Modeling Non-Uniform Distribution of Acoustic Sources and Wave Leakage in Sunspots

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Abstract. Observations show suppression of the amplitude of 5-min oscillations in sunspots. We developed a 3D numerical simulation code to model wave excitation and propagation in the upper convection zone and the atmosphere of the Sun. We model how suppression of acoustic sources affects the oscillation amplitude in sunspot regions. The calculations show that this suppression (due to strong magnetic field) significantly reduces the oscillation amplitude to a level comparable with the observed amplitude deficit. The precise value of the amplitude ratio outside and inside sunspots depends on the rate of wave leakage and damping in the lower atmosphere. We present the results of detailed investigation of this effect, including modeling of wave damping at various heights in the atmosphere and the frequency dependence of the amplitude ratio. These results show the importance of the wave energy leakage through the atmospheric layers of sunspots.

1. Introduction

It has been known for a long time that 5-minute solar oscillations have significantly lower amplitude (by a factor of 2–5) in sunspots and plages than in the quiet Sun (e.g., Woods & Cram 1981; Thomas, Cram, & Nye 1982; Title et al. 1992). Hindman (1997) enumerated four possible mechanisms to explain the observed power suppression: 1) reduction of excitation of p modes inside sunspots; 2) absorption of p modes inside sunspots; 3) the different height of spectral line formation due to the Wilson depression; 4) altering of p-mode eigenfunctions by the magnetic field. The precise contribution of these effects to the observed amplitude reduction is still unknown. The goal of this paper is to study the contribution to amplitude suppression of the first effect: changes in the oscillation amplitude due to suppression of acoustic sources, and investigate the sensitivity of this suppression to mechanisms of wave damping and source depth. Inside sunspots, strong magnetic field inhibits turbulent convective motions which are the source of the 5-min solar oscillations. Therefore, the waves in the 5-min period (3 mHz frequency) range, observed in sunspots, mostly come from the outside regions, and thus their amplitude is reduced in comparison to the quiet Sun. Our goal was to estimate the significance of this effect by 3D numerical simulation of wave fields in horizontally uniform background solar models with regions of reduced excitation. The main result is that the suppression of oscillation sources inside sunspots can make a substantial (from 60% to 80%, depending on source depth) contribution to the reduction of amplitude inside sunspots, and thus must be taken into account in sunspot seismology.
2. Physical Background

To simulate the acoustic wave field generated by multiple acoustic sources we have developed a code for solution of linearized adiabatic 3D Euler equations written in the conventional conservative variables

\[ \frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho_0 u') = 0 \]

\[ \frac{\partial}{\partial t} (\rho_0 u') + \nabla p' = g_0 \rho' + f(x, y, z, t), \]

where \( u' \) is the velocity perturbation, \( \rho' \) and \( p' \) are the density and pressure perturbations respectively, and \( f(x, y, z, t) \) is the function describing the acoustic sources. The pressure \( p_0 \), density \( \rho_0 \), and gravitational accelerations \( g_0 \) of the background reference model depend only on depth \( z \). We used the adiabatic relation \( \frac{\delta \rho}{\rho_0} = \frac{1}{\Gamma_1} \frac{\delta p}{p_0} \) between Lagrangian variations of pressure \( \delta p \) and density \( \delta \rho \). The adiabatic exponent \( \Gamma_1 \) was calculated from the realistic OPAL equation of state (Rogers et al. 1996).

Great attention has been paid to construction of the self consistent convectively stable background model in which reflecting properties at the top boundary are close to the properties of the standard solar model. The standard solar model is convectively unstable, especially in the superadiabatic subphotospheric layers where convective motions are very intense and turbulent. Using this convectively unstable model as a background model leads to the instability of the solution of the linear system. The standard solar model S (Christensen-Dalsgaard et al. 1996) with a smoothly joined model of the chromosphere of Vernazza, Avrett, & Loeser (1976) was used as a background model. The condition for stability against convection requires that the square of Brunt-Väisälä frequency \( N_0^2 \) is positive. To make the background model convectively stable we replaced all negative values of \( N_0^2 \) by zeros and recalculated the profiles of pressure and density from the condition of hydrostatic equilibrium. The remaining thermodynamic parameters were calculated from the realistic OPAL equation of state for the hydrogen \( X \) and heavy elements \( Z \) abundances of the standard model. The procedure guarantees convective stability of the background model. It was shown (Parchevsky & Kosovichev 2007) that the profile of acoustic cut-off frequency \( \nu_{ac} \) of the modified model is very close to the profile of \( \nu_{ac} \) in the standard solar model.

To prevent spurious reflections of acoustic waves from the boundaries we established non-reflecting boundary conditions based on the Perfectly Matched Layer method (Hu 1996) at the top and bottom boundaries. The top boundary was set in the chromosphere above the temperature minimum. This simulates a realistic situation when not all waves are reflected by the photosphere. Waves with frequencies higher than the acoustic cut-off frequency pass through the photosphere and are absorbed by the top boundary. This naturally introduces frequency dependence of the reflecting coefficient of the top boundary. The lateral boundary conditions are periodic.

The waves are generated by a source term \( f(x, y, z, t) \) in the right-hand side of equation (1). In our simulations we used spherically symmetric spatially localized sources of two types: a source of the vertical component of force and a pressure source. The sources have a finite lifetime and depend on time either
as one period of a sine function or as Ricker’s wavelet. The explicit expression for the source of $z$-component of force with Ricker’s time dependance is

$$f(x, y, z, t) = e_z A \left[1 - \left(\frac{r}{R}\right)^2\right]^2 (1 - 2\tau^2)e^{-\tau^2}, \quad r \leq R, \quad t_0 \leq t \leq t_0 + \frac{4\pi}{\omega}$$

with $r$ and $\tau$ given by

$$r = \sqrt{(x - x_{src})^2 + (y - y_{src})^2 + (z - z_{src})^2}, \quad \tau = \frac{\omega(t - t_0)}{2} - \pi,$$

where $e_z$ is the unit vector in the vertical direction, $x_{src}$, $y_{src}$, and $z_{src}$ are the coordinates of the center of the source, $R$ is the source radius, $\omega$ is the central frequency, $t_0$ is the moment of the source ignition, $A$ is the coefficient which is measured in dyn cm$^{-3}$, describes the source strength, and has a physical meaning of the force density. Sources were randomly distributed below the photosphere at a fixed depth and ignited at arbitrary moments of time.

We used non-dimensional variables

$$[x] = L, \quad [t] = \frac{L}{\bar{a}_0}, \quad [u] = \bar{a}_0, \quad [\rho] = \bar{\rho}_0, \quad [p] = \bar{\rho}_0 \bar{a}_0^2, \quad [g] = \bar{g}_0, \quad [A] = \bar{\rho}_0 \bar{a}_0^2 / L$$

where $L$ is the depth of the computational domain, quantities with the bar represent corresponding values of the background model at the top boundary.

### 3. Numerical Method

We use a semi-discrete numerical scheme. In the semi-discrete approach the space and time discretization processes are separated. First the spatial discretization using a uniform grid is performed, leaving the problem continuous in time. Then, the system of ODE thus obtained is solved by any stable time advancing scheme. This gives us flexibility in combining different spatial and time advancing schemes according to our preference in favor of accuracy or speed. We used a four-stage, 3rd-order strong stability preserving Runge-Kutta method with Courant number $C = 2$ as a time advancing scheme (Shu 2002). The high-order dispersion-relation-preserving (DRP) scheme of Tam & Webb (1993) was used for spatial discretization

$$\frac{\partial f}{\partial x} \bigg|_m \approx \frac{1}{\Delta x} \sum_{j=-3}^{3} b_j f_{m+j} = \frac{1}{\Delta x} \sum_{j=-3}^{3} b_j f(x_m + j\Delta x).$$

Coefficients

$$\begin{cases}
  b_0 = 0 \\
  b_{\pm 1} = \pm \frac{496 - 15\pi}{42(45\pi - 128)} \\
  b_{\pm 2} = \pm \frac{5632 - 1725\pi}{84(45\pi - 128)} \\
  b_{\pm 3} = \pm \frac{17(16 - 5\pi)}{14(45\pi - 128)}.
\end{cases}$$
of this finite difference (FD) scheme are chosen from the requirement that the error in the Fourier transform of the spatial derivative is minimal. It can be shown that the 4th-order DRP FD scheme describes short waves more accurately than the classic 6th-order FD scheme.

The efficiency of high-order FD schemes can be reached only if they are combined with adequate numerical boundary conditions. It is easy to derive non-symmetric boundary operators which approximate the first derivative near boundaries to high order. However, such approximations are often unstable. We follow Carpenter, Gottlieb, & Arbarbanel (1993) and use an implicit Padé approximation of the spatial derivatives near the top and bottom boundaries to derive stable 3rd-order boundary conditions consistent with the 4th-order DRP numerical scheme for interior points of the computational domain.

Waves with wavelength less than $4\Delta x$ are not resolved by the FD scheme. They lead to point-to-point oscillations of the solution that can cause a numerical instability. Such waves have to be filtered out. We used a 6th-order digital filter to eliminate the unresolved short wave component from the solution at each time step.

We performed numerous 1D and 3D tests of wave propagation in the hydrostatic isothermal ($p_0/\rho_0 = \text{const}$) background model, because it shows the characteristic behavior of realistic solution and yet is not too complicated and can be solved analytically. We compared numerical solutions with exact ones to validate the accuracy of the numerical scheme and the absorbing properties of the non-reflecting boundaries. Details of numerical realization and test results can be found in Parchevsky & Kosovichev (2007).

4. Results

Simulations were carried out in a rectangular domain of size $122 \times 122 \times 30$ Mm$^3$ using a uniform $816 \times 816 \times 630$ grid. The background model varies sharply in the region above the temperature minimum. Thus, to simulate propagation of acoustic waves into the chromosphere we chose the vertical spatial step $\Delta z = 50$ km in order to preserve the accuracy and numerical stability. The spatial intervals in the horizontal direction are $\Delta x = \Delta y = 150$ km. To satisfy the Courant stability condition for the explicit scheme, the time step equals 0.68 sec. A linear combination of source terms given by equation (2) was added to the right hand side of the $z$-momentum equation of system (1). Sources of $z$-component of force with random amplitudes and frequencies from range $2 \div 8$ mHz were randomly distributed at the depth of 350 km below the photosphere. The sources are ignited at random points of the horizontal plane (one source per time step).

To simulate the absence of acoustic sources inside sunspots we gradually decreased the source strength in the central region. Numerical experiments with different profiles of the source strength show in all cases that the simulated wave field profile averaged along the azimuthal angle has a shape similar to the profile of the acoustic source strength. We use a horizontal profile of the observed wave field (shifted and scaled to be in the range [0,1]) as an acoustic source strength profile.
Modeling Non-Uniform Distribution of Acoustic Sources

Figure 1. Amplitude maps obtained from observations of the sunspot in active region AR 8243 (a) and from simulations with source depth of 350 km, PML height of 500 km without explicit damping (b); source depth of 350 km, PML height of 1750 km with damping (c).

If the height of the top boundary is sufficiently high (in our simulations it was 1750 km) the waves are reflected back from the top boundary without noticeable damping. The modes with frequencies less than the acoustic cut-off frequency and turning points above the bottom boundary are trapped in the domain; their amplitude increases, and the rms (root mean square) oscillation amplitude does not reach an equilibrium state. This distorts the acoustic power spectrum and changes the amplitude ratio of trapped modes and modes that can be absorbed at the top and/or bottom boundaries. So, an additional damping term has to be introduced into the equations in case of a high top boundary. To model the subsurface (turbulent) damping we followed Gizon & Birch (2002) and added a friction-type term $-\sigma \rho v_z$ to the vertical component of momentum equation. The value of $\sigma$ was adjusted to match the observed line widths and relative amplitude of the peaks in the acoustic spectrum (Parchevsky & Kosovichev 2007).

Amplitude maps averaged in the frequency interval $\Delta \nu = 1.2$ mHz with central frequency $\nu = 3.65$ mHz for the height of 300 km, obtained from simulations for different source depth and damping mechanisms, are shown in Figure 1. The
simulated maps were filtered with a spatial Gaussian filter with FWHM of 1″5 to reflect the instrumental smoothing. Amplitudes outside and inside the masked region were calculated by averaging in the ring ($r_{\text{min}} = 45 \text{ Mm}$, $r_{\text{max}} = 50 \text{ Mm}$) and the central circle ($r_c = 9 \text{ Mm}$). Averaging regions in all cases were the same. Simulated ratios of amplitude outside and inside the masked region are the following: 2.3±0.4 for panel b) and 3.2±1.0 for panel c). Observational amplitude ratio equals 3.9±1.0 (panel a). Thus, in all cases our simulations show that more than a half of the amplitude suppression comes from the absence of acoustic sources inside the sunspot, and the results are not very sensitive to the mechanism of wave damping.

The average amplitude of the resulting wave field depends on the strength of the wave sources and their spatial density. Results of simulations for domain of size $30 \text{ Mm} \times 30 \text{ Mm} \times 10 \text{ Mm}$ (200×200×219 nodes) and different spatial density of the acoustic sources are shown in the left panel of Figure 2. Sources of vertical component of force (2) with dimensionless amplitude $A$ in the range $0 \div 1$ were distributed at the depth of 100 km below the photosphere and were ignited at random points at each time step (curve I) and at each second time step (curve II). Curve III corresponds to sources with amplitude in range $0 \div 3$ ignited at every second time step. The number of active sources in the domain is shown in the right panel.

5. Discussion

We have carried out numerical simulations of the effect of reduced excitation of solar oscillation in sunspot regions and studied the dependence of suppression of oscillation amplitude on the mechanism of wave damping and depth of the acoustic sources. The oscillations are excited by random sources of the vertical component of force. In sunspot regions, the wave sources are suppressed because the magnetic field of sunspots inhibits convective motions. The results of the
simulations show that for a wide range of sunspot diameters the wave amplitude suppression varies from 0.6 to 0.8 of the solar one. So, the absence of acoustic sources in sunspots plays an important role and must be taken into account.

Our simulations also showed that the oscillation amplitude in regions of suppressed excitation only weakly depends on the wave damping mechanism in the upper convection zone and atmosphere as long as the line widths in the simulated power spectrum are close to the observed ones. We modeled wave damping by two methods: introducing a friction-type term into the $z$-component of the momentum equation and putting the wave absorbing boundary at various heights in the chromosphere.

References