RESPONSE FUNCTIONS OF SPECTRAL LINES
SUITABLE FOR DIAGNOSTICS OF SOLAR ROTATION

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Abstract. The response functions for temperature and line-of-sight velocity of the medium-strong Fe II 523.5 nm, strong Fe I 543.4 nm and weak Ni I 543.6 nm spectral lines are examined. The lines were previously used for determination of the depth dependence of the solar rotation velocity. The positions of local maxima of the response functions to temperature and the calculated optical depths of formation of line cores are confronted. The close coincidence of these quantities is demonstrated and thus verifying the correctness of the optical depth scale of the investigated solar rotation profile.

Key words: Sun - rotation - line formation

1. Introduction

Response functions (RFs) were inferred as a result of application of the perturbation theory on the equation of radiative transfer (del Toro Iniesta, 2001). Each spectral line can be characterized by the set of RFs along with contribution functions well known from the past and frequently used for calculation of the depths of formation of spectral lines (Magain, 1986). On the contrary, the RFs carries information about sensitivity of a spectral line to the perturbation of some parameter of the atmospheric model, for example perturbation of temperature $T$, line-of-sight velocity $v_{\text{LOS}}$, micro-turbulent velocity $v_{\text{MIC}}$ or magnetic field $B$. Due to these features the RFs became the cornerstone of the inversion atmospheric code SIR (Ruiz Cobo and del Toro Iniesta, 1992) because they showed to be very useful tool in solution of many intricate problems regarding to the structure of the solar atmosphere (Bellot Rubio, 2001).
This paper follows the work of Balthasar (1983) who dealt with the depth dependence of the solar rotation velocity determined from Fraunhofer lines. The author selected 63 photospheric lines for which the depths of formation of line-cores in log $\tau$ were calculated using the atmosphere model HOLMU (Holweger and Müller, 1974). The velocity of solar rotation was determined from variation of line-centre positions measured along east-west solar diameter. As a result the dependence of the rotation velocity on the depths of formation of the line-cores was obtained, which is shown in Figure 1.

To gain a better insight into the process of spectral line formation throughout the solar photosphere the RFs to temperature and $v_{\text{LOS}}$ of three spectral lines are calculated and presented. This paper focuses on possible relation of the depth of formation of the line cores and maxima of the RFs.

2. Line Selection and Line Parameters

Although Balthasar (1983) used the set of 63 Fraunhofer lines only a sample of them will be investigated in the following. Some important characteristics of three selected spectral lines are listed in Table I. Intentionally this subset contains the weak line Ni I 543.6 nm, the medium-strong line Fe II 523.5 nm and the strong line Fe I 543.4 nm, which is a well known Doppler line with zero Landé factor. Table I indicates the depths of formation of the line cores log $\tau$ and corresponding rotation velocities $v$. Using these quantities the selected lines can be identified in Figure 1. The last two columns of Table I contain excitation potentials of the lower atomic level $E_{\text{P,low}}$ and logarithms of the multiplicity of the lower atomic level $g$ times the oscillator strength $f$. These values along with laboratory wavelengths $\lambda_{\text{lab}}$ and abundances are needed for the calculation of the RFs.

3. Calculations

For the purpose of this work, Stokes Inversion based on Response functions code (SIR) developed by Ruiz Cobo and del Toro Iniesta (1992) is used. The model HOLMU is adopted as suitable representative of one-component solar atmosphere models (Holweger and Müller, 1974).

The application of linearized perturbation theory on radiative transfer equation shows, that the modification of $i$-th atmospheric parameter $\delta x_i$ in-
Table I: Line parameters of investigated spectral lines. $\lambda_{lab}$ denotes the laboratory wavelength, EW the equivalent width, log $\tau$ computed optical depths of formation of the line cores using the model HOLMU, EP$_{low}$ the excitation potential of the lower level, and log$(gf)$ logarithm of the multiplicity of the lower atomic level $g$ times the oscillator strength $f$. These data have been taken from: $a$ – Vienna Atomic Line Database (Kupka et al., 1999), $b$ – Nave et al. (1994), EW – Moore et al. (1966), log $\tau$ and $v$ – Balthasar (1983), EP$_{low}$ and log$(gf)$ – Thévenin (1989).

<table>
<thead>
<tr>
<th>Element</th>
<th>$\lambda_{lab}$</th>
<th>EW</th>
<th>log $\tau$</th>
<th>$v$</th>
<th>EP$_{low}$</th>
<th>log$(gf)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe II</td>
<td>5234.625$^a$</td>
<td>81</td>
<td>$-1.88$</td>
<td>1.9952</td>
<td>3.22</td>
<td>$-2.31$</td>
</tr>
<tr>
<td>Fe I</td>
<td>5434.5238$^b$</td>
<td>184</td>
<td>$-4.816$</td>
<td>2.0304</td>
<td>1.01</td>
<td>$-2.22$</td>
</tr>
<tr>
<td>Ni I</td>
<td>5435.855$^a$</td>
<td>46</td>
<td>$-1.164$</td>
<td>1.9396</td>
<td>1.99</td>
<td>$-2.58$</td>
</tr>
</tbody>
</table>

Figure 1: Dependence of the rotation velocity on the formation depth of the line cores log $\tau$ for the Holweger and Müller (1974) atmosphere (Balthasar, 1983). The dots corresponding to the lines investigated by us are highlighted by the circles (see Table I).
duces the perturbation $\delta I_\lambda$ of the profile of the spectral line in wavelength $\lambda$ according Equation 1 (del Toro Iniesta, 2001):

$$\delta I_\lambda = \sum_{i=1}^{n} \int_{0}^{\infty} R_{x_i} \delta x_i d\tau,$$

where $n$ is the number of perturbed atmospheric parameters, $R_{x_i}$ is the response function (RF) to an atmospheric parameter $x_i$ ($T$, $v_{LOS}$, $v_{MIC}$, $B$...) and $\tau$ is the optical depth at 500 nm at continuum. The size of perturbation is determined by $R_{x_i}$ defined as:

$$R_{x_i} \equiv \frac{e^{-\tau \lambda}}{\alpha_5} \left\{ \alpha_\lambda \frac{\partial S}{\partial x_i} - \frac{\partial \alpha}{\partial x_i} (I - S) \right\},$$

where $e$ is the Euler number 2.71828..., $\alpha_5$ and $\alpha_\lambda$ are the monochromatic extinction coefficients (unit m$^{-1}$) at the optical depth $\tau$ at 500 nm and a given wavelength $\lambda$, respectively. $S$ is the source function, which in LTE equals Planck function $B_\lambda$, $I$ is the monochromatic intensity at the optical depth $\tau$ and $\tau_\lambda$ is the optical depth at a given wavelength $\lambda$. The calculations of the RFs to temperature $R_T$ and line-of-sight velocity $R_V$ were performed by Equations 3 and 4 in the range of optical depths from 1 to $-5$ and $\pm 15$ pm around line centres.

$$R_T(\tau, \lambda) \equiv \frac{e^{-\tau \lambda}}{\alpha_5(\tau)} \left\{ \alpha_\lambda(\tau) \frac{\partial B_\lambda(\tau)}{\partial T_\lambda(\tau)} - \frac{\partial \alpha_\lambda(\tau)}{\partial T_\lambda(\tau)} [I_\lambda(\tau) - B_\lambda(\tau)] \right\} \tau \ln 10$$

$$R_V(\tau, \lambda) \equiv \frac{e^{-\tau \lambda}}{\alpha_5(\tau)} \frac{\partial \alpha_\lambda(\tau)}{\partial v_{LOS}} [B_\lambda(\tau) - I_\lambda(\tau)] \tau \ln 10$$

The term $\tau \ln 10$ in Equations 3 and 4 reflects the discretization of atmospheric model using logarithmic scale sampled equidistantly with steps of $\Delta \log \tau = 0.1$. The effect of macroturbulent velocity $v_{MAC}$ on $R_T$ and $R_V$ is simulated by convolving Equations 3 and 4 with a Gaussian:

$$M(\lambda - \lambda_0, v_{MAC}) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\lambda - \lambda_0)^2}{2\sigma^2}},$$

where $\sigma \equiv \lambda_0 v_{MAC}/c$, $\lambda_0$ is the central wavelength of the transition and $c$ is the speed of light. Thus presented $R_T^*$ and $R_V^*$ are convolved RFs given by equations:

$$R_T^*(\tau, \lambda) = M(\lambda) * R_T(\tau, \lambda),$$

$$R_V^*(\tau, \lambda) = M(\lambda) * R_V(\tau, \lambda).$$


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The resulting $R^*_T$ and $R^*_V$ are absolute RFs in contrast to relative RFs (e.g. Ruiz Cobo and del Toro Iniesta, 1994). Thus they have units $K^{-1}$ and $(cm s^{-1})^{-1}$ respectively. Both, $R^*_T$ and $R^*_V$ are normalized to the HSRA continuum intensity (Gingerich et al., 1971) at the disk centre at the central wavelength of the line, therefore the energetic unit is omitted.

4. Results and Discussion

The resulting $R^*_T$ and $R^*_V$ for Fe II 523.5 nm, Fe I 543.4 nm and Ni I 543.6 nm spectral lines are shown in Figures 2, 3 and 4 respectively. The calculated RFs exhibit:

- the symmetry of the $R^*_T$ and the $R^*_V$ in respect to the line centre as the consequence of absence of $v_{LOS}$ gradients in the model HOLMU.

- the dominant rôle of temperature in the formation of the line profile. This is indicated by the ratio of the orders $R^*_T/R^*_V$.

- the global maxima of the $R^*_T$ in the layers of the formation of the continuum and one local maximum in upper layers in case of medium-strong line Fe II 523.5 nm and strong line Fe I 543.4 nm also discernible in $(\log \tau, \Delta \lambda)$ plane by contours.

- the concentration of the $R^*_V$ maxima into the optical depth $\log \tau \approx -1$. This is the possible reason of more pronounced upper parts of C-shaped bisectors of spectral lines.

- the good agreement of the optical depths of the formation of the line cores ($\log \tau$) and the positions of the local maxima of the $R^*_T$.

In general, the layers forming the line cores are well identifiable by the local maxima of the $R^*_T$ (Koza and Kučera, 2002). One can see that the line cores are sensitive to the perturbations of $T$ in upper photospheric layers and that the depths of formation of line cores almost coincide with the position of the local maxima of $R^*_T$. Also in case of the weak spectral line Ni I 543.6 nm we can identify all previously mentioned characteristics except one. There is missing a well pronounced local maximum of $R^*_T$. So the layers responsible for the formation of the line core are hardly identifiable.
Figure 2: The response function of the Fe II 523.5 nm line to temperature (top panel) and line-of-sight velocity (bottom panel). The value of 0 pm corresponds to the central laboratory wavelength of the line. The dashed line in (log τ, Δλ) plane in top panel helps to identify the optical depth of formation of the line core, which is −1.88 (see Table I).
Figure 3: Same as in Figure 2 but for the Fe I 543.4 nm line of which the optical depth of formation of the line core is $-4.816$. 
Figure 4: Same as in Figure 2 but for the Ni I 543.6 nm line of which the optical depth of formation of the line core is $-1.164$. 
5. Conclusion

This investigation points out the good coincidence of the depths of the formation of the line cores (log $\tau$) and the positions of the local maxima of $R_T^*$ in the case of the sample of three selected lines. This can be regarded as verification of correctness of the scale of the optical depths used in the work of Balthasar (1983). Conclusively, the obtained rotational velocities were assigned to the corresponding optical depths.

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References

ODAZIVNE FUNKCIJE SPEKTRALNIH LINIJA
POGODNE ZA DIJAGNOSTIKU SUNČEVE ROTACIJE

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Izlaganje sa znanstvenog skupa

Sažetak. Istražuju se odazivne funkcije temperature i brzine gibanja u doglednici srednje jake spektralne linije Fe II na 523.5 nm, jake linije Fe I na 543.4 nm i slabe linije Ni I na 543.6 nm. Ranije su te linije koristene za određivanje ovisnosti brzine Sunčeve rotacije o dubini. Položaji lokalnih maksimuma odazivnih funkcija sučeljavaju se s temperaturom i izračunatim optičkim dubinama stvaranja jezgri spektralnih linija. Pokazuje se bliska podudarnost veličina i time potvrđuje ispravnost mjerila optičkih dubina istraživanog profila Sunčeve rotacije.

Ključne riječi: Sunce - rotacija - stvaranje linija