USING SYNTHETIC EMISSION IMAGES TO CONSTRAIN HEATING PARAMETERS

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ABSTRACT

We have developed a steady-state energy balance model of the solar corona, which calculates coronal magnetic field structure and thermodynamics from a photospheric magnetogram. Our method involves a solution of energy and momentum equations along individual coronal loops, allowing for flows, gravity, non-uniform heating, and cross-sectional area variations. The model yields predicted plasma emissivities which are interpolated to a 3-D grid and used to create synthetic X-ray and EUV emission images. Such images can be generated using different heating approximations and compared with observed coronal images from past and future satellite missions to get observational constraints on coronal heating mechanisms.

Key words: coronal heating; coronal loops; heating scaling laws.

1. INTRODUCTION

One of the most challenging problems facing solar physics is understanding the nature and source of the 10^7 erg s^-1 energy flow required to maintain the multimillion-degree temperatures of active region solar coronae (Withbroe and Noyes, 1977). The primary difficulty in this pursuit is not a lack of plausible theories, but an inability to distinguish observationally between the myriad of suggested explanations.

One approach to discovering observational constraints involves scaling law relationships. Different theories predict different scalings of heating rate with quantities such as magnetic field strength, loop length, density, footpoint velocity, etc. (see, e.g., Mandrini et al., 2000, henceforth MKD, for a review of plausible heating mechanisms and associated scaling laws). These relationships can be compared to scalings inferred from observed coronal loops to provide an observational constraint on heating theories (e.g., Porter and Klimchuk, 1995; Mandrini et al., 2000; Démoulin et al., 2003).

In this paper, we take a similar approach, extended to consider whole active regions. We employ scaling laws from a given coronal heating theory to model emissions from all coronal loops in a region. The results take the form of a synthetic emission image, which represent a prediction for the emissions from the given active region heated according to the chosen scaling law. These predictions can be compared with the observed emissions from the active region. We strive to see if any current heating theories can accurately predict emissions.

Our approach has the advantage that it is not affected by the selection effects inherent in choosing loops to compare with theories. Loops must be distinct entities, distinguishable from background, and visible along much of their length in order to allow reliable observational diagnostics, but such loops may not be representative. Our method allows consideration of all the soft X-ray emission from a region, including loops that may not be observable individually.

Here we present preliminary results from four different scalings for the NOAA-designated Active Region 8210 (AR8210), observed on May 1, 1998. The method is described in Section 2. Synthetic emission images and the corresponding observed emissions are presented in Section 3. In Section 4., we note some additional applications of the model we have developed that may also be useful for coronal heating studies.

2. METHOD

Our method may be summarized as follows. We begin with the model’s only input: a magnetogram of the active region to be simulated. This field is extrapolated under the force-free-field assumption to determine the field in the corona. Once the magnetic field is known, we integrate magnetic field lines, selecting a sufficiently large number of lines with appropriate spatial distribution to represent the active region. The fieldlines are equated with coronal loops, along which we solve steady-state energy and momentum equations, using a given heating scaling law relationships for the input energy. The constant of proportionality in the scaling law is chosen such that the total X-Ray or UV emission from the region is equal to the total emission of the observations. Solving these equations yields temperature and density profiles along each loop, which we interpolate to a 3-D grid. We create synthetic emission images of the region by integrating emissivity over line of sight and convolving with instrument response function. We then compare the synthetic image from each scaling law to the observations from the chosen instrument.

2.1 The Magnetic Field

2.1.1 Magnetic Data

Photospheric vector magnetogram data for NOAA AR8210 are available from the Imaging Vector Magne-
Figure 1. A vector magnetogram of Active Region 8210 on May 1, 1998, 17:13. Grey scale contours indicate the strength and direction of the vertical component of the magnetic field, while arrows show the transverse component.

togram (Mickey et al., 1996; LaBonte et al., 1999) at University of Hawai‘i/Mees Solar Observatory. Five vector magnetograms were chosen with a three minute time cadence beginning at 17:07 UT and ending at 17:19 on May 1, 1998. We average these successive magnetograms to reduce random noise in the data. The transformation of the vector data to heliographic coordinates and the resolution of the 180° ambiguity in the component transverse to the line of sight were performed using the method of Canfield et al. (1993). These data are shown in Fig. 1.

2.1.2 Coronal Magnetic Field Solution

From the observed photospheric magnetogram, we solve for the three magnetic field components in the region’s corona. We use the non-constant alpha force-free-field calculation described in Wheatland et al. (2000). The technique involves minimizing the global departure of an initial field from a force-free and solenoidal state. Our initial field is a potential field extrapolation of the photospheric magnetic data. At the bottom box boundary, the field is set equal to the measured photospheric magnetic field, while other boundaries of the box are set equal to the results of the potential field extrapolation.

After a coronal field solution is calculated, we integrate to find a set of fieldlines that represents the field. Fieldlines are drawn from the photosphere, starting from pixels of the magnetogram with $|B_2|$ larger than a threshold value of 100 Gauss. Of the pixels above the threshold value, we choose points randomly as fieldline startpoints, with some weighting towards higher flux elements. These fieldlines are shown in Fig. 2. Of the resulting lines, we associate the fieldlines that close within the box with coronal loops for which we can solve an energy equation, as described in 2.2. Fieldlines that close on the edge of the box of our extrapolated coronal field are considered ‘open.’ These lines are ignored, leaving 1543 closed loops to model.

2.2 Loop Solutions

The basic equations to solve are the magnetohydrodynamic (MHD) equations for conservation of mass, momentum, and energy:

$$\frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{v} = 0$$

(1)

$$\rho \frac{D \mathbf{v}}{D t} + \nabla p - \mathbf{j} \times \mathbf{B} - \rho \mathbf{g} = 0$$

(2)

$$\frac{3}{2} \frac{DP}{Dt} - E_H + n^2 \Lambda(T) + \nabla \cdot \mathbf{F}_c + \frac{5}{2} P \nabla \cdot \mathbf{v} = 0,$$

(3)

where $n$ is particle density, $\mathbf{v}$ is plasma velocity, $\rho$ is mass density, $P$ is gas pressure, $\mathbf{j}$ is current density, $\mathbf{B}$ is magnetic field, $\mathbf{g}$ is acceleration due to gravity, $E_H$ is a volumetric heating rate due to an unknown coronal heating mechanism, $T$ is temperature, $\Lambda(T)$ is the radiative loss function of plasma at temperature $T$, $F_c$ is the thermal conductive flux, and $t$ is time. The thermal conductive flux is well approximated by the Spitzer (1962) formulation:

$$F_c = -\kappa_0 T^{5/2} \frac{dT}{ds}$$

(4)

with the coefficient $\kappa_0 \approx 10^{-6}$ in cgs units.

We assume that mass flux and thermal conduction perpendicular to the field is negligible in comparison to their magnitudes parallel to the field. Further accounting for the steady state assumption, the force-free assumption, magnetic flux conservation, and a fully-ionized,
Table 1. scalings with $B$ and $L$ and their corresponding heating mechanisms.

<table>
<thead>
<tr>
<th>Label</th>
<th>Scaling</th>
<th>Heating Theory</th>
<th>Mandrini et al. (2000) Model Number</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$B^2 L^{-2}$</td>
<td>Stochastic buildup</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Critical twist</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Current layers (stress)</td>
<td>6,7,8</td>
<td>3,4,5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Taylor relaxation</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>$B^2 L^{-1}$</td>
<td>Critical Angle</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Current sheets</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>$BL^{-1}$</td>
<td>Current layers (waves)</td>
<td>21</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>$BL^{-2}$</td>
<td>Reconnection $\propto v_A$</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>


Optically-thin plasma obeying $P = 2nk B T$, our mass, momentum, and energy equations reduce respectively to:

$$\frac{d}{ds} \left( \frac{n v}{B} \right) = 0$$

(5)

$$\frac{dP}{ds} = - \left[ \rho g \frac{\rho v^2}{P} \left( \frac{d \ln T}{ds} + \frac{d \ln B}{ds} \right) \right]$$

$$+ \left( 1 - \frac{\rho v^2}{P} \right)^{-1}$$

(6)

$$\frac{dF_c}{ds} = E_H - n^2 \Lambda(T) + v \frac{dP}{ds} +$$

$$F_c \frac{d \ln B}{ds} - 5 \frac{P v}{2} \frac{d \ln T}{ds}$$

(7)

These equations represent steady state loops heated by an arbitrary coronal heating mechanism, cooled by radiative losses, and moderated by thermal conduction and enthalpy flux along their axes. The loops have cross-sectional area profiles that vary with the magnitude of the magnetic field strength. They are acted on by gravity, and may have steady state flows, with constant mass flux from one footpoint to the other. These weak flows arise if the volumetric heating is asymmetric in the two sides of the loop (e.g. Craig and McClymont, 1986). This occurs in most loops, since most have at least some magnetic field profile asymmetry. For the weak flows exhibited in our loops, we find minimal differences between Equation (7) and the equation for hydrostatic equilibrium, $dP/\rho v = 0$, so we use the hydrostatic equilibrium approximation in our current calculations. This approximation will be eliminated in future studies.

2.2.1 The Heating Term

The choice of coronal heating mechanism determines the value of $E_H$ in Equation (7). Here we choose four scaling laws relating this volumetric heating rate to magnetic field strength $|\mathbf{B}|$ and loop length $L$. Table 1 shows our choices, with heating models from MDK that exhibit the same scalings with these two quantities. Other theories may also be consistent with these scalings, including some in MDK, with different choice of free parameters like power law spectra. We have considered only the parameters and theories treated in MDK.

We also note that the meaning of the parameters $B$ and $L$ can vary somewhat between the theories, making direct correspondence of these scaling laws with heating theories problematic. As MDK indicate, the parameter $L$ cannot be strictly interpreted as a loop length in their models 9 and 10, but rather designates a characteristic width of an assumed magnetic arcade. Interpretation of the magnetic field strength is problematic in that it may represent the base field strength, the local magnetic field strength, or some other kind of spatial or time average. For this presentation, we use the length averaged field strength, $\langle B \rangle = \frac{\int_B |\mathbf{B}| ds}{\int \rho v ds}$.

In addition to scaling laws, a proportionality constant must be chosen to determine the total rate of energy deposited in the loops. This constant is chosen such that the total soft X-ray emission from the synthetic image equals the total observed soft X-ray emission from the region. Some iteration is required to establish a relationship between the constant and total emissions. We typically find a near-power law trend, enabling relatively fast convergence to equality.

2.2.2 Radiative Losses

The radiative loss function $\Lambda(T)$ in Equation (7) is calculated using the CHIANTI atomic database (Dere et al., 1997; Young et al., 2003). We use abundance and ionization equilibria data supplied with version 4.2 of CHIANTI: the “extended” solar coronal abundances (Field-
man et al., 1992; Landi et al., 2002; Grevesse and Sauval, 1998), and the “extended” Mazzotta et al. ionization equilibria (Mazzotta et al., 1998; Landini and Fossi, 1991). Proton excitation rates are included: photoexcitation is not.

2.2.3 Numerical Technique for Loop Solutions

To calculate temperature and density profiles along each coronal loop, we integrate equations (6), (7), and (4) on each side of the loop from footpoint to loop ‘apex’, defined to be the point where the conductive flux equals zero. We use a fourth-order Runge-Kutta technique to accomplish the numerical integration. For a small percentage of loops, no equilibrium solution may be found (about 4% in this active region). For now, we simply discard these loops, assuming they make a negligible contribution to the overall emission.

Our loop calculations must extend to temperatures low enough such that 1) the neglected portion of the loop makes a negligible contribution to the total radiative cooling, and 2) we neglect none of the emission produced in wavelengths seen by the coronal imaging instruments with which we will compare observations. Loops must also extend low enough in the solar atmosphere for the conductive flux, which depends on both temperature and temperature gradients, to be negligible. Conversely, loops must terminate high enough that our fully-ionized and optically thin radiative cooling approximations remain valid. These conditions are met at typical chromospheric heights. For \( T_{\text{base}} \), we choose a chromospheric temperature that meets the above restrictions: \( T_{\text{base}} = 10^4 \) K. For \( F_C \) at the base, we would choose a bottom boundary of zero, but this would result in a singularity in the equations. Instead, we choose a very small, arbitrary value that is small enough to be effectively equal to zero as compared to typical values of \( F_C \): \( F_C(\text{base}) = -100 \) erg cm\(^{-2}\) s\(^{-1}\).

Two more boundary conditions are required from Equations (5) and (6): plasma velocity and gas pressure at the base. These values are related to two additional physical considerations: 1) the total length the loop, from the two separate loop-leg integrations, must equal the length of the original fieldline; and 2) the apex temperatures of the two loop legs must match, to prevent discontinuities at the apex.

To achieve these two physical restrictions, we perform a nested sequence of iterations with the boundary conditions for \( H_{\text{base}} \) and \( v_{\text{base}} \). Beginning first with a zero-velocity loop and a guess for the base pressure, we perform a Newton-Raphson iteration to find the base pressure that results in the actual loop length. If any asymmetric heating between the two legs exists, then the resulting apex temperatures from the two zero-velocity loop-leg integrations will be different. From the difference between these two apex temperatures, we derive a guess for the magnitude of the plasma flow driven from one leg to the other, and perform another Newton-Raphson iteration. At each velocity step in this second iteration, we must perform a series of pressure iterations to ensure that the loop length remains the same.

2.2.4 Interpolation

Solving the relevant equations for each loop yields temperature and density profiles along every closed fieldline chosen to represent the region. Interpolating the results to a regular, 3-D grid helps to remove some of the dependence on which fieldlines are chosen to represent the active region. It also enables quantitative, pixel-by-pixel comparisons with observations. We have chosen a simple first-order scheme to accomplish the interpolation, where the weight of a point depends on the volume of the portion of cell between it and the grid point.

2.3 Synthetic Images

We now create the image that a coronal observing instrument would see if it observed an active region with the calculated temperature and density values of our model. To do this, we calculate the instrument response (for the appropriate filter) and a line-of-sight emission measure. We use an SXT response recalculated using CHIANTI in order to be consistent with the radiative loss calculations used in solving the loop equations (see assumptions described in section 2.2.2). This response is shown in Fig. 3.

3. RESULTS

Synthetic SXT images for the AlMg filter are shown in Fig. 4, using the four scaling laws described in Table 1. The actual SXT observations for the region with the same filter are shown in Fig. 5.

In the observations, we label 4 major areas of emission: a bright spot near the left-central portion of the image, and 3 diffuse regions of emission surrounding the bright spot. The diffuse emission to the left of the bright spot (Region 2) is due to loops that connect to magnetic field outside of the field of view of our magnetogram. Since we model only closed loops, emission from loops that
Figure 4. Synthetic SXT images for AR8210, using four different heating scaling laws, from Table 1. The upper left image corresponds to scaling law 1 in Table 1. Scaling laws 2, 3, and 4 proceed it in clockwise order. These are based on magnetogram data taken at approximately 17:13UT on 1 May 1998.

close outside of the model box will never be reproduced, so none of the synthetic images will match that region of emission. As for the other three regions, they are each present in the synthetic images to varying degrees, with different relative brightness. Consulting the coronal field extrapolation, we find that Regions 1 and 3 are made up primarily of very short loops, while Region 4 is made up of a combination of a few short loops and many very long loops. Region 1 contains the loops with the largest magnetic field strength, while the other regions are made up of loops with weak field strength or loops with one footpoint in strong field and one in weak field.

We invite readers to make their own qualitative comparisons deferring discussion of quantitative differences to future papers. In our own brief qualitative analysis, we find that the $B L^{-1}$ scaling appears to predict too much emission on the long loops relative to the shorter loops (Region 4 too bright relative to Region 1), while the $B^2 L^{-2}$ predicts too much emission on the short, strong loops vs. the longer loops (Region 1 too bright relative to other regions). Trends in the other two scaling law predictions are not as clear. All of the synthetic images contain differences from the observations, particularly morphological differences in the structure and shape of the emission near Region 1. This could be due to fieldlines that close outside of our model box, to inaccuracies in the coronal field extrapolation, or to inaccuracies in the heating scaling laws, or to inaccurate assumptions in our physics. In future work, we will present additional analysis to help determine the source of discrepancies, such as exploring the effects of different magnetic field extrapolations.

4. ADDITIONAL APPLICATIONS

We note that a number of additional applications are feasible as a product of this type of modeling. Our model provides a relatively fast method of getting temperature, density, and magnetic field values in active region coronae. This information can be used for studying phenomena such as wave propagation and loop oscillations. For example, Fig. 6 shows a map of loop resonance frequencies (the inverse of Alfvén transit time). Comparing resonance frequency maps of this type with frequencies observed in TRACE movies of loop oscillations can enable studies of coronal seismology or provide an independent test on our model accuracy.

Figure 5. Left: Actual AlMg observations from Yohkoh SXT, taken at 17:16:53 UT on 1 May 1998. Right: Same image with 4 labeled regions of emission.

Figure 6. Loop resonance frequencies in Hz (equivalent to inverse Alfvén transit time).

5. CONCLUSIONS

We have presented a method for developing a steady-state energy balance model of an active region coronae. As preliminary results, we have presented four predictions of SXT emission images for AR 8210, corresponding to different scaling of coronal heating with magnetic field strength and loop length. We find that different scalings predict substantial differences in emissions. Thus
we conclude that this is a promising technique for providing observational constraints on coronal heating mechanisms.

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REFERENCES


