Hydrodynamic Modeling of Coronal Loops with Hinode and STEREO

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Abstract. The hydrodynamic evolution of impulsively-heated coronal loops and their subsequent cooling can now be modeled with multi-wavelength imaging instruments in soft X-ray (SXR) and extreme ultraviolet (EUV) wavelengths. Using analytical approximations to the hydrodynamic evolution of the density \( n(s, t) \) and temperature \( T(s, t) \) of an impulsively-heated loop (as a function of the loop length coordinate \( s \) and time \( t \)) we show an example how lightcurves observed with Hinode/XRT, EIS, GOES, and STEREO/EUVI can be modeled with a forward-fitting method in order to infer the maximum heating rate, the heating duration, and the cooling time of a heated loop during a small B1-class flare on 2007 February 1, previously analyzed by Warren et al. (2007).

1. Introduction

In this study we focus on the physical parameters of the hydrodynamic evolution of an impulsively-heated coronal loop that can be extracted from multi-wavelength observations from Hinode, using data from the EUV Imaging Spectrometer (EIS) (Culhane et al. 2007) and the X-ray Telescope (XRT) (Golub et al. 2007), as well as from the STEREO Extreme Ultra-Violet Imager (EUVI) (Wuelser et al. 2004). The hydrodynamic evolution of a coronal loop can only be sufficiently constrained with measurements from imaging instruments that cover the entire coronal temperature range in soft X-ray and EUV wavelengths, which are for the first time provided with high spatial resolution (on the order of \( 1''-2'' \)) with the combination of Hinode and STEREO. The high spatial resolution is required to isolate a single coronal loop from the myriad of ambient loops in the fore- and background corona, which all have their own time-dependent hydrodynamic evolution. This was not possible with previous spectroscopic imagers of lower spatial resolution, such as with the SOHO Coronal Diagnostic Spectrometer (CDS), for instance. The stereoscopic information of STEREO/A and B, in addition, provide the 3D geometry of a loop, in particular the accurate (unprojected) loop length and loop plane inclination angle that are important for hydrodynamic modeling. In this study we present a first attempt at multi-wavelength lightcurve modeling using Hinode and STEREO with a self-consistent hydrodynamic model. If a coronal loop is well-defined in all observed images in different wavelengths, this type of forward-modeling allows us to infer the heating function and cooling times, which constrain also whether the investigated loop is an elementary loop strand (with isothermal cross-section) or a composite loop system, and this way addresses a major issue of nanoflare heating models (Klimchuk 2006).
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2. Analytical Approximations of the Hydrodynamic Loop Evolution

There are two extremes of the hydrodynamic evolution of a coronal loop: either the loop is in a stationary equilibrium, which requires a constant heating rate to balance the conductive and radiative losses, or the loop is heated impulsively and cools down subsequently, a concept that is frequently used in nanoflare heating models. Since there is overwhelming evidence that most coronal loops show significant intensity changes during a time interval of $\approx 10 - 30$ min at highest spatial resolution (e.g., Winebarger et al. 2003), we consider here only the latter model of impulsive heating.

Impulsive heating is often parameterized with a Gaussian time profile and an exponential spatial dependence, either uniform, concentrated at the footpoint ($s=0$), or at the loop apex ($s=L$),

$$E_H(s,t) = H_m \exp \left( -\frac{(t - t_m)^2}{2\tau_{heat}^2} \right) \exp \left( -\frac{s}{s_H} \right) \left\{ \begin{array}{ll} s_H > 0 & \text{for footpoint heating} \\ s_H = \infty & \text{for uniform heating} \\ s_H < 0 & \text{for apex heating} \end{array} \right. \quad (1)$$

Figure 1. The hydrodynamic evolution of the heating rate $E_H(t)$ (top left panel), the apex temperature $T(t, s = L)$ (second left panel), the electron density $n(t, s = L)$ (third left panel), pressure $p(t, s = L)$ (forth left panel), and the evolutionary curve $T(n)$ (right panel) is shown from the computation of a hydrodynamic code (from Tsiklauri et al. 2004; thin curves) and from the analytical approximations described in text (thick solid curves).
were $t_m$ is the time of maximum heating, $\tau_{\text{heat}}$ the Gaussian width of the heating time interval, $s_H$ is the heating scale height, and $H_m$ is the volumetric heating rate at the footpoint. Such a heating time profile is shown in Figure 1 (top panel).

We derive a semi-empirical analytical approximation to the hydrodynamic evolution of the electron temperature $T(s, t)$ and electron density $n(s, t)$ in an impulsively-heated loop (with 1D spatial loop coordinate $s$), based on (1) the energy equation that contains the heating rate $E_H(s, t)$, thermal conductive loss rate $\nabla F_C(s, t)$, and radiative loss rate $E_R(s, t)$, (2) the Neupert effect during the chromospheric evaporation phase, and (3) the Jakimiec relationship $T(t) \propto n^2(t)$ during the cooling phase (Jakimiec et al. 1992). The approximation can be expressed with Gaussian and exponential functions in the two time domains of dominant heating ($t \leq t_p$) and dominant cooling ($t > t_p$),

\[
T(s = L, t) = \begin{cases} 
T_m \exp \left( -\frac{(t - t_m)^2}{7\tau_{\text{heat}}^2} \right) & \text{for } t \leq t_p \\
T_p \exp \left( -\frac{(t - t_p)}{\tau_{\text{cool}}} \right) & \text{for } t > t_p
\end{cases},
\]

\[
n(s = L, t) = \begin{cases} 
n_p \exp \left( -\frac{(t - t_p)^2}{7\tau_{\text{heat}}^2} \right) & \text{for } t \leq t_p \\
n_p \exp \left( -\frac{(t - t_p)}{2\tau_{\text{cool}}} \right) & \text{for } t > t_p
\end{cases},
\]

where the cooling time $\tau_{\text{cool}}$,

\[
\frac{1}{\tau_{\text{cool}}} = \frac{1}{\tau_{\text{cond}}} + \frac{1}{\tau_{\text{rad}}},
\]

is a combination of the conductive cooling time $\tau_{\text{cond}}$ and the radiative cooling time $\tau_{\text{rad}}$,

\[
\tau_{\text{cond}} = \frac{21n_p k_B L^2}{5\kappa T_p^{5/2}}, \quad \tau_{\text{rad}} = \frac{9k_B T_p^{5/3}}{5n_p \Lambda_0},
\]

with $k_B$ the Boltzmann constant, $L$ the loop half length, $\kappa = 9.2 \times 10^{-7}$ erg s$^{-1}$ cm$^{-1}$ K$^{-7/2}$ the thermal Spitzer conductivity, and $\Lambda_0 = 10^{-17.73}$ cm$^3$ s$^{-1}$ the radiative loss rate coefficient. The maximum temperature $T_m$ can be computed from the maximum heating rate $H_m$, which is

\[
T_m = \left[ \frac{7L^2 H_m}{4\kappa} \right]^{2/7},
\]

in the case of uniform heating. The peak density $n_p$ can be calculated from the RTV (Rosner et al. 1978) energy balance solution that is applicable at the time $t = t_p$ of the density peak,

\[
n_p = \frac{p_{\text{RTV}}}{2k_B T_p} = \frac{T_p^2}{2 \times 14000k_B L},
\]

or using Serio’s scaling law corrected for gravitational stratification and non-uniform heating (Serio et al. 1981). The pressure is approximately constant during the time interval $t_m < t < t_p$ between the temperature maximum and
the peak density, which yields a relation between the corresponding densities
and temperatures,
\[ p = 2n_m k_B T_m \approx 2n_p k_B T_p . \] (8)

Furthermore, because of the temporal symmetry of the heating function and
the Neupert effect (i.e., the density increases according to the time integral
of the heating rate that drives chromospheric evaporation), the peak density
approximately doubles from the time \( t_m \) of the temperature maximum to the
density peak time \( t_p \), which together with equation (8) defines the the peak time
\( t_p \),
\[ t_p = t_m + 2.2 \tau_{\text{heat}} , \] (9)

when the temperature drops to half of the maximum value (\( T_p = T_m/2 \)). An
example of the apex temperature profile \( T(t, s = L) \), density profile \( n(t, s = L) \),
and pressure profile \( p(t, s = L) \) is shown in Figure 1.

In order to validate our analytical approximation expressed in equations (1–
9) we show in Figure 1 a comparison with a numerical calculation using a hydro-
dynamic code described in Tsiklauri et al. (2004). The numerical simulation was
carried out for a maximum heating rate of \( H_m = 1.5 \text{ erg cm}^{-3} \text{ s}^{-1} \), a heating
time scale of \( \tau_{\text{heat}} = 164 \text{ s} \), and a loop half length of \( L = 5.5 \times 10^9 \text{ cm} \). The agree-
ment between the exact numerical solutions and our analytical approximation
is of order \( \approx 10\% \) of the mean temperature or density values (Fig. 1).
3. Forward-Fitting to *Hinode* and *STEREO* Light Curves

We apply now our analytical hydrodynamic model (eqs. 1–9) to observations from *Hinode/XRT*, EIS, *STEREO/EUVI*, and *GOES*. The evolution of a loop during a small GOES B-class flare on 2007 February 1, 15:45 UT, is visible in the set of 5 XRT and EUVI images shown in Figure 2. A relatively isolated loop brightens up and fades in the XRT images, while the corresponding western footpoint shows a similar brightening and fading in the EUVI images (marked with a square in the bottom row panels of Fig. 2). The *Hinode/XRT* and EIS

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**Figure 3.** The observed light curves (thin) of the 2007 February 1, 15:45 UT, flare are shown for 2 *GOES* channels (0.5–4 Å and 1–8 Å), one *Hinode/XRT* filter (Al-poly), six *Hinode/EIS* emission lines (262, 264, 202, 195, 258, 275 Å), and two *STEREO/EUVI* filters (195 and 171 Å), along with the analytical model fluxes (thick curves), the heating rate $E_H(t)$, temperature $T(t)$, and density profile $n(t)$.
observations and the extraction of light curves from the investigated loop are described in more detail in Warren et al. (2007). We show the same XRT and EIS light curves in Figure 3, along with light curves from GOES and EUVI 171 and 195 Å. The cadences are 3 s for GOES, 57 s for XRT, 31 s for EIS, and 600 s for EUVI.

Using the hydrodynamic model expressed in equations (1-9) with 6 free parameters ($t_m, H_m, \tau_{heat}, s_H, L, w$), we convolve the model emission measure

$$\frac{dEM(T[s], t)}{dT} = n^2(s[T], t)w^2(s)\frac{ds(T)}{dT},$$

with the instrumental response functions $R_w(T)$ in each wavelength band $w$,

$$F_w(t) = \int \frac{dEM(T, t)}{dT} R_w(T) dT,$$

and fit the model flux profiles $F_w(t)$ to the observed flux profiles. The fit shown in Figure 3 yields a uniform heating rate of $E_m = 0.26$ erg cm$^{-3}$ s$^{-1}$ and a Gaussian heating time scale of $\tau_{heat} = 140$ s. These parameters yield a maximum temperature of $T_m \approx 10.0$ MK and a peak density of $n_p = 1.2 \times 10^{10}$ cm$^{-3}$. The fit is not unique, but closely matches the time profiles at the highest temperatures (GOES and XRT). In the cooler EUV temperature bands, the fit approximately matches the rise and decay of the emission, as well as the peaks at higher temperatures ($T \gtrsim 2.0$ MK) in XRT and EIS. The width is consistently found to be $w \approx 2.5$ Mm in 6 (GOES, XRT, EIS) out of the 11 filters and emission lines, but there is a large discrepancy between the EIS emission lines and EUVI filters at the lower temperatures of $T \approx 1 - 2$ MK. It is not clear whether this discrepancy results from calibration errors, non-cospatial locations of the EIS scans, or partial obscuration by absorbing plasma for different stereoscopic aspect angles. We have also to be aware that the EUV temperature range of a hot SXR loop is confined to relatively small segments in the transition region, which are generally embedded into highly time-variable macrospicules.

This example illustrates the type of hydrodynamic forward modeling that can be accomplished for suitable loops identified in Hinode and STEREO images. More detailed studies with this promising method are forthcoming.

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References