CORONAL LOOP OSCILLATIONS

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ABSTRACT

Following Edwin and Roberts (1983), we carry out a study of the trapped modes of oscillation of a uniform flux tube embedded in a uniform environment. A magnetic flux tube which has a density enhancement is a wave guide for many modes of oscillation. Observations suggest that sausage and kink modes are commonly occurring, both as propagating and as standing waves. Therefore we study the principal modes of oscillation, determining the eigenfunctions for each mode, with the aim of matching these results with observational data.

Key words: Sun; Corona; Oscillations.

1. INTRODUCTION

Recent advances in solar observational technology have allowed increased spatial and temporal resolution both in space telescopes such as TRACE (Transition Region and Coronal Explorer; Handy et al., 1999) and SoHO (Solar and Heliospheric Observatory; Wilhelm et al., 1995) and in the ground-based telescope SECCIS (Solar Eclipse Corona Imaging System; Phillips et al., 2000). This has provided us with instruments able for the first time to make direct detections of MHD waves in the solar atmosphere. The first temporally and spatially resolved longitudinal waves were detected in plumes by the EUV Imaging Telescope (EIT) on board SoHO (Ofman et al., 1997). Direct detection of transverse oscillations of coronal loops were found by TRACE and were deemed to be flare excited standing waves of the fast kink mode, with a period of \(280 \pm 30\) s (Aschwanden et al., 1999) and decay time of roughly 14.5 min (Nakariakov et al., 1999). Standing slow waves in hot coronal loops were detected by the SUMER instrument (Wang et al., 2003a), and identified to be slow MHD waves (Ofman and Wang, 2002; Wang et al., 2003b).

Propagating slow modes in cool coronal loops have been observed in EUV (Nakariakov et al., 2000; Robbrecht et al., 2001; De Moortel, Ireland and Walsh, 2000). Outwardly propagating intensity fluctuations have been found to be a common occurrence in large, quiescent coronal loops (De Moortel et al., 2002a-c) and identified as slow magnetoacoustic waves (De Moortel et al., 2002b). These modes have two distinct periods: about 3 min for those loops above sunspots, and about 5 min for loops that are not above sunspots. They were not associated with flare-like events and are thought to be driven by movements of the loop footpoints (De Moortel et al., 2002b).

Finally, oscillations of period 6 s detected in SECCIS data (Williams et al., 2001) have been determined to be an impulsively generated fast magnetoacoustic wave which propagates along a coronal loop with a speed of \(2100\) km s\(^{-1}\) (Williams et al., 2002).

2. EQUILIBRIUM AND WAVE SPEEDS

We consider a radially structured equilibrium in a cylindrical geometry with a magnetic flux tube of radius \(a\) and plasma density \(\rho_0\). The tube acts as a wave guide for trapped modes of oscillation. Equilibrium quantities for \(r < a\) are denoted by a subscript ‘0’, and for \(r > a\) by a subscript ‘c’. The equilibrium magnetic field is \(B_0 e_z\) in the tube interior and \(B_c e_z\) in the environment, both aligned with the \(z\)-axis (see Fig. 1).

![Figure 1. Equilibrium magnetic flux tube model, consisting of a uniform plasma tube of radius \(r = a\) embedded in a uniform environment.](image)

Equilibrium pressure balance relates the sum of the plasma pressure \(P_0\) and magnetic pressure \(B_0^2/(2\mu)\) inside the tube to its value outside the tube:


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\[ P_0 + \frac{B_0^2}{2\mu} = P_e + \frac{B_e^2}{2\mu} \]  

(1)

This relation implies that

\[ \frac{\rho_0}{\rho_e} = \frac{2c_s^2 + \gamma v_{Ae}^2}{2c_s^2 + \gamma v_A^2} \]  

(2)

where \( c_0 = (\gamma P_0/\rho_0)^{\frac{1}{2}} \) and \( v_A = B_0/\sqrt{\mu_0 \rho_0} \) are the sound and Alfvén speed inside the tube, values \( c_e \) and \( v_{Ae} \) their values in the environment. The ratio \( \rho_0/\rho_e \) is estimated to be typically 2 - 7 in coronal loops (Aschwanden et al., 2003).

Within the basic model there are several typical wave speeds which occur. Firstly the local sound speed \( c_0 \), which has a typical coronal value of \( c_0 = 150 - 200 \) km s\(^{-1}\). The local coronal Alfvén speed \( v_A \) is generally thought to be in the range 1000 - 2500 km s\(^{-1}\). The tube (or cusp) speed \( c_T \) is subsonic and sub-Alfvénic, taking values in the range \( c_T = 140 - 200 \) km s\(^{-1}\). The kink speed \( c_k \) is important in the propagation of kink modes in a magnetic flux tube or surface waves on a magnetic interface (Roberts, 1981). Specifically,

\[ c_T = \frac{c_0 v_A}{(c_s^2 + v_A^2)^{\frac{1}{2}}} \]  

\[ c_k = \left( \frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2}{\rho_0 + \rho_e} \right)^{\frac{1}{2}} \]  

(3)

Taking a density enhancement of \( \rho_0 = 4.8 \rho_e \), for example, and a uniform magnetic field \( (B_0 = B_e) \) with Alfvén speeds \( v_A = 1000 \) km s\(^{-1}\) and \( v_{Ae} = 2250 \) km s\(^{-1}\) results in a coronal kink speed of \( c_k = 1300 \) km s\(^{-1}\).

The speeds introduced in this section can be separated into two categories: fast speeds (characterising the fast wave modes) and slow speeds (associated with the slow modes). Which speeds fall into each category is dependent on the situation. Under coronal conditions the Alfvén and kink speeds are the fast speeds and the sound and tube speeds are the slow speeds.

3. WAVES A UNIFORM FLUX TUBE

The ideal MHD equations (neglecting gravity and resistive terms) were perturbed about the above equilibrium state. The perturbed magnetic field and velocity are denoted by \( \mathbf{B}(= (B_r, B_\theta, B_z)) \) and \( \mathbf{v}(= (v_r, v_\theta, v_z)) \). Linearisation of the MHD equations for perturbations of the form

\[ f(r) \exp i(\omega t + n\theta + kz) \]  

(4)

leads to the second order differential equation

\[ \frac{\rho_0}{r} \frac{d}{dr} \left( k^2 v_A^2 - \omega^2 \right) \frac{dP_T}{dr} = \left( \frac{r}{r^2} - \frac{n^2}{r^2} \right) P_T = 0 \]  

(5)

where

\[ n_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_T^2 - \omega^2)}{(c_s^2 + v_A^2)(k^2 c_T^2 - \omega^2)} \]  

(6)

Equation (5) governs the total pressure perturbation \( P_T = P_1 + B_0 B_z / \mu \) in a radially structured medium; here \( P_1 \) denotes the perturbed plasma pressure. The integer \( n \) describes geometrically the various modes of oscillation of the tube; see Fig. 2. The wavenumber \( k \) describes the longitudinal behaviour of the mode.

In the case of a uniform flux tube embedded in a uniform environment, the magnetic flux tube is now defined by a discontinuity in the equilibrium density and temperature (Edwin and Roberts 1983) and (5) reduces to the Bessel equation:

\[ \frac{d^2 P_T}{dr^2} + \frac{1}{r} \frac{dP_T}{dr} + \left( \frac{n^2}{r^2} - \frac{n_0^2}{r^2} \right) P_T = 0 \]  

(7)

for the tube interior, with an equivalent equation for the environment.

4. DISPERSION RELATION

Equation (7) is solved subject to the condition that \( P_T \) is finite at \( r = 0 \) so that inside the tube \( (r < a) \) we have

\[ P_T = A_0 J_n(n_0 r) \]  

(8)

We do not consider outwardly propagating (leaky) waves (see Meerson, Sasarov and Stepanov 1978; Spruit 1982; Cally 1986; Tsap and Kopylova 2001), imposing instead the condition that \( P_T \to 0 \) as \( r \to \infty \). Then

\[ P_T = A_1 K_n(m_e r) \]  

(9)

where

\[ m_e^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_T^2 - \omega^2)}{(c_s^2 + v_A^2)(k^2 c_T^2 - \omega^2)} \]  

(10)

This solution has the restriction that \( m_e > 0 \).

It is required that the radial velocity \( v_r \) and the pressure perturbation \( P_T \) be continuous across the interface \( r = a \).
Figure 2. The undisturbed tube (shown solid) and its modes of oscillation (shown by broken curves): sausage mode \((n = 0)\), kink mode \((n = 1)\) and fluting modes \((n \geq 2)\).

Then the waves are governed by the dispersion relation (Edwin and Roberts 1983)

\[
\rho_e (k^2 v_{oe}^2 - \omega^2) n_0 J_n^2 (n_0 a) = \rho_0 (k^2 v_A^2 - \omega^2) h n_0 K_n^2 (m_e a) K_n (m_e a),
\]

(11)
a dash denoting the derivative. Equation (11) is a transcendental dispersion relation for the trapped modes \((m_e^2 > 0)\) of an oscillating magnetic flux tube, first derived in this form by Edwin and Roberts (1983). Equivalent forms have been discussed previously in Wilson (1980) and Spruit (1982).

Fig. 3 shows the dispersion curves of (11). We see two distinct ranges of phase speed for which trapped modes occur. The region of phase speed between \(v_A\) and \(v_{oe}\) contains the strongly dispersive fast body modes. All of the fast body modes possess a cut-off wavenumber, below which they may no longer propagate as a trapped mode. However, the principle kink mode propagates as a trapped mode for all wavenumbers and has a less dispersive nature than the other modes in the fast band. The second region is the acoustic range from \(c_T\) to \(c_0\) which contains the slow body modes; these modes propagate as trapped waves for all wavenumbers and show little dispersion.

5. FINDINGS

5.1. Kink mode eigenfunctions

In this section we discuss the eigenfunctions of the kink mode under coronal conditions, taking \(ka = 2.0\). The total pressure perturbation \(P_T\) is found to be zero at the tube axis and takes its maximum value on the tube boundary. Fig. 4(a) and 4(b) show \(v_x\) and \(v_\theta\), both taking their largest values in absolute terms at the centre of the tube. There is little variation in \(v_x\) across the tube, which is consistent with the tube radius remaining roughly unchanged by the propagation of the kink mode. The \(v_\theta\) component also shows little variation within the tube but it experiences a discontinuous reversal of sign at \(r = a\), which implies that the interior and exterior of the tube rotate out of phase with a shear layer near \(r = a\). The flow \(v_z\) along the tube is found to be zero on the tube axis and \(v_z\) is at its maximum value inside the tube close to the boundary, where \(v_z\) makes a discontinuous decrease in value, so at the boundary the flow of plasma in the \(z\) direction is larger on the inner surface of the boundary than on the exterior surface. The motions are predominantly radial and azimuthal rather then longitudinal, with an order of magnitude difference in these motions. The magnetic field perturbations \(B_r\), \(B_\theta\) and \(B_z\) are all multiples of the corresponding velocity components and so have similar behaviours. However, these multiples are
not constant across $r = a$, which results in $B_r$ being discontinuous at the boundary of the tube. All three components of the magnetic field are of comparable magnitude, unlike the velocity components. There is little variation in $B_r$ and $B_\theta$ across the tube.

The perturbed plasma pressure, temperature and density behave in a similar manner to each other, all being zero at the tube centre $r = 0$, with an approximately linear increase inside the tube and a discontinuous increase at the tube boundary. It should be noted that all of the perturbations for the principal kink mode have died away by a distance of $4a$ from the tube centre; therefore, the kink mode affects its environment up to a distance of about one wavelength from the tube centre.

**5.2. Sausage mode eigenfunctions**

Here we present plots of the eigenfunctions for the sausage mode under coronal conditions, for $ka = 2.0$. The total pressure perturbation for the sausage mode takes its largest value in absolute terms at the centre of the tube and changes sign inside the tube coming to a peak at the radius of the tube and then decaying within a distance of $4a$ outside the tube. The velocity component $v_r$ (Fig. 4(c)) for the sausage mode is zero at the centre of the tube, consistent with the prediction that the centre of the tube is fixed for all modes other than the kink mode. The radial velocity $v_r$ reaches its maximum value within the tube and does not change in sign. There is no azimuthal flow ($v_\theta = 0$), since $n = 0$. The longitudinal velocity $v_z$ (Fig. 4(d)) takes its largest value at the centre of the tube. As with $P_r$, $v_z$ experiences a change of sign inside the tube, so the plasma is flowing in the opposite direction along the loop in the outer section of the tube to the central part, and makes a discontinuous drop across the boundary of the tube (much as $v_z$ does for the kink mode). The maximum values of $|v_r|$ and $|v_z|$ are of the same order. As with the kink mode, the components of the magnetic field and the velocity exhibit a similar behaviour, apart from the discontinuity of $B_r$ ($v_r$ is continuous).

The perturbed plasma pressure $P_1$ has a very similar behaviour to $P_r$, taking its largest value at the centre of the tube, changing sign inside the tube and appearing to be continuous at the boundary of the tube. The absolute temperature and density perturbations, $|T_1|$ and $|\rho_1|$, both have their largest values at the centre of the tube, going through a change in sign inside the tube, making a discontinuous increase at the tube boundary and falling off rapidly outside. All of the perturbations have declined to zero by a distance of $4a$ outside the tube, but the most dra-
matic changes to the perturbations occur inside the tube.

6. CONCLUSIONS

Fast body modes each have a cut-off wavenumber below which they cannot propagate, with the exception of the principle kink mode which propagates for all wavenumbers. We examined the behaviour of the kink and sausage modes through their eigenfunctions. The kink mode, being an almost incompressible transverse oscillation of the loop, results in the loops exhibiting this mode of oscillation moving almost as a solid structure. The sausage mode is found to be a highly compressive pulsation, but with no rotation.

REFERENCES


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