Radiative Transfer for Polarized Radiation: a Personal, Historical Overview

E. Landi Degl’Innocenti

Dipartimento di Astronomia e Scienza dello Spazio, Università di Firenze, Italy

Abstract. Almost a century has elapsed since the first application of spectro-polarimetry to the diagnostic of solar magnetic fields. Since then, dramatic progress has been made in the instrumentation, which is now reaching unprecedented levels of sensitivity in the measurement of polarization signals in spectral lines. At the same time, the theoretical framework needed for the interpretation of polarimetric observations has steadily evolved from the pioneering methods based on simple formulae to a sophisticated structure which is nowadays used with success in the interpretation of solar observations. The present paper is intended to give a personal, historical perspective of the evolution of this research field and of its major achievements.

1. Introduction

Radiative transfer for polarized radiation is a rather peculiar branch of theoretical physics. The reason for this fact has to be sought in the following arguments: a) by definition, the validity of the theory can be checked only for optically thick media (in an optically thin medium there is no transfer...); b) it is practically impossible to realize in a laboratory an optically thick medium having a precisely specified run of the physical parameters along the ray-path; c) the only physical media that can be used for testing the theory are stellar atmospheres, in particular the solar atmosphere; d) the solar atmosphere is a passive medium; we cannot change its properties to perform an experiment; e) the solar atmosphere is an extremely complicated medium. Given these facts, it is obvious that, from one side, we need a well established theory in order to interpret the observations and to infer the physical properties of the sun, but, from the other side, these physical properties are too complicated to give us detailed feedbacks for upgrading the theory itself. These remarks, that may appear trivial, have to be brought in mind for a correct understanding of the historical evolution of this branch of astrophysics where observations are of little help in guiding the development of theoretical ideas.

2. The Early Days

The need for a theory of polarized radiation arises at the beginning of last century with the first spectro-polarimetric observations of Hale (1908). However, times are not yet mature because radiative transfer is still in a rudimentary phase. Moreover, Quantum Mechanics is just born, but one has to wait until 1913 for
the publication of Bohr’s theory of the hydrogen atom. Spectral lines are indeed observed in the sun, in stars and in the laboratory, but their interpretation is still an enigma. Atoms are described, in the framework of the Lorentz theory of the electron, as classical oscillators that are capable of vibrating at certain particular frequencies, whose quantitative values are taken as empirical parameters. In any case, the Lorentz theory is capable of predicting, at least for normal triplets, the polarization properties of the Zeeman components in optically thin media, and this allows to give the first interpretation of the polarized spectrum of the magnetic sun through the so-called Seares formulae (Seares 1913). These formulae give the intensities (normalized to unity) of the $\sigma$ and $\pi$ components of a normal Zeeman triplet observed through a filter which allows for circular polarization (the combination of a Fresnel prism and a Nicol, in the original experimental setting of Hale). Using the notations of Seares, the intensity in the violet component, $V$, in the middle component, $M$, and in the red component, $R$, are given by

$$V = \frac{1}{4} (1 - \cos \gamma)^2 , \quad M = \frac{1}{2} \sin^2 \gamma , \quad R = \frac{1}{4} (1 + \cos \gamma)^2 ,$$

(1)

where $\gamma$ is the angle between the direction of the magnetic field and the line of sight. With the help of Seares formulae, many important discoveries are made, like, for instance, the first and second Hale’s laws, the general organization of the solar magnetic field, the funnel shape of the magnetic field in sunspots, and so on. However, a soundly established theory, valid for optically thick media, is still missing.

The years between 1910 and 1930 are of crucial importance for the establishment of the theory of radiative transfer (for non-polarized radiation) thanks to the fundamental contribution of several distinguished astrophysicists such as Eddington, Milne, Schüster, Schwarzschild, and others. At the same time, Quantum Mechanics is developed. The origin of spectral lines is understood, spin is introduced in the description of spectra, the anomalies of the Zeeman effect are also understood. In the years 1930, radiative transfer (still for non-polarized radiation) is generalized to account for the presence of spectral lines (Unsöld and others) and their observed shape starts being derived from theoretical calculations. New important ideas and physical concepts are introduced in the lexicon of astrophysical research (saturation, curve of growth, atmospheric models, etc.).

Yet, in all these years, little progress is made in the theory of radiative transfer for polarized radiation. One has to wait until 1956 to have a real step forward thanks to a very important paper of the versatile Japanese astrophysicist Wasaburo Unno, also known for his work on stellar convection and on stellar non-radial oscillations. Indeed, from 1913 to 1956 there are (in the author’s knowledge) only two papers that bring some contribution to the field: one by Babcock (1949) and one by Hubenet (1954). Both papers deal with the problem of line formation in a magnetic field in the particular case where the three components of a (normal) Zeeman triplet can be treated as independent, either because the magnetic field is very intense (Babcock’s paper) or because it is parallel or perpendicular to the line of sight (Hubenet’s paper).
3. Innovative Ideas

The basic idea contained in the classical paper by Unno (1956) is to write a transfer equation for the Stokes parameters. The scalar equation for the intensity of the usual, non-polarized case generalizes into a vector transfer equation which involves a coupling among the Stokes parameters. Fig. 1 shows an extract of the original paper by Unno.

Unno’s equations are very innovative and they are immediately recognized as the crucial step for the development of the theory of line formation in a magnetic field. However, looking at Unno’s results with hindsight, it has to be remarked that Unno’s equations suffer from some deep limitations, namely:

a) they take into account only dichroism effects, anomalous dispersion effects being disregarded;

b) they only apply to normal Zeeman triplets;

c) they are based on the Local Thermodynamical Equilibrium (LTE) approximation;

d) the derivation of the equations is physically correct, but it is obtained by means of heuristic arguments, which avoids the possibility of generalizing them to more complex physical situations;

e) being derived under the LTE approximation, typical effects such as those due to stimulated emission or frequency redistribution are disregarded. In the following, we will see how these limitations have been overcome by subsequent work.

After few years from the publication of Unno’s paper, a further important step is accomplished by D.N. Rachkovsky, who recognizes that “something is missing” in Unno’s equations. Adopting a classical approach, Rachkovsky observes that in Unno’s equation only the imaginary part of the index of refraction of the spectral line is taken into account, whereas the real part is disregarded. The result is the publication of a paper in the “Izvestia” of the Crimean Astrophysical Observatory (Rachkovsky 1962). In the English translation of the abstract one can verbatim read: The radiative transfer equation in the magnetic field of sunspots is found, taking into account the refractive index of the spectral line. It is shown that for uniform magnetic field the obtained perfection of the existing theory on the formation of spectral lines does not practically affect the line profiles...

\[ \cos \theta \frac{dI}{d\tau} = (1 + \eta_I)I + \eta_Q Q + \eta_V V - (1 + \eta_I)B, \]

(24)

\[ \cos \theta \frac{dQ}{d\tau} = \eta_Q I + (1 + \eta_I)Q - \eta_Q B, \]

(25)

and

\[ \cos \theta \frac{dV}{d\tau} = \eta_V I + (1 + \eta_I)V - \eta_V B, \]

(26)

\[ \cos \theta \frac{dI}{d\tau} = (1 + \eta_I)I + \eta_Q Q + \eta_V V - (1 + \eta_I)B, \]

\[ \cos \theta \frac{dQ}{d\tau} = \eta_Q I + (1 + \eta_I)Q - \eta_Q B, \]

\[ \cos \theta \frac{dV}{d\tau} = \eta_V I + (1 + \eta_I)V - \eta_V B, \]

Figure 1. The original equations appearing in the paper by Unno (1956).

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I personally find somewhat intriguing the understatement contained in the last sentence. We know today that magneto-optical effects are important. Rachkovsky finds that they are needed...
Thanks to Rachkovsky’s contribution, the LTE radiative transfer equation acquires its modern form, namely

\[
\frac{dI}{d\tau} = \eta_I (I - B) + \eta_Q Q + \eta_U U + \eta_V V ,
\]

\[
\frac{dQ}{d\tau} = \eta_Q (I - B) + \eta_I Q + \rho_V U - \rho_U V ,
\]

\[
\frac{dU}{d\tau} = \eta_U (I - B) - \rho_V Q + \eta_I U + \rho_Q V ,
\]

\[
\frac{dV}{d\tau} = \eta_V (I - B) + \rho_V Q - \rho_Q U + \eta_I V .
\]

In the late 1960s, the Unno-Rachkovsky theory is generalized from normal triplets to arbitrary Zeeman multiplets (Beckers 1969a). This is accomplished by substituting the single \(\sigma_r\), \(\pi\), and \(\sigma_b\) components of the normal triplet with all the components belonging to each group, each component being multiplied by its own strength and being located at its own distance from line-center. At the same time, Beckers also publishes a thick volume (Beckers 1969b) containing the Zeeman patterns of all the possible transitions that are found in the solar spectrum. This volume has now only an historical interest, but it has been very helpful for a generation of solar physicists.

Notwithstanding these achievements, at the beginning of the years 1970s the situation is still rather confusing. In many papers the definition of the Stokes parameters is given in a rather cavalier fashion, and the same applies to the definition of the angles specifying the direction of the magnetic field and to the definition of the \(\sigma\) and \(\pi\) components, for which different notations are employed. The absorption profiles corresponding to the \(\sigma\) components, for instance, are denoted in the various papers, either with the symbols \(\eta_{-1}\) and \(\eta_1\), or with the symbols \(\eta_\ell\) and \(\eta_r\) (\(\ell\) and \(r\) standing for “left” and “right”, respectively), or with the symbols \(\eta_b\) and \(\eta_r\) (\(b\) and \(r\) standing for “red” and “blue”). Moreover, a large part of the literature concerning the argument is scattered in journals written in Russian (though with an English summary) with limited circulation.

In this situation, when my brother Maurizio and myself entered the field we tried to derive the transfer equations for polarized radiation directly from the first principles of Quantum Electrodynamics (Landi Degl’Innocenti & Landi Degl’Innocenti 1972). The idea revealed to give satisfactory results since the transfer equations for the Stokes parameters were derived as “a block” without introducing heuristic arguments and without the need of assembling “separate pieces”, taken from different sources and obtained through different formalisms. Obviously, we recovered the Unno-Rachkovsky equations (as generalized by Beckers for anomalous Zeeman patterns) but we also found some sign differences in the equations but tries to minimize the consequences of his finding. Was this due to the fact that some powerful “scientist” in his environment didn’t like these kind of “theoretical complications”?  

\[2\]Around 1970 the late Prof. G. Righini, at the time Director of the Arcetri Observatory, had the intention to install a magnetograph at the Arcetri Solar Tower. Unfortunately, the project never came to an end.
with respect to previous works, differences for which a sound assessment was difficult, given the somewhat careless fashion with which definitions had been given in several papers. We then decided to send the preprint of our paper to Prof. Jan Olof Stenflo who was already an authority in the field. This started an exchange of letters which lasted five months (from April to September 1972). At the end, all the sign ambiguities were clarified and the problem of line formation in a magnetic field resulted in being well established, at least for the LTE case.

Further theoretical developments took place few years later. From 1973 to 1975 I was holding an ESRO (now ESA) fellowship at the Utrecht Observatory under the supervision of Prof. Cornelis Zwaan. He had brought back from the Sacramento Peak Observatory a couple of photographic plates containing the circularly polarized spectrum of a big sunspot he had observed at the solar tower, and he had been intrigued by the peculiar appearance of some lines which were showing more Zeeman components than expected. In particular, he drew my attention to a vanadium line at $\lambda 6058$ (a purely umbral line) which showed in intensity five instead of four components. Prof. Zwaan suggested to me to check whether this behavior might be ascribed to the presence of hyperfine structure$^3$. Stimulated by this request, I generalized the theory of line formation in a magnetic field to account for hyperfine structure in the incomplete Paschen-Back effect regime (Landi Degl’Innocenti 1975, 1978). Unfortunately, the hyperfine structure components of the vanadium levels involved in the line were not available at the time. They became available some years later (Childs et al. 1979) and it is possible today to conclude that Zwaan’s conjecture was correct (see Fig. 2).

In the late 1970s, new theoretical developments took place in the field of spectro-polarimetry at the Meudon Observatory. To obtain a diagnostic tool for the magnetic field in solar prominences, Véronique Bommier and Sylvie Sahal-Bréchot introduced the formalism of the density-matrix to describe the excitation state of an atom embedded in a prominence magnetic field and pumped by the anisotropic radiation field coming from the underlying photosphere. They established a statistical equilibrium equation whose solution was then used to derive the broad-band scattered radiation (Sahal-Bréchot, Bommier, & Leroy 1977). The magnetic field was diagnosed by theoretically computing the modification that it introduces in the scattering polarization through the so-called Hanle effect.

The theory gave satisfactory results but it could be applied only to optically thin plasmas. It was indeed necessary to establish a coupling between the statistical equilibrium equations, written for the density-matrix elements, and the radiative transfer equations. This was not an easy task because the problem of finding the absorption and emission coefficient of a plasma is not a trivial one, even in the case of non-polarized radiation. Referring to classical textbooks does not help much under this respect. For instance, in Mihalas’ book Stellar $^3$Vanadium has a single stable isotope with mass number 51 and nuclear spin $I = \frac{7}{2}$.

$^4$The line belongs to multiplet n. 34 of V i. It originates from the transition between the lower level $3d^44s \ ^2D_{1/2}$ and the upper level $3d^44p \ ^3P_{1/2}$.
Figure 2. Left panel: strengths and splittings of the Zeeman components of the V\textsc{i} λ6058 line in a magnetic field of 3000 G (σ\text{red} components are drawn as dotted lines for the sake of clarity). Right panel: Unno-Rachkovsky solution for the intensity profile of V\textsc{i} λ6058 in a magnetic field of 3000 G directed along the line of sight. The full line, showing 5 components, is obtained accounting for hyperfine structure, the dashed line, with only 4 components, is obtained disregarding hyperfine structure. The solution is obtained for the following set of parameters (see Chap. 9 of Landi Degl’Innocenti & Landolfi 2004, for the meaning of the symbols): κ\text{L} = 2, a = 0.05, β = 5, Δλ\text{D} = 20 mÅ.

Atmospheres (Mihalas 1978) one finds a definition of the absorption coefficient\(^5\) which is totally empirical, being based on the equation

\[
dE = -\chi_\nu I_\nu dS ds d\omega d\nu dt ,
\]

where \(dE\) is the energy contained in the frequency band \(d\nu\) and propagating into the solid angle \(d\omega\) which is removed in the time interval \(dt\) from a beam of radiation having specific intensity \(I_\nu\) by an element of plasma having cross-section \(dS\) and length \(ds\). However, no “recipe” is given to find the value of \(\chi_\nu\), or, in other words, to relate it to the physical conditions of the plasma. In the same book one finds only some simple heuristic arguments like the following (quoting) \textit{“The extinction coefficient is the product of an atomic absorption cross section (cm}^2\text{) and the number density of absorbers (cm}^{-3}\text{) summed over all states that can interact with photons of frequency \(\nu\).”} This argument may be enough for finding the absorption (extinction) coefficient in simple situations. But how can it be used if one wants to find the coefficients entering the radiative transfer equations for the Stokes parameters in an “exotic” physical situation where the plasma is composed by atoms embedded in a magnetic field and pumped by anisotropic radiation? Obviously, a theoretical scheme is necessary.

\(^5\)The coefficient is called extinction coefficient in Mihalas’ book.
4. Latest Results

The solution to this problem was obtained in the early years 1980 (Landi Degl’Innocenti 1983) by means of an approach based on Quantum Electrodynamics which generalized the earlier work of Landi Degl’Innocenti & Landi Degl’Innocenti (1972). Starting from a physical system composed of a statistical ensemble of atoms and the (polarized) radiation field, the radiative transfer equations are derived from first principles by considering the time evolution of a pencil of radiation interacting with a large reservoir (consisting of an ensemble of atoms). Similarly, the statistical equilibrium equations are derived by considering the time evolution of an atom interacting with a large reservoir (consisting of the radiation field). The two derivations are obtained by means of the same formalism so that the resulting equations (radiative transfer equations and statistical equilibrium equations) are fully consistent.

The most important result contained in the paper just quoted is the establishment of the radiative transfer equation for the Stokes parameters, which acquires the matrix form

\[
\frac{d\mathbf{I}}{ds} = -\mathbf{K}^{(A)} \mathbf{I} + \mathbf{K}^{(S)} \mathbf{I} + \mathbf{\epsilon},
\]

where \(\mathbf{I}\) is the Stokes vector, \(\mathbf{K}^{(A)}\) is the propagation matrix due to absorption, \(\mathbf{K}^{(S)}\) is the propagation matrix due to stimulated emission (or negative absorption), and \(\mathbf{\epsilon}\) is the emission vector in the four Stokes parameters. The elements of the two matrices and those of the emission vector can be expressed in terms of the density-matrix components of the atomic system (obtained through the solution of the statistical equilibrium equations). In general, the resulting expressions are quite complicated but they can be computed (theoretically or numerically) for any given atomic system (atoms -with or without fine or hyperfine structure-, molecules, etc.), in an arbitrarily complex physical scenario, including the presence of magnetic (or electric) fields, an anisotropic, polarized radiation field, and collisions. Several applications concerning two-level atoms, multi-level atoms, multi-term atoms, and atoms with hyperfine structure can be found in Landi Degl’Innocenti & Landolfi (2004).

5. Conclusions

The theoretical scheme outlined above has been applied to the interpretation of spectro-polarimetric observations performed on various structures of the active sun (such as prominences, filaments, sunspots) and on the quiet solar disk close to the limb (second solar spectrum). Generally speaking, the results are satisfactory, although, in many cases, the fit to the observations is obtained by adjusting a number of free parameters. Also, it has not to be forgotten that there is a number of facts concerning the second solar spectrum that still defies any tentative of interpretation. This may be due to the inadequacy of the theory or to the fact, outlined in the Introduction, that the only laboratory on which the theory can be tested, the sun’s atmosphere, is an extremely complicated system which still conceals, at least in my opinion, several mysteries and unknowns.
Finally, we have to remark that the theory of radiative transfer for polarized radiation, as developed until today, is still incomplete. Notwithstanding a number of efforts that have been dedicated to the subject, the effects of redistribution in frequency on polarimetric line profiles cannot yet be accounted for within a fully satisfactory theoretical scheme. This limitation is not important for a large number of applications, but it restricts the possibility of interpreting with confidence the spectro-polarimetric profiles of the strongest lines, such as $\text{H}_\alpha$, the H and K lines of Ca II, the Ca I line at $\lambda 4227$, etc.. Further theoretical work in this field is urgently called for.

References

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