Polarimetric Properties of the D\textsubscript{2} Lines of Alkali Atoms

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Abstract. We present a theoretical investigation on the influence of a deterministic magnetic field on the linear and circular polarization of the Na I, K I, and Ba II D\textsubscript{2} lines. We describe the three ions by means of a three-level atomic model, and we take into account the hyperfine structure which is shown by some isotopes. We focus on the “solar prominence case”, in which an optically thin slab of chromospheric plasma, situated 7000 km above the visible solar “surface”, and permeated by a magnetic field of given strength and orientation, is illuminated from below by the continuum photospheric radiation field.

1. Introduction

The development of new spectropolarimetric instrumentation, able to reach high polarimetric sensitivity with high spectral and spatial resolution, is offering the chance of performing polarimetric measurements in spectral lines, within individual solar plasma structures. It is therefore worthwhile to carry out theoretical investigations on the magnetic sensitivity of the polarization signals shown by diagnostically interesting spectral lines, in the presence of magnetic fields of various configurations and intensities (e.g., Trujillo Bueno et al. 2002; Belluzzi et al. 2006). For example, the observation and theoretical modeling of the polarization signals that the joint action of the Hanle and Zeeman effects produce in the Ba II D\textsubscript{2} line provides a novel diagnostic window on the investigation of the magnetic fields of the solar photosphere and chromosphere (see Belluzzi et al. 2006). In this work we extend our attention to the Na I and K I D\textsubscript{2} lines, which can be described through the same atomic model adopted for the Ba II ion. We apply the quantum theory of polarization described in Landi Degl’Innocenti & Landolfi (2004) (hereafter LL04), through which we can investigate within a general and rigorous physical approach the role of resonance scattering and magnetic fields on the polarimetric properties of these lines. We consider an optically thin slab model, through which we are able to investigate in a rigorous way the essential physical mechanisms involved (resonance polarization, Zeeman, Paschen-Back and Hanle effects), avoiding complications due to radiative transfer effects.

2. Formulation of the Problem

We consider an optically thin slab of chromospheric plasma at 6000 K, located 7000 km above the $\tau_{5000}=1$ photospheric level. We assume the slab to be illuminated from below by the photospheric continuum radiation (i.e. by an
anisotropic, unpolarized radiation field, with cylindrical symmetry around the local vertical, without spectral structure (flat) over the typical width of the lines considered). The radiation field entering the Statistical Equilibrium Equations (SEEs), whose expression for an atomic system with hyperfine structure (HFS) can be found in Sect. 7.9 of LL04, is therefore known a priori. We calculate the intensity and the anisotropy factor of the photospheric continuum incident on the slab, at the frequencies of the lines under investigation, following Sect. 12.3 of LL04, and using the values for the disk-center intensities and limb-darkening coefficients given by Pierce (2000). Once the SEEs have been solved numerically, because of the optically thin slab assumption, the Stokes parameters of the radiation at frequency $\nu$, scattered in the direction $\vec{\Omega}$, can be simply found by the equation

$$S(\nu, \vec{\Omega}) = k \varepsilon_S(\nu, \vec{\Omega}),$$

where $S = I, Q, U, V$, $\varepsilon_S$ is the corresponding emission coefficient, and $k$ is a constant. The expressions of the emission coefficients for an atomic system with HFS can be found in Sect. 7.9 of LL04. In this work we will investigate the radiation scattered at 90° with respect to the local vertical, in the presence of a deterministic magnetic field of given orientation and strength. The reference direction for positive $Q$ is assumed to be parallel to the limb (see the left panel of Fig. 1).

3. The Atomic Model

To describe the three ions under investigation, we adopt a three-level model consisting of the ground level ($^2S_{1/2}$), the upper level of line D$_1$ ($^2P_{1/2}$) and the upper level of line D$_2$ ($^2P_{3/2}$). The abundance, nuclear spin, isotope shifts, and HFS constants of the various isotopes considered in this work are listed in Table 1. A Grotrian diagram describing qualitatively the HFS levels splitting for an isotope with nuclear spin $I = 3/2$ is shown in the right panel of Fig. 1.
### Table 1. Isotopes considered in this work

<table>
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<tr>
<th>Isotope</th>
<th>Abund. (%)</th>
<th>I</th>
<th>Isotope Shifts (MHz)</th>
<th>HFS Constants (MHz)</th>
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<tr>
<td>$^{23}$Na</td>
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<td>71.698</td>
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<td>reference isot.</td>
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1 A positive I.S. means that the line is shifted to higher frequencies with respect to the reference isotope. 2 The HFS constant $B$ is defined according to the convention of the american literature. 3 NIST on-line database; 4 Beckmann et al. (1974); 5 Hartmann (1970); 6 Schönberner & Zimmermann (1968); 7 Fueller (1976); 8 Bendali et al. (1981); 9 Ney (1969); 10 Wendt et al. (1984); 11 Wendt et al. (1988); 12 Becker et al. (1981); 13 Villemoes et al. (1993).

4. The Role of the Magnetic Field

In the presence of a magnetic field, each HFS level (or fine structure level, if the isotope is devoid of HFS) splits up into its magnetic sublevels. We describe this splitting within the framework of the general Paschen-Back effect theory (see LL04 Sect. 3.5).

As shown in Fig. 2, it is possible to distinguish three regimes: Zeeman effect regime (magnetic interaction weak with respect to the HFS interaction), incomplete Paschen-Back effect regime (magnetic and HFS interaction of the same order of magnitude) and complete Paschen-Back effect regime (HFS interaction weak with respect to the magnetic interaction). Note that the various regimes are reached by each of the three atomic systems under investigation for different magnetic field strengths. Hereafter, regardless of the particular regime, any polarization signal that originates from the splitting among the magnetic sublevels, will be referred to as Zeeman effect. The Zeeman effect produces in general elliptical polarization, which degenerates into linear polarization if the magnetic field lies on the plane perpendicular to the line-of-sight (LOS), and into circular polarization if the magnetic field lies along the LOS. The Zeeman effect dominates the polarization of the scattered radiation if the splitting among the magnetic sublevels is of the same order of magnitude or larger than the Doppler width ($\Delta \lambda_D$) of the line. For the ions under investigation, this gives the follow-
Figure 2. Splitting of the magnetic sublevels originating from the upper level of the D$_2$ line of three different isotopes with HFS, in the Zeeman effect regime (left column), incomplete Paschen-Back effect regime (central column) and complete Paschen-Back effect regime (right column). The zero of the energy scale is taken at the energy of the corresponding fine structure J-level.

ing critical values: 2500 G for Na I ($\Delta \lambda_D=41$ mÅ), 1500 G for K I ($\Delta \lambda_D=41$ mÅ) and 1300 G for Ba II ($\Delta \lambda_D=13$ mÅ).

However, if the magnetic field is not too weak, and if there are no other mechanisms that dominate the polarization, it is possible to identify Zeeman effect signatures on the Stokes profiles even for intensities much smaller than the critical one. It is well known that polarization signals can be produced through scattering processes (without the need of a magnetic field), whenever the atomic system is anisotropically excited, and the geometry of the scattering event has no remarkable symmetries. If an atomic system is anisotropically excited, the various magnetic sublevels are no longer evenly populated and are characterized by well defined phase relations (or interferences). The atomic system is then said to be polarized. It can be demonstrated that a magnetic field is able to modify the atomic polarization (more precisely the interferences among the magnetic sublevels), and therefore the polarization of the scattered radiation (Hanle effect). Depending on the configuration of the magnetic field and on the polarization state of the atomic system, interferences between different magnetic sublevels are modified, and different signatures of the Hanle effect are produced on the polarization profiles of the scattered radiation. An atomic system is sensitive to the Hanle effect for magnetic fields much smaller than the ones needed for the Zeeman effect to be appreciable. In particular, it can be demonstrated that the atomic polarization of a given level is significantly modified by the Hanle effect when the Zeeman splitting is of the same order of magnitude as the inverse
Figure 3. Theoretical Stokes profiles obtained in the presence of a vertical field (top row), of a horizontal field perpendicular to the LOS (second row) and of a horizontal field directed along the LOS (third and fourth rows). Note the non-zero linear polarization signals obtained in the absence of magnetic fields, and the modification towards the well known transverse Zeeman effect profiles for increasing magnetic fields. Note also that in each panel only the profiles corresponding to the more significative values of $B$ are plotted.

lifetime of the level. For the ions we are considering, this gives a Hanle effect sensitivity of the $D_2$ line upper level (which is much more polarized than the ground level) for magnetic fields between 0.5 and 50 G for Na I, between 0.3 and 30 G for K I, and between 10 and 100 G for Ba II.
5. Results and Conclusions

In the presence of a vertical magnetic field, the isotopes with HFS produce an enhancement of the linear polarization (see Trujillo Bueno et al. 2002). This is due to a special manifestation of the Hanle effect, usually referred to as “Anti-Level-Crossing effect” (see Bonnier 1980). Obviously, in the “solar prominence case” considered here, this enhancement of scattering polarization by a vertical field is much smaller for the case of barium, given the large abundance (82%) of isotopes without HFS (see top row of Fig. 3). However, it is important to emphasize that for the case of on-disk observations the “Anti-Level-Crossing effect” in the 18% of the barium isotopes endowed of HFS may produce a differential magnetic sensitivity of the three-peaks $Q/I$ profile observed in the D$_2$ line, which could be of great interest for the detection of vertical magnetic fields in quiet regions of the solar atmosphere (see Belluzzi et al. 2006). As the magnetic field strength increases, the transverse Zeeman effect becomes more important, and finally dominates the polarization of the scattered radiation as the critical intensity is approached. In the presence of a horizontal magnetic field, perpendicular to the LOS, or of a longitudinal magnetic field, both the isotopes with and without HFS are sensitive to the Hanle effect, which produces a decrease of the linear polarization (see second and third rows of Fig. 3). In the former case the Zeeman effect superimposes to the Hanle effect, in the latter it does not affect the linear polarization, but produces circular polarization as shown by the Stokes $V$ antisymmetric profiles (see last row of Fig. 3). We point out that in the presence of a longitudinal field the Hanle effect produces even a rotation of the plane of linear polarization, and therefore a non zero signal for the Stokes $U$ parameter (not shown here).

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References

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Ney, J., 1969, Z. Phys 223, 126