OSCILLATIONS OF CORONAL LOOPS WITH ELLIPTIC CROSS-SECTIONS

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ABSTRACT

Motivated by the Transition Region and Coronal Explorer (TRACE) observations of damped oscillations in coronal loops, Ruderman & Roberts [2002, ApJ, 577, 475] studied resonant damping of kink oscillations of thin straight magnetic tubes in a cold plasma. We extend their analysis for magnetic tubes with elliptic cross-sections. We found that there are two kink modes: one polarized along the small and the other polarized along the large axis of the elliptic cross-section. For moderate values of the ratio of the axis of the elliptic cross-section the damping time of the two kink modes due to resonant absorption does not differ very much from the damping time of kink modes in tubes with circular cross-section.

Key words: magnetohydrodynamics (MHD); plasma; Sun: corona; waves.

1. INTRODUCTION

This study was motivated by the recent TRACE observations of damped coronal loop oscillations (Aschwanden et al. 1999; Nakariakov et al. 1999; Schrijver & Brown 2000; Aschwanden et al. 2002; Schrijver et al. 2002). These observations may shed light on the puzzle of coronal heating and provide seismic information about the coronal plasma (e.g., Roberts 2000). Aschwanden et al. (1999) and Nakariakov et al. (1999) interpreted the observed oscillations in terms of the kink mode of oscillation of a coronal loop. Nakariakov et al. (1999) noted that the loop oscillations were strongly damped, decaying in about 14.5 minutes (compared with an oscillation period of 256 s). Ruderman & Roberts (2002) suggested that the damping of these oscillations is due to resonant absorption, which is the energy transfer from the global mode of oscillation of a coronal loop into quasi-Alfvénic oscillations (predominantly azimuthal) in a thin dissipative layer. This layer embraces an ideal resonant position where the frequency of the global mode matches the local Alfvén frequency. To study this process Ruderman & Roberts (2002) solved the initial value problem for a thin straight magnetic tube with a thin inhomogeneous layer at its boundary. They showed that a damped kink oscillation of this tube emerges from more or less arbitrary initial perturbations after the time of order of a few periods of this oscillation. The characteristic damping time is of order the oscillation period times the ratio of the thickness of the inhomogeneous layer and the tube radius. When this ratio is of order unity, the damping is so strong that there is not enough time for emerging a coherent kink oscillation from the initial perturbation.

Ruderman & Roberts (2002) considered the simplest unperturbed configuration. They neglected the twist of the magnetic field, the density stratification, and assumed that the magnetic tube has a circular cross-section. The aim of this study is to extend the analysis by Ruderman & Roberts for a magnetic tube with an elliptic cross-section. It follows from the analysis by Ruderman & Roberts that the asymptotic state of the tube oscillation is completely determined by the eigenmodes of the dissipative linear MHD equations describing the damped kink oscillations of the tube. This fact enables us not to consider the initial value problem and concentrate only on studying the damped eigenmodes.

2. FORMULATION

We consider oscillations of a straight magnetic tube in a cold plasma. The tube has an elliptic cross-section with the large half-axis $a$ and small half-axis $b$. Inside the tube the plasma density is $\rho_{1}$ and outside it is $\rho_{e}$. The two regions are connected by a thin layer where the plasma density varies monotonically from $\rho_{1}$ to $\rho_{e}$, $\rho_{1} > \rho_{e}$. The equilibrium magnetic field $B$ is everywhere uniform and in the $z$-direction, $B = B e_{z}$, where $e_{z}$ is the unit vector in the $z$-direction (see Fig. 1). The nonuniformity in plasma density $\rho(r)$ produces a nonuniform Alfvén speed, allowing resonant wave effects to occur. It is in such nonuniform layers that viscous effects are likely to be most important.

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Figure 1. A sketch of the equilibrium state, showing a magnetic flux tube with plasma density $\rho_i$ embedded in a plasma with density $\rho_e$. The equilibrium magnetic field everywhere has strength $B$. The equilibrium density varies in the thin layer from $\rho_i$ to $\rho_e$. The dashed lines show the perturbed magnetic tube in its kink mode of oscillation.

In what follows we adopt the elliptic coordinates $s$, $\varphi$, $z$ with the $z$-axis aligned with the equilibrium magnetic field. Cartesian $x$ and $y$-coordinate are related to $s$ and $\varphi$ by (e.g. Korn & Korn 1961)

$$x = \sigma \cosh s \cos \varphi, \quad y = \sigma \sinh s \sin \varphi,$$

where $\sigma$ is a constant with the dimension of length. The sketch of the elliptic coordinates in the $xy$-plane is shown in Fig. 2. The equation of the external boundary of the tube is $s = s_0$. The large and small half-axis of the tube cross-section, $a$ and $b$, are given by

$$a = \sigma \cosh s_0, \quad b = \sigma \sinh s_0.$$

The ellipse eccentricity is $\varepsilon = 1/\cosh s_0$. The inner boundary of the tube is determined by $s = s_0 - \delta$ with $\delta \ll s_0$. Hence, the thickness of the inhomogeneous layer monotonically increases from $\delta \delta$ in the $x$-direction to $a\delta$ in the $y$-direction. Dissipation is only important in a thin dissipative layer embracing an ideal resonant position inside the inhomogeneous layer. We are not going to solve the dissipative MHD equations inside the dissipative layer. Instead we use connection formulae to connect solutions to the ideal MHD equations to the left and the right of the dissipative layer. This enables us to use the system of ideal linearized MHD equations for cold plasmas in our analysis. After eliminating the perturbation of the magnetic field this system can be written in the form

$$\frac{\partial^2 v}{\partial t^2} - V_A^2 \frac{\partial^2 v}{\partial z^2} = -\frac{1}{\rho} \nabla_\perp \cdot \frac{\partial P}{\partial t},$$

and the perpendicular gradient given by

$$P = \frac{Bb_2}{\mu}, \quad V_A^2 = \frac{B^2}{\mu \rho}, \quad \nabla_\perp = \nabla - e_z \frac{\partial}{\partial z},$$

where $b_2$ is the $z$-component of the perturbation of the magnetic field and $\mu$ is magnetic permeability of empty space.

We assume that the magnetic tube is bounded at $z = 0$ and $z = L$ by dense ideal infinitely conducting plasmas with the magnetic field frozen in these plasmas. This implies the boundary conditions

$$v = 0 \quad \text{at} \quad z = 0, L.$$  (5)

Equation (5) coupled with (3b) implies that

$$P = 0 \quad \text{at} \quad z = 0, L.$$  (6)

We consider only the fundamental mode of the tube oscillation with respect to $z$. Then it follows from (5) and (6) that $v$ and $P$ are proportional to $\sin(\pi z/L)$. Our analysis remains applicable to an overtone if we regard $L$ as the distance between successive nodes along the loop. We also restrict our analysis to the normal modes and take $v$ and $P$ proportional to $e^{-i\omega t}$. Then equations (3) reduce to

$$\rho(\omega^2 - \omega_A^2)v = -i\omega \nabla_\perp P,$$

$$i\omega P = \rho V_A^2 \nabla_\perp \cdot v,$$

where $\omega_A = \pi V_A/L$ is the local Alfvén frequency.

3. DERIVATION OF THE DISPERSION EQUATION

In this section we outline the derivation of the dispersion equation. We derive this equation by steps. First we obtain the solution in the inhomogeneous layer ($s_0 - \delta < s < s_0$). Then we obtain the solution in the internal ($s < s_0 - \delta$) and external ($s > s_0$) homogeneous regions. Finally we match these solutions at the boundaries of the inhomogeneous layer.
3.1. Solution in the inhomogeneous layer

It can be shown that the relative pressure variation in the inhomogeneous layer is of order \( \delta s_0 \). Since \( \delta s_0 \ll 1 \), it follows that we can neglect the pressure variation across the inhomogeneous layer. Hence, the pressure is determined by its values on the boundaries and can be considered as a given quantity. Then, using the formulae for elliptic coordinates (e.g., Korn & Korn 1961)

\[
\nabla_\perp = \frac{1}{H} \left( e_s \frac{\partial}{\partial s} + e_\varphi \frac{\partial}{\partial \varphi} \right),
\]

\[\nabla_\perp \cdot \mathbf{v} = \frac{1}{H^2} \left[ \frac{\partial (Hu)}{\partial s} + \frac{\partial (Hv)}{\partial \varphi} \right],
\]

\[
H^2 = \sigma^2 (\sin^2 s + \sin^2 \varphi),
\]

(8a)
(8b)
(8c)

where \( e_s \) and \( e_\varphi \) are unit vectors in \( s \) and \( \varphi \) directions, and \( u \) and \( v \) are the \( s \) and \( \varphi \)-component of the velocity; we obtain from (7)

\[
\frac{\partial (Hu)}{\partial s} = \frac{i \omega}{\rho} \left( \frac{H^2 P}{V_A^2} + \frac{1}{\omega^2 - \omega_A^2} \frac{\partial^2 P}{\partial \varphi^2} \right).
\]

(9)

We will see that the real part of the eigenmode frequency, \( \omega_r \), satisfies the inequality \( \omega_A < \omega_r < \omega_A^\perp \). This implies that there is such an \( s_A \) that \( \omega_A(s_A) = \omega_r \). The equation \( s = s_A \) determines an ideal resonant line inside the inhomogeneous layer. At this line the Alfvén resonance takes place. We use the connection formulae to connect the ideal solution to the left and the right of the resonant line. These formulae read

\[
\begin{align*}
\langle P \rangle &= 0, \quad [u] = \frac{\pi |\omega_r|}{H|\Delta \rho_A} \frac{\partial^2 P}{\partial \varphi^2}, \\
\Delta &= - \frac{d \omega_A^2}{ds} \bigg|_{s = s_A},
\end{align*}
\]

(10)

where \( \langle f \rangle \) indicates the jump of the function \( f(s, \varphi) \) across the line \( s = s_A \), and all quantities in the expression for \( [u] \) are calculated at \( s = s_A \). With the use of (9) and (10) we can easily calculate the variation of \( u \) across the inhomogeneous layer:

\[
\begin{align*}
\frac{\partial}{\partial s} (u(s_0) - u(s_0 - \delta)) &= \frac{\pi |\omega_r|}{H|\Delta \rho_A} \frac{\partial^2 P}{\partial \varphi^2} + \frac{i \omega_r \delta}{\rho V_A^2} P \\
+ &\frac{i \omega_r \partial^2 P}{H \partial \varphi^2} \int_{s_0 - \delta}^{s_0} \rho(s) [\omega_r^2 - \omega_A^2(s)] ds,
\end{align*}
\]

(11)

where \( \mathcal{P} \) indicates the principal Cauchy part of the integral.

3.2. Solution in the inner region

Using (8) we reduce (7) to

\[
\nabla_\perp^2 P + \frac{\rho^2 - \omega_A^2}{V_A^2} P = 0,
\]

(12a)

\[
u = \frac{i \omega}{\rho H(\omega_A^2 - \omega^2)} \frac{\partial P}{\partial s}.
\]

(12b)

In what follows we look for the solution in the form of expansions with respect to \( \epsilon = \delta s_0 \ll 1 \):

\[
P = P_0 + \epsilon P_1 + \ldots, \quad u = u_0 + \epsilon u_1 + \ldots,
\]

\[
= \omega_0 + \epsilon \omega_1 + \ldots.
\]

(13)

We will see that \( \omega_1/\omega_r \ll 1 \). This estimate enables us to take \( \omega_0 \) to be real. Substituting (12) into the general solution to the equation periodic with respect to \( \varphi \) with the period \( 2\pi \) and regular at \( s = 0 \) is given by (Bateman 1955)

\[
P_i = C_i^0 C_0(s, \theta_i) c_0(\varphi, \theta_i)
+ \sum_{n=1}^{\infty} [C_n^0 C_n(s, \theta_i) c_n(\varphi, \theta_i)
+ D_n^0 S_n(s, \theta_i) s_n(\varphi, \theta_i)].
\]

(15)

Here \( c_0(\varphi, \theta_i) \) and \( s_0(\varphi, \theta_i) \) are the Mathieu functions, and \( C_n(s, \theta_i) \) and \( S_n(s, \theta_i) \) are the modified Mathieu functions, \( C_n^0 \) and \( D_n^0 \) are arbitrary constants, and the superscript ‘0’ refers to the inner region.

In the first order approximation we obtain

\[
\frac{\partial^2 P_i}{\partial s^2} + \frac{\partial^2 P_i}{\partial \varphi^2} + \frac{4H^2 \theta_i}{\sigma^2} P_i = -\frac{2\omega_0 \omega_1 H^2}{V_A^2} P_0^i.
\]

(16)

3.3. Solution in the external region

The motion in the external region is described by equations (12), however with \( \omega_A^\perp \) and \( V_A^\perp \) substituted for \( \omega_A \) and \( V_A \). We once again use the expansions (13) and arrive at the equation for \( P_0 \) obtained from (14) by substituting \( \theta_i \). The general solution to this equation periodic with respect to \( \varphi \) with the period \( 2\pi \) and vanishing as \( s \to \infty \) is given by (Bateman 1955)

\[
P_e^0 = C_0^0 F_{e0}(s, \theta_e) c_0(\varphi, \theta_e)
+ \sum_{n=1}^{\infty} [C_n^0 F_{en}(s, \theta_e) c_n(\varphi, \theta_e)
+ D_n^0 G_{en}(s, \theta_e) s_n(\varphi, \theta_e)],
\]

(17)

where \( F_{en}(s, \theta_e) \) and \( G_{en}(s, \theta_e) \) are the modified Mathieu functions, \( C_n^0 \) and \( D_n^0 \) are arbitrary constants, and the superscript ‘e’ refers to the external region.

In the first order approximation we obtain the equation similar to (13), however with \( \theta_e \), \( V_A \) and \( P_{e,0} \) substituted for \( \theta_i \), \( V_A \) and \( P_{0,0} \).
3.4. Matching solutions

It follows from the results obtained in Subsection 3.1 that the pressure is the same at \( s = s_0 + \delta \) and \( s = s_0 \). Then, using expansions with respect to \( \epsilon \), we obtain

\[
P_0 = P_0^i, \quad P_1 = P_1^i - s_0 \frac{\partial P_0^i}{\partial s},
\]

where all quantities are calculated at \( s = s_0 \).

The second boundary condition is the continuity of \( u \). It has to be satisfied at both boundaries of the inhomogeneous layer. Using (12b) and the similar equation for the inner region, together with expansions with respect to \( \epsilon \), we obtain from (11)

\[
\frac{1}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_0^i}{\partial s} = \frac{1}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_0^i}{\partial s}, \quad (19a)
\]

\[
\frac{1}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_1}{\partial s} = \frac{1}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_1}{\partial s}
\]

\[
= - \frac{s_0}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_0^i}{\partial s} - \rho_e \Delta \frac{\partial}{\partial s} \frac{\partial P_0^i}{\partial s}
\]

\[
+ \omega_1 \left[ \frac{\omega_{Ai}^2 + \omega_0^2}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_0^i}{\partial s} - \frac{\omega_{Ai}^2 + \omega_0^2}{\rho_e(\omega_{Ai}^2 - \omega_0^2)} \frac{\partial P_0^i}{\partial s} \right]
\]

\[
+ \frac{s_0 H^2}{\rho_e^2 A} P_0^e + \frac{s_0 \omega^2 P_0^i}{\delta \frac{\partial \sigma^2}{\partial s}} \int_{s_0 - \delta}^{s_0} \frac{ds}{\rho_0(\omega_0^2 - \omega_0^2)}
\]

where \( P_{0,1} \) and all their derivatives are calculated at \( s = s_0 \).

4. EIGENMODES OF A HOMOGENEOUS TUBE

Consider oscillations of a homogeneous magnetic tube, where there is no inhomogeneous layer, i.e., \( \delta = 0 \). In this case the solution is given by the first terms of the expansions (13). To obtain the dispersion equation determining \( \omega \) we use the expansions of Mathieu functions in the Fourier series:

\[
\mathcal{C}_{2m+j}(\varphi, \theta) = \sum_{r=0}^{\infty} A_{2r+j}^m(\theta) \cos[(2r + j)\varphi], \quad (20a)
\]

\[
\mathcal{S}_{2m+1+j}(\varphi, \theta) = \sum_{r=0}^{\infty} B_{2r+1+j}^m(\theta) \cos[(2r + 1 + j)\varphi], \quad (20b)
\]

\( m = 0, 1, 2, \ldots \) and \( j = 0, 1 \). The coefficients \( A_{2r+j}^m(\theta) \) and \( B_{2r+1+j}^m(\theta) \) are related by the recurrence relations that can be found, e.g., in Bateman (1955) and Ablowitz & Stegun (1964). Substituting (15) and (17) in (19), using (20), and collecting the terms proportional either to \( \cos(n\varphi) \) or to \( \sin(n\varphi) \) in the resulting equation, we obtain four systems of infinite linear homogeneous algebraic equations:

\[
\sum_{m=0}^{\infty} \mathcal{C}_{2m+j}(s_0, \theta_0) A_{2m+j}^m(\theta_0) C_{2m+j}^m = 0
\]

\[
\sum_{m=0}^{\infty} \mathcal{S}_{2m+1+j}(s_0, \theta_0) A_{2m+1+j}^m(\theta_0) C_{2m+j}^m = 0
\]

\[
\sum_{m=0}^{\infty} \mathcal{C}_{2m+j}(s_0, \theta_0) A_{2m+j}^m(\theta_0) C_{2m+j}^m = 0
\]

\[
\sum_{m=0}^{\infty} \mathcal{S}_{2m+1+j}(s_0, \theta_0) A_{2m+1+j}^m(\theta_0) C_{2m+j}^m = 0
\]

where \( j = 0, 1 \), \( n = 0, 1, 2, \ldots \), and the prime indicates the derivative with respect to \( s \). The system (21) is for variables \( C_{2m}^m \) and \( C_{2m+1}^m \) when \( j = 0 \), and for variables \( C_{2m+1}^m \) and \( C_{2m+2}^m \) when \( j = 1 \), where \( m = 0, 1, 2, \ldots \). The system (22) is for variables \( D_{2m+1}^m \) and \( D_{2m+2}^m \) when \( j = 0 \), and for variables \( D_{2m+2}^m \) and \( D_{2m+3}^m \) when \( j = 1 \), where once again \( m = 0, 1, 2, \ldots \). The eigenvalues \( \omega \) are determined by the condition that the infinite determinants of one of these four systems is zero.

In the long wavelength approximation \( (a \ll 1) \) we have \( \mid \theta_1, \theta_2 \mid \ll 1 \). The following asymptotic formulae are valid for \( \mid \theta_1, \theta_2 \mid \ll 1 \) (Abramowitz & Stegun 1964):

\[
A_{2r+j}^m \approx 2^{-1/2}, \quad B_{2r+1+j}^m \approx 2^{-1/2},
\]

\[
A_{2r+j}^m \approx O(\theta^{n+1}), \quad B_{2r+1+j}^m \approx O(\theta^{n+1}),
\]

\[
C_{2m+j} \approx O(\theta^{m+1}), \quad D_{2m+j} \approx O(\theta^{m+1}),
\]

\[
\mathcal{C}_0(x, \theta) \approx \mathcal{C}_0(x, \theta) \approx 2^{-1/2},
\]

\[
\mathcal{C}_m(x, \theta) \approx \cos(m \theta), \quad \mathcal{S}_m(x, \theta) \approx \sin(m \theta),
\]

\[
\mathcal{C}_m(x, \theta) \approx \cosh(m \theta), \quad \mathcal{S}_m(x, \theta) \approx \sinh(m \theta),
\]

\[
\mathcal{C}_0(x, \theta) \propto \ln \mid \theta_1, \theta_2 \mid, \quad \mathcal{C}_m(x, \theta) \propto \mathcal{S}_m(x, \theta) \propto e^{-m \theta},
\]
where \( n = 0, 1, 2, \ldots \), \( m = 1, 2, \ldots \), and \( j = 1, 2, \ldots, n/2 \), with \( [n/2] \) being the integer part of \( n \). We use these formulae and retain only the largest terms in equations (21) and (22). As a result, (21) splits in the infinite sets of independent systems of two equations for \( C_{n,k}^{\pm} \), and \( C_{n,k}^{\pm} \), and (22) in the infinite sets of independent systems of two equations for \( D_{n+1,k}^{\pm} \), and \( D_{n+1,k}^{\pm} \). After that we immediately obtain that, in the long wavelength approximation, there are two infinite sequences of eigenfrequencies, \( \{\omega_{n,c}\} \), and \( \{\omega_{n,s}\} \), where \( n = 1, 2, \ldots \). They are given by

\[
\omega_{n,c}^2 = \frac{\rho \omega_A^2 [1 + \tanh(ns_0)]}{\rho_1 + \rho_e \tanh(ns_0)},
\]

\[
\omega_{n,s}^2 = \frac{\rho \omega_A^2 [1 + \tanh(ns_0)]}{\rho_1 \tanh(ns_0) + \rho_e}.
\]

The sequence \( \{\omega_{n,c}\} \) is monotonically increasing and the sequence \( \{\omega_{n,s}\} \) is monotonically decreasing, and \( \lim_{n \to \infty} \omega_{n,c,s} = \omega_k \), where \( \omega_k \) is the kink frequency of a tube with a circular cross-section determined by

\[
\omega_k^2 = \frac{2 \rho \omega_A^2}{\rho_1 + \rho_e}.
\]

In the long wavelength approximation, \( \omega_k \) is the common frequency of all non-axisymmetric modes in a tube with a circular cross-section. When \( s_0 \to \infty \), the tube cross-section tends to a circle, and \( \omega_{n,c,s} \to \omega_k \) for any \( n \).

When \( n = 1 \), the oscillations with the frequencies \( \omega_{1,c} \) and \( \omega_{1,s} \) correspond to the kink oscillations. The kink mode with the frequency \( \omega_{1,c} \) is linearly polarized along the large axis of the elliptical cross-section of the tube, and the mode with the frequency \( \omega_{1,s} \) is linearly polarized along the small axis. The expression for the frequencies of these modes can be rewritten as

\[
\omega_{1,c}^2 = \frac{\rho \omega_A^2 (a + b)}{\rho_1 a + \rho_e b}, \quad \omega_{1,s}^2 = \frac{\rho \omega_A^2 (a + b)}{\rho_1 b + \rho_e a}.
\]

They satisfy the inequalities

\[
\omega_{1,c} < \omega_k < \omega_{1,s}.
\]

The perturbations of the boundary for the first three slow and fast modes are shown in Fig. 3.

5. **Resonant damping of kink modes**

In order to calculate the wave damping we have to find \( \omega_1 \). In the long wavelength approximation equation (16) determining \( P_1 \) in the inner region and a similar equation determining \( P_1 \) in the external region become the Laplace equations, and finding their solutions is straightforward:

\[
P_1 = U_n^0 + \sum_{n=1}^{\infty} [U_n^1 \cosh(ns) \cos(n\varphi) + W_n^1 \sinh(ns) \sin(n\varphi)],
\]

\[
P_1^0 = U_n^0 + \sum_{n=1}^{\infty} e^{-n s} \left[ U_n^1 \cos(n\varphi) + W_n^1 \sin(n\varphi) \right],
\]

where \( U_n^0 \) and \( W_n^0 \) are arbitrary constants. We substitute these solutions in the second equation (18) and in (19b), use the results obtained in Section 3, and collect the terms proportional either to \( \cos \varphi \) or to \( \sin \varphi \) in the resulting equations. This yields two systems of two linear inhomogeneous algebraic equations, one for \( U_n^1 \) and \( U_n^1 \), and the other for \( W_n^1 \) and \( W_n^1 \). The determinants of these systems are zero, so that they have non-trivial solutions only when their right-hand sides satisfy the corresponding compatibility conditions. This conditions give the equations determining \( \omega_1 \) for the slow and fast kink modes. The real part of \( \omega_1 \) gives only a small correction to \( \omega_0 \) and can be neglected. The imaginary part of \( \omega_1 \) is very important because it describes the resonant damping of the oscillations. Introducing the damping decrement \( \gamma = -\epsilon \omega_1 \), where \( \epsilon \) is the imaginary part of a quantity, we obtain

\[
\gamma_c = \frac{\pi a^2 \omega_A^2 (\rho_1 - \rho_e)^2}{2 \Delta (a + b)(a \rho_1 + b a)}.
\]

\[
\gamma_s = \frac{\pi a^2 \omega_A^2 (\rho_1 - \rho_e)^2}{2 \Delta (a + b)(b \rho_1 + a \rho_e)}.
\]

For a tube with a circular cross-section, where \( a = b \), \( \gamma_c = \gamma_s \), \( \gamma_c, \gamma_s \) being given by equation (56) of Ruderman & Roberts (2002). For not a very eccentric \( \gamma_c \) and \( \gamma_s \) are of the same order, so that the characteristic damping times of the two kink modes are the same.
6. DISCUSSION AND CONCLUSIONS

In this paper we have studied the damped oscillations of a thin straight magnetic tube with an elliptic cross-section in a cold ideal plasma. The damping of the oscillations is due to resonant absorption in a thin inhomogeneous layer at the tube boundary. Our main results are:

(i) There are two infinite sequences of the tube eigenmodes, \( \{ \omega_{nc} \} \) and \( \{ \omega_{ns} \} \) (n = 1, 2, ...), corresponding to the global modes of the tube. The sequence \( \{ \omega_{nc} \} \) is monotonically increasing, and the sequence \( \{ \omega_{ns} \} \) is monotonically decreasing. When \( n \to \infty \), \( \omega_{nc,s} \to \omega_k \), where \( \omega_k \) is the frequency of kink oscillations of a tube with a circular cross-section. The modes corresponding to \( \omega_{nc,s} \) have 2n nodes at the tube boundary;

(ii) The eigenfrequencies \( \omega_{1c,s} \) correspond to the two kink modes of the tube. They are given by equation (26). The kink mode with the frequency \( \omega_c \) is polarized along the large axis, and with the frequency \( \omega_s \) along the small axis of the elliptic cross-section.

(iii) The ratio of the mode decrements to the mode frequencies are of the order of \( \ell/a \), where \( \ell \) is the characteristic thickness of the inhomogeneous layer at the tube boundary. The decrements are given by equation (29). For moderate values of the ratio of the large, \( a \), and small, \( b \), half-axis of the elliptic tube cross-section (\( a/b \lesssim 2 \)) the decrements of the two kink modes do not differ very much from the decrement of the kink mode of a tube with the circular cross-section.

The results obtained in this paper may be important for interpretation of the observed coronal loop oscillations. When analyzing the results of the observations it is assumed that an oscillation of a coronal loop is a damped harmonic oscillation. Then the best fit is used to determine its amplitude, phase and decrement.

It follows from our analysis that coronal loop oscillations may be superpositions of two damped harmonic oscillations with different frequencies rather than a single damped harmonic oscillation. It would be rather interesting to analyze observations of coronal loop oscillations assuming that these oscillations are superpositions of two harmonic oscillations.

Our analysis is based on the assumption that the thickness of the inhomogeneous layer is much smaller than the tube radius. It is a severe restriction. Goossens et al. (2002) used observations of damped coronal loop oscillations obtained by TRACE to calculate the ratio of the thickness of the inhomogeneous layer \( \ell \) to the loop radius \( R \). They have used the expression for the resonant damping of kink oscillations of a tube with a circular cross-section obtained in the approximation of thin inhomogeneous layer (\( \ell \ll R \)). They analyzed eleven events and obtained that \( \ell/R \) varied from 0.16 to 0.49. Hence, in general, the condition \( \ell \ll R \) is not satisfied. Motivated by this result, Van Doorsselaere et al. (2003) calculated the frequencies and decrements of kink modes of inhomogeneous magnetic tube for arbitrary values of \( \ell/R \) numerically. Even for an extremely large value of \( \ell/R = 0.5 \) the difference between the analytical and numerical results does not exceed 20%. This result shows that the expression for the decrement obtained in the approximation of thin inhomogeneous layer can be used even when \( \ell \) is not small at all. We believe that the same result is valid for magnetic tubes with elliptic cross-sections, however a definite conclusion can be made only after corresponding numerical analysis.

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