function of frequency, for the 364-day and 182-day long cases, but only for the differences divided by the effective line width or the fitting uncertainty. Note how these changes remains a small fraction of the effective line width ($\delta\nu < 0.3\Gamma$), and barely significant ($\delta\nu < 2\sigma_\nu$).

2.1 Scaling with Activity & Latitudinal Localization

To parameterize the changes in frequency with activity, I have binned scaled frequency changes in the $(\nu, |\frac{m}{\ell}|)$ plane, and then modeled the changes as follows:
Figure 2. Top panels: results from fitting nine 728-day long segments. Left to right: (a) frequency differences for each epoch; (b) frequency differences multiplied by the normalized mode mass, \( E \); (c) frequency differences divided by the effective line width; and (d) frequency differences divided by the fitting uncertainty. All four panels show binned quantities as a function of frequency. Bottom panels: results from fitting 19 364-day long and 39 182-day long segments, similar to rightmost two top panels.

\[
\mathcal{D}_\nu(t, \nu, |m_\ell|) = \langle \delta \nu_{n,\ell,m}(t) E_{n,\ell} \rangle = A(t) G(|m_\ell|) F(\nu) = Q(t, |m_\ell|) F(\nu) \tag{1}
\]

where \( E_{n,\ell} \) is the normalized mode mass\(^1\), \( F(\nu) \) is an ad hoc function, while \( A(t) \) is the amplitude of a simple and constant function of \( |m_\ell| \) and \( \nu \). \( Q(t, |m_\ell|) \) is:

\[
Q(t, |m_\ell|) = c \int Q^2_{\ell,m}(x) |B|(t, x) \, dx + c_o \tag{2}
\]

where \( Q_{\ell,m}(x) \) is the latitudinal component of the spherical harmonic, and where \( |B| \) represents the magnitude of the magnetic field, averaged over longitude and epoch, and symmetrized over the latitude.

The coefficient \( c \) is a scaling factor (\( \text{i.e., } \mu\text{Hz}/\text{G} \)), while \( c_o \) is an offset, associated to the reference frequency set used in Equation 1. The contribution

\(^1\) Normalized by the mode mass for \( \ell = 0 \) and interpolated at 3 mHz.
Figure 3. Top: frequency change model amplitude versus time compared to scaled solar activity indices (Sunspot number, MgII core-to-wing ratio, 10.7cm radio flux, in red, blue and green respectively), for 728-, 364- and 184-day segments (l to r). Bottom: regression between observed and predicted change and latitudinal “localization” of frequency changes; curves: prediction from average magnetic field, \( Q \), averaged over \( \ell \); crosses: measured quantity \( D_\nu/F(\nu) \), offset for clarity. The average of the first and last epoch (low activity) was used as reference and accounted for so \( c_0 \approx 0 \).

of the reference set used can be removed using the value of \( Q \) corresponding to that reference. This has to be done iteratively, but converges and produces a nearly negligible value of \( c_0 \).

Figure 3 shows how \( A(t) \) scales with various activity indices, while it also shows how well the latitudinal dependence of the frequency changes scale with the magnetic field. This confirms the work of Howe et al. (2002) that showed that the frequency changes can be localized to the latitudinal distribution of the magnetic field, using a different approach. Note that the scaling is quite linear, as using this approach shows no indication of a change of sensitivity with latitude via some dependence on \( |m\ell| \).

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Figure 4. Predicted frequency change dependence on $|m/7|$, i.e., $Q(|m/7|)$ and observed scaled frequency changes, as a function of $|m/7|$, namely $D_\nu/F(\nu)$. Left to right: 728, 364 and 182-day segments. Each row corresponds to a different epoch.

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References

Korzennik, S. G. & Eff-Darwich, A. 2009, these proceedings.
Ben Brown and Rachel Howe discussing some new results during coffee break.