EVOLUTION OF TWISTED MAGNETIC FLUX ROPES EMERGING INTO THE SOLAR CORONA

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ABSTRACT

We present MHD simulations in both 2D axisymmetric and 3D spherical geometries of the evolution of a twisted magnetic flux rope emerging into the low-$\beta$ corona previously occupied by a potential arcade field. These simulations show two distinct stages of the evolution. The earlier evolution is quasistatic during which if the flux emergence is stopped, the flux rope settles to a neighboring equilibrium with stored free magnetic energy. Loss of equilibrium and eruption of the flux rope are found for both a 2D axisymmetric flux rope and a 3D line-tied flux rope, when too much twisted flux has been transported into the corona.

Key words: MHD — Sun: corona — Sun: magnetic fields.

1. INTRODUCTION

An appealing candidate for the basic underlying magnetic field configuration for CME precursors is a twisted magnetic flux rope with field lines twisting about each other by more than one wind between the two ends anchored to the dense photosphere (e.g. Low, 2001). Twisted magnetic flux ropes in the corona may form as a result of direct flux emergence from the interior (e.g. Fan, 2001; Magara & Longcope, 2001; Magara, 2004; Manchester et al., 2004). Models of twisted magnetic flux ropes can explain several observed features of CME precursors. First of all they contain free magnetic energy which is believed to be the main energy source for driving flares and CMEs. The dipped field lines contained in the twisted flux ropes naturally provide support for the dense cool prominence mass against gravity. Further more, the so called “bald-patch” separatrix surface (BPSS) of a partially emerged line-tied flux rope, comprising field lines that graze the photosphere at the polarity inversion line, is a site for current sheet formation under dynamic perturbations (e.g. Titov & Demoulin, 1999; Low & Berger, 2003; Fan and Gibson, 2004; Gibson et al., 2004). The current sheet tends to form along the BPSS because of a discontinuous transition in the dynamic behavior between the twisted field lines that graze the photosphere and the neighboring anchored field lines. The heating resulting from this current sheet can explain the observed X-ray sigmoids that are found to be often associated with the CME source regions. Direct MHD simulations by Fan and Gibson (2004) found that, at the onset of the kink instability of a line-tied twisted flux ropes, a sigmoid shaped current sheet indeed forms along the BPSS. The current sheet is of an inverse-S (forward-S) shape for a left-hand-twisted (right-hand-twisted) flux rope, consistent with the observed hemispheric preference for active region twist and X-ray sigmoid morphology.

The twisted magnetic flux ropes can exist in stable equilibria over long periods of time in the corona. A sudden catastrophic loss of equilibrium due to, e.g. the build-up of too much twisted flux with respect to the confining overlying field, can lead to the explosive release of the stored free magnetic energy, resulting in a CME (e.g. Priest & Forbes, 2002; Sturrock et al., 2001). Catastrophic loss of equilibrium of 2D flux rope configurations has been demonstrated analytically (e.g. Forbes & Priest, 1995; Lin et al., 1998). In this paper, we carry out MHD simulations in the low-$\beta$ regime to model the evolution of twisted magnetic flux ropes in the corona as they emerge quasistatically into a pre-existing coronal potential arcade field. We first consider an idealized 2D axisymmetric configuration with a toroidal flux rope circling around the sun, and then an arched 3D line-tied flux rope in a spherical geometry. We study both the initial quasistatic evolution during which the magnetic energy transported into the corona by flux emergence is being stored in a sequence of confined flux rope equilibria, and the eventual loss of equilibrium of the flux ropes when sufficient amount of twisted flux has been transported into the corona.
2. MODEL DESCRIPTION

The simulation domain is given by \( r = [R_\odot, 14.4R_\odot], \theta = [\pi/3, 2\pi/3] \) in the 2D axisymmetric case, and \( r = [R_\odot, 6R_\odot], \theta = [\pi/3, 2\pi/3], \phi = [-\pi/4.8, \pi/4.8] \) in the 3D case. The initial state in the domain is assumed to be an isothermal atmosphere in hydrostatic equilibrium with the density and pressure profiles given by:

\[
\rho = \rho_0 \exp \left( -\frac{R_\odot}{H_p} \left( 1 - \frac{R_\odot}{r} \right) \right) \tag{1}
\]

\[
p = \frac{RT_0 \rho}{\mu} \tag{2}
\]

where the temperature \( T_0 = 2\text{MK} \). Thus the isothermal sound speed is \( a_s = \sqrt{\frac{RT_0}{\mu}} = 128 \text{ km s}^{-1} \), the pressure scale height at the bottom of the domain, which corresponds to the base of the corona, is \( H_p = \frac{\sqrt{RT_0/\mu}}{(GM_\odot/R_\odot^2)} = 60\text{ Mm} \). The density at the base is \( \rho_0 = 8.365 \times 10^{-10} \text{ g cm}^{-3} \). The domain is initially occupied by a pre-existing potential arcade field whose normal field \( B_\nu(0, \theta, \phi) \) at the lower boundary is concentrated into two isolated bands. The peak field strength of the normal magnetic field at the lower boundary is \( 20G \), leading to an Alfvén speed of \( v_{a0} = B_0/\sqrt{\mu \rho_0} = 1951 \text{ km s}^{-1} \) which is more than a factor of 10 greater than the isothermal sound speed \( a_s \). In the domain, we solve the following isothermal MHD equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{3}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p + \rho \frac{GM}{r^2} \hat{\mathbf{r}}
+ \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}
+ \rho \nu \nabla^2 \mathbf{v}, \tag{4}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nu \times \mathbf{B}), \tag{5}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{6}
\]

\[
p = a_s^2 \rho. \tag{7}
\]

We greatly simplify the treatment of the thermodynamics of the coronal plasma by using an isothermal equation of state. Our focus is the evolution of the coronal magnetic field under the conditions of high electrical conductivity and low plasma \( \beta \). Since the isothermal sound speed \( a_s \) is much smaller than the characteristic Alfvén speed \( v_{a0} \), the Lorentz force is the dominant force in the momentum equation (eq. [4]). Along the magnetic field lines, in which direction the Lorentz force vanishes, plasma follows the hydrodynamics of an isothermal atmosphere with a pressure scale height of \( H_p = \frac{(RT_0/\mu)(GM_\odot/r^2)}{} \), which is \( \sim 0.087R_\odot \) near the lower boundary.

At the lower boundary of the domain, a twisted arched flux tube \( \mathbf{B}_{\text{tube}} \) is transported kinematically into the domain by imposing at the \( r = R_\odot \) boundary a time dependent transverse electric field \( \mathbf{E}_{\perp} |_{r = R_\odot} \) that corresponds to bodily lifting the flux tube at a velocity \( \mathbf{v}_0 \):

\[
\mathbf{E}_{\perp} |_{r = R_\odot} = \hat{\mathbf{r}} \times \left[ -\frac{1}{c} \mathbf{v}_0 \times \mathbf{B}_{\text{tube}}(R_\odot, \theta, \phi, t) \right] \times \hat{\mathbf{r}}, \tag{8}
\]

and the associate velocity field that is \( \mathbf{v}_0 \) in the area where the emerging tube intersects the lower boundary and zero in the rest of the area on the lower boundary. In the domain, we solve the dynamic equations (eqs. [3] through [7]) to model the evolution of the emerged field in the corona whose total twist and magnetic energy increases due to the imposed flux emergence at the lower boundary. We assume perfectly conducting walls for the side boundaries. For the outer boundary, we use a simple outward extrapolating boundary condition that allows plasma and magnetic field to flow through.

3. SIMULATION RESULTS

3.1. 2D axisymmetric flux ropes

Fig. 1 shows the evolution of the coronal magnetic field resulting from the emergence of a twisted toroidal flux rope (circling around the idealized sun) into a pre-existing poloidal potential arcade. Fig. 2 shows the evolution of the rise velocity and the height of the flux rope axis after emergence, and also the evolution of the magnetic energy, for three different cases in which the flux emergence is stopped at respectively \( t = 118R_\odot/v_{a0} \) for Case A, \( t = 114R_\odot/v_{a0} \) for Case B, and \( t = 112R_\odot/v_{a0} \) for Case C. The magnetic field evolution shown in Fig. 1 corresponds to Case A. We find that the evolution from \( t = 0 \) to about \( t = 114R_\odot/v_{a0} \) are nearly quasistatic during which if we stop the flux emergence, the system settles to a neighboring equilibrium. This is demonstrated in Case C (the dashed-dotted curves in Fig. 2), for which the flux emergence is stopped at \( t = 112R_\odot/v_{a0} \), and subsequently the rise velocity settles to zero and the magnetic energy remains constant. In other words, the free magnetic energy transported into the corona through flux emergence is being stored in the equilibrium flux rope. If the flux emergence is continued beyond about \( t = 114R_\odot/v_{a0} \), we find that the flux rope can no longer find a stable equilibrium and erupts, as can be seen in Cases A and B (the solid and the dashed curves in Fig. 2). In these cases, the flux rope accelerates to a peak velocity of about \( 0.4v_{a0} \), and the magnetic energy drops sharply. Because of the imposed azimuthal invariance, the entire flux rope erupts and all of the overlying arcade field is stretched out with the flux rope. A current sheet forms behind the flux rope.
and magnetic reconnection takes place which enables the flux rope to escape. Eventually, the reconnected post-flare loops reform a configuration that is similar to the initial arcade field, although some of the toroidal flux of the original flux rope is trapped in the reconnected field.

Fig. 3 shows the magnetic field structure and the current density distribution in a meridional cross-section of a stable flux rope, corresponding to $t = 114R_0/v_{ao}$ of Case C in which flux emergence has stopped (at $t = 112R_0/v_{ao}$) during the quasi-static stage of the evolution. The current density distribution within this stable flux rope (right panel of Fig. 3) show three major concentrations. In reference to the magnetic structure shown in the left panel of Fig. 3, the outer most layer of current concentration corresponds to the boundary surface between the emerging flux rope and the pre-existing potential arcade field. This is likely a current sheet or magnetic tangential discontinuity due to the discontinuous jump in the field-line direction between the two flux systems. This boundary surface of the flux rope may be identified with the boundary of the coronal cavity or filament channel – the expansion of the emerging flux rope due to its strong toroidal magnetic field may have produced the lower density in the cavity or filament channel. Within the flux rope (or the cavity) there is a second layer of current concentration, which appears to be barely necking off at the surface. This current layer coincides with the field line (or the flux surface) that just grazes the lower boundary surface (see the left panel of Fig. 3). It is the 2D version of the so-called bold-patch separatrix surface (BPSS), separating the field lines that are completely detached from those that are anchored. This separatrix surface may be another current sheet because the detached twisting field lines within the surface have very different dynamics compared to the short anchored field lines outside and, as a result, the two types of field lines relax differently in the corona to form magnetic tangential discontinuities. The central diffuse current distribution is simply the current that flows along the twisted flux rope. The two current sheets along the flux rope boundary and the BPSS respectively are likely to be dissipative and may produce significant heating. Specifically, the inner current sheet that forms along the BPSS may appear as a bright X-ray source within the filament channel or cavity, enclosing the cool prominence material that is located at the dips of the twisted field lines. Such a hot core seen in soft X-ray within a stable crown filament channel or cavity has been observed in several SXT observations (e.g. Hudson et al., 1999). This phenomenon of a bright cavity core in X-ray is termed “chewy nougat” (see http://solar.physics.montana.edu/YPOP/Nuggets/1998/981009/981009.html). In our interpretation, “chewy nougats” and X-ray sigmoid are of the same physical origin, namely current sheets forming along the BPSS of a twisted, partially emerged flux rope.

For a quasi-2D flux rope that may be associated with a crown filament, the BPSS forms a long “tunnel” enclosing the filament, whereas for a more compact 3D line-tied flux rope, with field-lines twisting about the axis by less than 2 winds between the anchored ends, the BPSS appears as a sigmoid shape (see e.g. Gibson et al., 2004) to be identified with the X-ray sigmoid brightening.

### 3.2. 3D line-tied flux ropes

Fig. 4 shows the 3D evolution of a line-tied twisted flux rope emerging into the corona previously occupied by a potential arcade field. The solid curves in Fig. 5 show the evolution of the rise velocity and the height of the apex of the emerged flux rope axis, and the evolution of the total magnetic energy, for two different runs where the flux emergence is driven until $t = 83R_0/v_{ao}$ (Run A) and $t = 96.5R_0/v_{ao}$ (Run B), when the field-line twist in the emerged tube reaches 1.88 and 1.6 winds, respectively. The magnetic field evolution shown in Fig. 4 corresponds to Run A. As is in the previous 2D case, there is an initial stage of quasi-static evolution (from $t = 0$ to roughly $t = 85R_0/v_{ao}$) during which if the flux emergence is stopped, the flux rope will settle into a neighboring equilibrium state. See for example in Run B, after the flux emergence is stopped, the flux rope settles to a mildly kinked equilibrium (see the structure shown the $t = 83$ panel of Fig. 4), with the rise velocity settling to zero and the magnetic energy remaining constant. However, if the flux emergence is continued, as in Run A, we find that after about $t = 85R_0/v_{ao}$, when the field line twist in the emerged tube reaches about 1.7 winds, the flux rope shows a significant acceleration (see Run A in Fig. 5), and erupts through the arcade field at a localized area (Fig. 4) with most of the arcade field remaining closed. The flux rope continues to accelerate after the flux emergence is stopped at $t = 96.5R_0/v_{ao}$, and it eventually moves out of the outer boundary ($r = 6R_0$) with a speed of about 0.3$v_{ao}$. The nonlinear evolution of the kink instability produces a significant rotation of the tube orientation at the apex, with the length-wise direction of the upward moving tube changing from being perpendicular to the arcade field to being parallel. As a result the upward intrusion of the flux rope becomes more localized and it becomes easier for the flux rope to push through the arcade field. This is a 3-dimensional effect that is described in Sturrock et al. (2001).

Fig. 6 shows an image produced by column integration of $j^2$, where $j$ is the current density, along the line of sight looking down on the erupting flux rope at $t = 102R_0/v_{ao}$. We find that an inverse S-shaped current sheet has formed as a result of the kink motion of the line-tied left-hand-twisted flux rope. Fan and Gibson (2004) and Gibson et al. (2004) have discussed in detail the close relation of the current sheet
Figure 1. Evolution of the coronal magnetic field from a 2D axisymmetric simulation of a twisted toroidal flux rope emerging into a pre-existing poloidal potential arcade (corresponding to Case A shown in Fig. 2, for which the flux emergence is stopped at t=118). The images show the magnetic field in a meridional plane with the contours showing the poloidal magnetic field lines and the gray scale intensity showing the toroidal magnetic field strength ($B_\theta$). The times are given in units of $R_\odot/v_\odot$. 
to the BPSS which separates the twisted field lines that graze the photosphere and the neighboring anchored field lines (Titov & Demoulin, 1999). The formation of the current sheet may explain the transient soft-X ray sigmoid brightenings that have been observed during the onset of some eruptive flares and CMEs (e.g. Sterling & Hudson, 1997; Moore et al., 2001).

4. SUMMARY AND CONCLUSIONS

We have carried out MHD simulations in spherical geometries to model the evolution of the coronal magnetic field as a twisted magnetic flux rope emerges slowly into a pre-existing coronal potential arcade field. We have considered both a 2D axisymmetric configuration of a twisted toroidal flux rope circling around the sun, and also a 3D flux rope with its ends line-tied to the lower boundary. We find that in both cases, there is an initial stage of quasi-static evolution where the magnetic energy transported into the corona by flux emergence is being stored in a sequence of confined flux rope equilibria. For these partially emerged flux ropes, current sheets may develop along the BPSS, due to the different dynamic behaviors between the fully emerged twisted field lines and the anchored arcade-like field lines. In the case of a partially emerged 2D axisymmetric toroidal flux rope, the BPSS within the flux rope takes up the shape of a long tunnel enclosing the prominence material supported by the field line dips. The current sheet formation along the BPSS in this case may explain the limb observation of a X-ray bright source at the filament location inside a stable filament cavity (the so-called “chewy nougat”, see e.g. Hudson et al., 1999). In the case of a 3D line-tied flux rope with a field-line twist of 1-2 winds between the line-tied ends, the BPSS is a sigmoid shaped ribbon, and may explain the hot X-ray sigmoids associated with CME source regions (e.g. Titov & Demoulin, 1999; Gibson et al., 2004).

Figure 2. The evolution of the rise velocity (top panel) and the height (middle panel) of the flux rope axis, and also the evolution of the magnetic energy (bottom panel), from 3 different 2D axisymmetric simulations for which the flux emergence is stopped at $t = 118$ (Case A, see Fig. 1), $t = 114$ (Case B), and $t = 112$ (Case C) respectively.
Figure 4. The 3D evolution of the coronal magnetic field resulting from the emergence of a twisted flux rope line-tied to the lower boundary, corresponding to Run A shown in Fig. 5. (from Fan, 2003)
Loss of equilibrium and eruption of the flux rope are found in both the 2D axisymmetric and the 3D cases when too much twisted flux is transported into the corona. In the 2D case, the imposed azimuthal invariance prohibits the kink motion. The entire flux rope erupts with all of the arcade field being stretched out with the flux rope. Reconnection takes place in a vertical current sheet that forms behind the erupting flux rope, allowing the flux rope to escape. In the 3D case, with the build-up of a moderate amount of twist ($< 2$ full winds of field line twist about the axis), the line-tied flux rope kinks and erupts through the arcade field at a localized area, with most of the arcade field remaining closed. A sigmoid-shaped current sheet forms below the flux rope during the eruption that may explain the transient soft X-ray sigmoid brightenings that have been observed during the onset of eruptive flares and CMEs (e.g. Sterling & Hudson, 1997; Moore et al., 2001).

ACKNOWLEDGMENTS

We thank B.C. Low for many helpful discussions. The National Center for Atmospheric Research is sponsored by the National Science Foundation. This work is supported in part by AFOSR grant F49620-
02-0191.

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