COMPARATIVE ANALYSIS OF COLLISIONAL AND VISCOUS DAMPING OF MHD WAVES IN PARTIALLY IONIZED SOLAR PLASMAS

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Abstract. A comparative study of the efficiency of MHD wave damping in solar plasmas due to collisional and viscous energy dissipation mechanisms is presented here. The performed estimations allow to conclude which damping mechanism is dominant in which regions. In general the correct description of MHD wave damping requires the consideration of all energy dissipation mechanisms via the inclusion of the appropriate terms in the generalized Ohm’s law, the momentum, energy and induction equations. Specific forms of the generalized Ohm’s Law and induction equation are presented that are suitable for partially ionized regions of the solar atmosphere.

Key words: Magnetohydrodynamics (MHD) - waves - Sun: photosphere - Sun: chromosphere - Sun: prominences

1. Introduction

Magnetohydrodynamic (MHD) waves are widely considered as a possible source of heating for various parts of the outer solar atmosphere. The heating effect of MHD waves is connected with a certain dissipation mechanism which converts the energy of damped MHD waves into the energy of the background plasma. Among the main energy dissipation mechanisms which play an important role under the solar conditions, are collisional dissipation.
(resistivity) and viscosity. The presence of neutral atoms in the partially ionized plasmas of the solar photosphere, chromosphere and prominences enhances the efficiency of both these energy dissipation mechanisms.

In previous publications collisional and viscous damping of MHD waves in partially ionized solar plasmas has been always considered separately. One group of researchers assumes that the friction between ions and the neutral fraction is the dominant dissipation mechanism (DePontieu et al., 2001). Another group of authors takes into account only the viscosity effects (Nakariakov et al., 2000; Ofman, 2002). Because of the different physical nature of the above two mechanisms of MHD wave dissipation, in each particular case a certain comparative study and physical reasoning are needed in order to decide which of the two dissipation mechanisms dominates in a given part of the solar atmosphere. In the general case, the correct description of MHD wave damping requires the consideration of both dissipation mechanisms via the inclusion of the appropriate terms in the generalized Ohm’s Law, as well the momentum and energy equations.

Here we give some elementary estimations concerning the efficiency of the frictional and viscous damping of MHD waves in a partially ionized plasma of the lower solar atmosphere and prominences.

2. MHD wave damping in the linear approximation

In this section we summarize the results from Braginskii (1965) on the linear damping rates of MHD waves, obtained from the calculation of the energy decay time using the local energy dissipation rates \( Q_{\text{visc}} \) and \( Q_{\text{frict}} \). These results are applied in later sections. The decay of the wave amplitude is described by the complex frequency \( \omega - i\delta \) so that the energy in a particular wave mode will decay as \( e^{-t/\tau} \), where \( \tau = (2\omega\delta)^{-1} \) is a characteristic wave damping time and \( \delta (<< 1) \) is the logarithmic damping decrement.

The MHD wave modes under consideration are the Alfvén wave, fast magnetoacoustic, and slow magnetoacoustic waves. The slow magnetoacoustic wave degenerates in the limit \( C_s \ll V_A \) to a pure acoustic wave distorted by the magnetic field (\( \omega = C_s k_\parallel \)). The condition \( C_s \ll V_A \) holds true for magnetic fields \( B_0 > 10 \text{ G} \) everywhere above the middle chromosphere, as well as in the majority of prominences. In the opposite limit of \( C_s \gg V_A \), which is valid in the non- or weakly magnetized regions of the chromosphere and photosphere, fast magnetoacoustic waves transform to the usual acous-
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tic wave \((\omega = C_s k)\), while the slow magnetoacoustic wave behaves like the Alfvén wave \((\omega = V_A k_{\|})\). Here \(V_A = \frac{B_0}{\sqrt{4\pi \rho_0}}\) and \(C_s = \left(\frac{\gamma p_0}{\rho_0}\right)^{1/2}\) are the Alfvén and the sound speed in a plasma with total density \(\rho_0\), pressure \(p_0\), external magnetic field \(B_0\), and adiabatic constant \(\gamma\).

For the damping decrements of Alfvén, fast magnetoacoustic, and acoustic waves propagating in a partially ionized plasma, Braginskii (1965) gives the following expressions:

**Alfvén wave** (A.w.):

\[
2\omega \delta_{Joule}^{A.w.} = \frac{1}{\tau_{Joule}^{A.w.}} = \frac{c^2}{4\pi \sigma} k_{\perp}^2 + \frac{\xi_n c^2}{4\pi \sigma C_s^2} k_{\parallel}^2 = \frac{c^2}{4\pi \sigma} k^2 + \frac{c^2}{4\pi} \frac{\xi_n^2 B_0^2}{\alpha_n c^2} k_{\parallel}^2,
\]

(1)

**Fast magnetoacoustic (or magneto sonic) wave** (f.ms.w.):

\[
2\omega \delta_{Joule}^{f.ms.w.} = \frac{1}{\tau_{Joule}^{f.ms.w.}} = \frac{c^2}{4\pi \sigma} k^2 + \frac{c^2}{4\pi} \frac{\xi_n^2 B_0^2}{\alpha_n c^2} k_{\parallel}^2.
\]

(2)

**Acoustic (or sound)** wave (s.w) (in the case \(m_i = m_n\)):

\[
2\omega \delta_{frict}^{s.w.} = \frac{1}{\tau_{frict}^{s.w.}} = \frac{c^2}{4\pi \sigma} \frac{C_s^2}{V_A^2} k_{\perp}^2 + \frac{\xi_n^2 C_s^2 \rho_0}{\alpha_n} \left( \frac{k_{\parallel}^2 n_e^2}{\rho_0^2} + \frac{k_{\perp}^2 (n_i + n_n)^2}{n_0^2} \right).
\]

(3)

Here \(k_{\parallel}\) and \(k_{\perp}\) are components of the wave vector relative to the magnetic field direction; \(\xi_n = \frac{m_n n_n}{m_n n_n + m_i n_i}\) is the relative density of neutrals; \(\alpha_n = m_e n_e \nu_{en} + m_i n_i \nu_{in}\), where \(\nu_{kl} = \frac{m_l}{m_l + m_k} \nu_{kl}, k = e, i, l = i, n\) are the effective collisional frequencies.

We use in Equations (1) and (2) the notation \(\delta_{Joule}\) in place of \(\delta_{frict}\) because the collisional friction energy dissipation rates for the Alfvén and fast magnetoacoustic waves (when \(C_s \ll V_A\)) in a partially ionized plasma have a form of Joule dissipation: \(Q_{frict} = Q_{Joule} = \frac{\dot{J}_{\parallel}^2}{\sigma} + \frac{\dot{J}_{\perp}^2}{\sigma C}, \) where \(\sigma\) and \(\sigma_C\) are Coulomb and Cowling conductivities of the plasma. They are related to each other as \(\sigma_C = \frac{\xi_n^2 B_0^2}{\alpha_n c^2 \sigma}\) (Cowling, 1957). In a strong

enough magnetic field $\sigma / \sigma_C \approx 1 + \xi_n^2 \max \{\nu_{ei}^*, \nu_{en}^*\} \nu_{in}^* \gg 1$. Figure 1 shows $\sigma / \sigma_C$ variation in the low solar atmosphere calculated for the VAL C model (Vernazza et al., 1981) and different $B_0$.

The viscous damping decrements in a partially ionized plasma in the majority of cases can be calculated using the fully ionized plasma expressions

$$2\omega \delta_{\text{visc}}^{A.w.} \equiv \frac{1}{\tau_{\text{visc}}^{A.w.}} = \frac{\left(\eta_1 k_{1}^2 + \eta_2 k_{||}^2\right)}{\rho_0}$$

$$2\omega \delta_{\text{visc}}^{f.m.s.w.} \equiv \frac{1}{\tau_{\text{visc}}^{f.m.s.w.}} = \frac{1}{\rho_0} \left[\left(\frac{\eta_0}{3} + \eta_1\right) k_{1}^2 + \eta_2 k_{||}^2\right]$$

$$2\omega \delta_{\text{visc}}^{s.w.} \equiv \frac{1}{\tau_{\text{visc}}^{s.w.}} = \frac{1}{\rho_0} \left(\frac{4}{3} \eta_0 k_{||}^2 + \eta_2 k_{1}^2\right)$$

but with the viscosity coefficients $\eta_0$, $\eta_1$, $\eta_2$ modified appropriately to include the neutral gas.

3. Application to the Sun

In relation to the phenomena on the Sun, the case of longitudinal (||) with respect to the magnetic field ($k_{1} = 0$, $k_{||} \neq 0$), propagation of MHD waves appears to be the most important.

3.1. MHD wave damping in the solar photosphere/chromosphere

Equations. (4), (5), (1), (2), give us the ratio of the collisional friction and viscous damping times of ||-propagating in the partially ionized solar plasmas Alfvén and fast magnetoacoustic waves:

$$\frac{\tau_{\text{Joule}}}{\tau_{\text{visc}}} \bigg|_{A.w.(||)}^{f.m.s.w.(||)} = \frac{4\pi \eta_2 \sigma_C}{c^2 \rho_0}.$$  

In Figure 2 the dependence of $\frac{\tau_{\text{Joule}}}{\tau_{\text{visc}}} \bigg|_{A.w.(||)}^{f.m.s.w.(||)}$ on height, calculated according the VAL C model (Vernazza et al., 1981) for different values of the
magnetic field $B_0$, is presented. Figure 2 shows that damping of Alfvén and fast magnetoacoustic waves in the low solar atmosphere due to the frictional energy dissipation (ion-neutral collisions mainly) is much stronger than due to viscosity. The latter, therefore, can be neglected (DePontieu et al., 2001).

Based on Equations (3) and (6), using the definitions for $\eta_0$, $\alpha_n$ and $C_s^2$ we obtain for the $\parallel$-propagating acoustic wave:

$$\frac{\tau_{\text{frict}}}{\tau_{\text{visc}}}|_{\text{s.w.} (\parallel)} = \frac{4}{3} \frac{\eta_0 \alpha_n n_0^2}{\xi_n^2 C_s^2 \rho_0^2 n^2} = \frac{0.64 \tau_\nu \nu_{in} (2 + n_n/n)}{\gamma \xi_n^2 (1 + n_n/n)}.$$  \hspace{1cm} (8)

Here and below we assume $n_e = n_i = n$.

The variation of $\frac{\tau_{\text{frict}}}{\tau_{\text{visc}}}|_{\text{s.w.} (\parallel)}$ with height in the solar chromosphere defined by Equation (8) is presented in Figure 3. Here we limit our consideration to the height interval of the middle and upper chromosphere and again use the plasma parameters provided by the VAL C model (Vernazza et al., 1981). Figure 3 shows that the frictional and viscous damping of acoustic waves in the chromosphere are of the same order of magnitude with a slight domination of the frictional damping. Thus, none of these mechanisms can be neglected in the models.
Figure 2: $\frac{\mathbf{\tau}_{\text{Joule}}}{\mathbf{\tau}_{\text{visc}}}$ versus height in the low solar atmosphere for different $B_0$: 1) 5 G; 2) 10 G; 3) 100 G; 4) 1000 G.

3.2. MHD WAVE DAMPING IN PROMINENCES

An important example of solar partially ionized plasmas are prominences. The prominence material is a relatively cold ($T = (6-10) \times 10^3 \text{K}$) and dense ($n = (1 - 50) \times 10^{10} \text{ cm}^{-3}$) partially ionized ($\frac{n_m}{n} = 0.05 - 1$), magnetized ($B \sim 10 \text{ G}$) medium, with the majority of ions provided by the ionized hydrogen.

The Equations (7) and (8) can be directly used for calculation of the Alfvén, fast magnetoacoustic and acoustic waves collisional and viscous damping times ratios.

A range of the damping time ratios defined by Equations (7) and (8) for the plasma densities and temperatures typical for prominences, is presented in Figure 4. We assumed $B_0 = 10 \text{ G}$ and $\frac{n_m}{n} = 1$ in these calculations. Figure 4 indicates that for all $||$-propagating plasma MHD waves in the prominence considered here the collisional damping mechanism dominates above the viscosity damping.

Of interest for prominences is also the transverse ($\perp$) propagation of MHD waves with respect to the background magnetic field ($k_\perp \neq 0, k_\parallel = 0$).
Based on Equations (5), (6), (2) and (3) we obtain:

$$\frac{\tau_{Joule}}{\tau_{visc}}_{f.ms.w.(\perp)} = \frac{4\pi(\eta_0/3 + \eta_1)c^2\rho_0}{\eta_2} \approx \frac{\eta_0/3 + \eta_1}{\eta_2},$$

$$\frac{\tau_{frict}}{\tau_{visc}}_{s.w.(\perp)} \approx \frac{\eta_0/3 + \eta_1}{\eta_2} \frac{n_0^2}{C_{s}^2\rho_0^2(n + n_n)^2} = \frac{3}{4} \eta_2 \left(1 + \frac{n_n}{n}\right)^{-2},$$

Under the prominence plasma conditions the factor $\frac{\eta_0/3 + \eta_1}{\eta_2}$ in Equation (9) is $>1$, whereas the factor $\frac{3}{4} \eta_2 \left(1 + \frac{n_n}{n}\right)^{-2}$ in Equation (10) is always $<1$.

Thus, the relative role of the collisional damping of fast magnetoacoustic waves, as compared to the viscosity damping, is smaller for $\perp$-propagating waves than for $||$-propagating ones. At the same time, the role of collisional damping of acoustic waves compared to their viscous damping in the case of $\perp$-propagation is even stronger than in the $||$-propagation case.

Typical values of the collisional and viscous damping time ratios for $\perp$-propagating fast magnetoacoustic and acoustic waves in the prominence plasma calculated for $B_0 = 10$ G and $\frac{n_n}{n} = 1$ are presented in Figure 5.
Thus, the collisional damping of MHD waves in the prominence plasma is always stronger than their viscous damping. It should be considered as the main mechanism of the MHD wave energy dissipation in prominences.

4. Generalized Ohm’s Law and magnetic induction equations in the low solar atmosphere

Under certain conditions (some cases of prominences; chromospheric plasma) more realistic modelling could require inclusion of both, collisional and viscous, mechanisms of MHD waves damping. For the self-consistent description of MHD waves damping in this case the generalized Ohm’s Law (Cowling, 1957; Braginskii, 1965; Khodachenko & Zaitsev, 1992)

$$E^* = \frac{\varepsilon G - \nabla P_e}{en} + \frac{j}{\sigma} + \frac{(1 - 2\varepsilon n)}{en} [j \times B] - \frac{\xi_n}{\rho_\alpha n} \left\{ \xi_n \left[ \frac{[j \times B] \times B}{c} - [G \times B] \right] \right\}$$

(11)
Figure 5: a) $\frac{\tau_{\text{Joule}}}{\tau_{\text{visc}}} f, \text{s.w.}(\perp)$ and b) $\frac{\tau_{\text{frict}}}{\tau_{\text{visc}}} s, \text{w.}(\perp)$ in the prominence plasma.

where $E^* = E + \frac{\nabla \times B}{c}$; $\mathbf{V} = \xi_n \mathbf{n}_n + \xi_i \mathbf{n}_i$; $\epsilon = \frac{n_e m_e v'_{en}}{\alpha_n} = \frac{\sqrt{\epsilon'_{en}}}{\sqrt{\epsilon'_{en} + \left(m_i/m_e\right) \epsilon'_{in}}}$; $\Phi = \xi_n \nabla (P_e + P_i) - \xi_i \nabla P_n$ (with $\xi_i = \frac{m_i n_i}{m_n n_n + m_i n_i}$) should be included into the modelling set of MHD equations. By this, the pressure terms in Equation (11) should contain the generalized pressures $P_k = p_k + \pi^k_{\alpha \beta}$, $k = e, i, n$, which include both, the kinetic pressures $p_k$, $k = e, i, n$ and the viscous stress tensor $\pi^k_{\alpha \beta}$.

Application of the generalized Ohm’s Law equation (11) changes the form of the magnetic induction equation and the Joule heating term $(E^* \mathbf{j})$ in the energy equation. In the easiest case of a cold strongly magnetized plasma and relatively slow processes the corresponding induction equation will have the following form:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{V} \times \mathbf{B}] + \eta \Delta \mathbf{B} + \left(\eta_C - \eta\right) \nabla \times \left(\frac{[\nabla \times \mathbf{B} \times \mathbf{B}] \times \mathbf{B}}{B_0^2}\right),$$

(12)
where \( \eta = \frac{c^2}{4\pi \sigma} \) and \( \eta_C = \frac{c^2}{4\pi \sigma_C} \) are the Coulomb and Cowling coefficients of magnetic diffusion. Besides the usual convective and magnetic diffusion terms, the induction equation (12) in a partially ionized plasma contains an additional term, which plays a role of an anisotropic magnetic diffusion resulting from the strong dissipation of the transverse electric current \( j_\perp \) due to ion-neutral collisions. The ratio of the traditional and the newly appeared anisotropic magnetic diffusion terms in Equation (12) is \( D = \frac{\sigma_C}{\sigma - \sigma_C}. \)

Similar to the traditional magnetic Reynolds number \( Re_m = \frac{4\pi V_0 L_0}{c^2} \sigma \), where \( V_0 \) and \( L_0 \) are a characteristic speed and spatial scale respectively, a new, partially ionized plasma magnetic Reynolds number

\[
\tilde{R}_e = \frac{4\pi V_0 L_0}{c^2} \frac{\sigma \sigma_C}{\sigma - \sigma_C},
\]

characterizing the relative role of the convective and the ion-neutral collisional dissipation terms in the magnetic induction equation, can be introduced.

In the partially ionized plasma of the solar chromosphere, where \( \sigma \gg \sigma_C \) (see Figure 1) and \( Re_m \approx 10^5 - 10^7 \), the partially ionized plasma magnetic Reynolds number is \( \tilde{R}_e \approx \frac{4\pi V_0 L_0}{c^2} \sigma_C = \frac{\sigma_C}{\sigma} Re_m \ll Re_m \) and the parameter \( D \approx \frac{\sigma_C}{\sigma} \). This means that the traditional magnetic diffusion term can always be safely neglected, whereas the convective and the anisotropic magnetic diffusion terms will be of different importance in different regions of the chromosphere. Relatively small values of \( \tilde{R}_e \) in the chromosphere sometimes can make possible numerical simulation of processes without artificial increase of the numerical resistivity while using the realistic quantities. Furthermore, due to \( \sigma_C \ll \sigma \), damping of \( j_\perp \) as compared to \( j_\parallel \) will naturally result in an evolutionary transformation of the non-force-free (non-potential) photospheric and chromospheric magnetic structures to the force-free (potential) coronal ones.

5. Conclusion

1. The main conclusion: The collisional friction damping of MHD waves in the solar partially ionized plasmas is usually more important than the
viscous damping.

2. The expressions used above are valid only if the damping decrements \( \delta \ll 1 \) (linear approximation). Thus, the performed analysis is correct only for waves with frequency \( f = \omega/(2\pi) \ll f_c \). Depending on plasma parameters, dissipation mechanism and particular MHD mode, the critical frequency \( f_c \) varies from \( \sim 0.1 \text{ Hz} \) till \( 10^{11} \text{ Hz} \). For more details see Khodachenko et al., 2004.

3. In this paper we did not consider another important mechanism of MHD waves energy dissipation – thermoconductivity. Preliminary rough estimations indicate that in the low solar atmosphere the thermoconductivity effects could be of the same importance for MHD waves damping as the collisional friction, and appear to be more important than viscosity effects.

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References

KOMPARATIVNA ANALIZA SUDARNOG I Viskoznog Prigušenja MHD Valova u Djelomično Ioniziranoj Sunčevoj Plazmi

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Izlaganje sa znanstvenog skupa

Sažetak. Prikazuje se komparativna analiza učinkovitosti prigušenja MHD valova sudarnim i viskoznim mehanizmima dispacije energije. Procijenjuje se koji je mehanizam prigušenja dominantan u pojedinim područjima. Ispravan opis prigušenja MHD valova zahtijeva da se uzmu u obzir svi mehanizmi dispacije energije uključivanjem odgovarajućih članova u poopćeni Ohmov zakon te jednadžba za impuls, energiju i indukciju. Prikazuju se specifični oblici poopćenog Ohmovog zakona i jednadžbe indukcije koji su prikladni za djelomično ionizirana područja Sunčeve atmosfere.

Ključne riječi: magnetohidrodinamika (MHD) - valovi - Sunce: fotosfera - kromosfera - prominencije