TIME-DISTANCE HELIOSEISMOLOGY AND THE MAGNETIC ATMOSPHERE OF THE SUN

Y. Taroyan¹, R. Erdélyi², and J.G. Doyle¹

¹Armagh Observatory, College Hill, Armagh BT61 9DG, Northern Ireland, email: yat,jgd@arm.ac.uk
²Space and Atmosphere Research Group, Department of Applied Mathematics, The University of Sheffield, Sheffield S3 7RH, England, email: Robertus@sheffield.ac.uk

ABSTRACT

In time-distance helioseismology the travel time of the acoustic waves between different points on the solar surface is measured to infer the local structure and properties of the subsurface layers of the Sun. The presence of the magnetic canopy in the solar atmosphere is expected to modify the behavior of the acoustic waves. The travel time changes due to the atmospheric magnetic field are evaluated theoretically. The uniform magnetic field in the atmosphere shortens the travel time of the acoustic waves due to enhanced rigidity of the interface where the waves are reflected. This effect increases rapidly with increasing frequency and harmonic degree. The effect of the magnetic field on the acoustic wave travel time is compared with the corresponding effect of large-scale sub-surface flows. It is suggested to use the time-distance analysis as a diagnostic tool in determining the local properties of the Sun’s atmospheric magnetic field.

Key words: Sun; time-distance helioseismology; magnetic canopy.

1. INTRODUCTION

Helioseismology studies the internal structure and properties of the Sun by examining the properties of the oscillations observed on the solar surface. The measurements are carried out by observing oscillations in the intensity or in the Doppler shift of a spectral line as the plasma producing the spectral line oscillates back and forth along the line of sight. These oscillations are known to have a discrete spectrum. Each mode is described by the eigenfunctions of the wave equation of a spherical system and is characterised by a spherical harmonic degree l, radial order n, and azimuthal order m. In the ω – l diagram the eigenmodes reside on parabolic ridges. The p-mode ridges have been detected from degree l = 0 up to degree l = 4000. Radial orders up to n = 35 have been identified. The lowest-order ridge with n = 0 is the fundamental, or the f-mode, characterised with the dispersion relation of surface waves in deep water ω² = gk₁. The mode structure of the oscillations has been thoroughly investigated to study the internal structure of the Sun.

By measuring the frequencies of the modes, which extend over all longitudes, traditional helioseismology infers the rotation speed, sound speed and other parameters inside the Sun as functions of radius and latitude. However, it provides no information about the longitudinal variation of the properties of the Sun. Local helioseismology and, in particular, time-distance helioseismology has been useful for determining local properties of the subsurface layers of the Sun. The basic idea proposed by Duvall et al. (1993) is to measure the acoustic travel time between different points on the solar surface using the signal cross-correlation technique and then to use these measurements for inferring variations of internal properties such as the sound speed, magnetic field and flow velocities along the wave paths connecting the surface points. In the present paper the relationship between the time-distance helioseismology and the magnetic atmosphere of the Sun is clarified.

2. THE RAY THEORY APPROACH

Time-distance helioseismology departs from the traditional modal approach and regards the solar oscillations as wave packets travelling along ray paths. The ray path is specified by the group velocity of the central frequency of the wave packet. The wave packet enters the Sun and is refracted by the increasing sound speed until it turns around and returns to the surface, where it is reflected due to the density gradient and resumes its journey through the solar interior. Waves originated at one point of the solar surface may reach the other point directly or after one bounce at the surface, two bounces at the surface, and so on. The sound speed is higher in the deeper layers of the Sun and, therefore, the direct
waves arrive first followed by the waves which have bounced at the surface. Strictly speaking the ray path approach is valid only in the case of large radial orders \( n \) and large spherical harmonic degrees \( l \). According to Bogdan (1997), in the case of low radial orders the actual \( p \)-modes are poorly localised along the nominal ray path and could be some 10-30 Mm away from it. This raises some concerns as to how one should interpret the results of the ray theory approach.

3. SIGNAL CROSS-CORRELATION

Time-distance helioseismology is based on the idea of signal cross-correlation. The cross-correlation function of the oscillation signal \( f(t, \mathbf{r}) \) between two different points on the solar surface is

\[
\Psi(t, \Delta r) = \int f(\tau, \mathbf{r}_1) f^*(\tau + t, \mathbf{r}_2) d\tau,
\]

where the integral is taken over the time of observation, \( \Delta r \) is the distance between the points \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) on the solar surface, \( t \) is the delay time, * denotes the complex conjugate. The signal \( f \) is the velocity or intensity variation. One can represent it as a Fourier integral and use the cross-correlation theorem (the cross-correlation in the time domain is equivalent to the cross-correlation of the inverse Fourier transform of the signal in the frequency domain) to obtain

\[
\Psi(t, \Delta r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A^2(\omega)e^{i\mu(\omega)t} d\omega,
\]

where \( A(\omega) \) is the distribution of the amplitude over the frequency,

\[
\mu(\omega) = \omega - \frac{1}{t} \int \mathbf{k} \cdot d\mathbf{r}
\]

and the line integral is taken along some path from \( \mathbf{r}_1 \) to \( \mathbf{r}_2 \). D’Silva (1996) has shown that if the amplitude \( A(\omega) \) is a Gaussian, or, in general, is appreciable only in a small frequency interval around some \( \omega_0 \), then the cross-correlation function attains its maximum along the path

\[
\frac{d\mathbf{r}}{dt} = \frac{\partial \omega}{\partial \mathbf{k}}|_{\omega_0}
\]

determined by the group velocity of the wave packet at the central frequency \( \omega_0 \). In other words, the peak of the cross-correlation function obtained by cross-correlating the signals at two different points provides the time taken by the wave packet to travel between them. This time is called the group travel time and is given by

\[
t_g = \int \frac{\partial \mathbf{k}}{\partial \omega}|_{\omega_0} \cdot d\mathbf{r}.
\]

The phase peak of the cross-correlation function provides the phase travel time of the two wave packets. In a nondispersive system these two times are equal to each other.

3.1. Example

In order to illustrate how this technique works consider the simple example of a Doppler shifted acoustic wave with a dispersion relation

\[
\omega = c(\mathbf{r})k(\mathbf{r}) + \mathbf{k}(\mathbf{r}) \cdot \mathbf{V}(\mathbf{r}).
\]

The travel time between two reflection points is determined by the local sound speed \( c(\mathbf{r}) \) and the flow velocity \( \mathbf{V}(\mathbf{r}) \):

\[
t = \int \frac{dr}{c(\mathbf{r}) + \mathbf{n}(\mathbf{r}) \cdot \mathbf{V}(\mathbf{r})},
\]

where the integration is carried along the path between the two photospheric reflections and \( \mathbf{n}(\mathbf{r}) \) is a unit vector tangent to the path. Note that both the group and phase travel times are given by the same formula. Let us represent the sound speed in the form

\[
c(\mathbf{r}) = c_0(\mathbf{r}) + \delta c(\mathbf{r})
\]

and assume that both \( \delta c(\mathbf{r}) \) and \( \mathbf{n}(\mathbf{r}) \cdot \mathbf{V}(\mathbf{r}) \) are small compared to \( c_0(\mathbf{r}) \), the sound speed of a reference model. Then a series expansion of the integrand leads to the following expression:

\[
t = \int \frac{dr}{c_0(\mathbf{r})} - \int \frac{\delta c(\mathbf{r}) + \mathbf{n}(\mathbf{r}) \cdot \mathbf{V}(\mathbf{r})}{c_0^2(\mathbf{r})} dr.
\]

As first pointed out by Gough and Toomre (1983) the effects of the sound speed and the flow can be separated by measuring the travel times \( t^\pm \) of signals propagating in opposite directions. By taking the sum and the difference of \( t^+ \) and \( t^- \) one can derive the following formulae:

\[
\frac{1}{2}(t^+ + t^-) = \int \frac{dr}{c_0(\mathbf{r})} - \int \frac{\delta c(\mathbf{r})}{c_0^2(\mathbf{r})} dr,
\]

\[
\frac{1}{2}(t^+ - t^-) = - \int \frac{\mathbf{n}(\mathbf{r}) \cdot \mathbf{V}(\mathbf{r})}{c_0^2(\mathbf{r})} dr.
\]

These are two separate inverse problems for the two unknowns \( \delta c(\mathbf{r}) \) and \( \mathbf{V}(\mathbf{r}) \). In this simple example one can see that the internal flow and sound speed structures can be determined by measuring the travel times of the signals propagating in opposite directions and by solving the two inverse problems.

4. TRAVEL TIME IN THE PRESENCE OF AN ATMOSPHERIC MAGNETIC FIELD

However, other effects should also be taken into account. We ask the question: how important is the role of the magnetic atmosphere in time-distance helioseismology and how much could it modify the travel times of the solar oscillations? Below the photosphere the magnetic field resides in the form of flux tubes which form sunspots or remain isolated from
one another as intense tubes. Above the photosphere the flux tubes spread out into the atmosphere creating a horizontal magnetic canopy. The field strength within such canopies and their heights depend upon whether they are in active regions associated with sunspots or in quiet regions far from them.

In order to evaluate the travel time in the presence of an atmospheric magnetic field consider a plane parallel equilibrium model in which the solar interior (\( z > 0 \)) is permeated by a uniform equilibrium flow along the \( x \)-axis \( \mathbf{V} = (V, 0, 0) \) and has a temperature profile linearly increasing with the depth. Gravity is constant in the interior and acts downwards. Above the interior lies the isothermal atmosphere permeated by a uniform magnetic field along the \( x \)-axis \( \mathbf{B}_0 = (B_0, 0, 0) \). The temperature in the isothermal atmosphere (\( z < 0 \)) is assumed to be equal to that at the photospheric top of the field-free convection zone.

The travel time is determined by the dispersion relation of the medium in which the waves propagate and therefore in order to be able to calculate the travel time in the presence of an atmospheric magnetic field we have to derive a dispersion relation for such a medium. The wave motion is described by the following set of linearised ideal MHD equations:

\[
\begin{align*}
\frac{d\rho}{dt} &= -\rho_0 \nabla \cdot \mathbf{v}, \\
\rho_0 \frac{d\mathbf{v}}{dt} &= \frac{\mathbf{B}_0}{\mu_0} \cdot \nabla \mathbf{b} - \mathbf{V} \left( \frac{p + \mathbf{B}_0 \cdot \mathbf{b}}{\mu_0} \right) + \rho \mathbf{g}, \\
\frac{d\mathbf{b}}{dt} &= (\mathbf{B}_0 \cdot \nabla) \mathbf{v} - \mathbf{B}_0 (\nabla \cdot \mathbf{v}), \\
\frac{dp}{dt} &= -c_s^2 \rho_0 \nabla \cdot \mathbf{v},
\end{align*}
\]

where \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \) is the substantial derivative, \( p \) is the pressure perturbation, \( \rho \) is the density perturbation, and \( c_s \) is the sound speed. For simplicity we consider waves propagating only in the \( x \)-direction.

4.1. The Dispersion Relation

We briefly describe the derivation procedure of the dispersion relation skipping the details. The perturbed quantities are Fourier analysed and taken proportional to \( \sim \exp i(\omega t - k_z z) \), where \( k_z \) represents the horizontal wavenumber related to the spherical harmonic degree by the formula \( k_z = \sqrt{l(l + 1)} / R_0 \). The equations are then reduced to second order ODEs and solved separately in the convection zone and in the atmosphere. The solutions must be selected in such a way that the wave energy density remains finite in the entire space. The normal component of the Lagrangian displacement and the total pressure perturbation are required to be continuous across the interface separating the two regions. The condition for the existence of a nontrivial solution yields the dispersion relation

\[
2ak_z^2 U(1 - a, m + 3, 2k_z z_0) \frac{g \omega_D^2}{\omega_D^2} - k_z^2 \omega_D^2 = 0,
\]

where \( \omega_D = \omega - k_z V \) is the Doppler shifted frequency, \( \gamma \) is the adiabatic index, \( c_0 \) is the sound speed at the temperature minimum, \( c_{AC} \) and \( c_CR \) are the Alfvéén and cusp speeds in the atmosphere, the parameter \( \alpha \) is given by

\[
a = \frac{m + 1}{\gamma \frac{\omega_D^2}{2gk_z}} + \left( \frac{m - 1}{\gamma } \right) \frac{c_0^2}{2\omega_D^2} - \frac{m - 1}{2},
\]

and

\[
m = \frac{\gamma g z_0}{c_0^2} - 1
\]

is the polytropic index, \( z_0/(1 + m) \) is the pressure scale height at \( z = 0 \).

\[
\varphi = 1 - \frac{pq A_1 A_3}{k_x r A_2} \frac{F \left( p + 1, q + 1, r + 1, -\frac{A_1}{A_2} \right)}{F \left( p, q, r, -\frac{A_1}{A_2} \right)},
\]

where \( A_1 = A_1(k_z, \omega) \) are some quantities depending on \( k_z \) and \( \omega \), \( U \) is the confluent hypergeometric function of the second kind, and \( F \) is the generalised hypergeometric function. In a static equilibrium the obtained dispersion relation is reduced to the one derived by Evans and Roberts (1990).

4.2. The Group Travel Time

We can now go back to the travel time formula and using the obtained dispersion relation calculate the travel time in the presence of a large-scale sub-surface flow and an atmospheric magnetic field. By representing \( dx \) in the form

\[
\frac{dx}{dz} = \left( \frac{c_s}{\omega_D} \right)
\]

we may rewrite the travel time formula in the form

\[
t_g = \int_{x_1}^{x_2} \left( \frac{\partial x}{\partial \omega} + \frac{\partial k_z}{\partial \omega} \frac{dx}{dz} \right) dx,
\]

where \( x_1 \) and \( x_2 \) are the \( x \)-coordinates of the two reflection points on the surface. The travel path is determined by \( \frac{dx}{dz} = \frac{\partial x}{\partial k_z} \). Substituting this expression in the previous formula we arrive at

\[
t_g = 2\Delta x \frac{\partial \omega}{\partial k_z},
\]

where \( \Delta x = x_2 - x_1 \) is the distance between the reflection points on the surface. The dependence
of $t_g$ on $\Delta x$ is not linear since $k_x$ is a function of $\Delta x$. By integrating the $x$-component of the equation \[ \frac{dr}{dt} = \frac{\partial \omega}{\partial k_x} \] and using the last expression for the group travel time $t_g$, we obtain

\[ \Delta x = C_1 t_g^2, \] (23)

where $C$ is a constant. This expression has been established observationally by Duvall et al. (1993) and it states that the cross-correlation between two intensity fluctuations on the solar surface separated by a distance $\Delta x$ is maximal along a parabolic curve of the form $\Delta x \propto t_g^2$ known as the time-distance curve. In a nondispersive medium the expression for the travel time is reduced to

\[ t = \frac{2\Delta x}{\omega/k_x}, \] (24)

which has been derived by Price (2000).

5. AN ASYMPTOTIC RESULT

In order to calculate the travel time in a dispersive medium one has to calculate the derivatives of the hypergeometric functions with respect to their parameters which is a difficult task. However, it turns out that under certain conditions it is possible to reduce the dispersion relation to a simple asymptotic expression for $\omega$ which is much easier to deal with than the original dispersion relation. The measure of the pressure scale height $z_0 \sim 0.25$ Mm, so that the argument of the hypergeometric functions $k_x z_0$ is small for not very large horizontal wavenumbers. This allows us to expand both sides of the dispersion relation into Taylor series. After some algebra we obtain the following asymptotic result

\[ \frac{\omega}{\sqrt{g k_x}} = \Omega_n + \frac{V}{c_0} \sqrt{\frac{2}{m+1}} \sqrt{\frac{\gamma}{\Gamma(1+\gamma)}} (2k_x z_0)^{-\frac{1}{2}} \]

\[ + \frac{\gamma(1+\gamma)}{\Gamma(1+\gamma)} \left[ \frac{2m+1}{\gamma} - m \right]^{-1} \times \left( \frac{1 - \frac{4}{n} - 20\Omega_n^2}{n} (2k_x z_0)^{m+1} \right), \] (25)

where $\Gamma$ is the $\Gamma$-function, $\beta = c_A^2/c_A^2$ is the square of the ratio of the sound and Alfvén speeds in the atmosphere and $\Omega_n$ is determined from the equation

\[ \frac{m+1}{\gamma} \frac{\Omega_n^2}{n} + \left( \frac{m+1}{\gamma} \right) - m = 2n. \] (26)

The derivation details of the asymptotic result are given in Erdélyi and Taroyan (2001). The $x$-component of the group velocity can be determined as follows:

\[ \frac{\partial \omega}{\partial k_x} = \frac{1}{2} \sqrt{g} k_x \Omega_n + V + \frac{\gamma(m+3/2)}{\Gamma(1+m+n)} \]

\[ \times \sqrt{\frac{(1-\frac{4}{n}) - 20\Omega_n^2}{n}} (2k_x z_0)^{m+1} k_x^{m+\frac{1}{2}} \]

\[ \times \frac{2m+1}{\gamma} - m \Omega_n (1 - \frac{4}{n}) (\gamma + 2\beta). \] (27)

6. NUMERICAL RESULTS

In the numerical results presented below we have set $\gamma = 5/3, m = 3/2, g = 274$ km/s, $z_0 = 0.25$ Mm. The temperature at the interface separating the convection zone and the atmosphere is $T = 4170$ K. The angular frequency $\omega$ is replaced by the cyclic frequency $\nu = \omega/(2\pi)$. The solar radius is $R_\odot = 696$ Mm.

Fig. 1a shows the relative difference $D = 100(\theta_0 - t_g)/t_g$ of the travel times in nonmagnetic and magnetic states as a function of the magnetic field strength when the flow speed $V = 0$. Here $\theta_0 = t_g(V = 0, B = 0)$ is the travel time in the absence of the flow and magnetic field. The horizontal distance $\Delta x$ is kept fixed and the values for the harmonic degree and frequency are $l = 200, \nu \approx 2.1 - 4.2$ mHz. It can be seen that the magnetic field reduces the travel time by speeding up the wave propagation. This effect is due to the greater rigidity at the interface introduced by the uniform magnetic field. For the given parameter values the magnetic field of $B_0 = 20$ G may shorten the travel time between two surface points by up to 10%. For comparison we plot $D$ as a function of the flow speed $V$ measured in km/s (Fig. 1a). Here the atmosphere is assumed to be nonmagnetic while other parameters are the same. The flow is assumed to be in the same direction as the wave propagation and therefore reduces the travel time. The horizontal axis covers a wide range of flow speeds from those typical for meridional flows (10-20 m/s) to the equatorial rotation speed of 2 km/s.

The next plot (Fig. 2a) shows the relative time difference $D$ as a function of the magnetic field strength when $l = 500$ and $\nu \approx 3.4 - 6.7$ mHz. It can be seen that the effect of the magnetic field has become even stronger compared to the case with $l = 200$. For a field strength of $B = 20$ G the travel time between two surface points is shortened by 50%. Meanwhile, a flow with a speed of $V = 2$ km/s shortens the travel time by only 12% (Fig. 2b).

The last pair of diagrams shows that as the degree $l$ increases the magnetic effect on the travel time grows much more rapidly than the flow effect. The horizontal distance between the surface points is changing correspondingly.

We might have slightly overestimated the role of the magnetic field by taking zero height of the magnetic canopy and assuming a strongly uniform magnetic field. However, our results indicate that the magnetic field in the atmosphere should have a considerable effect on the travel time of the acoustic waves inside the Sun. The comparison with the flow effects shows that the magnetic effects should be measurable. On the other hand, by measuring and comparing the travel times in quiet and active regions one could in principle gain some information about the structure and the properties of the magnetic field of the Sun’s atmosphere.
Figure 1. The relative time difference $D = 100(t_0^0 - t_0)/t_0^0$, where $t_0^0 = t_0(V = 0, B = 0)$; (a): as a function of the magnetic field strength when $V = 0$; (b): as a function of the flow speed when $B_0 = 0$. The horizontal distance $\Delta x$ is kept fixed and the chosen parameter values are $l = 200$, $\nu \approx 2.1 - 4.2$ mHz.

Figure 2. The relative time difference $D = 100(t_0^0 - t_0)/t_0^0$, where $t_0^0 = t_0(V = 0, B = 0)$; (a): as a function of the magnetic field strength when $V = 0$; (b): as a function of the flow speed when $B_0 = 0$. The horizontal distance $\Delta x$ is kept fixed and the chosen parameter values are $l = 500$, $\nu \approx 3.4 - 6.7$ mHz.

Figure 3. The relative time difference $D = 100(t_0^0 - t_0)/t_0^0$, where $t_0^0 = t_0(V = 0, B = 0)$; as a function of the harmonic degree $l$ when (a): $V = 0$ km/s, $B = 20$ G; (b): $V = 2$ km/s, $B = 0$ G.
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