Time-Distance Helioseismic Imaging of a Numerically Simulated Solar Tachocline

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Abstract. The solar tachocline, located near the bottom of the convection zone, is a very important region for solar dynamics and the solar dynamo. We develop a time-distance helioseismology technique, including both surface- and deep-focusing measurements, together with inversions, to derive the interior image of the sound speed perturbation at the tachocline with a latitudinal dependence. We test the technique on numerically simulated global wavefields and find that the technique is able to recover the major features that are preset in the numerical model, though a bit more widespread into the deeper interior. This measurement and inversion technique will be applied to MDI observations to derive the structures of solar tachocline.

1 Introduction

The solar tachocline, first defined by Spiegel & Zahn (1992), is an area located between the solar convection zone and radiative zone. It is a transition zone where the latitudinal differential rotation throughout the convection zone changes into a rigid rotation within the radiative zone, as found by numerous helioseismology studies of the solar interior rotation (e.g., Thompson et al. 1996). It is also close to where the helioseismically inferred sound speed perturbation profile exhibits the largest difference relative to a standard solar model—Model S (Christensen-Dalsgaard et al. 1996), as revealed by a number of helioseismology studies as well (e.g., Kosovichev et al. 1997). The solar tachocline is also widely believed to be the location for the solar dynamo operation, because the sharp rotational gradient observed near this area may build up a strong toroidal field nearby (Parker 1993). Therefore, it is clearly important to understand the structure, dynamics, and cyclic variations of the tachocline to better understand the transition of solar convection zone to the radiative zone, to better understand the solar magnetism generation and amplification, and to better understand the periodicity and evolution of solar cycles.

In this paper, we plan to examine the capability of time-distance helioseismology to image the solar tachocline area based on numerically simulated global wavefields. We develop two time-distance measurement schemes, surface- and deep-focusing, as well as inversion codes corresponding to these two different measurement techniques, and apply these codes on numerically simulated

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datasets to test our ability to reconstruct the sound speed perturbations that are preset at the tachocline area in the numerical model.

2 Technique

2.1 Surface- and Deep-Focusing Measurements

Time-distance helioseismology measures the travel times of acoustic waves that propagate from one solar surface location to another along curved paths in the solar interior by computing the cross-correlation between acoustic signals observed at the two separate locations.

The measurement procedure is as follows. A dataset with certain time sequences, either numerically simulated or observed, is remapped onto a Postel's projection coordinate system with the center of the projected map at the solar disk center, and with a pixel size of 0.6° (here and after, degree means heliographic degrees), consistent with the MDI medium-ℓ data sampling resolution at disk center. For the surface-focusing scheme (see left panel of Figure 1), every location in the dataset would serve as a central point, and for each central point, acoustic signals would be averaged from the annulus around this central point. Caution must be used during the procedure of selecting annulus pixels because the data are already remapped. The radius of the annuli ranges from 6.0° to 84.0° with a step of 0.6°, hence a total of 131 annuli. At each location, cross-correlations are computed between the signals from the central point and signals from each annulus, respectively. To further improve the signal to noise ratio, we again average all cross-correlations at the same latitude range, with a width of 5.0°, to obtain one set of cross-correlations at one latitude, which ranges from −60° to 60° with a step of 0.6° and some overlapping. Then, a Gabor wavelet fitting is performed to derive the mean acoustic travel time for all annuli at each latitude. For the deep-focusing scheme (see right panel of Figure 1), the measurement procedure is basically the same as for surface-focusing,
except that the signals for cross-correlation computation are selected based on the scheme as shown in the figure.

2.2 Inversion

After the acoustic travel times are measured, inversions are done to infer the sound speed structure inside the Sun. Sensitivity kernels in spherical coordinates are constructed using the ray approximation equations given by D'Silva & Duvall (1995). The computation of sensitivity kernels is similar to how the acoustic travel times are measured. Three-dimensional (two dimensions for horizontal and one for vertical direction) kernels are first computed for all different annuli, and then the three-dimensional kernels are collapsed along the longitudinal direction so that the kernel only depends on latitude in the horizontal direction. Kernels for surface- and deep-focusing schemes are made separately but by similar approaches.

The inversions are performed by solving a series of linearized equations in the sense of least squares. Generally, two different inversion algorithms have been used in time-distance helioseismology: the LSQR algorithm (Zhao, Kosovichev, & Duvall 2001) and Multi-Channel Deconvolution, also known as MCD (e.g., Jacobsen et al. 1999). In this paper, we employ the MCD technique to perform the inversions. Although the inversions are best performed in spherical coordinates, MCD is still useable when the interior vertical grids are divided along the radial direction, and when the boundaries are properly taken care of.

3 Results

3.1 Sound Speed Perturbation Model for Tachocline

A numerical simulation code has been developed to simulate solar wave fields on global scales in spherical coordinates (Hartlep & Mansour 2008). The simulation solves linearized Euler equations describing wave propagation, and in practice, the simulation is limited to contain spherical degrees, $\ell$, ranging from 0 to 170, which is sufficient for most global scale studies. Comparing with observations obtained from MDI medium-$\ell$ program, this simulation is very accurate in the locations of the power ridges in the $k - \omega$ diagram, as well as in time-distance acoustic travel times (see Figure 1 in Hartlep et al. 2008).

An artificial model of the tachocline is constructed from small-amplitude perturbations of the sound speed relative to Model S, as shown in Figure 2. The perturbed structure has a two-dimensional Gaussian shape in each hemisphere, and is symmetric relative to the equator. For each Gaussian shape, the sound speed perturbation amplitude, $\delta c/c$, is 0.006, which is the same order of magnitude derived from global helioseismology analysis (e.g., Kosovichev et al. 1997). In the radial direction, each Gaussian shape has a perturbation centered at $0.70 R_\odot$ with a FWHM of $0.082 R_\odot$ (the variance of the Gaussian function is $0.035 R_\odot$). In the latitudinal direction, the perturbation is centered at $30^\circ$ latitude with a FWHM of $35^\circ$ (the variance of Gaussian function is $15^\circ$). A 1024-min numerical simulation of global wave fields is constructed using this modified solar model, and time-distance studies are carried out on the photospheric oscillation data to examine how well the features that are preset in the model can be recovered by our technique.
3.2 Surface-Focusing Results

For the surface-focusing measurements, the measured acoustic travel times are shown in Figure 3 after the measured travel times from the original Model S simulation are subtracted, displayed as a 2D image as a function of latitude and annulus radius. It is apparent that the travel times are smaller on the right hand side of the image, when larger than 46° or so in annulus radius. This can be clearly seen in the 1D curve obtained after a latitudinal average, also shown in Figure 3.

The inversion of the sound speed perturbation for the surface-focusing scheme are also shown in Figure 3. For the 2D image, the sound speed perturbation bump preset in the model near the tachocline area is very nicely recovered after the time-distance measurements and inversions are done, though not perfectly matching the two Gaussian shaped structures in both hemispheres. The location and amplitude of the perturbed area match quite well. However, it is also clear, especially in the latitudinally averaged 1D sound speed perturbation curve shown in the lower right panel of Figure 3, that the inverted sound speed perturbation is wider than the numerical model, especially in the deeper interior. A negative sidelobe close to the surface can also be seen.

3.3 Deep-Focusing Results

The measured acoustic travel times from the deep-focusing scheme are shown in the upper row of Figure 4 after the corresponding travel times measured from the simulation without a tachocline are subtracted. Similar to the measurements from surface-focusing, a sharp drop in the travel time can be seen at an annulus radius of about 45° and beyond in both the 2D image and the 1D latitudinally averaged curve. It is noteworthy that at higher latitudes and large annulus

![Figure 2.](image)

Left: Sound speed perturbation model near the tachocline used in the numerical simulation of global wave fields. The strongest perturbation is 0.006, centered at 0.70R⊙ and 30° in both hemispheres. The white dashed line indicates a radius of 0.70R⊙. Right: Latitudinally averaged sound speed perturbation of the tachocline model in the left panel.
Figure 3. **Upper:** Left panel: measured acoustic travel times from the numerical simulation with a tachocline relative to the measured times from a simulation with no tachocline, using the surface-focusing measurement scheme. Right panel: Latitudinally averaged 1D acoustic travel time. **Bottom:** Left panel: Time-distance inverted results of sound speed perturbation, displayed using the same color scale as in Figure 2. Right panel: Latitudinally averaged 1D sound speed perturbation from inversion results. Dotted curve shows the 1D perturbation model. Errors bars are displayed only at selected points after a magnification of 2 for better visibility.

Radii, the acoustic travel time fitting often fails to give reliable measurements, therefore travel times in these cases are discarded so that the inversion results are not contaminated by bad fitting points.

The inversion results from the deep-focusing scheme are shown in the lower row of Figure 4. Similar to the inversion results from the surface-focusing scheme, the sound speed bump preset in the tachocline area is successfully recovered, although the shape does not perfectly match the model. Again, the inverted sound speed is spread into areas beyond the tachocline area, in particular, strong perturbations are producing visible scattering in areas beneath the convection zone.

4 Conclusion

We have developed surface- and deep-focus time-distance measurement schemes, as well as inversions corresponding to these schemes. We apply these techniques
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Figure 4. Same as Figure 3, but for the deep-focusing scheme. Note that travel times cannot be measured or inverted at all latitudes and annulus radii.

on numerically simulated global wave fields, which have sound speed perturbations preset at the tachocline. We find that our technique can well reconstruct the major features preset in the model. We will apply this technique to MDI observations to derive the two-dimensional picture of the solar touchline.

References

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