TURBULENCE CONVECTION AND OSCILLATIONS IN THE SUN

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ABSTRACT

We evaluate the possible use of three-dimensional, time-dependent simulations for the generation of artificial data to be used to develop and calibrate helioseismology inversion methods. We analyze flow fields from the simulations of Stein & Nordlund for the upper convection layer by splitting the velocity field into a rotational component and a potential component. The aim is to separate the turbulence from the acoustics. In general, turbulence is defined to be vortical while acoustics is potential. We find that the two fields are weakly coupled. The vorticity temporal normalized power shows a weak resonance at the fundamental frequency. We find that most of the (kinetic) energy is in the vortical component indicating that the potential component can be treated as a small perturbation relative to the vortical field. However, for a given mode we find that the temporal power is mostly in the potential component. These results indicate that we can treat the potential component as a small perturbation. They also show why helioseismology works even in a highly turbulent environment. To generate artificial helioseismic data, we plan to propagate acoustic waves through frozen fields from simulations of the solar convection zone. We show preliminary results where we follow an acoustic wave as it interacts with a frozen speed of sound perturbation in full spherical geometry. A series of near-circular scattered waves diverge from the perturbation, extracting energy from the original wave.

Key words: convection—stars:oscillations—methods:numerical.

1. INTRODUCTION

Recent achievements in helioseismic mission observations, both ground-based (GONG) and airborne (SOHO/MDI), provide large amount of data to be analyzed. These data are being used to develop, calibrate and verify existing numerical and analytical models. In turn, models are important for predicting effects and events not (yet) observed directly, and for explaining the observations. Therefore it is vital to develop accurate models of solar convection and oscillations to obtain a clear picture of the origin of solar oscillations due to turbulent convection and to understand the subsequent interplay between turbulence and oscillations.

Three-dimensional time dependent simulations of solar convection are becoming increasingly more realistic. Simulation results of the shallow upper layer of the solar convection by Stein & Nordlund (Stein & Nordlund 2000) demonstrate an excellent agreement with existing analytical theories and observations. For instance, comparison of velocity eigenmodes from these simulations with the S model of Christensen-Dalsgaard (Christensen-Dalsgaard et al. 1996), as well as comparison of the rates of stochastic energy input to solar modes from these simulations with GOLF observations (Roca Cortés et al. 1999) show good agreement (see Stein & Nordlund (2001), their Figs 2 and 7, respectively).

Numerical models represent a powerful tool of solar data analysis because they do not require the simplifying assumptions and free parameters commonly found in their analytical counterparts, yet they render the underlying physical picture of a phenomenon rather tractable. We pursue various goals while im-
implementing numerical models. One of our aims is to provide artificial data of solar-like oscillations for testing helioseismology inversion techniques. Another aspect of our investigation is to use realistic simulations of the convective zone to better understand the interactions between the mean field, turbulence, and acoustics.

2. SEPARATING TURBULENCE FROM ACOUSTICS

Stein & Nordlund (private communication) looked at two characteristics of the velocity field, its vorticity and divergence. They find that vorticity $|\mathbf{\omega}| = |\nabla \times \mathbf{u}|$ is concentrated primarily in strong, fast, turbulent downdrafts, formed in intergranular lanes, whereas divergence $d = \nabla \cdot \mathbf{u}$ occurs in smooth, slow, laminar upflows (Fig. 1). Recent theories, supported by observations, suggest that the excitation of solar oscillations originates in the intergranular lanes due to their turbulent motions. As a first step toward an attempt to separate acoustic signals from turbulence, we look at the temporal power spectra of simulated solar acoustic modes in the divergence and vorticity signals, as well as their density-weighted counterparts (Fig. 2). Divergence of velocity shows a rather flat profile, with prominent modes, whereas divergence of density-weighted velocity (mass flux) demonstrates well-defined modes with an underlying slope (background noise), with an interesting feature at low frequencies, common for density power spectra. Enstrophy, or vorticity squared, of either velocity or mass flux, shows the background noise with its distinct slope, but no modes except for the first mode peak. This suggests coupling between the divergence and vorticity signal at that particular frequency.

Without loss of generality, we can split the mean velocity field into potential and rotational components:

$$u_i = \epsilon_{ijk} \psi_{k,j} + \phi_{,i} = u_i^R + u_i^P$$ \hspace{1cm} (1)

We take the divergence of the above, getting:

$$\phi_{,ii} = u_{i,i} = u_{i,i}^P$$ \hspace{1cm} (2)

and solve the resulting Poisson equation to obtain the potential velocity component, $u_i^P = \phi_{,i}$. The rotational component is then the difference between the total velocity and its potential component.

We apply this decomposition to the velocity field snapshots from the Stein & Nordlund code and present vertical (Figs 3, 4, 5) and horizontal (Figs 6, 7, 8) slices of the resulting flow fields. We find that the rotational velocity component is dominant, closely resembling the total velocity, whereas the potential component is rather weak and featureless. This conclusion is reinforced by the comparison of spatial energy spectra of the three velocity components (Fig 9): the energy content is much lower in the potential velocity component, and it
Figure 3. Mean vertical velocity slice at a fixed arbitrary horizontal coordinate, showing strong turbulent downdrafts.

Figure 4. Vertical slice of the potential velocity at the same coordinate as in previous figure. This component looks rather structureless, but it is larger in the slow laminar upflows.

Figure 5. Vertical slice of the rotational velocity looks similar to the total velocity.

Figure 6. Horizontal slice of the total velocity at the visible surface $\tau = 1$. Vortex structures in the intergranular lanes are prominent.

Figure 7. Horizontal slice of the potential velocity at the visible surface. There are no sharp features, and potential velocity is smaller in the intergranular lanes and larger in the granules.
Figure 8. Horizontal slice of the rotational velocity again closely resembles the total velocity field, with sharp intergranular vortical features.

Figure 9. Spatial power spectra of the total velocity and its potential and rotational components. Energy content is much lower in the potential component, and its characteristic spatial scales are larger than those of the rotational or total components.

Figure 10. Temporal power spectra of different velocity components for modes $l = 740$. Total and potential velocity show oscillation mode peaks, while the rotational velocity signal shows only background noise.

Figure 11. Temporal power spectra of mass fluxes of different velocity components for modes $l = 740$. Here, the mass flux of the rotational velocity demonstrates a prominent f-mode peak, as strong as the f-mode peaks in the total and potential velocity mass fluxes.

peaks at lower spatial wavenumber (larger characteristic spatial scale) than the vortical (or total) velocity with its dominant sharp small-scale turbulent features. Temporal power spectra of the velocity components show that the potential component has prominent acoustic modes, whereas the rotational component has the characteristic slope of the background noise, with no mode signals (Fig. 10). The situation is somewhat different when we compare the density-weighted velocity components, or mass fluxes (Fig. 11). Here the rotational component strongly peaks at the fundamental mode, but shows no signs of other acoustic modes on top of the usual background noise. In the last two figures, we plot the temporal power spectra of non-radial modes. These figures show that for these non-radial modes, the vortical component does not contribute to the temporal
power spectra, but that the density, on the other hand, has a strong contribution. This is not surprising because the density is strongly coupled to the acoustic modes.

3. MODELING WAVE PROPAGATION

The analysis of the Stein & Nordlund fields has shown that we can treat the acoustic component as a perturbation. We expect that this will be particularly true in the convection zone where the Mach number is extremely small. In addition, at low Mach number, the sound will propagate at high speed compared to the convection speed, so that the convection field will appear frozen relative to sound waves.

By expanding all dependent variables in the compressible Navier-Stokes equations into a base flow, denoted by $()$, and a perturbation of order $\epsilon$, denoted by $(\cdot)'$, one will find that to zeroth order in $\epsilon$, the base flow satisfies the compressible Navier-Stokes equations, and to first order the equations for the perturbation satisfy linear equations with non-constant coefficients that depend on the base flow. Two implicit assumptions in this split are that (1) we can separate acoustic perturbations from the base flow, and (2) that the amplitude of these perturbations is small compared to the amplitude of the base flow. In this case, the acoustic perturbations do not modify the base flow.

In general, we can take the base flow to be solutions of the compressible Navier-Stokes equations, as in the flowfields of Stein & Nordlund. If we assume that the Mach number of the base flow is small, and that we can ignore the acoustic waves generated by the base flow, we can approximate the base flow using the anelastic decomposition as in the flowfields of Miesch et al. (2000) and Brun & Toomre (2002).

In our developmental effort, we start with a simplified case, assuming that the base flow is steady with zero velocity, infinite Reynolds number, no radiation, no entropy gradient, no rotation, and no gravity. In this case the equations for the perturbation reduce to the following equations for mass and momentum conservation:

$$\rho_{i,i}^t + m_{i,i}^\prime = 0, \quad (3)$$

and

$$m_{i,i}^t = -c_s^2 \rho_{i,i}^\prime \quad (4)$$

where

$$m_i^\prime = \dot{\bar{\rho}} u_i^\prime, \quad (5)$$

is the mass-weighted perturbation velocity and $c_s$ is the speed of sound of the base flow. In an effort to generate “artificial helioseismology observations,” Werne et al. (2004) solve the above equations in cartesian coordinates. In what follows we show results of wave propagation in spherical coordinates.

Working in a spherical domain, we solve equations (3)–(4) using a pseudo-spectral technique with spherical-harmonic basis functions and a leap-frog time differencing scheme. One of our first test cases is illustrated in Figures 12–14. As a starting point we consider a 2-D spherical surface and initialize our system with a specified density perturbation represented by the spherical harmonic mode of degree $\ell = 10$ and order $m = 5$ (Fig. 14). The initial velocity potential is given a similar structure but with a phase lag appropriate for an eastward-propagating wave. Thus, if the background sound speed is homogeneous, the “soccer ball” pattern in the left frame of Fig. 14 will simply propagate to the right, keeping its spatial structure indefinitely.

We now introduce an inhomogeneity. The background temperature field is given a local enhancement centered at a latitude of $10^\circ$N and a longitude of $0^\circ$ as shown in the right frame of Fig. 12. The form of the enhancement is Gaussian, with an e-folding length of $10^\circ$ and a peak value which is twice the background temperature. Thus, the local sound speed is increased by a factor of $\sqrt{2} = 1.4$. This can be regarded as a crude (and exaggerated) first approximation to a sunspot or active region, which are generally associated with localized thermal variations.

The evolution of the simulation is illustrated in Fig. 13. The eastward-propagating acoustic wave scatters off the temperature enhancement, sending out a series of circular acoustic pulses. The distortion of the acoustic mode is apparent, but the density field is still dominated by the $\ell = 10$, $m = 5$ mode which is imposed as the initial condition and which drives the dynamics.

The structure of the acoustic pulses can be seen more clearly if we subtract out the primary driving mode as shown in Figure 14. The temperature enhancement acts as a scattering center, sending off divergent circular waves with a continuously varying phase. These waves extract energy from the primary mode, eventually causing it to decay.

4. SUMMARY AND CONCLUSIONS

We have analyzed the flowfields of Stein & Nordlund by splitting the velocity field into a potential and a rotational component. This split is consistent with classical definitions of turbulence as a chaotic phenomena that causes mixing. Attributing the poten-
Figure 12. Initial conditions for one of our acoustic wave simulations. The density perturbation is chosen to be a spherical harmonic mode with \( \ell = 10, m = 5 \), propagating eastward (toward the right) at a rate equal to the sound speed. The background temperature is uniform except for a Gaussian enhancement (by a factor of two) located at a latitude of 10° N and a longitude of 0°.

tial component to the acoustic field is not straightforward. It should be recognized that the acoustic component is only part of the potential component of a flowfield, and that the vorticity field will generate potential fluctuations that do not propagate as acoustic waves. We find that the vortical field and the potential field are weakly coupled. The vorticity temporal normalized power shows a weak resonance at the fundamental frequency. We find that most of the (kinetic) energy is in the vortical component indicating that the potential component can be treated as a small perturbation relative to the vortical field. However, for a given mode, we find that the temporal power is mostly in the potential component. This explains why helioseismology works even in a highly turbulent environment.

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Figure 13. The density field is shown as the simulation evolves. As in Figure 12, bright areas denote density enhancements and dark areas rarefactions. The time (in minutes) is indicated to the upper right of each frame. The interval between frames is 400 seconds (6.66 min).
Figure 14. Similar to Figure 13 but with the $\ell = 10, m = 5$ component subtracted out.