Two Fundamental MHD Problems in Solar Physics

Takashi Sakurai

National Astronomical Observatory, 2-21-1 Osawa, Mitaka, Tokyo
181-8588, Japan

Abstract. Two fundamental MHD problems in solar physics are discussed. The first one is the so-called Parker problem, namely the behavior of magnetic fields when their fieldline footpoints are moved around by flows. The expected incapability of settling the field into static equilibrium is a theoretical basis for the microflare model of coronal heating. The second one is the so-called Aly-Sturrock conjecture, stating that the magnetic field stressed by footpoint motions attains energy but may never exceed the energy of the open field. This is thought to be an anti-CME theorem.

1. Introduction

In this short note, I would like to discuss two fundamental problems in magnetohydrodynamics (MHD) which are also important in solar physics. The first one is the so-called Parker’s problem, and the second one is Aly and Sturrock’s conjecture.

2. Parker’s Problem and Parker’s Hypothesis

2.1. Statement of the Problem

We consider a closed rectangular volume filled with infinitely conducting medium and permeated by uniform magnetic fields through the top and bottom boundaries (Parker 1972). We then impose random motions on the two boundaries. Because of the frozen-in condition, the magnetic field lines will be deformed in responding to the prescribed boundary motions. For this system to be in equilibrium, the gas pressure must be constant along each field line (the gravity is here neglected). However, the pressure on the two boundary planes can be specified as any function of position, and the field line footpoints may move to anywhere on the boundaries, Parker conjectured that generally the equilibrium may not be maintained. Specifically, when the boundary motion does not conserve the winding pattern of field lines (Fig. 1), clearly the pressure equilibrium cannot be achieved. Therefore, a discontinuity may arise in the system, leading to the so-called ‘braided flux tube’ configuration. The discontinuity, or a current sheet, will dissipate its associated energy by e.g. magnetic reconnection.

2.2. Interpretation

A simplest case in which the winding pattern of field lines change from one boundary plane to the other is when the boundary motions themselves contain...
Figure 1. Parker’s problem: change of winding pattern of field lines.

Table 1. Parker’s problem and its consequences.

<table>
<thead>
<tr>
<th>Magnetic field</th>
<th>Boundary motion</th>
<th>Create discontinuity?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Uniform</td>
<td>Singular</td>
<td>Yes</td>
</tr>
<tr>
<td>(b) Discontinuity</td>
<td>Regular</td>
<td>Yes</td>
</tr>
<tr>
<td>(c) With nulls</td>
<td>Regular</td>
<td>Yes</td>
</tr>
<tr>
<td>(d) Dipping field lines</td>
<td>Regular</td>
<td>Yes</td>
</tr>
<tr>
<td>(e) Uniform</td>
<td>Regular</td>
<td>No</td>
</tr>
</tbody>
</table>

discontinuity (Table 1, case (a)). Other more subtle cases are also listed in Table 1. Case (b) arises, for example, discrete flux tubes embedded in a field-free medium are moved around. Because the motion of flux tubes are independent, generally discontinuity is created at the interface. Case (c) is when a magnetic null point exists in the configuration. Since the field lines coming to the vicinity of the null will be diverted to points widely separated at distant locations, a smooth motion on one boundary surface is mapped out to discontinuous motion on another boundary plane (Fig.2, left). This ‘singular mapping’ arises without the existence of magnetic nulls, and case (d) with dipping field lines is such a case (Fig.2, right).

The central problem here is case (e), namely, when both the magnetic fields and the boundary motions are continuous. Parker’s hypothesis is that even in this case the discontinuity will be created. Here we argue contrary to his hypothesis.

When displacements on the boundary are small, we can show that Parker’s problem always has a solution (Sakurai & Levine 1981; van Ballegooijen 1985). For displacements of finite amplitude, numerical simulation is necessary to study the evolution of the system. An initial study by Mikic, Schnack, & van Hoven
(1989), and more elaborate studies by Galsgaard & Nordlund (1996) and Craig & Sneyd (2005) all indicate that, although the field is more and more contorted as the footpoints are moved around on the boundaries, no discontinuity is created. The electric currents grow and become concentrated in thin layers, but such current layers have finite thickness and were resolved in the numerical computations. Although the magnetic system may eventually become dynamically unstable after it is highly deformed, a loss of equilibrium and the creation of braided flux tubes are not observed.

In real life, magnetic resistivity is finite, and when the initial uniform magnetic fields are deformed by boundary motions, magnetic nulls will be created by magnetic reconnection. Once the nulls are created, according to the scenario of Table 1(c) current sheets may form because of singular mapping of field lines. Parker’s hypothesis expects magnetic reconnection as a result of a loss of equilibrium. However, a true scenario is that first magnetic nulls are created by magnetic reconnection, and then singular mapping of field lines will greatly enhance the sheet currents and hence the dissipation.

3. Stressed Force-Free Fields and Aly-Sturrock Conjecture

3.1. Statement of the Problem

If we continue stressing the magnetic fields, eventually the system may evolve into an unstable situation. In order to realize an eruptive flare or ejection of a CME, the system at the time of the instability may need to have an energy in excess of the final state, by the amount of energy to be imparted to thermal and kinetic energies of eruption. An extreme case of the final state is that the ejected plasma blows off the magnetic field lines, and all the field lines become ‘open’, namely, one foot of the field line remains at the solar surface but the other end is dragged out to the interplanetary space (and out to infinity mathematically). In the initial state, we assume that all the field lines have their both footpoints rooted on the solar surface, i.e. ‘closed field lines’. More specifically we can write
\[ W(\text{preflare}) = W(\text{post-flare}) + \text{CME kinetic energy} + \text{other form of flare energy} \]
\[ W(\text{closed field}) = W(\text{open field}) + \text{CME kinetic energy} > W(\text{open field}) \]

As a matter of fact, Barnes & Sturrock (1972) presented a result of numerical simulation supporting this scenario. However, based on the integral inequality

\[ W \leq \frac{1}{4\pi} \left[ \int_{z=0} (x^2 + y^2) B_z^2 dx dy \right]^{1/2} \left[ \int_{z=0} B_z^2 dx dy \right]^{1/2}, \quad (1) \]

Aly (1984) noticed that the footpoint motions cannot inject energy indefinitely, because there is an upper limit to \( W \) for a given distribution of magnetic flux. (The lower limit is given by the energy of the potential (vacuum) field.) He further conjectured that the maximum energy state will be attained by the open magnetic field, as follows. For a closed flux tube, one can always increase its energy by stressing (twisting) its footpoints. Therefore a field configuration with closed field lines cannot be the state of maximal energy. On the other hand, open field lines cannot be further stressed because they have only one footpoints on the solar surface. Hence the open field might be expected to be a maximal energy state (Aly 1985, 1991; Sturrock 1991). This is the so-called Aly-Sturrock conjecture.

The numerical solution given by Barnes & Sturrock (1972) showed an energy exceeding the open field energy. Now it is interpreted that the boundary effect artificially pushed the energy above the open field energy.

Figure 3. Evolution of stressed force-free fields: possible scenarios.
3.2. Interpretation

Recently Choe & Cheng (2002) performed an interesting numerical simulation. They considered a system made of two flux tubes, and followed the evolution when the footpoint motion makes the two tubes interlocked. They found that in such a case the energy exceeds the open field energy. However, the role of non-magnetic medium separating the two flux tubes is not clear. We would obtain a similar results if the volume between the tubes is filled with (weak but finite) magnetic fields. It is necessary to clarify under what kind of condition the energy of the stressed field can exceed the open field energy.

An exception to the Aly-Sturrock constraint is when floating field lines exist. Floating field lines or levitated flux tubes are configurations with their field lines closed within themselves and have no footpoints, and might be considered to be a model for filaments/prominences. The configuration with floating flux tubes can have energy larger than the energy of the open field (without floating flux tubes), as has been demonstrated by Li & Hu (2003) and by Zhang, Hu, & Wang (2005).

4. Discussion

Parker’s problem was first proposed for a system in a closed box, but later development also includes configurations in half-open space, like the situations considered in Aly-Sturrock’s conjecture. Therefore, the two problems are related. A difference is that small-scale, random footpoint motions are postulated in Parker’s problem, while large-scale shear flows are assumed in Aly-Sturrock problem.

Parker’s problem is also taken as the basis for the so-called micro/nano-flare theory for the coronal heating mechanism (Parker 1982). The idea is that a large number of miniature flares might be able to provide energy required to heat the solar corona. The number distribution of flare energy \( E \) is well approximated by a power-law distribution

\[
f(E) = AE^{-\gamma},
\]

and if \( \gamma > 2 \), the total amount of energy contributed from small flares dominates (Hudson 1991). However, the observed distribution of flares has shown that \( \gamma < 2 \). For example, Shimizu (1995) derived from Yohkoh soft X-ray observations of small flares \( \gamma = 1.5–1.7 \). What matters is not only \( \gamma \) but also \( A \), but Shimizu (1995) concluded that microflares provide only 20% or less of the energy required to heat the corona.

Small flares may escape from being detected, so that the observations might underestimate the number of small events. Namely the exponent \( \gamma \) may become larger than 2 for flares too small to be detected. However, Aschwanden (1999) warns that as the event energy decreases, the geometrical size of the event also decreases. As the flare loop becomes shorter and the loop volume as a whole becomes closer to the cool solar surface, the temperature of the flare is reduced. At the small end of ‘flare’ population, the event can never give rise to a flare temperature. In any case a different value of \( \gamma \) means different physical mechanism, and such a population at low energies with \( \gamma > 2 \) should, if exist, be driven by a mechanism different from flares.
Let me conclude this short report by pointing out that to settle the longstanding problem of coronal heating mechanism(s) is the most important research target of the Solar-B mission.

References