Figure 4A during the relatively quiet phase of the cycle, one can see that the streaming flows are systematically deflected by the magnetic complexes that are present. With increasing activity (Figs. 4B, C), many larger active regions now appear as zones of flow convergence and possible subduction at depths such as 7 Mm and shallower. Some major active complexes exhibit even stronger diverging flows at the greater depth of 14 Mm, whereas others at both depths have flows converging toward the major axis of the active region. Yet in others such converging flows and a strong shear line only extend to depths of about 4 Mm from the surface, below which the flows appear to meander with little regard for the active regions evident at the surface, suggesting that the magnetism there may be confined close to the surface. Clearly there is a wide range of subsurface flow behavior associated with active complexes.

Such subsurface flows can mechanically twist and displace field lines, possibly leading to unstable magnetic configurations that may flare or erupt as CMEs. Preliminary helioseismic probing has revealed that at least one recent active region complex possessed strong inflowing streams just below the surface and prominent diverging flows at greater depths during an extended interval of flaring and CMEs, as discussed here by Haber et al. (2003). It is highly likely that such flows and magnetic fields are broadly linked in their evolution. The ongoing observations with SOI-MDI and GONG+ provide opportunities to understand the types of shearing flow structures that may develop in the surroundings of magnetic complexes. One should especially look for patterns of flow behavior that may precede rapid magnetic reconfiguration. We do not really know whether the surrounding large-scale flows of SSW serve to contribute to the possibility of flaring and CMEs, or whether the real culprits are far smaller scales of turbulent flow that we cannot detect with the current ring-diagram sampling of SSW. On the other hand, there could be gradual evolution in the magnetic topology by advection and diffusion until a highly unstable configuration is attained, which then proceeds to have rapid reconnection. Although we do not know ‘who does what to whom’ in terms of flows in which major magnetic structures are embedded, it is now evident that the large-scale flows of SSW show complex signatures in the surroundings of magnetic complexes, and this clearly deserves careful scrutiny.

4. ROTATING COMPRESSIBLE CONVECTION IN SPHERICAL SHELLS

Substantial progress has been achieved recently in modelling the interaction of turbulent convection and rotation within the bulk of the solar convection zone using 3-D numerical simulations on the latest breed of massively-parallel supercomputers. In particular, the Anelastic Spherical Harmonic (ASH) code permits study of differential rotation profiles established within full spherical shells of rotating turbulent convection, using spherical harmonic Ym expansions up to degree ℓ ~ 680 (Elliott et al. 2000; Miesch et al. 2000; Miesch 2000; Toomre et al. 2001; Brun & Toomre 2001, 2002).

These models of convection in spherical shells are intended to be a faithful if highly simplified description of the solar convection zone. Solar values are taken for the heat flux, rotation rate, mass and radius, and a perfect gas is assumed since the upper boundary of the shell lies well below the H and He
ionization zones; contact is made with a real solar structure model for the radial stratification being considered. The computational domain extends from about 0.63$R$ to 0.98$R$, where $R$ is solar radius, thereby including in some of our simulations a region of stable stratification of thickness 0.07$R$ below the primary unstable zone in which effects of penetrative convection can be studied. Such shells have an overall density contrast in radius of about 60, and thus compressibility effects are substantial. We have softened the effects of the very steep entropy gradient close to the surface that would otherwise favor the driving of very small granular and mesogranular scales of convection, since these require a spatial resolution at least ten times greater than presently available. The flux of enthalpy by the unresolved eddies near the surface is explicitly taken into account with subgrid-scale (SGS) terms.

**Rapid Evolution in Convection Patterns** The resulting convection is highly time dependent and the flows are intricate. The convection is dominated by intermittent plumes of upflow and stronger downflow, some possessing a distinctive cyclonic swirl: the dominant role of coherent plumes, first revealed in planar geometry, is now becoming apparent as we study more turbulent flows. Many downflows extend over the full depth of the zone, changing from wavy downflowing sheets to distinct plumes at greater depths (Figs. 6A, B). The convection patterns also evolve over fairly short time scales compared to the solar rotation period, and are advected and sheared by the strong differential rotation that they drive (Figs. 7A-C). This suggests that the largest global scales of convection are unlikely to be recognizable after one full rotation, as in observing a subsequent Carrington rotation. Equally important, this may provide an explanation for the changes in SSW seen from one rotation to the next, or even over an interval of a few days (Fig. 3).

**Strong Differential Rotation** These 3-D simulations of solar convection with ASH are making serious contact with helioseismic findings about differential rotation within the deep interior of the sun. The time-averaged angular velocity $\Omega$ in one of our simulations is nearly constant on radial lines throughout much of the convection zone at mid-latitudes (Fig. 1C), and there is a systematic decrease of rotation rate with latitude from the equator to the poles (Brun & Toomre 2001, 2002). The equator to pole contrast in $\Omega$ is of order 30%, much like in the sun.

We can understand many aspects of the resulting $\Omega$ profiles in terms of the angular momentum transport achieved by the modified mean Reynolds stresses established by the turbulent convection, coupled with the effects of the fairly complex meridional circulations that are present.

**Complex Meridional Circulations** The meridional circulations typically involve several cells in latitude within a given hemisphere, and often two layers of cells in depth across the convection zone. The latter appears to be crucial in obtaining the particularly slow rotation at high latitudes now being realized in our simulations (Fig. 1C); this is a prominent feature in the helioseismic findings for $\Omega$ (Fig. 1A, B) that has previously been difficult to replicate. Further, the two hemispheres in our simulations can exhibit differing meridional circulation patterns as deduced from averages formed over several rotations, emphasizing that such broken symmetries are typical rather than an exception. There is also noticeable evolution in what is deduced to be the meridional circulation from such longitudinal and temporal averaging. Something akin is seen in SSW in Figure 5, though one is there sampling regions closer to the surface than can be captured by the current ASH simulations. It should be emphasized that the models have not yielded single meridional circulation cells that span all latitudes and depths within a hemisphere, as is sometimes favored in estimates of transport that might be achieved by such circulations.

**Structures in Enstrophy and Temperature** The global convection has quite a different appearance if viewed in terms of the flow enstrophy (squared vorticity scalar) shown in Figure 7D. Here the prograde propagating fronts of strong enstrophy (aligned roughly with the rotation axis, or north-
south) are strikingly visible at low latitudes. They are replaced at higher latitudes by more isotropic, swirling cyclonic structures (of smaller scale) that are retrograde propagating. This emphasizes that the large-scale convection has a different intrinsic character at low and high latitudes (with a delineation roughly at 40°). One might well expect SSW to likewise show such behavior; the higher spatial resolution afforded by time-distance analyses possibly provides a path for examining such questions. In another respect, the fluctuating temperature field (Fig. 7E) exhibits prominent zonal banding, with overall variations of about 6K at this depth. Such banding arises from systematic variations with latitude in both the radial and latitudinal enthalpy (heat) fluxes carried by the convection, and through heat transported by its associated meridional circulations. The presence of such latitudinal temperature contrasts contributes to a thermal wind component (baroclinicity) to explain some of the differential rotation that results, as analyzed in some detail in Brun & Toomre (2002). Such temperature banding may be detectable by helioseismic probing, though the effects of surface magnetic fields offer major obstacles.

**Multi-Scale Near-Surface Convection** To examine some of the convective dynamics that may yield the near-surface rotational shear layer (Figure 1B), we have carried out preliminary studies with ASH within thin spherical shells, such as positioned between 0.90\(R\) and 0.98\(R\) (DeRosa & Toomre 2000; DeRosa, Gilman & Toomre 2002). Figures 6C, D show the level of complexity in the turbulent convection of many scales realized in those simulations, with the smallest resolvable cellular flows now close to that of supergranulation. These small cells (15 to 30 Mm across) tend to be advected laterally by the horizontal outflow motions associated with the broader cells (about 200 Mm across) possessing a connected network of downflow lanes near the top of the layer. These simulations are the first to exhibit a decrease in angular velocity \(\Omega\) with radius at low and mid latitudes that are in the spirit of the helioseismic findings, strongly encouraging the deep shell simulations that we have now begun which are capable of both resolving supergranulation and of dealing with angular momentum redistribution within the full shell.

5. INTERCOMPARISON OF SSW FLOWS

**Approaches to Measure Solar Flows** Five approaches are now being used to measure local horizontal flows within the sun: direct Doppler measurements (e.g. Duvall 1979, Hathaway et al. 1996, Ulrich 1998), correlation tracking of mesogranules (DeRosa & Toomre 1998; DeRosa, Duvall, & Toomre 2000; Lisle, DeRosa & Toomre 2000; Shine, Simon & Hurlburt 2000), ring-diagram analysis, time-distance tomography, and acoustic holoography. Each technique possesses strengths and weaknesses. Direct Doppler measurements and correlation tracking yield superb spatial resolution, but they only sample the flow at the surface.

Of the helioseismic methods, time-distance tomography (and the related acoustic holography) can achieve the finest horizontal resolution and the deepest penetration. However, the time-distance inversion procedures are computationally intensive and near-real-time analyses over a majority of the solar disk are currently infeasible. Ring-diagram methods have the advantage that the analysis procedures can be executed relatively rapidly, making this technique promising for continuous monitoring of horizontal flows. They possess the drawback that the spatial resolution and the maximum sampling depth are limited by the tiling size. Multi-scale tiling strategies and new 3-D inversions will provide substantial improvements, and both are being pursued.

A major challenge in all such probing is the diverse range of dynamical scales that is clearly present in the near surface shear layer, as Figure 8 shows in comparing the vigorous scales of supergranulation being mapped by correlation tracking (CT) of meso-
granules with the much larger flow scales being deduced by the ring-diagram methods in the surroundings of a major active region. It is comforting that spatial averaging of the CT velocity fields does yield flow patterns that are very similar to those obtained from ring-diagram probing, though the flow amplitudes are greater. Time-distance probing can have a horizontal spatial resolution comparable to correlation tracking, while probing over a range of depths. As the subsequent talks on local helioseismology will address, the apparent complexity of the subsurface flow fields is quite daunting, with a very wide range of temporal and spatial scales all operating simultaneously within this turbulent layer.

Comparisons Between Several Local Helioseismic Methods Local helioseismic probing of SSW flows has so far been carried out both with ring-diagram and time-distance analyses. The application of these procedures to large amounts of solar data has only occurred quite recently, and thus few detailed intercomparisons have been performed with regard to specific flow features in common data sets. These two approaches involve quite different averaging of flows and thermal structures by either the acoustic wave field itself or by how the wave field is sampled. Furthermore, the scattering and absorption of waves as they propagate through a magnetized region is measured in a fundamentally different fashion by the two techniques. Thus the two procedures are expected to have differing sensitivities.

Since both approaches provide the means to sample intricate flows that interact with magnetic complexes, it is essential to begin making explicit comparisons between their subsurface flow deductions both near active regions and in the surrounding quiet sun. We anticipate that such comparisons will help us to sort out the sensitivities of both techniques. Such critical comparisons are likely to also stimulate improvements in each of the data analysis procedures and in the complex inversions used to interpret their scientific meaning. By intercomparing the results from a variety of flow measurement techniques using common data sets, it should be possible to distinguish effects of truly solar origin from those introduced by the analysis procedures. So far little systematic testing of this sort has been performed.

It is clearly very desirable that such intercomparisons be undertaken as an issue of high priority, much in the spirit of the preliminary comparisons discussed here by Hindman et al. (2003). The continuing helioseismic observations forthcoming from SOI-MDI and GONG+ in the current descending phase of this solar cycle are likely to reveal new aspects of the interaction of convection, rotation and magnetism, such as may occur in the vicinity of persistent equator-crossing coronal holes that are now becoming prevalent. The complementary probing of the subsurface flows using the different helioseismic analysis approaches, accompanied by careful intercomparison of the results, holds out the greatest promise of revealing the possible underlying role of the convection zone in producing such remarkable structures that exhibit almost no differential rotation.

6. A BRIGHT FUTURE

We have the opportunity to refine the local helioseismic techniques so as to have them ready for the very intensive observational data streams at fourfold higher resolution to be provided by HMI on SDO. Thus intercomparison efforts should continue in earnest. We likewise expect significant advances in the 3-D simulations of solar convection zone dynamics, both through rapid advances in supercomputing technology and slower advances in understanding aspects of highly nonlinear turbulence. The interplay between such modeling of turbulent MHD and the refined deductions being drawn from local helioseismology holds out a future that is most likely to be very fascinating as we approach the SDO era.

Local helioseismology is now entering an interpre-
tive stage in the analysis of subsurface flows. It will be important to have dynamical models available to provide a perspective for the interpretations. For instance, the prominent evolution and propagation of the large-scale convection patterns in Figure 7, obtained through the 3-D spherical shell simulations, are beginning to provide the means to understand many of the flow changes seen within SSW. This includes the symmetry breaking and evolving multicell character exhibited by the meridional circulations in Figure 5. Magnetism appears to play an important role in some of the deflected or converging flows seen near active complexes. The inclusion in a self-consistent manner of large-scale magnetism into the convection simulations is now becoming feasible. Thus we foresee many improvements in how the theoretical models will be used in the interpretation of SSW. The likely advances in such theoretical approaches, combined with refined helioseismic probing forthcoming from GONG+ and HMI, holds out a bright future for studying the very rich dynamics of the solar convection zone.

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HELIOSEISMIC IMAGING OF THE FARSIDE AND THE INTERIOR

D. C. Braun and C. Lindsey
NorthWest Research Associates, Colorado Research Associates Division, 3380 Mitchell Ln, Boulder, Colorado, 80301, USA

ABSTRACT

Helioseismic holography is a highly efficient and flexible procedure with a wide range of utility, from mapping sound wave travel times over the entire far solar hemisphere to imaging small scale scatterers and flows beneath solar active regions. Seismic images covering the entire far hemisphere of the Sun have been constructed using data from the recently upgraded Global Oscillation Network Group (GONG+) network and compare favorably with those made using simultaneous data from the Michelson Doppler Imager (MDI) onboard the Solar and Heliospheric Observer (SOHO). We are also continuing our comprehensive exploration of diffraction-limited seismic imaging of active regions. We have recently extended our applications of helioseismic holography to include Doppler diagnostics of active regions and quiet Sun. A major finding presented here is that the horizontal velocity field in supergranules and sunspot moats appears to be concentrated at the surface. Another recent, but vital, contribution to local helioseismology has been a study of what is termed the “showerglass effect”. Magnetic fields in the photosphere produce large, local amplitude and phase perturbations to the observed acoustic wave field which may be quantified and removed prior to the holographic computations. Removal of the showerglass from local helioseismic images of active regions is proving to be a crucial step in the detection of compact subsurface scatterers.

Key words: local helioseismology; holography; active regions; supergranulation; farside.

1. INTRODUCTION

The basic principle of what we call helioseismic holography is the phase-coherent reconstruction of the acoustic field within the solar interior from $p$-mode disturbances observed on the surface. The acoustic field at the surface, observed over a limited region $P$, called the pupil, is computationally regressed through the solar interior to a focal point at some distant location to express the acoustic egression and ingression. The egression and ingression are incomplete, but coherent, estimates of the acoustic disturbances propagating out of and into the focal point, respectively. In a “space-frequency” context, the egression and ingression are computed by integrals of the form:

$$H_\pm(r, z, \nu) = \int_P d^2r' G_\pm(r, r', z, \nu) \psi(r', \nu)$$

where $H_+$ and $H_-$ are the monochromatic egression and ingression, and $\psi$ is the local acoustic disturbance at temporal frequency $\nu$. $G_+$ and $G_-$ are Green’s functions that express how a monochromatic point disturbance at a position $r'$ on the surface propagates backwards and forwards in time to the focus at $r$ and depth $z$ respectively. A recent summary of many technical aspects of helioseismic holography is given by Lindsey & Braun (2000b). Our goal here is to report on progress in the development and application of “phase-correlation” seismic holography, which offers an appraisal of the phase shifts due to refractive and Doppler perturbations in the solar interior. A review of the scientific results achieved from earlier explorations in the domain of “acoustic power holography” may be found in Braun & Lindsey (2000a).

In section 2 below, we compare phase-correlation images, sensitive to travel-time perturbations, made from simultaneous data from the GONG+ network and the MDI instrument on SOHO, to assess the utility of the GONG+ images for local helioseismology, including farside imaging. In section 3, we report results obtained from Doppler-sensitive holographic computations from MDI observations. Depth-sensitivity of our phase-correlation computations is achieved by considering imaging configurations like that shown in Figure 1. In the final section, we summarize recent progress in understanding and correcting the phase and amplitude perturbations introduced locally to the surface wave field by magnetic regions, which act as an acoustic “showerglass”.

Figure 1. Seismic imaging with the focus placed at a depth below the solar surface. Here, the radiation sampled by the pupil illuminates the focal point from angles inclined up to ±45° from the horizontal direction. This configuration allows the construction of seismic images with the maximum possible horizontal resolution.

2. GONG+/MDI COMPARISONS

Comparisons of holographic analyses between datasets obtained from different instruments are needed to assess the sources of noise, and to judge the reliability of the results. Helioseismic data from the recently-upgraded ground-based GONG+ network provides a natural source for comparison with the MDI observations. Currently, seismic images of the solar far side are updated daily on the MDI website (http://soi.stanford.edu/data/farside), and have been helpful to a broad variety of users ranging from solar astronomers planning upcoming observing sessions to amateur radio enthusiasts. However, MDI is now exceeding its projected lifetime of six years. It is important to assess the ability of ground-based instrumentation, such as the GONG+ network, to continue the farside program, and other helioseismic projects, until new space-based instruments become available.

We performed similar holographic analyses of active regions observed both on the front and far sides of the Sun by the GONG+ network and the MDI instrument. The observations span a 24 hour time interval starting on 2001 September 29. GONG+ images made at the Big Bear, Learmonth, El Teide, and Cerro Tololo sites were used to achieve approximately 97% temporal coverage. The GONG+ and MDI full-disc images were interpolated to form Postel's projections spanning approximately 60°, and tracked at the Carrington rotation rate. Figure 2 shows a comparison of the power spectra, integrated over temporal frequencies, \( \nu \), between 2 and 6 mHz. Of primary importance for local helioseismology is the sensitivity of the instruments to \( p \)-modes of high degree (\( \ell \)). The fall-off of the power with \( \ell \) observed in the GONG+ images, relative to that of MDI, clearly reflects the effects of atmospheric seeing and attenuation.

From both data sets, we computed phase-correlation seismic images, sensitive to travel-time perturbations, using the methods described by Braun & Lindsey (2000b). In phase-correlation holography, we consider correlations between the egression and ingress:

\[
C(r, z, \nu) = H_+(r, z, \nu)H_-(r, z, \nu),
\]

where the asterisk denotes the complex conjugate. The phase of the correlation is

\[
\phi(r, z) = \arg \left( \frac{C(r, z, \nu)}{\Delta \nu} \right),
\]

where the brackets indicate an average over a bandwidth \( \Delta \nu \). In the temporal domain, we may relate a mean travel-time difference, \( \delta t \), between the egression and ingress to the correlation phase above by

\[
\delta t(r, z) = \phi(r, z) / 2\pi\nu_o,
\]

where \( \nu_o \) is the central frequency of the bandwidth \( \Delta \nu \).

Depth diagnostics are achieved by using pupils similar to that shown in Figure 1, with the focal plane moved to various depths \( z \) below the surface. Figure 3 shows some sample images from this analysis, computed with \( p \)-modes over a 1 mHz bandwidth centered at 4 mHz. In general, the mean travel-time perturbations associated with the active regions shown here are sharpest at focal depths closest to the surface, and appear to defocus with depth. This is consistent with the ideas put forth by Braun & Lindsey (2000b), who suggest that most of the observed travel-time perturbations are confined to the surface and very likely due to a Wilson-like depression of the \( p \)-mode cavity in magnetic regions.

As Figure 3 illustrates, the travel-time images made with GONG+ appear noisier than the MDI images, especially for focal depths close to the solar surface. As the focus is moved downward, the acoustic radiation used to construct the images decreases in wavenumber. Below depths of approximately 10
Mm, propagating modes have wavenumbers (ℓ) below 300 and the phase-correlation travel-time images from GONG+ and MDI have similar noise characteristics. At this and lower depths, the similarity of the MDI and GONG-derived images demonstrates that the r.m.s. fluctuations in these images are solar in nature. At shallower depths, atmospheric seeing introduces excess noise, which increases with mode degree, to the images derived from GONG+ data.

Imaging the far side of the Sun was conceived as one of the first potential applications of helioseismic holography well over a decade ago (Lindsey & Braun, 1990) The successful application of phase-correlation holographic procedures to farside imaging (Lindsey & Braun, 2000a) introduced the immediate possibility of a synoptic monitor of far-side activity for purposes of space weather forecasting. This tool has long been needed for studies of active-region evolution as well. Our initial application of imaging the far surface of the Sun relied on what we call 2-skip/2-skip phase-sensitive holography. This scheme is essentially blind to active regions more than about 50 degrees from the antipode of disk center. Braun & Lindsey (2001) subsequently showed how travel-time perturbations may be mapped over the entire portion of the Sun facing away from the Earth, including the polar regions. This is achieved by performing ingress−egression correlations computed over pupils illuminated by acoustic radiation which propagate one and three skips.

Images showing the acoustic travel-time perturbations over the entire far hemisphere obtained from the GONG+ and MDI data are essentially identical (Figure 4). This demonstrates that the GONG+ network may be used as the basis of a synoptic farside imaging program of comparable quality to that now provided by MDI. The GONG+ project is now seeking resources to implement this program.

3. HORIZONTAL DOPPLER DIAGNOSTICS

Phase-correlation holography typically involves the assessment of travel-time perturbations derived from the observed temporal correlations between the egression and ingressions. Diagnostic procedures sensitive to flows may be developed in a variety of
Figure 4. A comparison of seismic images of the solar far side made from data from the MDI and from the GONG+ network, and a synoptic magnetogram for the subsequent solar rotation. For far-side imaging, we use p-modes with temporal frequencies between 2.5 and 4.5 mHz, and spherical harmonic degrees between 20 and 40.

ways. The method employed here divides the pupil $P$ into four quadrants, each spanning 90 degrees and oriented in the North, South, East and West directions. Using just the North and South quadrants gives p-mode travel-times which are sensitive to wave advection caused by north or south motions. Likewise, the egressions and ingressions determined from the East and West quadrants are used to infer east-west motions. Denoting the quadrant’s areas as $N$, $S$, $E$, and $W$, we compute the four egressions and four ingressions:

$$H_{x,y,z,w}^{N,S,E,W} = \int_{N,S,E,W} d^2r' G_{\pm}(r,r',z,\nu) \psi(r',\nu)$$

and compute the four correlations $C^{E\rightarrow W}$, $C^{W\rightarrow E}$, $C^{N\rightarrow S}$, and $C^{S\rightarrow N}$, where, for example,

$$C^{E\rightarrow W} = H_{x}^{E} H_{x}^{W*}, \text{etc.}$$

(For convenience, we omit the arguments (r, z, \nu), which are understood.) Defining four correlation phases, given by

$$\phi^{E\rightarrow W} = \arg \left( \langle C^{E\rightarrow W} \rangle_{\Delta \nu} \right), \text{etc.,}$$

we interpret the phase-differences between opposite directions to be caused by horizontal flows:

$$V_x \propto -\frac{1}{2}(\phi^{E\rightarrow W} - \phi^{W\rightarrow E}),$$

$$V_y \propto -\frac{1}{2}(\phi^{S\rightarrow N} - \phi^{N\rightarrow S}).$$
Doppler-sensitive phase-difference images, when averaged over longitude, show the signatures of both differential rotation and meridional circulation with 24 hours of observations (Figure 5). We use published measurements of the photospheric differential rotation to calibrate Equations 8 and 9. Our simple calibration scheme assumes that the rotation rate does not change with depth over the range considered here, ignoring the gradients inferred by global helioseismology which are, at most, on the order of a few percent. Caution must be exercised in assessing and interpreting travel-time perturbations with observations that use uncorrected Doppler measurements over magnetic regions, due to phenomena which are now collectively called the “showerglass” (see Braun (1997); Lindsey & Braun (2002) and section 4 below). In this analysis, we simply mask out regions within sunspot umbrae and penumbrae, excluding completely their contribution to the holographic computations.

![Figure 5](image)

Figure 5. Averaging the E-W and N-S correlation phase signatures (at a focal depth of 7 Mm) over longitude shows clear evidence of differential rotation (top panel) and the meridional circulation (bottom panel) in 24 hours of observations. The velocity scale (y axis) is set by fitting the E-W correlation phase differences to published measurements of photospheric differential rotation (shown by the smooth curve in the top panel).

Figure 6 illustrates a sample of our results using MDI observations from the 24 hr period discussed in §2 above. We performed the analysis using p-modes at 3, 4, and 5 mHz, with 1 mHz bandwidths. All three sets of modes showed similar velocity fields, which were then combined by averaging to improve the statistics. Our Doppler-sensitive images generally reveal strong, localized outflows from sunspots, such as that around the spot group NOAA 9636 shown in Figure 6. These patterns strongly resemble the “moat” floats commonly observed in the photosphere (Brickhouse & LaBonte, 1988), which have similar magnitude (∼500 m/s) and spatial extent. Similar outflows have also been previously seen in inversions of time-distance observations (Zhao, Kosovichev & Duvall, 2001). As Figure 6 shows, the velocity field derived with the focus placed below 10 Mm appears to be anticorrelated with the near surface field. In general, the divergence images are all significantly correlated with the horizontal velocity divergence at the shallowest depth (3 Mm), with the correlation coefficient becoming negative below a depth of 10 Mm. We can quantitatively describe the behavior of the velocity divergence with depth by making scatter plots of the divergence values observed at a given depth with the (near-surface) value at 3 Mm depth. The “velocity ratio” is defined as the slope of a line fit to the scatter. The variation of the velocity ratio with the focus depth is shown in Figure 7.

![Figure 7](image)

Figure 7. The velocity ratio is the slope of a linear fit to a scatter plot of the horizontal velocity- divergence image at a given depth to the divergence image at 3 Mm. The open squares are from the holographic observations, while the line connects values derived from a control computation in which the Doppler signatures at the surface are simply averaged over the pupil quadrants.

A major goal of our helioseismic Doppler diagnostics is to probe the variation with depth of flows caused by supergranulation and other convective effects. It is tempting to interpret the reversal of the horizontal divergence signal at 10 Mm as a subsurface “return-flow” in supergranule cells which penetrate below this depth. However, careful consideration shows that in fact, the reversal of the horizontal velocity patterns appears to be consistent with the expectations of a horizontal velocity pattern which is highly concentrated in the near-surface layers. As the focal plane is submerged, the pupils used in the holographic computations become more spatially extended (examples are shown in the middle three columns in Figure 8). The apparent reversal of the flows with depth may be caused by the increasing contribution of oppositely directed surface flows as the pupils increase in size. We test this hypothesis in the following fashion. Using the observed velocity field at 3 Mm as a best estimate of the actual surface field, we compute the weighted average of the surface velocity with each quadrant for every pupil employed in our holographic analysis. The weighting function is the same as that used in the egression and ingression computations (see § 8.1 of Lindsey & Braun (2000b)) Differences between the East and West, and North and South, quadrant averages provide estimates of the expected surface contribution (hereafter referred to as the “control”) to our seismic measurements. The divergence of the control velocities is shown in the right three panels in Figure 8.
The control velocity patterns resemble the observations (left panels) to a high degree. We also computed velocity ratios for the control velocity fields in the same manner as performed for the seismic measurements. The slopes derived from the control are shown connected by a solid line in Figure 7, which are in remarkable agreement with the observed values (open squares).

4. THE SHOWERGLASS EFFECT

It is known for some time that the photospheres in magnetic regions introduce strong local amplitude and phase perturbations in the acoustic wave field. The shifts in the phase due to surface magnetism act as a "showerglass" that makes the imaging of subsurface acoustic perturbations immediately below the showerglass particularly difficult, and sometimes impossible. In this section, we briefly summarize our recent progress in assessing the showerglass effect and developing a magnetic proxy that will permit its removal from the surface wave field prior to helioseismic imaging. The reader is referred to another paper in these proceedings which discusses these issues in more detail (Lindsey & Braun, 2002).

To assess the phase errors introduced by surface magnetic fields, we examine the statistics of the sets of correlations between the egressions (and ingression), computed with the focal plane at the surface, and the local wave amplitude. These correlations are designed to compare an extrapolated acoustic field, propagated backward (or forward) in time from the measured egression (or ingression), with the actual surface wave field, and hence assess the local distortion in the latter due to surface activity. Plots of the mean phase and amplitude of these correlations, as functions of the surface magnetic flux density, are shown in Figure 9. The difference between the phases of the egression and ingression correlations at the strongest observed magnetic flux density is substantial, and correspond to travel-time perturbations on the order of 40 seconds or more between inward and outward propagating waves. This is comparable to the travel-time asymmetries in center-annuli correlations, centered on sunspots, reported previously by Duvall et al. (1996) and Braun (1997).

Where the phase errors are below a radian or so, the
Figure 8. A comparison of the observed horizontal divergence, derived from seismic holography (left-most panels) with those derived from a control computation which assumes the Doppler signatures are concentrated at the solar surface (right-most panels). The middle panels show the weighted pupils used in the analysis.

Born approximation may be regarded as valid. In this case, it may be reasonable to consider modeling (e.g. inverting) uncorrected helioseismic data to reconstruct an acoustic perturbation which consists of both the surface showerglass and a (potentially more interesting) subsurface contribution. The regions below weak plages, for example, may be amenable to such modeling. However, given the strong perturbations introduced by active regions, we have found that the unambiguous detection of compact subsurface scatterers within 10 Mm or less below magnetic regions requires a careful assessment and removal of the phase errors introduced at the surface. We have recently begun to explore the use of the observed magnetic flux density as a proxy for the phase errors, using measurements of the egression and ingresson correlations like those shown in Figure 9. Preliminary results are encouraging, and include the discovery of compact features below large active regions, some of which are invisible or degraded in seismic images in which the showerglass is not removed.

5. CONCLUSIONS

This report represents a snapshot of our recent research efforts in a variety of local helioseismic topics. Our main findings may be summarized as follows:

- Helioseismic images of the far side obtained with data from the GONG+ network are essentially identical to those obtained with the MDI instrument, supporting the proposal to add a synoptic farside imaging program to the GONG+ project.

- Comparisons of seismic imaging applied to simultaneous GONG+ and MDI data suggest that GONG+ data is well suited for local helioseismic analysis, particularly at depths of 10 Mm and deeper below the convection zone.

- Doppler-sensitive holography shows supergranulation flows and outflows from sunspots, consistent with surface measurements, and most likely confined to the near-surface layers of the convection zone.

- Correlations between the local acoustic wave field and the extrapolated egression and ingressions demonstrate the existence of an “acoustic showerglass” in magnetic regions, which severely scrambles the phase of impinging radiation. The removal of the showerglass by means of a magnetic proxy appears crucial in the identification and modeling of compact acoustic scatterers below active regions.
Further information and reprints of recent articles on our program in local helioseismology may be found at http://colorado-research.com/~dbraun.

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THE SHOWERGLASS EFFECT IN SEISMIC DIAGNOSTICS OF ACTIVE REGION
SUBPHOTOSPHERES

Charles Lindsey and D. C. Braun
NorthWest Research Associates, Colorado Research Associates Division, 3380 Mitchell Lane, Boulder,
Colorado 80301, USA

ABSTRACT

A major obstacle that encumbers local seismic diagnostics of the shallow subphotospheres of strong active regions is phase errors introduced by overlying surface magnetic fields. These errors function as a sort of “acoustic showerglass” that obscures subphotospheric acoustic anomalies, scrambling computational images of these derived by phase-coherent seismic reconstruction. We develop a proxy based on the surface magnetic field to correct the showerglass phase errors and image acoustic scatterers beneath it. Preliminary applications of this correction give us signatures that appear to signify strong, sharply outlined acoustic anomalies 3–9 Mm beneath large growing active regions. Correction of the showerglass correction appears to be important, if not essential, for diffraction-limited diagnostics of acoustic anomalies in the shallow subphotospheres of strong active regions.

Key words: local helioseismology; helioseismic holography.

1. INTRODUCTION

Over the past decade helioseismology has begun to focus on solar interior diagnostics from the local perspective, now recognized as “local helioseismology.” A major subject of interest to local helioseismology has been physical anomalies in the subphotospheres of active regions (e.g. Braun et al., 1992; Duvall et al., 1996). The interaction of active regions with acoustic waves in the p-mode spectrum is now well established. Active regions are known to exhibit strong absorption of solar interior p-modes that reflect from them (Braun, 1995) and to reduce travel times by up to a minute as compared to waves that reflect from the quiet solar photosphere (Braun, 1995; Fan et al., 1995; Duvall et al., 1996; Kosovichev, 1996; Hindman, 2000). Computational holographic images of active regions show “acoustic moats” surrounding large sunspots (Lindsey & Braun, 1998; Braun et al., 1998), and “acoustic glories,” stochastic regions of enhanced high-frequency acoustic emission surrounding large, growing magnetic regions (Braun & Lindsey, 1999; Donea, Lindsey & Braun, 2000). These acoustic phenomena seem to be relatively superficial, characterizing the medium within a few Mm of the base of the active region photosphere (Braun & Lindsey, 2000a).

One of the major diagnostic utilities that local helioseismology borrows from electromagnetic applications is what can be regarded as the “optical perspective.” The optical perspective is the basis of computational seismic holography, a diagnostic based on phase-coherent seismic imaging of the solar interior acoustic field that is very much the analogy of the function of our eyes and other optical accessories with respect to light and other electromagnetic radiation. While it would be a dangerous mistake to suggest that local helioseismology is computational acoustic optics, we believe that the optical perspective is critical to local solar interior diagnostics, and that fundamental diagnostic limitations imposed by elementary optics are bound to apply to helioseismic diagnostics in general. A simple example is that of the effects of diffraction, which limits spatial discrimination of compact subphotospheric anomalies in the same way that optical diffraction limits the resolution of an optical microscope. As far as we are presently aware, the diffraction limit is fundamental to wave mechanics, and cannot be circumvented in practical terms by known alternative diagnostics.

The purpose of this report is to address the diagnostic impact of seismic phase errors introduced at the solar surface by magnetic fields. We propose to quantify the phase errors and establish the need for a magnetic proxy to correct them. This analysis will be expressed essentially in terms of the optical perspective.

The basic principle of computational seismic holography is to extrapolate the surface acoustic field into the solar interior, into the neighborhoods of local acoustic anomalies. This is accomplished either forward or backward in time by computing fields of the
form

\[ H_\pm(r, \nu) = \int d^2 r' G_\pm(r, r', \nu) \psi(r', \nu), \]

(1)

where \( \psi(r', \nu) \) represents the complex amplitude of the surface acoustic field at frequency \( \nu \) and surface location \( r' \), and \( G_\pm(r, r', \nu) \) represents the Green's function that propagates an acoustic disturbance at \( r' \) to the "focal point," \( r \), in an acoustic model devoid of anomalies. In these computations, \( r' \) ranges over a region \( \mathcal{P} \) called the "pupil" of the computation (see Lindsey & Braun, 2000). The focal point, \( r \), of the computation most conveniently ranges over any submerged "focal plane" chosen by the analyst or a range of such planes covering depths of particular interest. The forward extrapolation, \( H_- \), in time is called the "acoustic ingestion," whereas the time-reverse, \( H_+ \), is called the "acoustic egression." In practice, maps of the egression power, \( |H_+(r, \nu)|^2 \), show clear signatures of compact acoustic emitters or absorbers, the latter rendered as silhouettes, when these occur at or near the depth the focal plane.

Anomalous acoustic absorbers are well established in association with active regions, and even anomalous emission at relatively high frequencies. These show up clearly as silhouettes in birefringence-power maps. However, these seem to be quite superficial, and there is little evidence for the existence of either strong acoustic emitters or absorbers more than about a Mm beneath the solar photosphere. Besides these, the two most familiar prospective local acoustic anomalies recognized by local helioseismology are those characterized in terms of (1) "sound-speed" anomalies and (2) flows, to which we also refer as "Doppler anomalies." These anomalies, which act as scatterers, are invisible to simple acoustic power holography for lack of contrast. They simply replace acoustic radiation that they block by radiation scattered from some other direction. However, because these anomalies shift the phase of radiation passing through them, they can be clearly rendered by phase-correlation statistics. These are recognized by the terms "time-distance helioseismology" and "phase correlation holography." Efforts to model subphotospheric sound-speed and Doppler anomalies based on time-distance helioseismology include Duvall et al. (1996); Kosovichev (1996); Zhao, Kosovichev & Duvall (2001). The optical variation of this diagnostic, phase-correlation holography, is based on maps of the correlation

\[ C(r, z) \equiv \langle H_+(r, z, \nu) H_-^*(r, z, \nu) \rangle_{\Delta \nu}, \]

(2)

averaged over an appropriate range, \( \Delta \nu \), in frequency.

A compact submerged acoustic scatterers will generally show a strong localized signature in \( C(r, z) \) when the focal plane of the computations is at the same depth, \( z \), as the scatterer. When the focal plane is moved substantially above or below such an anomaly, the signature becomes defocused, and therefore diffuse, but does not otherwise entirely disappear. It is important to keep in mind the distinction between the foregoing exercise of extrapolating the acoustic field from the solar surface and modeling of the various acoustic anomalies that contribute to the acoustic signatures. Indeed, the holographic extrapolation expressed by equation (1) is based on the assumption that no such anomalies exist. While we believe that the problem of inverting holographic signatures to derive models of acoustic anomalies in terms of sound-speed and Doppler perturbations is straightforward under realistic conditions, we will not attempt to prescribe how to do this here. We are convinced that the basic diagnostic requirements for physical modeling of acoustic anomalies are essentially the same as those for coherent holographic signatures—even if modeling is to be based on principles that circumvent holographic signatures.

We will now proceed with a brief summary of some elementary practical considerations that confront modeling of phase anomalies based on optical signatures.

1.1. The Born Approximation

Holographic diagnostics of a local anomaly are essentially based on how the anomaly shifts the phase of radiation that encounters it, as compared with the phase with which the radiation would have arrived at the surface if the anomaly had been absent. If the radiation arrives at the solar surface with a scrambled phase because it has passed through a swarm of other intervening anomalies, then the analyst is confronted with the basic problem of trying to "see" through a cloud or a fog bank. For purposes of seismic diagnostics of active region subphotospheres, we will say that the "Born approximation" is satisfied if a substantial fraction of acoustic radiation propagates from the focal plane of the acoustic extrapolation to the solar surface without undergoing further significant scattering. This means that the phase errors are kept comfortably less than a radian for a usable fraction of the radiation.

The function of a showerglass is to introduce stochastic phase variations in excess of a radian such that modeling based on the Born approximation is prohibitive. In the practical realm, it appears that modeling of any sort is prohibitive when the Born approximation is thoroughly violated. We have generally been forced to treat the problem of diagnostics through a showerglass as hopeless, at least within the confines of the optical context. We either must resort to extra-optical resources to remove the showerglass or flatten it (the function of windshield wipers, for example) or defer to an alternative diagnostic that is not encumbered with prohibitive phase errors.

1.2. Assessing the Phase Errors

Consider acoustic radiation impinging from the solar interior upward into the photosphere. How is the phase of that radiation shifted if a magnetic field is

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imposed at the surface? To make an assessment of this, we compute acoustic extrapolations, $H_-(r, \nu)$ and $H_+(r, \nu)$, integrating over pupils that are confined, as substantially as possible, to the quiet Sun, and whose focal points are located also at the surface but in regions with various magnetic field strengths. We then examine the respective correlations between $H_\pm$ and the local amplitude, $\psi$, at $r$:

$$C_\pm(r, \nu) = \langle H_\pm(r, \nu) \psi(r, \nu) \rangle$$

Statistics of these correlations are plotted in Figure 1. In this exercise, we did not attempt to confine the pupil to regions of quiet Sun. As a result, the statistics plotted in Figure 1 may be somewhat of an underestimate of the actual phase errors. In any case, the phase errors plotted in Figure 1 strongly suggest that fields in the neighborhood of 200 Gauss are sufficient to endanger the Born approximation.

The differences between the phases of $C_+$ and $C_-$ are substantial, and consistent with travel time asymmetries previously reported by Duvall et al. (1996). They and other authors proposed that the travel-time asymmetries are the Doppler signature of rapid downflows beneath magnetic regions (Kosovichev, 1996). There is some question as to whether the phase inequality could be the result of other, non-Doppler effects. While this is an important issue, we do not propose to address this question here. In either case, it is evident (Braun, 1997) that the phase inequality is, at least in large part, a relatively superficial phenomenon, originating within a few Mm beneath the photosphere, and that it is significant over a broad range of frequencies for moderate or greater magnetic fields. On that basis we will proceed here to treat it as part of the acoustic showerglass with more of a regard at present for the problem of seeing beneath it than for settling its origin in physical terms.

Figure 2 shows egression phase errors for NOAA AR 8179 mapped in projected relief as prescribed by the egression phase plot in Fig 1a (solid curve).

The acoustic showerglass is only significant for diagnostics of the relatively shallow subphotospheres of strong magnetic regions. For these diagnostics relatively compact pupils are needed, which cannot avoid the overlying surface magnetic regions.

1.3. Correcting the Phase Errors

The correction of the showerglass by use of the magnetic proxy is fairly straightforward in principle. The observed local acoustic amplitude, $\psi$, at the solar surface is multiplied by the reciprocal of the correlation between egression, $H_+$, and local acoustic amplitude, $\psi$, as expressed by the magnetic proxy shown in Fig 2a. The egression is then computed as if there would be the amplitude observed if there were no showerglass. The local acoustic amplitude is likewise multiplied by the reciprocal of the magnetic

Figure 1. Correlations $C_\pm$ between acoustic egression, $H_+$, (acoustic extrapolations backward in time) and ingestion, $H_-$, (forward in time) and local acoustic amplitude, $\psi$, as functions of the magnitude of the surface magnetic field. The foregoing statistics were derived from helioseismic observations by SOHO/MDI of NOAA AR 8179 over the 24 hr period beginning on 1998 March 15 in the 4.5--5.5 mHz spectrum, and on line-of-sight magnetic observations also by the MDI instrument in the same timeframe. The full vector magnetic fields were reconstructed from the line-of-sight component under the assumption that the overlying magnetic field is the gradient of a potential. Panel a shows the phases, arg $\{C_\pm\}$, of the correlations. Panel b shows the difference in the phases plotted in Panel a. Panel c shows the amplitudes of the respective correlations.
Showerglass Amplitude

Line-of-Sight Magnetic Field

100 Mm

Figure 2. The acoustic showerglass for NOAA AR 8179. Panel a shows a map of the vertical magnetic field taken from SOHO/MDI on 1998 March 15 at 16:00 UT. Panel b shows a projected relief map of the egression correlation phase errors as prescribed by the curves plotted in Figure 1.

proxy of the ingestion-local amplitude correlation before the ingestion is computed. We then proceed as prescribed by equation (2) to make correlation maps of the subphotosphere.

A careful analysis comparing phase-correlation maps over a range of depths with and without the showerglass correction is under preparation for a forthcoming publication (Lindsey & Braun, 2002). The method used is an adaptation of that summarized above. Results at this point are preliminary, and we will only summarize them here. Correction of the acoustic showerglass in large active regions generally renders a considerable degree of fine detail in correlation maps up to 9 Mm in depth that is not substantially visible without the correction. Figure 3 shows an example: a showerglass corrected phase-correlation map of the egression-ingression correlation, $C$, of NOAA AR 8179 at depth 5 Mm. The active region signature at this depth is characterized by a strong seismic signature with a fairly sharp boundary spanning approximately 160 Mm from east to west and 65 Mm from south to north. The correlation between the $C$ and the overlying surface magnetic field is actually quite weak on a fine scale. The acoustic signature extends far outside of the sunspots the east and west ends of the active region, and otherwise offers minimal acknowledgment of their existence.

Phase correlation maps of the subphotospheres of isolated sunspots generally show a somewhat less horizontally extended anomaly than that of a large active region, with radii ranging from 20–30 Mm. However, this extension is invariably far outside the outer boundary of the penumbra of the sunspot. An

AR 8179

Correlation Real Part

Correlation Imaginary Part

Depth 5 Mm

Figure 3. Phase correlation map of AR 8179 at depth 5 Mm computed after application of showerglass phase and amplitude correction. The top frame shows a magnetograph of the region. The underlying two frames show the real and the imaginary parts of the correlation function, respectively, integrated over the frequency range 4.5–5.5 mHz and over the 24-hr period beginning on 1998 March 15, 11:00 UT.
example is seen to the isolated sunspot (NOAA AR 8185) north-east (above and to the left) of AR 8179 in Figs 3b and 3c.

Preliminary control computations to determine the signature of a superficial anomaly are accomplished by moving the showerglass correction to a region of quiet Sun and computing the correlation maps as before. The result is a signature that is discernible at 8.4 Mm, but far out of focus and diffuse. The relative sharpness of detail in the correlation map shown in Fig 3b suggests that a considerable part of the acoustic signature appearing in Figs 3b and 3c is due to sharply defined acoustic anomalies in the neighborhood of that depth.

Figure 4 shows egression-ingression correlation maps of NOAA AR 9169 at depth 8.4 Mm with and without the showerglass correction. The active region signature in Fig 4b shows a strong seismic signature with point-like condensations and other fine detail in the near periphery of a large sunspot at the east (right) side of the active region (lower middle of frame). These details are essentially invisible when the phase correction of the showerglass is removed (Fig 4c), and still further suppressed if the amplitude correction also is omitted.

1.4. Conclusions

It is evident that the effects of phase and amplitude errors introduced by strong surface magnetic fields degrade the coherence of acoustic radiation from subphotospheric acoustic anomalies. We have begun to explore the use of the photospheric magnetic field as a proxy for phase errors imposed by the acoustic showerglass. The proxy we have introduced here is relatively crude, but suggests that a realistic magnetic proxy is practical. It should be kept in mind that the assessment of the showerglass errors plotted in Figure 1 include anomalies that are submerged considerably beneath the photosphere along with the effects of the magnetic field at the extreme surface. The magnetic proxy presented here therefore removes not only the phase errors due to the showerglass but includes a sort of overall average of what lies beneath the showerglass when the phase errors are measured. At the same time, contamination of the pixel by magnetic regions during the error measurements may result in an underestimate of the actual errors. The function of the correction applied in this exercise is more rather that of replacing the showerglass with glass that is relatively flat but of an unknown thickness than that of removing the showerglass entirely. It simply makes it possible to see through the showerglass, thereby to reconstruct sharp images of underlying local structure that was otherwise obscured by the strong stochastic surface anomalies.

Figure 5 shows a plot of the phase of the correlation signature imaged in Figure 3 along the north-south direction in a strip 10 Mm wide that crosses

![Figure 4](image_url)

Figure 4. Correlation amplitude of AR 9169 at a depth of 8.4 Mm computed with and without the showerglass correction. Panel a shows a magnetogram of the region. Panel b shows the real part of the complex correlation amplitude, C(r, 8.4 Mm), integrated over the 4.5–5.5 mHz frequency band and over the 24-hr period beginning on 2000 September 23, 13:00 UT. Panel c shows the same with the phase correction of the showerglass removed. Panel c shows the correlation map with neither a phase correction nor an amplitude correction.
Figure 5. The phase of the correlation signature, $C'$, of AR 8179 along the north-south direction in a 10 Mm wide strip that crosses the sunspot on the east end of AR 8179. The left of the plot represents south of the sunspot, at abscissa zero, with the right representing north.

show considerable stochastic fine structure particularly underlying the peripheries of the sunspots. The acoustic signatures we find, together with published statistics of phase anomalies that characterize acoustic moats, are roughly consistent with a fractional sound-speed enhancement of order 1% over the depth range 3—9 Mm. This assessment should be regarded as tentative. A careful examination including further control work is needed to judge whether such an interpretation is realistic.

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CONSTRAINT ON PARTICLE PHYSICS FROM HELIOSEISMOLOGY

Hiromoto Shibahashi and Satoru Watanabe
Department of Astronomy, University of Tokyo, Tokyo 113-0033, Japan

ABSTRACT

Helioseismology used to be used to see what is the cause of the solar neutrino problem. Recent progress in experiments of neutrino detection showed, however, that neutrino oscillation accounts for this long-lived issue. This fact does not mean at all that helioseismology is not useful for elementary particle physics. Rather, combination of helioseismology and recent experiments of neutrino detection enables to give stringent constraint on some aspects of fundamental particle physics. We discuss what we should do and what we can do in this situation for elementary particle physics from the helioseismological side. We will demonstrate some examples; — limits on the neutrino parameters, the axion parameter, and the pp-nuclear cross section.

Key words: helioseismology; neutrino; axion; nuclear cross section.

1. INTRODUCTION

The Nobel Prize in Physics for this year, 2002, was awarded with one half jointly to the two pioneers of neutrino astronomy, Dr. Raymond Davis, Jr., and Dr. Masatoshi Koshiba, for their pioneering contribution to astrophysics, in particular for the detection of cosmic neutrinos. Detection of neutrinos from the sun has been desired to confirm that the energy source of sunshine is nuclear reaction occurring in the sun. The photons by which we can see the sun right now do not tell us the physical state of the present solar core, since the photons interact so frequently with matter in the sun that it takes $\sim 10^4$ years for them to reach the solar surface from the solar center. The only possible way to confirm that nuclear reactions are indeed occurring in the sun as being expected is to detect the neutrinos produced by nuclear reactions and to measure the neutrino fluxes from the sun, since neutrinos interact so little with matter, unlike photons, that they reach the earth only eight minutes after they are generated at the solar center. The fact that neutrinos hardly interact at all with matter is the source of difficulty of detection of neutrinos from the sun. Dr. Davis and his team constructed a completely new detector to overwhelm this difficulty, and succeeded in detecting neutrinos from the sun. With another new gigantic detector, Dr. Koshiba and his team also succeeded in detecting neutrinos from the sun, and they were also able to detect neutrinos from a distant supernova explosion, SN1987A (Koshiba 1987). The pioneering experiments led by Davis and by Koshiba to detect solar neutrinos have, however, surprisingly shown that the capture rate of solar neutrinos is substantially smaller than the theoretically expected number. This discrepancy is known as the solar neutrino problem, and has been one of the big problems concerning astrophysics and particle physics (for reviews, see, e.g. Bahcall & Davis 1982). Since various aspects of the theory of stellar structure and evolution, on which most of our understanding of the universe is based, are calibrated with the sun, the solar neutrino problem and the resultant questions about solar structure have been one of the big problems in astrophysics.

Neutrinos are generated in the sun mainly through three different nuclear processes in the pp-chain; i.e., the pp-neutrinos produced in the pp-I branch, the $^7$Be-neutrinos produced in the pp-II branch, and the $^8$B-neutrinos produced in the pp-III branch. Both of the neutrino detection experiments led by Davis and by Koshiba are sensitive mainly to $^8$B-neutrinos, of which generation rate is highly sensitive to the temperature near the solar center. It seemed that some slight reduction of the temperature near the solar center, keeping the solar photon luminosity, would be able to explain the discrepancy. Indeed, various attempts have been made along this line, but none of them has succeeded in solving the problem without any contradiction in the micro-physics or various observational data of the sun.

Helioseismology is a unique diagnostic tool of the invisible solar internal structure. With the development of the ground-based observation network (GONG) and the launch of a satellite (SOHO) dedicated to helioseismology, the eigenfrequencies of p-modes with a wide range of spherical degrees are now systematically measured with relative accuracies of the order of $10^{-5}$. With the combination of a wide range of the eigenfrequencies and spherical de-
degrees, we can now diagnose the invisible solar interior independently of the solar neutrino detection experiments. Helioseismology has been thus expected to be a useful key to solving the solar neutrino problem.

Recent progress in experiments of neutrino detection showed, however, that neutrino oscillation accounts for the solar neutrino problem. The apparent reduction of the solar neutrinos is now explained by transformation of the electron-neutrinos generated in the solar core to those of other kinds (tau-neutrinos and muon-neutrinos) as a consequence of interaction with electrons in the sun (MSW effect; Mikheyev and Smirnov 1986, Wolfenstein 1978). The SNO and Super-Kamiokande data clearly shows that the grand sum of the detected fluxes of all kinds of neutrinos is in agreement of the theoretical expectation and that the solution of the solar neutrino problem does not lie with the sun (Ahmad et al. 2001, 2002).

In this situation, what can helioseismology do concerning the neutrino problem? In this paper, we discuss what we should do and what we can do in this situation for elementary particle physics from the helioseismological side. We will demonstrate some examples; —limits on the neutrino parameters, the axion parameter, and the pp-nuclear cross section.

2. NEUTRINO OSCILLATION

According to the standard theory of particle physics, neutrinos are massless particles. If the mass is non-zero, however, it is expected that the mass-eigenstate of neutrinos is not identical with the flavor-eigenstate — that is, the apparent states of electron-neutrino, muon-neutrino, and tau-neutrino should be described as three different mixed states of the three mass-eigenstates. The change from one flavor-eigenstate to another may occur, being dependent on the parameters; this then is the phenomenon called neutrino oscillation. Neutrino oscillation was hypothetical for a long time, but its reality was recently justified at Super-Kamiokande (Fukuda et al. 1998) by the careful observation of neutrinos produced by cosmic rays in the terrestrial atmosphere.

Although the neutrino oscillation found in the atmospheric neutrinos cannot be applied to the solar case directly, as the original flavor-eigenstate is different, it seems unreasonable to reject the possibility of neutrino oscillation only in the case of the solar neutrinos.

How many electron-neutrinos generated in the sun are changed to other types of neutrinos is dependent on the energy of the neutrinos, the electron density profile in the sun, and the characteristics of neutrinos, which are the differences in the squared mass among the three mass-eigenstates and ‘the mixing angles’. The energy spectrum of the generated neutrinos and the electron density profile are well defined by the solar internal structure. However, the mass differences and the mixing angles are not predicted theoretically at this stage and they are treated as free parameters. Since the pp-, $^7$Be-, and $^8$B-neutrinos produced in the sun have different energy spectra, the reduction rates are different among these three kinds of neutrinos for a given set of the neutrino parameters. By comparing each of the currently running detection of solar neutrinos (Homestake, Super-Kamiokande, GNO, SAGE, and SNO) with the corresponding theoretical prediction, we can limit the allowed regions in the parameter space. The problem is then to find the common regions for these allowed regions. The physical cause of neutrino oscillation is evidently the non-zero mass of neutrinos. Hence, its impact is strong in particle physics as well as in other fields of astrophysics, such as cosmology.

The Super-Kamiokande (SK) experiment (based on the water-Cerenkov method) is sensitive not only to electron-neutrinos but also to muon-neutrinos and tau-neutrinos, but the sensitivity to the latter types of neutrinos is only $\sim 15\%$ of that to electron-neutrinos:

$$\phi_{SK} \simeq \phi(\nu_e) + 0.15\phi(\nu_{\mu,\tau}). \quad (1)$$

While SK tries to catch Cerenkov light emitted by electrons running in light water, $H_2O$, as a consequence of elastic scattering by neutrinos, the Sudbury Neutrino Observatory (SNO) uses heavy water, $D_2O$, to measure neutrino fluxes via the different ways in which neutrinos interact with the heavy water. One of the neutrino reactions in heavy water is the “charged current reaction” (CC), in which, as the electron-neutrino approaches the deuterium nucleus, a $W$ boson is exchanged and this changes the neutron in deuterium to a proton and the neutrino to an electron that emits Cerenkov light. Hence the neutrino flux measured in this way is that of electron-neutrinos only:

$$\phi_{SNO}^{CC} = \phi(\nu_e). \quad (2)$$

Another type reaction is the “neutral current reaction,” in which the deuterium nucleus is broken by neutrino hitting, and gamma-rays are emitted when the liberated neutron is captured by another nucleus:

$$\phi_{SNO}^{NC} = \phi(\nu_e) + \phi(\nu_{\mu,\tau}). \quad (3)$$

The third reaction is elastic scattering of electrons by neutrinos (the same as in the SK):

$$\phi_{SNO}^{ES} \simeq \phi(\nu_e) + 0.15\phi(\nu_{\mu,\tau}). \quad (4)$$

The SNO data is the grand sum of these three reaction processes. These reactions are different each other in energy sensitivity and in direction sensitivity, and hence the gross capture rate is decomposed into the three components. The first data release of SNO data were concerned with the ES and the CC reaction. The SNO data of elastic scattering are, however, less accurate than the SK data, as the total volume of water used in the SNO experiment is less than that used in SK and the integration time has not been long enough, hence the event number.
at SNO has been less than that at SK. Therefore, a combination of the SK data and the SNO CC data is used to estimate with a reasonable accuracy the original $^8$B-neutrino flux;
\[ \phi_\odot (\nu_e) \approx \phi_{\text{SNO}}^{\text{CC}} + (\phi_{\text{SK}} - \phi_{\text{SNO}}^{\text{CC}})/0.15. \]  
(5)

The result is in good agreement with the theoretically expected value based on the evolutionary model of the sun, and this news was sensationaly released as evidence for neutrino oscillation (Ahmad et al. 2001). Later, the SNO NC data was carefully analyzed and it was also sensationaly released to confirm the neutrino oscillation (Ahmad et al. 2002);
\[ \phi_{\text{SNO}}^{\text{NC}} \approx \phi_{\text{SNO}}^{\text{CC}} + (\phi_{\text{SK}} - \phi_{\text{SNO}}^{\text{CC}})/0.15. \]  
(6)

At this stage, the long-standing issue of the solar neutrino problem was eventually solved.

3. HELIOSEISMOLOGY AND THE NEUTRINO PROBLEM

Together with evidence for neutrino oscillation obtained from atmospheric neutrinos that are described in Section 2, the consistency of the SNO-SK combined data with the theoretical estimate of the neutrino flux strongly implies that the solar neutrino problem should be interpreted in terms of neutrino oscillation. The next thing to do is deduction of the neutrino parameters which determine the probability that neutrinos oscillate from one type to another, ‘mixing angles’ and masses, by comparing the theoretically expected values and the observed ones. In this situation, what can astrophysicists do concerning the neutrino problem? It should be noted here that we need a good solar model to derive the neutrino parameters since they are obtained by comparing the observed neutrino fluxes with the theoretical expectation. It should also be noted that the deviation in the sound-speed profile and in the density profile of the evolutionary solar models from the helioseismically determined profiles is larger than the errors in helioseismology. Therefore, it is expected that we can construct a solar model, by using helioseismic information efficiently, which matches the seismic data better than the evolutionary models.

3.1. Evolutionary solar model vs seismic solar model

Let us review how to construct the standard evolutionary solar models. The basic assumptions are

- the model is in hydrostatic equilibrium,
- the model is in thermal equilibrium,
- chemically homogeneous at zero age,
- no mass loss and no mass accretion during evolution,
- the updated microphysics is used in calculations of nuclear reaction, opacity, diffusion, and convection,
- abundance changes are caused only by nuclear reactions and diffusion,
- the present age of the Sun is $t_\odot = 4.5 \times 10^9$ yr,
- $M = 1 M_\odot$, $R = 1 R_\odot$, and $L = 1 L_\odot$ at $t = t_\odot$, and
- the initial abundance ratio is chosen so that the surface abundances at $t = t_\odot$ match the present abundances observed at the photosphere.

Among them, the first assumption and the values of the present solar mass, radius, and luminosity should be definitely accepted. In comparison with these, the other assumptions have less experimental support. For example, there is no way to justify observationally the assumed past history of the sun.

The luminosity and the surface abundances of the model are mainly dependent on the initial abundances of hydrogen $X_0$ and of heavy elements $Z_0$. The radius of the model is strongly dependent on the treatment of convection. However, the theory of convection is still incomplete and we have to introduce a free parameter to express the efficiency of convective energy transport, — usually the mixing length, $\ell$. In summary, three parameters ($X_0, Z_0, \ell$) are introduced and they are chosen so that the last two requirements listed above are satisfied. It can be said, therefore, that theoretical uncertainties are parameterized in constructing evolutionary models.

In the following, let us depart from the standard construction of a solar model and try to reconstruct a solar model by using only the experimentally well-measured quantities. These quantities are the mass $M_\odot$, the radius $R_\odot$, the photon luminosity $L_\odot$, and the sound-speed and the density profiles obtained from helioseismology. In summary, the assumptions in constructing a seismic solar model are as follows:

- $M = 1 M_\odot$, $R = 1 R_\odot$, and $L = 1 L_\odot$,
- the sound-speed profile $c(r)$ is that obtained from helioseismology,
- the density profile $\rho(r)$ is that obtained from helioseismology,
- the model is in hydrostatic equilibrium,
- the model is in thermal balance,
- the updated microphysics is adopted concerning the nuclear reactions, opacity, and the equation of state, and
- the envelope is chemically homogeneous, and $Z/X$ there matches the spectroscopically determined value.
Note that, in order to estimate the neutrino fluxes from the sun, we need the temperature and the chemical-composition profiles in the sun. This requires more than the sound-speed profile $c(r)$ and the density profile $\rho(r)$, both of which are primarily determined from helioseismology.

The basic equations for constructing a seismic solar model with the above assumptions are formally identical with those used in constructing evolutionary models. The only difference is that the independent variable used in the evolution calculation is $m$ which is a Lagrangian coordinate while the Eulerian coordinate $r$ is adopted in the present case:

$$dm/dr = 4\pi r^2 \rho,$$  
$$dP/dr = -Gm\rho/r^2,$$  
$$dL_r/dr = 4\pi r^2 \rho e,$$  
$$dT/dr = \begin{cases}  
-3k\rho L_r/(16\pi acT_3^2) & \text{if radiative} \\
(1-1/\Gamma_2)Td\ln P/dr & \text{if convective} 
\end{cases}$$

with the boundary conditions $m = 0$ and $L_r = 0$ at $r = 0$ and $m = M_\odot$ and $L_r = L_\odot$ at $r = R_\odot$, where the symbols have their usual meanings ($P$: pressure, $T$: temperature, $m$: mass inside the radius $r$, $L_r$: luminosity at the radius $r$, $e$: rate of nuclear energy generation, $\kappa$: opacity, $a$: Stefan constant, $c$: the speed of light, $G$: gravitational constant, $\Gamma_2$: adiabatic exponent).

In addition to these basic equations, we need auxiliary equations — that is, the equation of state, the equations for the opacity and for the nuclear reaction rates:

$$\rho = \rho(P,T,X_i),$$
$$\kappa = \kappa(P,T,X_i),$$
$$\varepsilon = \varepsilon(P,T,X_i).$$

It may be instructive to remember that, in constructing evolutionary models, the chemical composition profiles are given at each time step by following their temporal evolution

$$\partial X_i(t,m)/\partial t = (\partial X_i/\partial t)_{\text{nuclear}} + (\partial X_i/\partial t)_{\text{diffusion}}$$

assuming chemical homogeneity at zero age, $\partial X_i(t = 0,m)/\partial m = 0$, and then the basic equations can be solved as a closed system. A suspected instability and the resultant mixing may introduce an additional term in the right-hand side of equation (14), but such a term is ignored in the standard evolution scenario.

If we distinguish only hydrogen and helium separately as $X$ and $Y$, respectively, and treat all other elements collectively as heavy elements, $Z$, then the sound-speed and the density can be regarded as functions of the chemical composition, $X$ and $Z$, and any two other thermodynamical quantities such as $P$ and $T$:

$$c = c(P,T,X,Z)$$

$$\rho = \rho(P,T,X,Z).$$

It should be noted here that the sound-speed profile and the density profile in the solar interior have already been determined from helioseismology. Hence equations (15) and (16) inversely relate the hydrogen abundance, $X$, and the heavy element abundance, $Z$, at a given $r$ to the pressure, the temperature, the sound-speed, and the density; $X = X(P,T,c_{\text{inv}},\rho_{\text{inv}})$ and $Z = Z(P,T,c_{\text{inv}},\rho_{\text{inv}})$, where $c_{\text{inv}}(r)$ and $\rho_{\text{inv}}(r)$ denote the seismically determined sound-speed and density profiles, respectively. The opacity and the nuclear reaction rates are, in turn, given in terms of $(P,T,c_{\text{inv}},\rho_{\text{inv}})$ by equations (12) and (13), respectively. Thus all the variables appearing in the right-hand side of equations (7) – (10) can be expressed in terms of the variables in the left-hand side, and hence these equations are solvable. Note that in this way we obtain directly a model of the present-day sun. Note also that we do not need to make assumptions about the chemical composition profiles in the sun, but obtain the $X$, $Y$, and $Z$ profiles as a part of the solutions of equations (7) – (10).

The depth of the base of the convection zone, $r_{\text{conv}}$, is well determined from helioseismology. Following Takata & Shibahashi (1998), we shift the outer boundary from $r = R_\odot$ to $r = r_{\text{conv}}$ and required that the radiative temperature gradient matches the adiabatic temperature gradient there. This cutting off of the convective outer 30% of the sun has little effect on the solar interior where the neutrinos are generated. By setting the outer boundary at the base of the convection zone and treating only the radiative core, we do not need to worry about the treatment of convection.

In practice it is fairly hard to determine the $Z$ profile directly as outlined in the previous section, since the dependence of the equation of state on $Z$ is weak. Alternatively, we construct a series of solar models with various $Z$ profiles by imposing the constraint of the seismically determined sound-speed profile, and search among them for the model for which the density profile fits best with the seismically determined density profile $\rho_{\text{inv}}(r)$. If $Z(r)$ is given, $X$ is represented in terms of $(P,T,c_{\text{inv}},Z)$ by equation (15), and in turn the density, the opacity and the nuclear reaction rates are given as well in terms of $(P,T,c_{\text{inv}},Z)$, then equations (7) – (10) can be solved. The problem is changed to finding, among various seismic solar models, the model which minimizes $F = \int_0^{r_{\text{conv}}} (\rho_{\text{inv}}(r) - \rho_{\text{model}}(r))^2\sigma_r^2 dr$, where $\sigma_r$ is the standard error of the helioseismically determined density. In applying the recipe for the realistic seismic data we restrict ourselves to trying stepwise $Z$ profiles, varying the height with a step of 0.001; the step width $\Delta r/R_\odot$ is 0.1 except for the innermost step which is taken to be 0.2. Nevertheless the very important point is that even the $Z$ profile can also be deduced from helioseismology.

It should be noted here that the main source of the error is not the error in the helioseismically determined profile of the sound speed but that of the den-
sity, but the nuclear cross-section of the pp-reaction ($S_{11}$-factor). We can say therefore that, with the help of knowledge of microphysics used in stellar evolution, helioseismology works well to construct a very accurate model of the present-day sun.

3.2. Neutrino flux estimate based on seismology

Once a good seismic solar model is constructed, one can estimate the neutrino fluxes based on it. Figure 1 shows the theoretically expected neutrino fluxes based on the present seismic model for the gallium experiment (GALLEX, GNO and SAGE), the chlorine experiment (Homestake), and the $^8$B-neutrino (SK and SNO). In each group, the theoretical value based on the solar model is shown as the second left bar and the numerical value is given at its top with the units of SNU for the gallium and the chlorine experiments and $10^6$cm$^{-2}$s$^{-1}$ for the $^8$B-neutrino. (A SNU = solar neutrino unit - is defined to be $10^{-36}$ interactions per second per target atom.) The amount of contribution from each neutrino source is shown with different color tones. The error bars correspond to 1-$\sigma$ level. For reference, the theoretical estimate based on the evolutionary solar model by Bahcall et al. (2001) is shown as the left-most bar in each group. The values themselves are quite close to each other. However, it should be stressed again here that concepts of these two models are different — the seismic solar model is more directly constructed by using the experimentally well-measured quantities about the present-day sun. The error bars are apparently longer in the seismic solar model than the evolutionary model. As described previously, the main source of the error is the uncertainty in the $S_{11}$-factor. This is also true in the case of the evolutionary model; here, in order to keep the luminosity, the slight change in the $S_{11}$-factor can be compensated by changes in temperature and in density. On the other hand, in the case of the seismic solar model, since the density is constrained, the change in the nuclear reaction rate can be compensated only by changes in temperature. Hence, the uncertainty in the central temperature due to the uncertainty in the nuclear reaction rate is apparently larger in the case of the seismic solar model than the evolutionary solar model, and this leads to the apparently large error bars concerning the neutrino fluxes. In Figure 1, the observed fluxes are also shown. As concerns the $^8$B-neutrino flux measurements, the flux measured at SNO through the charged current reaction, $\phi_{^8\text{B,SNO}}$, and the fluxes measured at SK and SNO through elastic scattering of electrons are also shown. Furthermore, based on the MSW effect hypothesis, the original $^8$B-neutrino flux estimated from the combination of the SNO CC data and the SK data (see equation (5)) is also shown. As seen in Figure 1, the combined data of the SK and SNO is consistent with the seismic solar model, which justifies the assumption about the thermal equilibrium of the model. The next thing to do is deduction of the neutrino parameters which determine the probability that neutrinos oscillate from one type to another, ‘mixing angles’ and masses, by comparing the theoretically expected fluxes and the observed ones. The seismic solar model provided by helioseismology should also be used in this process.

4. CONSTRAINT ON THE PP-NUCLEAR CROSS SECTION

The nuclear fusion process of two protons producing a deuteron is the basis for the whole pp chain, but the rate of this primary reaction is too slow to be measured in the laboratory at relevant energies. The theoretically calculated nuclear cross section for this process, based on the theory of low-energy weak interactions, still has about 5% uncertainty. It is this uncertainty that is the main source of the uncertainty of the theoretical estimate of the neutrino fluxes based on the seismic solar model. As seen in Figure 1, the uncertainty of the observed $^8$B-neutrino flux (based of the combined data of SK and SNO) is only about a half of the theoretical uncertainty based on the seismic solar model. Hence, as far as we accept all the other micro-physics as they are, we can get a constraint on the uncertainty of the pp-nuclear cross section ($S_{11}$-factor) by a factor two.

5. AXION AND HELIOSEISMOLOGY

The axion is a light pseudoscalar particle introduced to solve the strong CP problem (Peccei and Quinn 1977, Weinberg 1978) and one of the most likely candidates for the dark matter. Therefore the axion is an extremely important particle for particle physics and astronomy. In the interior of the sun, blackbody photons can convert into axions in the fluctuating Coulomb fields of the charged particles in the plasma, $\gamma + (e^-, Ze) \rightarrow (e^-, Ze) + a$, and this reaction is called as the Primakoff process. Large number of current and proposed experiments are trying to detect thus-created solar axions (e.g., Avignone et al.)
1998, Moriyama et al. 1998). The axion interacts so weakly with other particles that it escapes freely from the sun once it is produced. Therefore it functions as an energy-loss mechanism.

The axionic energy-loss rate by the Primakoff effect can be written in the form
\[
\varepsilon_{\text{axion}} = 0.892 \times 10^{-3} g_{10}^2 T_7^7 \rho_2^{-1} F(\kappa^2) \text{ erg g}^{-1} \text{s}^{-1},
\]
\[
F(\kappa^2) = \frac{\kappa^2}{(2\pi^2)} \int_0^\infty dx \frac{x}{(e^x - 1)} \times \left[ (x^2 + \kappa^2) \ln(1 + x^2 \kappa^{-2}) - x^2 \right],
\]
\[
\kappa^2 \approx 8.28 \rho_2 T_7^{-3} (3 + X),
\]
where \( g_{10} \equiv g_{\alpha \gamma \gamma}/10^{-10} \text{ GeV}^{-1}, \ T_7 \equiv T/10^7 \text{ K}, \ \rho_2 \equiv \rho/10^2 \text{ g cm}^{-3}, \) and \( X \) is the mass fraction of hydrogen (Raffelt 1996). We calculate a series of seismic solar models following the recipe described in Section 3.1 by introducing the additional term given by equation (17) this time into the RHS of equation (9) with varying \( g_{\alpha \gamma \gamma} \), which determines the amount of axionic energy-loss. The models constructed in this way are naturally consistent with heliosismic data. We do not try to explain how these models with axionic energy-loss can be realized in the evolutionary process, and such an investigation is beyond our scope. Our purpose is to clarify what can be derived if we construct self-consistent solar models with axionic energy-loss, which are faithfully consistent with almost all observations except for the neutrino fluxes and free of severe subjective restrictions on the evolutionary history.

An increase in the axionic energy-loss must be compensated by an increase in the nuclear energy generation, which leads to an increase in the theoretically expected neutrino fluxes. An increase in nuclear energy generation is realized with increases in \( T \) and \( \rho \) near the center of the solar models. However, \( \rho \) of the seismic solar models is constrained by observations, so \( T \) should increase particularly. A higher \( T_{\text{core}} \) means a steeper temperature gradient, which is proportional to the opacity. A higher opacity is realized with a higher \( Z \). The density constraint and a non-uniform Z-profile make \( T_{\text{core}} \) of the seismic solar models very sensitive to \( g_{\alpha \gamma \gamma} \). Because the nuclear reaction rates are mainly controlled by \( T_{\text{core}} \), the theoretically expected neutrino fluxes of the models are very sensitive to \( g_{\alpha \gamma \gamma} \). Thus an increase in \( g_{\alpha \gamma \gamma} \) leads to an increase in the theoretically expected \( ^8\text{B} \) neutrino flux of the seismic solar model. Because a higher \( g_{\alpha \gamma \gamma} \) than a critical value makes the model’s neutrino flux too high to be consistent with that determined from SNO and SK, we can limit \( g_{\alpha \gamma \gamma} \). From the comparison of these neutrino fluxes, we set a limit \( g_{\alpha \gamma \gamma} < 4.0 \times 10^{-10} \text{ GeV}^{-1} \). This limit is about a factor of 3 improvement over the previous theoretical limit (Schlattl et al. 1999) and a more severe limit than the solar axion experiments (Avignone 1998, Moriyama 1998). Therefore this limit is the most stringent limit on solar axions.

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SOLAR TORSIONAL OSCILLATIONS: HELIOSEISMIC MEASUREMENTS VERSUS DYNAMO MODELLING

S. V. Vorontsov¹,², E. Covas¹, D. Moss³, R. Tavakol³

¹Astronomy Unit, Queen Mary, University of London, Mile End Road, London E1 4NS, UK
²Institute of Physics of the Earth, B.Gruzinskaya 10, Moscow 123810, Russia
³Department of Mathematics, The University, Manchester M13 9PL, UK

ABSTRACT

We compare the variations in the solar internal rotation ("torsional oscillations"), as measured with SOHO MDI and GONG data, with predictions of a nonlinear axisymmetric mean-field dynamo model in a spherical shell, in which the only nonlinearity is due to action of the Lorentz force on the angular velocity. The overall picture of the torsional oscillations reveals a number of striking similarities between the model predictions and the seismic observations.

Key words: Sun:interior – Sun: oscillations.

1. INTRODUCTION

Accurate measurements of the rotational splittings of solar p modes are now available from SOHO MDI and GONG projects for more than a half of a solar activity cycle. They allow, with a proper inverse analysis, inference of variations in the solar internal rotation—the so-called solar torsional oscillations (Howe et al 2000; Antia & Basu 2000; Vorontsov et al 2002a,b; and references therein), first discovered as migrating bands of faster and slower rotation at the solar surface by Howard and LaBonte (1980).

We compare the helioseismic measurements with torsional oscillations produced in a nonlinear axisymmetric mean-field dynamo model, in which the only nonlinearity comes from the direct action of the dynamo-generated magnetic field on the azimuthal component of the velocity (Moss & Brooke 2000; Covas et al 2000; Tavakol et al 2002). The preliminary results of such a comparison were quite encouraging (Vorontsov et al 2002c). Here we extend the previous analysis by including a plausible radial density stratification in the dynamo model, which gives torsional oscillations with more realistic amplitudes.

In the helioseismic inversion, we analyze 30 individual 72d data sets using an 18th degree fit to the frequency splittings (Schou 1999) of the SOHO MDI measurements now available from 1 May 1996 to 22 August 2002, and of consecutive 36d subsets of classical GONG data with a 10th degree fit, covering a 6 yr period from May 1995 to May 2001. The inversion is performed with the adaptive regularization technique described in Strakhov and Vorontsov (2001). We now supplement the 2D inversion for the rotation as a function of depth and latitude, applied separately to the consecutive time “frames”, with a constrained 3D inversion, using time as a third variable, and with the time dependence approximated by a harmonic function with an 11 yr period.

Throughout this paper, we use a single particular run of the numerical dynamo simulation with parameters calibrated to produce the 22 yr periodicity in the butterfly diagram, and torsional oscillations which possess reasonably large surface amplitudes. We note however that similar behaviour is produced for a range of model parameters.

Figure 1. Variation of the solar internal rotation relative to the solar minimum, measured from SOHO MDI data (6 years, top), and as predicted by the model (11 years, bottom).
2. THE MODEL

The model employed here (see Moss and Brooke, 2000; Tavakol et al 2002 for details; also Covas et al 2002) is an axisymmetric mean-field dynamo model, with the standard mean-field dynamo equation for the magnetic field \( \mathbf{B} \), in which the velocity field is taken to be of the form \( v = v_0 + v' \), where \( v_0 = \Omega_0 \tau \sin \theta \) and \( \Omega_0 \) is given by an interpolation on the MDI data obtained from 1996 to 2002 (Vorontsov et al 2002b). The component \( v' \) satisfies the Navier-Stokes equation and the source of nonlinearity in the problem is the nonlinear action of the azimuthal component of the Lorentz force of the dynamo generated magnetic field on \( v' \).

In our calculations, the region of the numerical calculations was \( 0.64 \leq r \leq 1 \). The model possessed radial dependence in both the unquenched \( \alpha \) term and the turbulent diffusion coefficient \( \eta \): the \( \alpha \) profile was chosen in the form \( \alpha = \alpha_r(r) \sin^2 \theta \cos \theta \), where \( \alpha_r = 1 \) for \( 0.7 \leq r \leq 0.8 \) with cubic interpolation to zero at \( r = 0.64 \) and \( r = 1 \). The \( \eta \) profile was chosen in order to take some cognizance of the likely decrease of \( \eta \) in the overshoot region, by allowing a simple linear decrease by 50% from \( r = 0.8 \) to a constant value in \( r < 0.7 \).

The model parameters employed were the following, with the convention that \( \alpha_r > 0 \): \( R_\alpha = -4.5 \), \( R_\eta = 52500 \), the turbulent Prandtl number \( P_\tau = 1.5 \) and the ratio of the densities at the top (\( r = 1 \)) and the bottom (\( r = r_0 \)) of the computational domain \( \rho_{\text{rat}} = 10000 \). The outer boundary condition was semi-open and the inner boundary condition was of an overshoot type (see Tavakol et al 2002 for details).

We have checked that the type of behaviour reported here is robust and does occur for ranges of parameters of the model. In particular it remains qualitatively the same with or without density stratification, with open or semi-open outer boundary conditions, and with different \( \alpha \) and \( \eta \) profiles and values of the parameters \( R_\alpha \), \( R_\eta \) and \( P_\tau \).

3. 2D INVERSION

Figure 1 compares the model predictions with variation in the solar internal rotation obtained from the SOHO MDI measurements. The 2D inversion was applied to each individual 72d data set. To reduce the effects of systematic errors, we employed differential inversion, first subtracting the rotational splitting coefficients of the first year of observations (360d data set). The variation of the internal rotation is thus measured relative to the first year, which is centered near the solar activity minimum. Here and below, the angular velocity is represented in the inversion by polynomials of degree 80 in acoustic radius and degree 9 in \( \cos^2(\theta) \), the degree range in angular coordinate being governed by the degree of the polynomial fit to the multiplet splittings.

Two branches of zonal flows are clearly seen in both the solar rotation and in the model prediction, and these propagate from the mid-latitudes towards the poles and towards the equator.

To address the oscillation pattern in the deeper interior, Fig. 2 shows a similar comparison, but for the variation of the rotation rate from minimum to maximum of the solar activity cycle. In order to improve the signal to noise ratio, the MDI data were averaged over one year (five 72d data sets). The entire convective envelope, down to the bottom, is apparently involved in the oscillations, in both the model prediction and in the seismic data. The model results exhibit however richer and more complicated behaviour, with prominent flows in opposite directions around the base of the convection zone. No similar features are seen in the solar inversion; however, we cannot claim definitely that they are not there. In seismic inversions with noisy data, these features could well be hidden below the noise level.

The inversion technique which we apply is based on the iterative minimization of the residuals, which is mathematically equivalent to consecutive descents with conjugate gradients. The number of iterations \( k \) plays the role of a regularization parameter. With a proper choice of the 2D polynomials which describe the rotation profile, the response of the solution to the random noise in the data is set to be nearly uniform over all the meridional plane in the first iterations (Strakhov & Vorontsov 2001; just a few iterations appear to be enough when using solar data due to the relatively small signal-to-noise ratio).

When going to a larger number of iterations (relaxing regularization), the real features in the solution develop gradually (and then saturate) in regions where they are not overwhelmed by the noisy component of the solution, which grows rapidly with \( k \). Since the sensitivity of seismic data to the internal rotation decreases rapidly when going to greater depths, the real features of the solution in the deeper interior may appear with their amplitudes significantly
4. CONSTRAINED 3D INVERSION

The variation of the internal solar rotation with time, as seen from the consecutive 2D “frames”, appears to follow with quite adequate accuracy the 11 yr harmonic behavior (Vorontsov et al. 2002a,b). We indeed have some interesting systematic signatures of anharmonicity in the near-surface layers, which might be attributed to the 3rd harmonic of the 11 yr period, but their magnitude is an order of magnitude smaller than the dominant variation. In the torsional oscillations predicted by the dynamo modelling, any deviations from the harmonic behaviour are undetectably small.

The assumption of purely harmonic behaviour of the 11 yr torsional oscillations can be used by the seismic inversion as additional a priori information, to improve the stability of the results to random errors in the rotational splitting measurements. In this way, we arrive at a single global inversion using all the data collected over the six years as input. Such a constrained 3D inversion, with time as a third variable, was performed with both the solar and the artificial data. In the artificial inversion, the torsional oscillations predicted by the dynamo model were used to mimic the rotational splittings of each mode in each individual data subset, adding the Gaussian noise corresponding to the reported observational errors of individual measurements.

The amplitudes of the torsional oscillations in the meridional plane, inferred from the SOHO MDI data as well as from the artificial inversion, are shown in Figs 5 and 6. The solar inversion (Fig. 5) is shown with $k = 7$ iterations (as in the artificial inversion), and also with $k = 10$, when the regularization was intentionally overrelaxed to address the behaviour of the solution when it is allowed more freedom. The artificial inversion (Fig. 6) helps us to investigate the visibility of the torsional oscillations in the seismic data: when approaching the base of the convection zone, their amplitude becomes significantly underestimated by the inversion (cf Fig. 4).

A more adequate comparison of the seismic inversion with theoretical predictions thus necessarily involves using an artificial inversion. The results of two artificial inversions are shown in Figs 3 and 4. In the first simulation presented, the artificial rotational splitting coefficients (of the same modes as available in the 360d MDI data set) were inverted without adding any noise, in order to investigate the inherent spatial resolution of the seismic inversion. The small-scale features below the base of the convection zone at high latitudes appear to be smeared away, and rotation near the pole distorted, due to insufficient angular resolution (9 polynomials in $\cos^2 \theta$). Otherwise, the inversion looks nearly perfect. When random noise is added to the artificial splittings to mimic the observational errors (Fig. 4), the result is very different. All the prominent features below the base of the convection zone appear to be hidden below the noise level; we can not even detect the boundary where variations drop abruptly to zero ($0.64 R_\odot$, the inner boundary of the dynamo-modelling domain).
Variations with time and latitude not far below the solar surface (at $r = 0.98R_\odot$) are shown in Fig. 7. In the artificial inversion (the two lower panels), the zonal flows are substantially distorted near the poles due to insufficient angular resolution (cf Figs 3 and 4). Otherwise, the artificial inversion appears quite adequate (this test was targeted essentially at the ability of the global inversion to reproduce the torsional oscillations when having only 6 years of rather noisy data). The low-latitude zonal flows seen in the inversion with real data (top panel) are in good agreement with the model predictions, including the prominent modulation pattern; the only noticeable difference is their slightly smaller amplitude. The high-latitude flows, however, differ significantly: in the model, their slope is nearly the same as that of low-latitude flows, while in the solar inversion it is much steeper.

Radial cuts into the deep solar interior at $20^\circ$ latitude are illustrated in Figs 8–10. The results are shown for two different values of the regularization parameter $k$, and for the inversions using both SOHO MDI and GONG data. The solar torsional oscillations apparently penetrate all the way (or almost all the way) down to the bottom of the convection zone. The torsional oscillations produced by the model (lower panel of Fig. 10) have nearly the same (slightly smaller) amplitudes. The slope of the zonal flows in the time-depth coordinates is opposite in the solar inversion and in the model, but as with other particular features, this responds to the particular tuning of the parameters of the dynamo simulation. The artificial inversion (Fig. 10) indicates that the prominent reverse flow, which is predicted by the model just below the base of the convection zone, is probably marginally detectable in the seismic inversion with the data currently available. There is an impression that a similar reverse flow is seen in the solar inversions (Figs 8 and 9), but it is a matter of speculation whether or not it is a real feature. If so, it appears to be more pronounced in the inversion with the GONG data. A rather modest improvement of data quality (or the length of the observations) is needed to answer this question.
Figure 8. The variation of the internal rotation versus time and depth at 20° latitude, obtained from the SOHO MDI data with $k = 7$ iterations (top) and with $k = 10$ (bottom). Contour lines are shown at 0.5 nHz intervals.

Figure 9. The same as Fig. 8, but from the GONG data inverted with $k = 5$ (top) and $k = 7$ iterations.

Figure 10. The same as Fig. 8, but for the artificial inversion. The bottom panel shows the exact variations in the model. The artificial set of the rotational splitting coefficients mimic the SOHO MDI data.
5. CONCLUSIONS

Despite all the simplifying assumptions of the axisymmetric mean-field dynamo model used here, the nonlinear simulations predict solar torsional oscillations which appear to be in good general agreement with helioseismic measurements. The features in common include the two branches of zonal flows propagating with growing amplitude in opposite direction from the mid-latitudes; the prominent amplitude modulation of the low-latitude flows; and the fact that torsional oscillations penetrate all the way down to the bottom of the convection zone. These agreements suggest that helioseismic measurements now provide quite rich direct observational constraints for improving our understanding of the solar MHD dynamo.

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Helioseismic Imaging

Chair: B. Fleck
SUPERGRANULATION SUPPORTS WAVES

L. Gizon¹ and T. L. Duvall, Jr.²

¹W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA
²Laboratory for Astronomy and Solar Physics, NASA/GSFC, Greenbelt, MD 20771, USA

ABSTRACT

Supergranulation on the surface of the Sun is a pattern of horizontal outflows with a distinct scale of 30 Mm and an apparent lifetime of 1 day, outlined by a network of small magnetic features. The dynamics of the supergranulation is poorly understood and there is as yet no explanation for the observation that the supergranular pattern appears to rotate faster than the magnetic features. In this paper we show that supergranulation undergoes oscillations and supports waves with periods of 6-9 days. The nature of supergranulation appears to be travelling-wave convection. Waves are predominantly prograde, which explains the apparent superrotation of the pattern. We also show that supergranular flows have a net kinetic helicity, which is negative in the northern hemisphere.

Key words: supergranulation, convection, magnetic fields, helioseismology.

1. INTRODUCTION

Convective-like motion on the solar surface consists of two main components: granulation and supergranulation. Granules, with a typical size of 1.5 Mm, are well understood as a convective phenomenon and can be studied with realistic numerical simulations (Stein & Nordlund, 2000). Supergranules, however, have remained puzzling since the early observations of Hart (1954). Hart (1956) described local variations with a spatial periodicity of 26 Mm and an rms velocity of 0.3 km s⁻¹, but rejected a convective instability as an explanation on the basis that the scale of the phenomenon was too “large”. In a classic paper, Leighton et al. (1962) reported “large cells of horizontally moving material distributed roughly uniformly over the entire solar surface” that are outlined by the chromospheric network. It is worth citing the physical description of supergranulation given by Simon & Leighton (1964) nearly forty years ago, as it summarizes the current paradigm:

“This observed cellular flow pattern of the large-scale motions is strongly suggestive of convective motions, and we tentatively propose that we are dealing with a larger scale version of the familiar photospheric granulation – a supergranulation. The observed dimensions of the large cells (32000 km) suggest that they originate in the Sun’s convective envelope which extends from the bottom of the photosphere to depths of 5000-10000 km. Although the lifetime (20 h) of the supergranulation appears very long at first glance, if one considers the small velocity and the large dimensions one finds that the matter in all probability does not circulate more than once. This fact, coupled with the very irregular size and shape structure of the cells, suggests that we are observing an example of non-stationary convection, rather than the stationary laminar convection of classical Bénard cells. Since we are able to observe the supergranulation only from above, we have no means of determining the geometrical shape of the convection pattern; it may be either a circulating flow or a columnar convection such as a cloud or plume... In view of these considerations it seems proper to ask whether there is a possible mechanism in the Sun which would select cell depths of roughly 5000 km. A possible answer lies in the ionization zones of either neutral He atoms or singly ionized He⁺, or perhaps both.”

“The observed horizontal motions provide a mechanism for building up relatively strong fields in a narrow network pattern, as is observed. These magnetic channels in turn suggest an explanation for the origin of the Ca II emission network... Magnetic fields would tend to be swept to the cell boundaries by the horizontal currents, and concentrate there in strengths several times greater than the average field.”

This description makes a lot of sense, although some points still need confirmation. Despite several studies (e.g. Simon & Weiss, 1968) it remains to be shown that a convective instability due to the recombination of ionized Helium is the origin of the distinct supergranular scale. The depth of the supergranulation layer is largely unknown. It has been suggested that the properties of convective motions in a highly stratified atmosphere may imply that supergranules are a deep phenomenon, with depths in excess of their horizontal diameters (Parker, 1973). There have been a number of studies related to the...
Figure 1. Near-surface horizontal flows inferred from f-mode time-distance helioseismology (Gizon et al., 2000). (a) An 8-hr average of MDI Dopplergrams in the high-resolution field of view. The arrow points toward disk center. (b) Line-of-sight projection of the inferred horizontal velocity field (obtained by inverion of f-mode travel-times). It is assumed that vertical flows are small. (c) Horizontal divergence of the inferred flow showing supergranular cells. (d) Scatter plot of (b) versus (a). The line with slope 0.8 is a fit assuming equal errors in both coordinates. The correlation coefficient between the inferred and measured line-of-sight velocity is 0.7.

The influence of supergranular flows on magnetic fields. It has been shown that a stationary cellular flow tends to expel the magnetic field from the regions of fluid motion and concentrate the flux into ropes at the cell boundaries (Parker, 1963; Galloway et al., 1977; Galloway & Weiss, 1981). For obvious reasons of simplicity, analytical or semi-analytical studies often picture supergranulation as laminar convection, although, as noted above by Simon & Leighton (1964), the flows must be highly turbulent and non-stationary. Recent numerical simulations of stratified convection at high Rayleigh number have revealed a very complex picture. It is now accepted that heat and momentum transport in solar-like turbulent convection is controlled by a network of coherent cyclonic plumes sinking from the thermal boundary layer (Julien et al., 1996; Brummell et al., 1996). The dynamics of individual plumes is dominated by strong vortex-vortex interactions with neighboring plumes (Julien et al., 1996). In these proceedings, Rast (2002) claims that the scale of supergranulation has its origin in the interaction and merging of individual granular plumes (see also Pioner et al., 2000). A somewhat related model was proposed earlier by Rieutord et al. (2000, 2001) whereby supergranulation is the result of a nonlinear large-scale instability of the granular flow, triggered by exploding granules. In both these models, supergranulation is not a proper scale of thermal convection, and the depth of supergranulation is determined by how deep network plumes can remain stable (Rast, 1997). Realistic numerical simulations will be crucial in understanding the nature of supergranulation. Yet, the solar convection zone is so highly turbulent and stratified that numerical modeling at supergranular scales has remained elusive. One of the most promising calculations is due to DeRosa et al. (2002).

On the observational side, the original work of Leighton and coworkers has been refined. A variety of methods have been used to characterize the distribution of the cell sizes. A characteristic scale can be obtained from the spatial autocorrelation function (e.g. Hart, 1956; Simon & Leighton, 1964; Duvall, 1980), the spatial Fourier spectrum (e.g. Hathaway, 1992; Beck, 1997), and segmentation or tessellation algorithms (Hagenaar et al., 1997). Although definitions vary, average cell sizes are in the range 15-30 Mm. The topological properties of the pattern have been studied by Schrijver et al. (1997). It is unclear whether there is a variation of cell sizes with latitude: Rimmele & Schröter (1989) and Komm et al. (1993a) report a possible decrease with latitude, Berrilli et al. (1999) an increase, and Beck (1997) no significant variation. The typical lifetime of the supergranular/chromospheric network is found to be

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in the range 1-2 day (e.g. Rogers, 1970; Worden & Simon, 1976; Duvall, 1980; Wang & Zirin, 1989). An important observation by Beck & Duvall (2001) shows that the temporal autocorrelation of the supergranulation pattern does not follow a simple exponential decay with time but becomes negative after a few days. Thus e-folding lifetimes may be misleading (we will come back to this point later). The typical rms horizontal velocity of supergranular flows is known to be about 0.3 km s$^{-1}$ (e.g. Hathaway et al., 2000). However, the vertical component of the flows has been extremely difficult to measure (e.g. Giovaneli, 1980) or infer (November, 1989). Miller et al. (1984) caution that Doppler velocity measurements at the cell boundaries may be polluted by the network field. The best estimate is perhaps due to Hathaway et al. (2002) who find that the vertical flows have speeds of about 10% of their associated horizontal flows or about 30 m s$^{-1}$, although the topology of the vertical flows is largely unknown. Perhaps even more difficult to measure are the related temperature fluctuations. Observers have searched for the thermal signature of a convective process, i.e. rising hot material at the cell centers and sinking cool material at the cell boundaries. Unfortunately, answers vary too widely (see Lin & Kuhn, 1992, and references therein).

Local heliososseismology opens prospects for mapping the structure of supergranular flows below the surface. It is estimated that the correlation between internal flows and surface flows switches sign at depths of 5-10 Mm (Duvall, 1998; Zhao, 2002; Braun & Lindsey, 2002), suggesting the existence of a "return flow" below these depths. Duvall & Gizon (2000) have used time-distance heliosismology and surface gravity waves (f modes) to probe flows at a depth of 1 Mm. The interpretation of f-mode travel times is relatively straightforward as f modes propagate horizontally. Furthermore, the horizontal flows inferred from this technique can be assessed by direct comparison to the line-of-sight Doppler images at the photosphere (Fig. 1; Gizon et al., 2000).

Duvall (1980) showed that the rotation of the supergranular pattern is faster than the photospheric plasma by approximately 5%. This anomaly has since been confirmed by Snodgrass & Ulrich (1990) who used a local correlation tracking method with a time-lag of $\Delta t = 24$ hr. Figure 2 shows the apparent rotation and meridional circulation that we obtained as a function of $\Delta t$. The rotation rate increases rapidly with $\Delta t$ and the meridional circulation is very small. It has been speculated (e.g. Corbard & Thompson, 2002) that the fast rotation of the pattern may be a consequence of convective cells deeply rooted in the shear layer below the surface (rotation increases inward at low latitudes). However, Beck & Schou (2000) remark that rotation measured from correlation tracking ($\Delta t = 24$ hr) is faster than the rotation of the solar plasma measured at any depth in the interior. It is especially puzzling that the rotation of the magnetic network (Komm et al., 1993b) is also less than that of the supergranular pattern, since magnetic fields are expected to be strongly advected by supergranular flows. To date, a definitive explanation of the excess rotation speed of the supergranular pattern has not been presented.

In this paper, we study a long time series of flow maps of the solar supergranulation, obtained from f-mode time-distance heliosismology. These data enable us to characterize, for the first time, the dynamics of the supergranulation. We find that the power spectrum appears to be consistent with a spectrum of traveling waves (Gizon et al., 2002). We also study the vertical vorticity at supergranular scales and confirm the detection of a Coriolis effect (Duvall & Gizon, 2000).
2. SUPERGRANULAR WAVES

Turbulent solar convection is commonly characterized by an autocorrelation function that exhibits an exponential decay in time (Harvey, 1985; Kuhn et al., 2000), leading to a power spectrum that is a decreasing function of temporal frequency. That supergranulation may not follow such a simple model has been hinted at previously. For example, the observed autocorrelation function becomes negative after 60 hr (Beck & Duvall, 2001) and new supergranular cells appear to form near the boundaries of decaying cells (Shine et al., 2000). The negative excursion of the autocorrelation function leads to a power spectrum peaked at a low non-zero frequency (Duvall & Gizon, 2000), suggesting an underlying long-range order.

To study supergranulation, we use a 60-day sequence of Doppler velocity images obtained in 1996 by the Michelson Doppler Imager (Scherrer et al., 1995) on board SOHO. MDI full-disk Dopplergrams are observed at a one-minute cadence with a spatial sampling of 0.12" at disk center. Images are tracked at the Carrington angular velocity ($\Omega_C = 2.87$ $\mu$rad s$^{-1}$) to remove the main component of solar rotation. We apply the techniques of time-distance helioseismology (Duvall et al., 1993) to obtain every 12 hr a 120° x 120° map of the horizontal divergence of the flows in a 1 Mm-deep layer beneath the surface (Duvall & Gizon, 2000). Unlike raw Doppler images, the divergence signal has uniform sensitivity across the solar disk and is subject to few systematic errors. Supergranules appear as cellular patterns of horizontal outward flow in the

Figure 3. Power spectrum of the supergranulation signal near the equator ($\lambda = 0^\circ$). Cuts are shown at constant wavenumber $k = 120/R$ where $R$ is the solar radius. (a) The thick line is the power spectrum versus frequency, $\nu$, for $k = (k_0, 0)$ pointing in the direction of solar rotation, $\psi = 0$ (West). There are two peaks at frequencies $\nu_-$ and $\nu_+$. The frequency resolution is given by the power spectrum of the temporal window function (thin line). (b) Cylindrical cut, $P_k(\nu, \psi)$, in the power spectrum at constant $k$ versus $\nu$ and the direction of $k$, $\psi$. By construction, $P_k(\nu, \psi) = P_k(-\nu, \psi - \pi)$. Power peaks in two ridges at frequencies $\nu_-(\psi)$ and $\nu_+(\psi)$. For each $\psi$, we measure $\nu_\pm$ by fitting the sum of two independent Lorentzian functions to the power. The fits take into account the convolution by the window function. The sinusoidal variation of $\nu_\pm$ with $\psi$ is due to advection by a background flow $u = (u_x, u_y)$. The double lines show the fit $\nu = \pm\nu_0 + (k u_x \cos \psi + k u_y \sin \psi)/2\pi$ to $\nu_\pm(\psi)$, where $\nu_0$ is a constant frequency. At the equator we find $u = (43, 0)$ m s$^{-1}$. The velocity $u_x$ is measured in a frame co-rotating with the Sun at the Carrington rotation rate. (Gizon et al., 2002)
The divergence maps are obtained by measuring the time it takes for solar $f$ modes to propagate from any given point on the solar surface to a concentric annulus around that point. The difference in travel times between inward and outward propagating waves is a proxy for the local horizontal divergence of the flow field. Images were interpolated onto Postel’s azimuth equidistant projection (Pearson, 1990) centered at latitude $0^\circ$ and Carrington longitude at image center. Data cubes go through a three-dimensional Fourier filter to isolate the $f$-mode ridge and cut off the power below 2 mHz (supergranulation noise). The temporal signal at a given pixel is cross-correlated with the signal in a concentric annulus of thickness $0.12^\circ$. The cross-correlation function contains information about waves propagating outward and inward from the central pixel depending on the sign of the correlation time lag. To enhance the signal, cross-correlations are averaged on a $2 \times 2$ grid of origins, corresponding to a spatial sampling of $0.24^\circ$ at image center. Travel times for inward and outward propagating waves are measured by fitting a Gaussian wavelet to the cross-correlations. Travel-time differences are then averaged for a range of annuli (mean radius 15 Mm). The divergence maps are finally interpolated onto a Carrington longitude-latitude grid with a resolution of $0.24^\circ$ in both coordinates.

For any given target latitude, $\lambda$, we extract a longitudinal section of the data $10^\circ$ wide in latitude centered about $\lambda$. The divergence signal is Fourier transformed in three dimensions to make power spectra as a function of frequency, $\nu$, and horizontal wavevector, $k = (k_x, k_y)$, where $k_x$ and $k_y$ are in the East-West and South-North directions respectively. In cylindrical coordinates, $k$ is uniquely specified by its magnitude, $k$, and its direction, $\psi$, such that

$$k_x = k \cos \psi$$

and

$$k_y = k \sin \psi .$$

Figure 3 shows cuts in the equatorial power spectrum at a constant $k$ typical of the supergranulation. For each azimuth $\psi$, the power has two broad peaks at frequencies $\nu_+$ and $\nu_-$ (Fig. 3a). No Galilean transformation can cause these peaks to coalesce, at zero frequency or otherwise. This implies that the supergranulation undergoes oscillations.

For each azimuth, we measure the frequencies $\nu_+$ and $\nu_-$ by fitting the sum of two Lorentzian functions to the power (Fig. 3b). Observations show that the difference $\nu_+ - \nu_-$ is essentially independent of azimuth, and $\nu_\pm$ have a sinusoidal dependence with $\psi$ of the form:

$$\nu_\pm = \nu_0 + \nu_1 \cos(\psi - \psi_0) .$$

We interpret $\nu_1$ to be a Doppler frequency shift,

$$\nu_1 = k \left| \mathbf{u} \right| / 2 \pi ,$$

produced by a horizontal background flow $\mathbf{u}$ pointing in the direction $\psi_0$, as one does in helioseismological ring analysis (Schou & Bogart, 1998). The nearly linear relationship measured between $\nu_1$ and $k$ in the range $40 < kR < 180$, where $R$ is the solar radius, is consistent with this interpretation. The latitudinal dependence of $\mathbf{u}$ is shown in Figure 4. The inferred rotation (Fig. 4a) and meridional circulation (Fig. 4b) are both remarkably similar to that of the small magnetic features (Komm et al., 1993b,c). This property is consistent with the view

\[ Figure 4. Flows, $\mathbf{u}$, inferred from the advection of the supergranulation spectrum versus latitude, $\lambda$. (a) Flow in the direction of solar rotation, $u_r$ (black solid line). The gray line shows the rotation of the small magnetic features (Komm et al., 1999b) and the dashed line is for the photospheric rotation (Snodgrass & Ulrich, 1990). The dotted line shows the pattern rotation obtained by tracking supergranular features with a 24 hr delay, in agreement with an earlier measurement (Snodgrass & Ulrich, 1990). (b) Northward meridional flow, $u_y$ (black solid line). The meridional flow of the magnetic features from Komm et al. (1998c) (gray) is again similar. The dotted line shows the anomalous results obtained by tracking the supergranulation pattern with a 24 hr delay. (Gizon et al., 2002) \]
that magnetic fields are advected by supergranular flows.

The dynamics of the supergranulation are best studied once the background flow, \( \mathbf{u} \), has been removed. In a co-moving frame, each spatial component oscillates at a characteristic frequency \( \nu_0 \). We find a clear relationship between \( \nu_0 \) and the wavenumber \( k \), well described by a power law (Fig. 5a). This is a fundamental relationship as it is measured to be independent of both \( \psi \) and \( \lambda \). The data are consistent with a spectrum of travelling waves with a dispersion relation \( \nu = \nu_0(k) \). The waves have a rather low quality factor, as can be seen in the azimuthally averaged power spectrum (Fig. 5b). The shape of the power spectrum is described accurately by the sum of two Lorentzian functions. From the measured line widths (Fig. 5a) we find that the lifetime of supergranules is about 2 days at \( kR = 100 \).

Since \( \nu_0 \) and the dominant size (cf. Sect. 1) of supergranules are observed to be essentially independent of latitude, the general dynamics determining the time scale and the spatial scale of supergranulation is not affected by the Coriolis force associated with the large scale vorticity (rotation). We observe, however, a pronounced anisotropy in the azimuthal distribution of wave power at fixed \( k \) (Fig. 6a). The power is maximum in the direction of rotation and toward the equator in both hemispheres (Fig. 6b). The pattern therefore senses the effect of rotation. A snapshot of the divergence field would not reveal this as the sum of the powers measured in opposite directions is nearly isotropic (Fig. 6a); the vorticity field, on the other hand, is slightly sensitive to the effect of the Coriolis force as we will show later.

As mentioned in the introduction (cf. Beck, 2000), earlier estimates of supergranulation rotation (Duvall, 1980; Snodgrass & Ulrich, 1990), obtained by tracking the supergranulation pattern from one image to the next, were systematically found to be higher than the rotation of the magnetic network (Fig. 4a). This apparent super-rotation of the pat-
tern can now be understood since waves are predominantly prograde. The East-West motion of the pattern is effectively a power-weighted average of the true rotation and the non-advective phase speed $v_p = 2\pi v_0 / k \sim 65$ m s$^{-1}$. Similarly, the excess of wave power toward the equator is reflected in the meridional motion of the pattern (Fig. 4b).

We have shown that supergranulation displays a high level of organization in space and time. Perhaps this order has its origin in the network of coherent cyclonic plumes that controls solar-like turbulent convection. The prograde excess of wave power is most likely due to the influence of rotation that breaks the East-West symmetry, allowing for new instabilities to propagate. Recent numerical simulations of solar convection (Miesch et al., 2000) show patterns that move prograde relative to the local rotation at low latitudes, and may help explain the observations. Convection in oblique magnetic fields (Hurlburt et al., 1996) also exhibits solutions that take the form of travelling waves, where the tilt of the convection cells, their wave speed, and direction depend on the strength and obliquity of the field. Supergranulation would appear to be a rare known example of travelling-wave convection in a very highly turbulent fluid, a non-linear phenomenon which has been observed in laboratory and numerical experiments (e.g. Zhong et al., 1991; Walden et al., 1985) under conditions with much weaker turbulence.

3. VORTICITY

F-mode time-distance helioseismology not only provides maps of the horizontal divergence of the flows, but it also provides information about the two individual components of the horizontal velocity (Duvall & Gizon, 2000). This is achieved by correlating a central pixel with quadrants centered on the four cardinal directions. The horizontal vector flow, $\mathbf{v} = (v_x, v_y)$, is estimated by measuring the difference in travel time for waves propagating in opposite directions. Instead of a full inversion (Gizon et al., 2000) we use a simple calibration to convert from travel-time perturbations to velocities. For two points separated by a distance $\Delta = \Delta \hat{n}$, the relative perturbation in phase travel-time, $\delta \tau / \tau$, due to a flow $\mathbf{v}$ is given by

$$\frac{\delta \tau}{\tau} \simeq -\frac{\delta c}{c} = -\frac{\hat{n} \cdot \mathbf{v}}{c},$$

where $c$ is the phase velocity of the f mode at 3 mHz. This calibration gives

$$v_x \simeq -\frac{c}{2\tau} (\delta \tau^W - \delta \tau^E),$$

$$v_y \simeq -\frac{c}{2\tau} (\delta \tau^N - \delta \tau^S),$$

where, for example, $\delta \tau^N$ is the travel-time perturbation for waves propagating North. From each 12 hr vector flow image we subtract the mean image, to remove rotation and field effects. Like before, we consider longitudinal strips centered at latitude $\lambda$ and 10$^\circ$ wide in latitude. The vertical component of the vorticity, $\omega = \nabla \times \mathbf{v}$, is given by

$$\text{curl} = \partial_x v_y - \partial_y v_x,$$

where spatial derivatives are approximated by a first-order centered difference. Note that differential rotation does not contribute to the vorticity as it has been removed. The divergence of the flow field, denoted by div, is directly calibrated from the “divergence signal” used in section 2 (difference in travel-times for f modes propagating between a central point and an annulus).
Figure 7. Effect of the Coriolis force on supergranular flows. (a) Plot of the correlation coefficient, $C(\lambda)$, between the vertical vorticity, $\text{curl}$, and the horizontal divergence, $\text{div}$. (b) Horizontal averages of the vorticity, $\langle\text{curl}\rangle_+$ (solid) and $\langle\text{curl}\rangle_-$ (dashed), over regions with $\text{div} > 0$ and $\text{div} < 0$ respectively. A vorticity of 1 $\text{m}^2\text{s}^{-1}$ corresponds to an angular velocity of $2.5^\circ \text{day}^{-1}$ or a typical circular velocity of 10 $\text{m s}^{-1}$. Note that there is a small difference between the total area covered by regions of positive divergence and negative divergence (Duvall & Gizon, 2000). (c) Estimate of the slope, $s$, in the linear fit $\text{curl} = s \text{div}$. The solid line shows $s(\lambda)$ versus $f(\lambda)$ in the case when the ratio of errors $r = \sigma_{\text{div}}/\sigma_{\text{curl}}$ is assumed to be zero. The dashed line is for the case $r = 2$. (d) Plot of $\langle\text{curl div}\rangle$ versus $f(\lambda)$.

It is not straightforward to predict the statistical properties of the vorticity in rotating turbulent convection (cf. Hathaway, 1982). Vorticity production is due to the effect of the Coriolis force and to vortex stretching and tilting mechanisms. The importance of the Coriolis force is characterized by an inverse Rossby number, or Coriolis number, defined by

$$Co = 2\tau_c \Omega \cdot \hat{g},$$

where $\hat{g}$ is a unit vector in the downward direction and $\tau_c$ is a characteristic correlation time of the turbulence. Linear theory predicts $\text{curl} \sim Co \text{ div}$ (order of magnitude), and we expect the latitudinal variations of the Coriolis effect to go like

$$f(\lambda) = \frac{\Omega(\lambda) \sin \lambda}{\Omega_{eq}},$$

where $\Omega_{eq}$ is the equatorial solar angular velocity. Away from the equator, the magnitude of Co is greater than unity in most on the convection zone, except near the surface were it can be very small. Taking $\tau_c = 2$ day we find that $Co \simeq -0.98 \sin \lambda$ for supergranulation.

Despite the fact that the vorticity field is very noisy, we detect a significant correlation of a few percent between the vertical vorticity and the horizontal divergence (Fig. 7a). The correlation coefficient at latitude $\lambda$ is defined by

$$C(\lambda) = \frac{\langle\text{div curl}\rangle}{\sqrt{\langle\text{div}^2\rangle \langle\text{curl}^2\rangle}},$$

where the angle brackets denote the spatial average over the area of a $10^6$ band centered around $\lambda$. In the North, positive (negative) divergence is correlated with clockwise (anticlockwise) vorticity. The correlation changes sign in the South. Thus, away from the
equator, the number of right-handed cyclones is not equal to the number of left-handed cyclones. The sign and the latitudinal variation of \( C(\lambda) \) are both characteristic of the effect of the Coriolis force on the flows. This confirms the preliminary detection by Duvall & Gizon (2000).

Figure 7b shows horizontal averages of the vertical vorticity, \( \langle \text{curl}\rangle_\pm \), versus \( f(\lambda) \), where the averages \( \langle \cdot \rangle_+ \) and \( \langle \cdot \rangle_- \) are restricted to the regions of positive and negative divergence respectively. We observe a nearly perfect linear relationship between \( \langle \text{curl}\rangle_\pm \) and \( \mp f(\lambda) \), given by \( \langle \text{curl}\rangle_\pm \propto \mp 3f(\lambda) \text{ Ms}^{-1} \). This is again consistent with the interpretation as a Coriolis effect. In principle, a linear fit of the form

\[
\text{curl}(\lambda) = s(\lambda) \text{ div}(\lambda) \quad (12)
\]

can be extracted from the data at each latitude, \( \lambda \). This operation is not trivial as it requires a knowledge of the errors in the observations, \( \sigma_{\text{curl}} \) and \( \sigma_{\text{div}} \). A first approximation is to assume that div is mostly signal, and curl is mostly noise:

\[
s \rightarrow \frac{\langle \text{div}\text{curl} \rangle}{\langle \text{div}^2 \rangle} \quad \text{as} \quad r = \frac{\sigma_{\text{div}}}{\sigma_{\text{curl}}} \rightarrow 0. \quad (13)
\]

In this limit we find \( s(\lambda) \simeq -0.048 f(\lambda) \) (Fig. 7c), or \( s(\lambda) \sim \text{Co}(\lambda)/20 \) in terms of the Coriolis number quoted above.

Figure 7d shows that the latitudinal variations of the horizontal average (\text{curl} div) are well described by \( \langle \text{div}\text{curl} \rangle \simeq -3f(\lambda) \times 10^{-10} \text{ s}^{-2} \). This observation may be compared directly to a prediction by Rüdiger et al. (1999), who used the mixing length theory to estimate the effect of rotation on convection:

\[
\langle \text{div}\text{curl} \rangle \sim \frac{8\alpha^2}{35\gamma^2} \frac{\Omega_{\text{eq}}}{\tau_c} f(\lambda), \quad (14)
\]

where \( \alpha \) is the mixing length parameter and \( \gamma \) is the ratio of specific heats. For \( \alpha = 1.5, \gamma = 5/3, \) and \( \tau_c = 2 \text{ day} \), the prediction is \( \langle \text{div}\text{curl} \rangle \sim -3f(\lambda) \times 10^{-12} \text{ s}^{-2} \) for supergranulation, i.e. two orders of magnitude smaller than the measured value. This disagreement is perhaps not too surprising as there is some freedom in choosing \( \alpha \) and \( \tau \) in Eq. (14) and the mixing length theory may oversimplify the problem. Furthermore the measurement errors are not quite understood and we need to do some more work to derive the spatial spectrum of the vorticity field. We also note that accurate flow measurements can only be obtained from a proper inversion of the travel times (Gizon & Birch, 2002).

Parker (1955) proposed that the magnetic field can grow in a moving medium with a net kinetic helicity, \( \mathcal{H}_{\text{kin}} = (\mathbf{v} \cdot \mathbf{\omega}) \neq 0 \). Cyclonic convection is often invoked as an important mechanism for generating a large-scale poloidal field from an initial toroidal field. Although we only observe the two horizontal components of velocity, Rüdiger et al. (1999) suggest to use a proxy for the average kinetic helicity,

\[
\mathcal{H}_{\text{kin}} \sim H_m \langle \text{div}\text{curl} \rangle, \quad (15)
\]

where \( H_m = -\langle \partial_z \ln|\rho_0| \rangle^{-1} \) is the average vertical momentum scale height, expected to be positive. Hence, our measurements would suggest a negative kinetic helicity in the North for supergranulation. However, it is not clear if this approximation works. Turbulence may also be driven by magnetic buoyancy (Brandenburg & Schmitt, 1998; Rüdiger et al., 2001) and it may be of interest to search for solar cycle variations in the quantity \( \langle \text{div}\text{curl} \rangle \).

4. SUMMARY

The spectrum of the horizontal divergence of supergranular flows is consistent with a spectrum of traveling waves with a dispersion relation \( \nu \simeq 1.65(kR/100)^{0.45} \mu\text{Hz} \). Waves have a low quality factor and propagate in all directions. The distribution of wave power is anisotropic with increased power in the direction of the rotation and toward the equator, explaining the super-rotation of the pattern. The rotation of the plasma through which the pattern propagates is consistent with the rotation of the magnetic network. The main conclusions presented here have been confirmed by Schou (2002a,b) in direct Doppler data.

Away from the equator the vertical vorticity of supergranular flows is spatially anti-correlated with the horizontal divergence. The latitudinal variation of the correlation is a signature of the effect of the Coriolis force. This observation gives hope for a direct measurement of the kinetic helicity in the upper convection zone that would perhaps help constrain the mechanism of the solar dynamo.

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ACOUSTIC IMAGING AND TIME-DISTANCE ANALYSIS OF TON

Dean-Yi Chou\(^1\), Alexander Serebryanskiy\(^1\), Ming-Tsung Sun\(^2\), and The TON Team\(^3\)
\(^1\)Department of Physics and Institute of Astronomy, Tsing Hua University, Hsinchu, Taiwan
\(^2\)Department of Mechanical Engineering, Chang-Gung University, Kwei-San, Taiwan
\(^3\)The TON Team members are shown in the footnote *.

ABSTRACT

We review the development of acoustic imaging, including the recent results on the inversion of phase-travel-time perturbation to obtain the threedimensional distribution of wave-speed perturbation. We discuss the solar-cycle variations of meridional flows measured with time-distance analysis, which relates to the active latitudes. We present the result from a recent study using solar-cycle variations of p-mode travel time of different wave packets to probe magnetic fields at the base of convection zone. The result shows the evidence of the magnetic fields at the base of the convection zone.

Key words: Solar Magnetic Fields; Active Regions; Solar Cycle.

1. INTRODUCTION

One of major goals of helioseismology is to study the magnetic fields inside the Sun. Magnetic fields in the solar interior could modify the solar p-modes in a complicated way. The presence of magnetic fields would modify the dispersion relation and generate various magneto-acoustic modes. Magnetic fields could also modify the thermal properties of plasma, such as sunspots; it in turn changes the sound speed \(c = \sqrt{\gamma p / \rho}\). The influence of the former is of order of \(1 / \beta = B^2 / 8 \pi p\). The effect of the latter is difficult to compute. However, \(\delta c / c\) should not be too much different from \(1 / \beta\). The value of \(1 / \beta\) decreases rapidly with depth because the gas pressure \(p\) increases rapidly with depth. It is of order of unity near surface inside sunspots, but drops to about \(10^{-5} B_{25}^{1/2}\) at the base of the convection zone, where \(B_2\) is the field strength in units of \(10^5\) gauss. Thus it is difficult to detect the signal of magnetic fields deep in the convection zone (CZ). Here we discuss the recent results using acoustic imaging to probe near-surface magnetic fields, and time-distance analysis to probe the magnetic fields deep in the solar CZ; especially we discuss a recent study using the multiple-bounce travel time of different wave packets to probe the magnetic fields at the base of the convection zone.

2. ACOUSTIC IMAGING

Chang, Chou & LaBonte (1997) first used the measured acoustic signals at the surface to construct the signals at subsurface points. They named the method acoustic imaging. The acoustic signal at a target point in the solar interior is constructed by coherently adding the signals measured at the surface (Chou et al. 1999; Chou 2000)

\[
\Psi_{\text{out, in}}(t) = \sum_{\tau = \tau_1}^{\tau_2} W(\tau, \theta) \cdot \Psi(\theta, t \pm \tau),
\]

where \(\Psi_{\text{out}}(t)\) and \(\Psi_{\text{in}}(t)\) are the constructed signals at the target point at time \(t\), \(\Psi(\theta, t \pm \tau)\) is the azimuthal-averaged signal measured at the angular distance \(\theta\) from the target point at time \(t \pm \tau\), where \(\tau\) and \(\theta\) satisfy the time-distance relation, and \(W(\tau, \theta)\) is the weighting function. The positive sign corresponds to \(\Psi_{\text{out}}\) which is constructed with the waves propagating outward from the target point. The negative sign corresponds to \(\Psi_{\text{in}}\) which is constructed with the waves propagating inward toward the target point. The range of \(\tau\) in equation (1) corresponds to an annular region in space, called the aperture of computational acoustic lens in analogy to the optical lens in optics. Since each point on the time-distance curve corresponds to a wave packet formed by the modes with the same \(\omega / \lambda\), each aperture corresponds to a range of \(\omega / \lambda\) used to construct the signal, which determines the spatial resolution of acoustic imaging. The constructed signal contains information on intensity and phase. The intensity at the target point can be computed by summing \(|\Psi_{\text{out,in}}(t)|^2\) over time. The constructed intensity in magnetic regions is smaller than the quiet Sun. Chen...
et al. (1998) first used the cross-correlation function between \( \Psi_{in}(t) \) and \( \Psi_{out}(t) \), computed as
\[
C(t) = \int \Psi_{in}(t') \cdot \Psi_{out}(t' + t) dt' ,
\]
to study the phase of constructed signals. The phase of \( C(t) \), which is determined by a fit, relates to the phase travel time of the wave packet. The phase travel time of active regions is shorter than that of the quiet Sun; the difference is defined as the phase shift (change in phase travel time), \( \delta \tau \). Beside the phase shift, the envelope of \( C(t) \) also shifts in magnetic regions. The envelope shift relates to the change in the group travel time of the wave packet. The phase shift and envelope shift provide different information on the physical conditions of plasma along the ray path of the wave packets (Chou et al. 2000; Chou & Duvall 2000). One can form the 3-D maps of intensity, phase shift, and envelope shift. The discussion of the these maps was reviewed in Chou (2000).

Although the goal of acoustic imaging is to construct the signal at the target point, the constructed signal contains both the signal at the target point, and the signals along the ray path (Chou et al., 1999). The contribution of nonlocal effect depends on the density of the ray distribution in the ray approximation. With the ray approximation, the phase shift at a target point, \( \delta \tau(\vec{r}) \), can be expressed in terms of the sound-speed perturbation \( \delta c \) (Sun & Chou 2002)
\[
\delta \tau(\vec{r}) \approx - \int K(\vec{r}, \vec{r'}) \frac{\delta c(\vec{r'})}{c(\vec{r'})} d^3r'
\]
where the kernel \( K(\vec{r}, \vec{r'}) \) is proportional to the number of rays per unit area of the cross section perpendicular to the rays, and weighted by power of each ray and \( 1/c \). The dimension of the kernel is cm\(^{-2}\). The kernel can be computed with a standard solar model (Christensen-Dalsgaard et al. 1996) and the ray theory.

With equation 3, one can do the forward problem: given an assumed distribution of \( \delta c(\vec{r}) \) and then compute \( \delta \tau(\vec{r}) \) to compared with the observed values (Chou & Sun 2001). One can also invert the measured phase shift \( \delta \tau \) to obtain the 3-D distribution of sound-speed perturbation. For a small region, it reduces to one-dimensional inversion problem in the vertical direction in the Fourier domain (Jensen et al. 1998; Jensen et al. 2000). Sun & Chou (2002) used a regularized least-square inversion method to obtain the three-dimensional distribution of sound-speed perturbation. The result for NOAA 7981 is shown in Figures 1 and 2.

3. SOLAR-CYCLE VARIATIONS OF MERIDIONAL FLOWS

Giles et al. (1997) first showed with time-distance analysis that the meridional flow penetrate into the entire CZ. Chou & Dai (2001) used the TON data from 1994 to 2000 to study the solar cycle variations of meridional flow in the CZ. They found that the velocity of meridional flow increased when the solar activity decreased from 1994 to 1997. As solar activity increased from 1997 to 2000, a new divergent component of meridional flow was created in each hemisphere as shown in Figures 3 and 4. The center of the new divergent flow moves toward the equator.
Figure 3. Time difference $\Delta t$ versus latitude for 1997-2000 (from Chou and Dai 2001). It is averaged over angular distance $\Delta = 6 - 10^\circ$. The results are from the data taken at Big Bear. Here only the error bars of one curve are shown, and the error bars of other curves are similar.

from 1998 to 2000. Moreover, the latitude of the center of the new flow coincides with the centroid of the active latitudes along the solar cycle. The velocity of the new component increases with depth to about 20 m/s at a depth of 0.1 $R_\odot$, the limit of the study. Recently, Beck et al. (2002) used the MDI data to show that this new divergent component relates to the torsional oscillation.

These results suggest that the new divergent flow is associated with magnetic fields. If it is created by the magnetic fields deep in the CZ, this new flow could be used as an indirect probe for the magnetic fields deep in the CZ. The equipartition field strength corresponding the velocity of new divergent flow at 0.9$R_\odot$ is about 1600 gauss.

4. MAGNETIC FIELDS AT THE BASE OF THE CONVECTION ZONE

4.1. Motivation

How and where solar magnetic fields are generated is a long standing unanswered question in astronomy (Cowling 1934; Parker 1955; Babcock 1961). It is generally believed that magnetic fields are generated at the boundary between the radiative zone and CZ by a dynamo mechanism. However, until now no clear evidence of the magnetic field in this region has been found. Recently, Chou and Serebryanskiy (2002) measured solar-cycle variations of travel time of wave packets, penetrating into different depths, to probe the magnetic fields at the base of the convection zone (BCZ). The modes with the similar phase velocity form a wave packet. Different wave packets penetrate into different depths: the wave packet with a larger phase velocity penetrates into a greater depth as shown in Figure 5. If a magnetic field is present at the BCZ, it would affect the wave packets penetrating into the BCZ, while leave other wave packet intact. If the magnetic fields at the BCZ vary with the solar cycle, travel time is expected to vary with the solar cycle as well. However, the change in travel time due to the magnetic fields at the BCZ is small because $1/\beta$ is very small. To improve the S/N, one can measure multiple-bounce travel time because the change in travel time is linearly proportional to the number of bounces. Chou and Serebryanskiy (2002) measured the time for a wave packet which take $N$ bounces to travel around the Sun to come back to the same spatial point. Thus the problem becomes measuring solar cycle variations of travel time with the auto-correlation function of the time series at the same spatial point. Chou and Serebryanskiy (2002) use two different approaches: (1) the direct computation of auto-correlation function (MBTFA), and (2) the power spectrum simulation analysis (PSSA), to measure the multiple-bounce travel time of wave packets. The near-surface magnetic fields also influence
the travel time of wave packets. How to separate the influences of near-surface magnetic fields from the influences of the magnetic fields at the BCZ is a key to probe the magnetic fields at the BCZ. The travel time of wave packets described above can separate the influences of these two sources. The ray paths of different wave packets are similar near the surface because they are almost vertical near the surface. Thus the influences of near-surface magnetic fields on one-bounce travel time for different wave packets are approximately the same. This property gives the one-bounce travel time an advantage to probe the magnetic fields at the BCZ.

4.2. Multiple-Bounce Travel Time Measurements

The detail of data analysis to measure multiple-bounce travel time was described in Chou & Serebryanskiy (2002). In the first approach (MBTTA), the MDI Doppler images are filtered with a phase-velocity filter to isolate the signals in a range of phase velocity. The center of filter is selected such that the one-bounce travel distance is $360^\circ / N$, where $N$ is an integer. The auto-correlation function of time series at each spatial point is computed. The phase-velocity filter is necessary to remove the interference between different wave packets in auto-correlation function as shown in Figure 6. The auto-correlation functions are then averaged over the solar disk. The phase travel time $\tau_N$ is determined from the averaged auto-correlation function.

The correlation function is the inverse Fourier trans-

form of the power spectrum of p-modes. Thus the signal corresponding to the travel time perturbation detected in MBTTA should also exist in the mode frequencies which are determined from the power spectrum. In the second approach (PSSA), the measured mode frequencies from MDI and GONG data are used to construct the power spectrum in $(l, \omega)$ with assumed line widths and relative mode amplitudes. The constructed power spectrum is filtered with the same phase velocity filter as in MBTTA prior to computing the cross-correlation function with the inverse Fourier transform. The cross-correlation function at zero travel distance corresponds to the auto-correlation function averaged over all spatial points, which is used to determine the travel time $\tau_N$ of the wave packet.

To study the solar-cycle variations of travel time, one-bounce travel time $\tau_N / N$ is averaged over several periods along the solar cycle. The length of periods is about one year for MDI and about two years for GONG. For MBTTA, we have analyzed only two periods at minimum and maximum. The change in one-bounce travel time relative to minimum $\delta \tau_N / N$ versus $N$ for different periods is shown in Figure 7. The interesting feature in Figure 7 is that $\delta \tau_N / N$ is approximately constant except a drop at $N = 8$. The
drop corresponds to a shorter travel time relative to the solar minimum. The magnitude of the decrease in travel time at $N = 8$ increases with solar activity as shown in Figure 8. Since the wave packet corresponding $N = 8$ has the lower turning point at the BCZ, the results in Figures 7 and 8 suggest that there exists the perturbation related to magnetic fields at the BCZ.

The constant value of $\delta\tau_N/N$ is caused by near-surface perturbations, and is well correlated with the frequency change. The fraction of change in $\delta\tau_N$ versus the fraction of change in frequency is shown in Figure 9. It is of interest to note that the fraction of change in $\delta\tau_N$ is slightly less than the fraction of change in frequency. We do not know the cause of it.

![Figure 7. Change in one-bounce travel time relative to solar minimum versus number of bounces $N$, which corresponds to different wave packets (from Chou and Serebryanskiy 2002). The result from the multiple-bounce travel time analysis (MBTTA) is denoted by the open circle in the left panel. The result from the power spectrum simulation analysis (PSSA) is denoted by the filled circle. The left panel is computed from the MDI mode frequencies, and the right panel from the GONG mode frequencies. The sequence of the averaging periods is indicated by the number associated with each curve. The range of each period is indicated by the horizontal bar of each point in Figure 8. The error bar of PSSA is estimated from Monte Carlo simulations using the errors in mode frequencies.](image)

4.4. Tests of Data Analysis Procedure

To test whether the anomaly at $N = 8$ is caused by the analysis procedure, we did the following test with the MDI mode frequencies. The frequency difference between solar maximum (period 6 in Figure 8) and minimum is smoothed by a fit in the $(l, \nu)$ domain. The mode frequencies at solar maximum are simulated by adding this smooth function to the mode frequencies at minimum. Applying the PSSA procedure to the measured frequencies at solar minimum and the simulated frequencies at solar maximum to
First, we make the ray approximation. Second, the measured change in travel time is caused only by the near-surface perturbations. Third, each near-surface perturbed element causes the same change in travel time for different wave packets which pass through this element. With these assumptions, the change in travel time obtained from the auto-correlation function at \( \delta \tau_N(\vec{r}) \), can be expressed as a 2-D integral over the entire surface

\[
\delta \tau_N(\vec{r}) = \int K(\vec{r}, \vec{r}') S(\vec{r}') \, d^2 \vec{r}'
\]

where \( S(\vec{r}) \) is the near-surface perturbation, and \( K(\vec{r}, \vec{r}') \) is the corresponding kernel which is a function of \( |\vec{r} - \vec{r}'| \) only. The average \( \delta \tau_N \) over the entire surface is

\[
< \delta \tau_N > = \int \int K(\vec{r}, \vec{r}') S(\vec{r}') \, d^2 \vec{r}' \, d^2 \vec{r}
\]

\[
= \int \int K(\vec{r}, \vec{r}') d^2 \vec{r}' S(\vec{r}) \, d^2 \vec{r}
\]

Here we have used the symmetry of the kernel: \( K(\vec{r}, \vec{r}') = K(\vec{r}', \vec{r}) \). Since the Sun is spherically symmetric, the integral in the bracket is independent of \( \vec{r} \) and is proportional to \( N \). Equation (5) becomes

\[
< \delta \tau_N > \propto N \int S(\vec{r}) \, d^2 \vec{r} \propto N
\]

Thus, \( < \delta \tau_N > \) is dependent on the total amount of the near-surface perturbations, \( \int S(\vec{r}) \, d^2 \vec{r} \), but independent of how the near-surface perturbation \( S(\vec{r}) \) is distributed. Moreover, the one-bounce travel time, \( < \delta \tau_N > /N \), is independent of \( N \).

In our analysis the spatial average is over only the visible part of the sphere. However, we average the travel time over a period of many solar rotations. It is equivalent to averaging over the whole sphere. Therefore, we conclude that the measured anomaly at \( N = 8 \) can not be caused by the organized pattern of the near-surface perturbations.

4.6. Comparison with Model Computation

To understand how the measured anomaly at \( N = 8 \) relates to the perturbations at the BCZ, we compare the measurements with model simulation. We model the perturbations at the BCZ by perturbing the sound speed \( c \) in the standard solar model for the region near the surface and the BCZ. The distribution of \( \delta c/c \) is a Gaussian for both regions. Although this simplified perturbed model is not a consistent model for all physical variables, it is a starting point for investigating the relation between the sound-speed perturbation and the change in travel time. The travel time of each wave packet is computed with the ray theory for both perturbed and unperturbed models. Then the change in one-bounce travel time,
To quantify it, we average the measured $\delta \tau_N / N$ over $N = 9-14$ and $N = 4-7$. The difference of these two average values increases with the sunspot number as shown in Figure 12.

Now we turn to the inconsistency. Although the drop in computed $\delta \tau_N / N$ is the greatest at $N = 8$, there is also a significant drop for the adjacent wave packets, $N = 7$ and 9. However, the measured $\delta \tau_N / N$ seems to oscillate around $N = 8$ for all periods shown in Figure 7. This discrepancy is probably caused by the ray approximation in our model computation. The oscillatory feature around $N = 8$ may be caused by the wave property of travel time sensitivity kernel (Birch & Kosovichev 2000). The study with model simulation may help understand it.

4.7. Estimates of the Field Strength at the BCZ

Although we believe that the anomaly at $N = 8$ is caused by the magnetic fields at the BCZ, finding the relation between the magnetic fields at the BCZ and measured anomaly at $N = 8$ is difficult because the magnetic fields could change the travel time in a complicated way as discussed in Section 1. This complication makes the detailed interpretation of the measurements of the change in travel time difficult. Despite of difficulties, Chou & Serebryanskiy (2002) tried to estimate the magnetic field strength at the BCZ based on the assumption that the change in wave speed $w$ is only caused by the change in the dispersion relation of magneto-acoustic waves: $\delta w / w \sim 1/\beta$. Although they did not include the change in sound speed in $\delta w / w$, the contribution of $\delta c / c$ to $\delta w / w$ due to the presence of magnetic fields is probably also of order $1/\beta$.

The fraction of change in travel time $\delta \tau_N / \tau_N \sim f \cdot \delta w / w$, where $f$ is the fraction of travel time the wave packet spends in the magnetic region at the BCZ, which depends on the vertical dimension of the magnetic region and the horizontal filling factor of the magnetic fields at the BCZ. If the vertical dimension of the magnetic region is about the width of the tachocline and the horizontal filling factor is unity, $f \sim 0.1$, then the field strength is about $4 \times 10^5$ Gauss from the measured decrease in travel time at $N = 8$. If the magnetic fields at the BCZ are concentrated near the equator, the horizontal filling factor is less than unity and the estimated field strength will be greater. Using different azimuthal degree $m$ would provide information on the latitudinal distribution of the perturbations causing the anomaly at $N = 8$.

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TIME-DISTANCE: WHAT DOES IT TELL US?

Jesper Munk Jensen
Theoretical Astrophysics Center, Danish National Research Foundation, DK-8000 Aarhus C, Denmark

ABSTRACT

Time-distance helioseismology has become a very important tool for investigating phenomena in the solar convection zone such as convection cells, meridional flows and active regions. Using time-distance helioseismology it has become possible to study these phenomena in unprecedented detail. In this review I will discuss some of the results obtained using time-distance helioseismology and how they compare with results obtained from other local-area helioseismic methods. When more than one of local-area helioseismic methods, e.g. time-distance, holography and ring-diagram analysis, can be applied to the same problem the information obtained is increased. One of the future goals of local-area helioseismology will be to interpret the same phenomena using all the available methods.

Time-distance data have been inverted using tomographic techniques to reconstruct the conditions in the solar interior. The accuracy of this reconstruction depends both on the sensitivity kernels and the inversion methods employed. I will show some results obtained using different kernels and inversion methods. The similarities between the results ensure that what we are seeing are not artifacts introduced by the inversion procedure. To understand better the inversion results, averaging kernels and estimations of the model uncertainty are needed; these illustrate the resolution and precision that may be achieved with the inversion methods. I will show examples of such resolution analysis. To test the inversion methods independent forward modeling is needed. Preliminary results from an acoustic solar-like model will be presented.

1. INTRODUCTION

The field of helioseismology has been studying the Sun using seismic methods for the past decades. It has been very successful in obtaining knowledge about the solar interior and is almost the only observational window that directly can give information about the inner workings of the Sun. The vast majority of the results from helioseismology has been obtained by study of the global-mode oscillations of the Sun. Global helioseismology has been extremely successful in determining the large scale structures of the Sun and has helped improve the understanding of the physics of the Sun and thereby also of other stars. In the last decade the field of local-area helioseismology has emerged. Here the waves traveling inside the Sun are studied locally and it is therefore possible to study phenomena that the global approach cannot resolve. Time-distance helioseismology is one of several different local-area helioseismic techniques. Other local area techniques include ring-diagram analysis and helioseismic holography. Time-distance helioseismology employs travel times observed between different surface location on the Sun. These travel times are obtained by cross correlating the observed surface oscillations. Because of the similarity between these data and seismic travel-time data used in geophysics is possible to use methods from terrestrial seismology dealing with similar data. Time-distance helioseismology has the potential to help answering still unsolved questions about the workings of the Sun and its magnetic activity. The origin of the solar magnetic fields and its variation in time known as the 11 year solar cycle still poses unanswered questions which the local-area methods hopefully can help resolve.

2. TIME-DISTANCE MEASUREMENTS

In time distance helioseismology travel times for wave packets traveling through the solar interior is obtained from cross correlations of the observed oscillations on the solar surface (Duvall et al. 1993). The cross correlations are given as

\[ \Gamma(\tau, \Delta) = \int_0^T f(t, \mathbf{r}_1) f(t + \tau, \mathbf{r}_2) dt, \]  \( (1) \)

where \( f(t, \mathbf{r}) \) is the oscillation pattern at time \( t \) observed at the surface point \( \mathbf{r} \), \( \Delta \) is the angular separation between the surface points \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \), \( \tau \) is the delay time and \( T \) is the length of the period of observations. An example of a cross correlation obtained from solar data is shown in Figure 1. The data used to obtain this cross correlation was obtained at the Earth’s South Pole by Jeffries et al. (1994) and...
the cross correlation is averaged over the whole solar disk. Cross correlation functions obtained from MDI data have higher spatial resolution. An example of such a cross correlation function is shown in Kosovichev, Duvall & Scherrer (2000). The cross correlation shows several ridges corresponding to the direct arrival and the arrival of waves that have been reflected once or more at the solar surface. By going to larger distances and longer times waves can be followed further and Duvall et al. (1993) were able to track waves coming back after being reflected at the backside of the Sun.

3. RAY APPROXIMATION

In the ray approximation the travel-time perturbation is given to first order by the integral of the perturbation to the medium along the unperturbed ray path. This is also known as Fermat’s Principle (e.g., Gough 1993). For the phase travel-time the perturbation is given as

$$\delta \tau = \frac{1}{\omega} \int_{\Gamma_0} \delta k \cdot d\mathbf{r},$$

where $\Gamma_0$ is the ray path in the unperturbed medium, $\delta k$ is the perturbation to the wavenumber and $d\mathbf{r}$ is directed along the ray path. By perturbing the dispersion relation one finds that the travel-time perturbation is given as (Kosovichev & Duvall 1997, Paper IV)

$$\delta \tau = - \int_{\Gamma_0} \frac{\mathbf{n}_0 \cdot \mathbf{U}}{c_0^2} + \frac{\delta c}{c_0} + \frac{1}{2} \left( \frac{c_A^2}{c_0^2} + \frac{\mathbf{n}_0 \cdot \mathbf{c}_A}{c_0^2} \right) S ds,$$

where $\mathbf{n}_0$ is a unit tangent vector to the ray, $\mathbf{U}$ is the flow velocity and $S = k_0/\omega$ is the phase slowness. Here the effects of $\mathbf{U}$, $\delta c$ and $\mathbf{B}$ are considered to be perturbations to the reference model.

By taking means and differences of waves traveling in opposite directions along the same ray path it is possible to isolate the effects of flows from the effects of sound-speed perturbations and magnetic field. This gives

$$\delta \tau^{diff} = \delta \tau^+ - \delta \tau^- = -2 \int_{\Gamma_0} \frac{\mathbf{n}_0 \cdot \mathbf{U}}{c_0^2} ds$$

and

$$\delta \tau^{mean} = \frac{1}{2} (\delta \tau^+ + \delta \tau^-) = - \int_{\Gamma_0} \frac{\delta u}{u} S ds,$$

where $\delta u/u$ given as

$$\frac{\delta u}{u} = \frac{\delta c}{c_0} + \frac{1}{2} \left( \frac{c_A^2}{c_0^2} - \frac{\mathbf{n}_0 \cdot \mathbf{c}_A}{c_0^2} \right),$$

is the wave speed perturbation. Here + and − denote the waves traveling in opposite directions.

Two special cases are worth mentioning, waves traveling along and perpendicular to the magnetic field lines. In these two cases $c_f$ becomes

$$\left( \frac{\delta u}{u} \right)_{||} = \frac{\delta c}{c_0}, \quad \left( \frac{\delta u}{u} \right)_{\perp} = \frac{\delta c}{c_0} + \frac{c_A^2}{c_0^2},$$

where the subscripts $||$ and $\perp$ denote the parallel and perpendicular case respectively. Here we see that the waves traveling perpendicular to the field lines are most affected by the presence of the field, whereas the waves traveling along the field lines do not feel it at all. This anisotropic behavior could potentially be used to separate the effect of the magnetic field from the temperature variations which gives an isotropic perturbation.

Bogdan (1997) showed that a wavepacket traveling in the solar interior is not confined to the raypath but samples a large volume surrounding the raypath. Wave-theoretical sensitivity kernels which takes this finite wave-length effect into account have been derived by Jensen et al. (2000a), Birch & Kosovichev (2000) and Jensen & Pijpers (2002).

4. RESULTS

Since Duvall et al. (1993) published the first helioseismic time-distance measurements many different phenomena, such as convection cells, meridional flows and active regions, have been investigated using this technique. Here I will give a review of some of the results obtained from time-distance helioseismology and how they compare with results obtained using other methods. The comparison of results from different methods is an area that has only just started and will be an important area of research in the future.
4.1. Meridional Flow

Meridional flows from the equator towards the poles have been investigated by time-distance measurements by Giles et al. (1997). They found a poleward flow of $\approx 20$ m/s extending down to 26 Mm. Giles (1999) extended the analysis and found that the flow obtained from the data without additional constraints was poleward throughout the convection zone. By including mass conservation as an additional constraint in the inversions Giles (1999) found an equatorward return flow of $\approx 3$ m/s in the lower third of the convection zone.

The meridional circulation has also been investigated using ring-diagram analysis by several groups (Schou & Bogart 1998, Gonzalez et al. 1999, Basu et al. 1999, Haber et al. 2000). They all find poleward flows of around 20 - 40 m/s in 1996 and 1997. The findings agree well with the time-distance results of Giles et al. (1997). Figure 2 shows the result of Schou & Bogart (1998) compared with the result from Giles et al. (1997). The results have been averaged over all the different offsets for the time-distance data and over all the modes present at all latitudes for the ring-diagram data. The results agree very well except at high latitude. The reason for the discrepancy here is not well understood.

Giles (1999) also investigated the time evolution of the meridional flow from 1996 to 1998 and found evidence of an equatorward flow from the north pole to 60° latitude emerging in 1998 which gave rise to a two-cell flow pattern in the northern hemisphere coinciding with the increase in magnetic activity as the Sun goes from magnetic minimum to maximum. Figure 3 shows the time variation of the meridional flow from 1996 to 1998. Haber et al. (2002) also found a double cell pattern emerging in the northern hemisphere in 1998 using ring-diagram analysis.

Giles et al. (1998) used time-distance measurements to determine the solar rotation rate with depth. The result found agreed quite well with the results obtained from inversion of frequency splittings of eigenmode frequencies due to rotation. Giles (1999) confirmed this agreement by taking into account the averaging kernels of the different inversions. Figure 4 shows the results for the solar rotation obtained using time-distance and global mode inversions. The inversion results from time-distance data have been convolved with the averaging kernels from the inversion of the frequency splittings and vice versa to obtain two models with the same resolution. The models compare quite well except for close to the surface and at high latitudes. The region close to the surface is difficult to resolve both with global-mode measurements and the time-distance data used in this inversion. The shortest offset is 3° which gives a lower turning point of 0.98 R$_\odot$. For high latitude the data coverage becomes a problem.

Giles et al. (1998) and Giles 1999 also investigated the zonal flows. Bands of fast and slow rotation. Their results agree well with results from global modes near the surface but Giles (1999) was unable to confirm that the zonal flows extend down to 70 Mm as found by Toomre et al. (2000) and Howe et al. (2000).

4.2. Solar Rotation

Duvall & Gizon (2000) used time-distance measurements of the solar f-mode to investigate near-surface flows. The f-mode (or fundamental mode) consists of surface-gravity waves similar to water waves and is therefore only sensitive to near surface horizontal flows. They found the divergence of the flows to correlate well with super granulation. Figure 5...
Figure 4. Comparison between results from inversion of eigen-mode frequency and time-distance data at different latitudes. Shown are the inversion result from frequency splittings (dotted), and the inversion result from time-distance data (solid). The dashed lines indicated the $\lambda_0$ level for the time-distance inversions. The arrows show the surface rotation rate. (From Giles 1999)

shows how the divergence of the flow correlates with the measured magnetic field which is concentrated in the lanes between supergranulation cells.

4.4. Convection

Duvall et al. (1997) and Kosovichev & Duvall (1997) studied convection using time-distance data. They found patterns of converging and divergent horizontal flows organized in cells of 20-30 Mm in size, where the cell boundaries coincide with lanes of enhanced magnetic activity. This is in agreement with surface observations of supergranular flows (e.g. Title et al. 1989). Vertical down flows were found in colder areas with lower wave speed and up flows in hotter areas (see Figure 6). The granulation patterns were only found down to a depth of 2-3 Mm indicating the supergranulation could be a near surface phenomenon. Duvall (1998) correlated the horizontal flow patterns at the surface with the ones obtained at different depths and gave a different estimate for the depth of the super granular layer of 8 Mm.

4.5. Active Regions

Duvall et al. (1996) used forward modeling in the ray approximation to detect a down flow under a sunspot. They found that a down flow of 2 km/s extending to 2 Mm below the surface explained the difference data. They also estimated that a magnetic field of around 2-4 kG in the upper 600 km was consistent with the mean data. The same data was inverted by Kosovichev (1996) using a tomographic approach. He found a convergent down-flow of around 1 km/s around a newly formed sunspot and an increase in wave speed of around 5% down to 32 Mm below the sun spot. An older decaying region showed a wave-speed decrease and a divergent up-flow. Kosovichev (1996) gave an estimate of the magnetic field strength at approximately 50 kG at a depth of 24 Mm. The data used in these analyses was obtained from Earth-based observation and the resolution ($\sim$ 15 Mm) did not allow detailed study of the magnetic structures. The structure of sunspots has later been studied using high resolution data from the MDI instrument aboard the SOHO satellite which allows time-distance data to be obtained with a spatial resolution of $\sim$ 1.65 Mm. Kosovichev et al. (2000) and Jensen et al. (2001) investigated an emerging active region and found wave-speed structures extending down to 20 Mm of a typical amplitude of around 0.4-0.5 km/s. Just below the surface there was typically a wave-speed decrease of 0.2-0.3 km/s which could be linked to the lower temperature observed in sun spots. Kosovichev et al. (2000, 2001) also showed results for a more developed sunspot which showed wave-speed perturbation of up to 3 km/s. Kosovichev et al. (2000) found "fingers" connecting magnetic pores with the sunspot below the surface. The pores that were connected were of the same polarity whereas pores of the opposite polarity were not connected to the spot. Kosovichev et al. (2000) decreased the temporal intervals used in the
data analysis to try to better resolve the emerging flux and estimated the emergence speed as around 1.3 km/s. Using these data Kosovichev et al. (2001) were able to see a strong wave-speed perturbation at 20 Mm below the surface which was much stronger than what was observed closer to the surface at the time.

Zhao et al. (2001) investigated flows around sunspots using MDI data. Figure 7 shows some of their results. They found a converging down flow centered on the sunspot in the upper 3 Mm of 0.5-1 km/s. Divergent outflows from the sunspot were found at a depth of 6-9 Mm which were stronger than the convergent flows at the surface. Up flows were seen further below the sunspot which can account for the increased mass flows. At a depth of 9-12 Mm a flow across the sunspot is observed. Zhao et al. (2001) argue that the presence of this flow supports the cluster model for sunspots (Parker 1979) as opposed to a monolithic structure. The convergent flow near the surface is not consistent with the surface observation of an outflow from the inner of the sunspots penumbra to the surrounding photo sphere (the Evershed flow first observed by Evershed 1909). This could indicate that the Evershed flow is a very shallow phenomena.

Putting together the results for structure and flows around a sun spot gives a possible model for the dynamics of a sun spot. Figure 8 shows a sketch of the inferred dynamics. The concentrated magnetic field in the sun spot blocks the convective upflow of hot material from the solar interior. This leads to cooling of the material near the surface while the trapped hot material increases the temperature further down. The cold material near the surface plummets down which give rise to an inflow that holds the sun spot together. Below the sun spot the up welling hot ma-

Figure 6. Sound-speed perturbations (gray scale) and flows connected to convective motions as inferred by inversions of time-distance data. The maximum velocity is around 1.5 km/s. Downflows and upflows seem to coincide with cold and hot regions of decreased and increased sound-speed respectively. (From Kosovichev & Duvall 1997.)

Figure 7. Cross section through the flow field inferred below a sunspot. The longest arrow correspond to a velocity of 1.4 km/s. The extent of the sunspots umbra and penumbra is indicated at the top. (From Zhao et al. 2001.)

Figure 8. Artist impression of inferred flows and wave-speed structure below sun spot. The dark gray area just below the surface is colder material whereas the lighter gray area further down corresponds to hot upwelling material. Adapted from NASA Space Science Update Nov. 6, 2001.

5. COMPARISON OF INVERSIONS

Both Kosovichev et al. (2000) and Jensen et al. (2001) studied an active region that emerged in January 1998. Kosovichev et al. (2000) used ray-theoretic sensitivity kernels and a conjugate-gradient solver (LSQR) while Jensen et al. (2001) used wave-theoretical kernels and a Fourier domain based inversion technique (MCD). The LSQR method is described by Paige and Saunders (1982) while the MCD method is described by Jacobsen et al. (1999). Jensen and Pijpers (2002) describes the wave-theoretical kernels employed. Figure 9 shows three dimensional visualizations of the inversion results from Figure 2 of Jensen et al. 2001. Shown are the three cross sections corresponding to time step 2, 3 and 4. The cross sections of Figure 9 are taken parallel to the cross sections of Jensen et al. 2001 in a distance of 6 Mm. Also shown are horizontal
are connected further below the surface. In the last of the cross sections some "wiggly" structure is seen near the surface. These structures are similar for the two cases. Also a deep wave-speed increase is present in both inversions. Thus this feature seems to be independent of the choice of sensitivity kernels and inversion technique but could still be due to the additional noise at this time.

The similarity of the results is encouraging in that it shows that both methods are getting similar results for the same data set. This indicates that the methods seem to work and what is seen are not artifacts from the specific method. The differences between the results are interesting but it is not clear what is causing them. Differences in regularization, kernels or method could all contribute. No noise estimation or averaging kernels have been shown for the inversion results of Kosovichev et al. (2000). Therefore it is difficult to discuss the resolution of the different methods.

6. UNCERTAINTY ESTIMATION

The resolution power of helioseismic time-distance analysis is an area where more work is clearly needed. Jensen et al. 2002a presents a first analysis of the uncertainty estimates on the inversion results for subsurface structures. Here both uncertainty on the vertical localization of the averaging kernels and the error estimates on the inferred model are considered. The noise analysis of Jensen et al. 2002a is the first noise analysis for helioseismic time-distance inversion taking the spatial correlation of the noise into account. Baudin & Korzennik (1998) showed results of trying to estimate errors for the travel times and found them to be strongly correlated. Previous investigations of the noise propagation have used uncorrelated noise (Giles 1999, Korzennik 2001). Gough & Sekir (2002) investigated the effect of correlated noise on global inversions and found that in some cases correlated noise could strongly influence the results. For time-distance data Korzennik (2001) found that the uncorrelated noise could be removed due the differences in spatial scales between the correlated noise and the real data, this would not be the case for noise with the right statistics.

Figure 10 shows averaging kernels obtained by Jensen et al. (2002a). The kernels have been integrated in the two horizontal directions to give an indication of the localization with depth of the model estimates. Here we see that close to the surface the kernels are well localized around the target depth. At 20 Mm the kernel becomes very wide and have large oscillations near the surface. Figure 11 shows the estimated model errors obtained from the model covariance matrix. Here the data covariance matrix has been estimated from the observed data and takes the spatial correlation of the data into account. Also shown is the error estimate obtained when assuming that the noise is uncorrelated. This leads to an underestimation of the standard deviation of the model.
Figure 10. Averaging kernels obtained for four different target depths. The target depths are indicated by the vertical dotted line. The averaging kernels have been horizontally integrated to show the depth localization. (From Jensen et al. 2002a.)

Figure 11. Error estimates as a function of depth for time-distance inversion. Shown are the obtained error estimates when the data noise is assumed spatial correlated (solid line) and uncorrelated (dashed line). If the noise is assumed uncorrelated the model error is significantly underestimated. (From Jensen et al. 2002a.)

Figure 12. Estimated wave-speed perturbation below an active region including errorbars. (From Jensen et al. 2002a.)

Figure 12 shows a plot through wave-speed perturbation below the sunspot with errorbars inferred from the diagonal of the model covariance matrix and the width of the averaging kernels. The averaging kernels below 30 Mm are not really localized at all, therefore width of the kernel makes little sense and the small horizontal errorbars here do not indicate a good depth resolution. Near the surface we see a negative wave-speed perturbation while further down the wave-speed perturbation is positive. The reason for this behavior could be that close to the surface the waves are traveling mostly in vertical direction and therefore along the magnetic field lines. As discussed in Eq. 6 this means that they do not feel the presence of the magnetic field. Thus close to the surface the temperature effect could dominate the wave-speed perturbation and since sunspots are colder than the surroundings the wave-speed would decrease. The observed wave-speed decrease is -0.3 km/s. If this is taken to be a temperature effect due to the cold sunspot we find that it corresponds to a temperature decrease of around 700K. The wave-speed perturbation increases up to around 0.8 km/s at a depth of 6 Mm. To explain this wave-speed a magnetic field of around 25kG or a temperature increase of 2500K should be present at this depth. The wave-speed perturbation then decreases down to 25 Mm below which we see no significant anomaly.

Averaging kernels for helioseismic time-distance inversions have previously been presented by Giles (1999) for inversions concerning subsurface flows. Figure 13 shows examples of averaging kernels obtained by Giles (1999). The kernels for the rotation rate were obtained from data with offsets between 3° and 19.2° while the data used in the meridional inversions had offsets of up to 45°. The turning point for the deepest rays in the inversion for rotation was around 0.88R ≈ 83 Mm and for the meridional circulation 0.71R ≈ 201 Mm. Thus the data for the meridional flows spans the entire convection zone. All the averaging kernels shown are for a latitude of 2.5°. The deepest averaging kernels for the rotation is for a target depth of ~ 55 Mm and a good localization is obtained around the target depth. For the very surface-near target depth the localization is weak because of lack of data for very short offsets. Since the meridional data have rays with deeper turning points it should be possible to obtain focused averaging kernels for deeper targets depth. The averaging kernel for the deepest target depth has very little amplitude at the target depth. It seems difficult to obtain localized kernels below ~ 70 Mm even with the offsets used. Averaging kernels for the inversions done using the LSQR method (e.g. Kosovichev & Duvall 1997, Zhao et al. 2001) have not been published and comparisons of the resolution between the different inversion results are therefore difficult. The averaging kernels of Figure 10 shows that localization can be obtained down to around 20 Mm. Since the averaging kernels of Figure 13 are localized much deeper it should be possible to increase this depth
using longer offsets.

7. INVERSION TESTS

A way to obtain information about the resolution of the inversion is by performing test inversion on artificial data. Most often the data inverted has been calculated by forward calculation using the same sensitivity kernels employed in the inverse procedure (e.g. Kosovichev & Duvall 1997, Zhao et al. 2001, Korzennik 2001). Jensen et al. (2000a) calculated synthetic data using an independent forward calculation and performed the inversion using both ray approximation kernels and wave-theoretical kernels. The forward calculations were done by 2D finite difference modeling of the full wave field. The synthetic data was contaminated with 10% white noise before inversion. The results showed an improvement in the inferred averaging kernels for the wave-theoretical kernels, although the models inferred were similar. Other test inversions (Kosovichev & Duvall 1997, Zhao et al. 2001) have used noise-free data calculated using the same kernels to test the inversion results using the LSQR method. They find that flow patterns can be reconstructed well although it is difficult to reconstruct deep vertical flows since the wave propagation is primarily horizontal here. Korzennik (2001) performed test inversion on flow data using both the LSQR method and a truncated SVD method. He investigated both noise free data and data contaminated with white noise. He found that the flow pattern could be recovered in the upper 10 Mm except for the vertical component which could only be recovered very near the surface. Further investigation concerning the resolution of time-distance inversions would be very useful in the interpretation of data and comparisons of methods. Inversion of synthetic data calculated and inverted using the same kernels but contaminated with realistic noise as described in Jensen et al. (2002b) would be one way of improving the synthetic test. Another way would be to model the full 3D wave field using finite-differences, perform the correlation analysis on these data and then invert the obtained travel times. Figure 14 shows preliminary results of such an inversion test performed by Jensen et al. (2002b). The forward modeling was done by three dimensional finite-difference modeling of an acoustic wave field using a stochastic source. The model used in the calculations was 160 Mm by 160 Mm by 75 Mm. The top panel of Figure 14 shows the perturbations placed in a solar atmosphere for the modeling. From 8hrs of the surface oscillations obtained in the modeling time-distance data was calculated and then inverted. The inversion was carried...
out using the MCD methods and wave-theoretical kernels. The regularisation was found by using an L-curve criterion. In Figure 14 the bottom panel shows the result of the inversion. We see that it is possible to recover the general structure of the anomaly. Both the sound speed decrease near the surface and the increase further down are recovered. There was no noise added to data so much of the noise in the inferred model must be from the limited time used to calculate the data. Future studies using synthetic data calculated by finite-difference modeling can help to understand the properties of both time-distance data and inversion.

8. "SOLAR SURFACE TOPOGRAPHY"

Braun & Lindsey (2000) suggested that much of the reduction in travel times could be due to an "acoustic Wilson depression", where the upper turning point of the waves is shifted down. A down shift of 300 km could give a travel-time reduction of around 40 seconds. This interpretation is different from the linearized approach used to calculate sensitivity kernels for time-distance inversion. Brüggen & Spruit (2000) investigated the effect of temperature perturbations on travel times taking both the increased sound speed and the effect on the upper turning point into account and found that a increase in temperature could in some cases lead to an increase in travel time in their model in contrast to what just an increased sound speed would suggest. The interaction of acoustic waves with magnetic fields is complex and not fully understood. Therefore it is necessary to apply approximations in order to interpret the data. To be able to do linearized inversions the effect of the magnetic field has to be approximated as a perturbation to the reference model. This approach could, along with the concept of an acoustic Wilson depression, very well be too simplified to account for the interaction between sunspot and acoustic waves. The exact conditions in sunspots and their interactions with the incoming acoustic waves is an area of ongoing research (e.g. Bogdan 2000) and progress in this field will be necessary in order to better interpret the results of the local helioseismic methods.

9. SUMMARY

The time-distance method provides an opportunity to study a wide variety of solar phenomena in great detail. In this review I have shown some of the results obtained using time-distance measurements and how they compare with results from other methods. The results obtained for solar flows compare well with results from other measurements thus giving a good validation of the time-distance method. Both the solar rotation and meridional flows have been studied using time-distance helioseismology. The results for rotation agree with results from inversion of frequency splittings for global modes, while results for the meridional circulation agree well with inferred flows for ring-diagram analysis.

An important application of time-distance helioseismology is the study of active regions. Results for the same data set and different inversion methods have been discussed. The similarity of the results shows that the inversion procedures seem to be robust and what we are seeing are not artifacts introduced in the inversion.

In the future, studies of the uncertainty and reliability of time-distance inversions will be very important. Here results were presented for the resolution of the inversion results showing averaging kernels and model-error estimates. The spatial correlation of the measurement errors was taken into account and shown to be important. If the errors are assumed uncorrelated the model errors become underestimated. This can make inversion methods, such as MCD, that can handle the full covariance matrix very important in order to obtain proper uncertainty estimates. Another way of investigating the properties of the inversion results is by inversion of synthetic data. Figure 14 show preliminary results from inversion of synthetic data obtained from three-dimensional finite-difference modeling of the wavefield. Such studies will be very useful in the future for improving the understanding of the results from time-distance inversions.

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HELIOSEISMIC PROBING OF THE SOLAR DYNAMO

Alexander Ruzmaikin
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA, E-mail: aruzmaik@pop.jpl.nasa.gov

Charles Lindsey

ABSTRACT

According to theoretical predictions, the solar dynamo operating in the convection zone generates maximal magnetic fields near the base of the convection zone. Detection of this field is a challenging task for helioseismology. We discuss the ways of probing the magnetic field in the solar interior and estimate the magnitude of the field that can be detected with presently achievable accuracy. It is easier, however, to detect the flows that drive the dynamo. We describe the major flow parameters related to the dynamo flows and present the requirements for measurement of these parameters with local helioseismic techniques.

1. MAGNETIC FIELD IN SOLAR INTERIOR

Solar activity originates in interplay between magnetic fields and fluid motions within the solar convection zone (Parker, 1979). Fields emerge at the solar surface, diffuse, reconnect, and are transported by the solar wind into the Heliosphere. The source of the major magnetic field is expected to be near the base of the convection zone (tachocline). A challenging task for helioseismology is to detect the magnetic field there.

A simple estimate of the field strength can be obtained from the balance of magnetic and Coriolis stresses, which involves azimuthal and radial components of the field: \( B_B = 4 \pi \rho \nu \Omega l \), where \( \rho, \nu, \Omega, l \) are the density, convective velocity, angular velocity and a characteristic scale of the field at the bottom of the convection zone (Zeldovich et al., 1983). Inferred from surface observations (associated with the sunspot fields) the mean azimuthal field exceeds the mean radial field by a factor of 100. Using \( \rho = 0.2 \) g/cm\(^2\), \( \Omega = 3 \times 10^{-6} \) s\(^{-1}\), and \( \nu = 10^5 \) cm/s, \( l = 0.05 \) x depth of convection zone = 2 \( 10^7 \) cm, the mean azimuthal field is estimated to be about 310 \( \mu \)G. This number is very rough due to uncertainties in \( \nu \) and \( l \), so that the field could be larger or smaller than 10\( \mu \)G.

A magnetic field strength close to 10\( \mu \)G or larger is required for magnetic flux tubes formed near the tachocline to emerge at the solar surface (c.f. Caligari et al., 1995; Longcope et al., 1996). Observational estimates a close value for the field follow from the knowledge of the observed flux (about 10\(^2\) Mx per active region), latitudinal distribution of active regions (< 40°) and tilts of the lines connecting leading and following sunspots (D’Silva and Choudhuri, 1993).

Magnetic fields break the spherical symmetry of the Sun resulting in splitting the frequencies of acoustic oscillations. The magnetic splitting is quadratic in the field (compared to the rotational splitting proportional to the angular velocity). Attempts have been made to detect the splitting using the global helioseismology (Dziembowski and Goode, 1989; Gough, 1990; Basu, 1997). The best estimate obtained in this way is an upper limit of about 2-3MG.

2. DYNAMO FLOWS

The magnetic field is generated by motions (flows) of the conducting plasma in the convection zone. These motions are called dynamo. We will show that it is easier to detect dynamo flows inside the Sun than the field. Let us first describe the main dynamo flows.

2.1. Differential Rotation

One type of dynamo motions is the differential rotation, that converts a poloidal field into the toroidal one. The knowledge of the gradient is more important than the angular velocity itself, although dynamo waves basically propagate along the two-dimensional surfaces of \( \Omega(r, \theta) \). Global helioseismology, using \( p \)-waves, has established the distribution of differential rotation inside the convection zone and gave evidence on its time variations (Tomczyk et al., 1995; Howe et al., 2001).

The validation of these variations and more accurate determination of gradients of \( \Omega(r, \theta) \) near the tachocline are still needed.

2.2. Mean Kinetic Helicity

Another element of the dynamo is the mean kinetic

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helicity $\text{KH} = \langle \nabla \times \mathbf{v} \rangle$. It arises because the number of cyclonic vortices in the rotating convection zone is different from the number of anti-cyclonic vortices (Zeldovich et al., 1983). Like the rotation, the mean kinetic helicity is a 2-D smooth function that may be (but has not yet been) determined by global helioseismological methods. The latitudinal distribution of the KH is determined by the Coriolis force so that KH has opposite signs in the northern and southern solar hemispheres. The radial distribution of the KH (peak at the tachocline, in the middle of the convection zone, or near the solar surface?) is presently unknown. Different locations of KH result in different locations of the maximal magnetic field generated by the dynamo (Bigazzi and Ruzmaikin, 2003). Thus the determination of the position of the field maximum inside the Sun will discriminate between different dynamo models. Note that the quantity $\alpha = \tau \langle \nabla \times \mathbf{v} \rangle$, where $\tau$ is a convective turn-over time, is directly involved in the dynamo. It has the dimension of speed and expected to be in the range $10^{-100}$ m/s (Zeldovich et al., 1983).

2.3. Turbulent Diffusion

Because of the extremely high plasma conductivity in the convection zone the mean-field dynamo cannot effectively work without destruction and transport of the field by turbulent diffusion. The turbulent diffusivity is proportional to the mean square of the convective velocity, $\langle \mathbf{v}^2 \rangle$. Estimates of the surface diffusivity, obtained from studies of supergranulation motions, are in the range $(1.2-6) \times 10^8$ m$^2$/s (Wang and Shelley, 1993; Simon et al., 1995; Ruzmaikin et al., 1996). According to mixing-length convection models the diffusivity does not vary much in depth within the convection zone, but the abrupt change in diffusivity by about 8 orders of magnitude that occurs at the tachocline directly affects the dynamo (Zeldovich et al., 1983).

2.4. Meridional Circulation

Another type of dynamo related motion is meridional circulation. This flow is not required for a dynamo to work but it can provide an important transport of the field to the sources of generation. Meridional flows confined to the convection zone are included in some solar dynamo models (Durney, 1997; Dikpati and Charbonneau, 1999). To explain the observed latitudinal distribution of sunspots, Nandy and Choudhuri (2002) developed a dynamo model with a meridional flow that transports the field into the radiative zone to 0.6 $R_\odot$, i.e. well below the tachocline. A meridional flow of about 20 m/s is directly measured on the solar surface. First attempts have been made to follow it below the photosphere (Giles et al., 1997) but available data (from SOHO and GONG) do not provide sufficient accuracy to definitively establish the pattern of meridional flows deep within the convection zone. The expected magnitude of the flow at the base of the convection zone near the equator is $3 \text{m/s}$.

3. METHODS OF MEASUREMENTS OF THE FIELD AND FLOWS

Here we outline the major methods for measuring the dynamo fields and flows that use 3-D (local) helioseismology.

A simple geometric optics approach is to measure the acoustic travel time of a wave packet between different points on the solar surface, $r_1 - r_2 = D$, and then to infer the flows and fields from the variation of these times (Kosovichev and Duvall, 1997). Travel times are extracted from the maxima of the cross-correlation function $\langle f(t,r)f(t+\tau,r+D) \rangle$ of Doppler (or intensity) signal $f(t,r)$ presented in the form

$$\Psi(\tau,\Delta) \sim \cos[\omega_0(\tau - \Delta / \nu)] e^{-|\delta\omega(\tau - \Delta / \nu)|^2},$$

where $\omega_0 = 2\pi v_0$ and $\delta\omega$ are the central frequency and width of the filtered signal, and $\nu, u$ are the phase and group velocities of the chosen acoustic mode.

Variations of $\tau$ due to magnetic and flow effects can then be studied. Thus a variation of the travel time of a fast magneto-acoustic wave passing through a magnetic field is

$$\delta T = -\frac{1}{2} \int \left( \frac{2\delta \mathbf{c}}{c} + \frac{\mathbf{v}_A^2}{c^2} - \frac{v_A^2}{k^2 c^2} \right) k ds,$$

(Kosovichev and Duvall, 1997), where $c$ is the speed of sound and $v_A$ is the Alfvén speed, and the integral is taken along the path of the wave defined by $v_A(k, \mathbf{k})$. The effect maximizes for the wave propagating perpendicular to the magnetic field.

There is also an Alfvén mode propagating along the field lines. Consider a magnetic loop that, in accord with the standard (Babcock) model, emerged at solar surface to form an active region field and is connected to a toroidal field at the tachocline. Then

$$r_A^2 = \frac{-1}{2} \int ds \frac{2 \pi R_e + 2h}{v_A} \frac{v_A^2}{v_A},$$

where $R_e = 0.7 R_\odot$ and $h = 0.3 R_\odot$ are the radius and the depth of the convection zone ($R_\odot$ is the solar radius). An accurate measurement of this time would give the value of B at the tachocline. For B = 10$^8$ G, $v_A = 10^4$ m/s at the tachocline. With $v_A = 10^4$ m/s at the surface, the travel time is longer (about a month) at the tachocline. Hence, active regions that live longer than a solar rotation are needed for this purpose.

A flow of speed v produces a variation

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\[ \delta \tau_v = -2 \int \frac{vdS}{c^2(r)} \]

(Kosovichev and Duvall, 1997).

A more sophisticated method is to locate acoustic sources inside the Sun as regions of convergence of waves propagating backward in time from the solar surface (similar to the use of the focal point in optics to locate the source position). The measurable quantity is the phase-correlation \[ C(r, \nu) = \langle H_\nu(r), H_\nu^*(r) \rangle \]
between the amplitude of acoustic radiation at frequency \( \nu \) coherently focused into a point \( r \) inside the Sun and amplitude of the same radiation emerging back to the solar surface in a conjugate pupil after passing through the neighborhood of \( r \) (Lindsey and Braun, 2000). The travel time along the wave path is defined as

\[ \tau = \frac{1}{2\pi \nu_0} \arg \int_{\Delta \nu} C(r, \nu) d\nu \]

(Lindsey and Braun, 2000), where \( \Delta \nu \) is a bandpass centered at \( \nu_0 \). The determination of correlations in both cases requires statistical averaging over a number of spatial \( N_s \) and frequency (time) \( N_r \), resolution elements.

4. MEASUREMENT REQUIREMENTS

The acoustic responses to the undersurface magnetic field and flows are weak and have to be extracted from a noisy environment. Let us briefly estimate the major technical requirements to the measurements.

The error in time delay can be estimated as

\[ \delta \tau = \frac{\delta \phi}{2 \pi \nu} = \frac{1}{2 \pi \nu (N_s N_r)^{1/2}} \]

where \( \delta \phi \) (in radians) is the error in phase and \( \nu \) is the signal frequency. The number of frequency-resolution elements is \( N_s = \delta \nu T \), where \( \delta \nu \) is the bandwidth of selected frequencies inversely proportional to the time cadence of imaging, and \( T \) is the time interval over which the correlation function is integrated. The number of spatial resolution elements is proportional to the size of the region used for imaging and can be expressed as \( N_r = S / \lambda^2 \), where \( S \) is the area of the resolution pupil, and \( \lambda = c / \nu \) is the wavelength which determines the diffraction-limited resolution. We see that the time delay resolution is inversely proportional to \( \nu^2 \), i.e. it is favorable to use higher frequencies.

For a rough numerical estimate let us take \( 1 / \nu = 150 s \), resolving a size \( \lambda = 75 \) Mm at the tachocline, where speed of sound is \( c = 0.5 \) Mm/s. With an image area 60° by 60°, bandwidth 2 MHz, and an integration time \( T \)

= 1 day we get \( \delta \tau = 0.2 \) s. Repeating the time series say for 100 days and increasing the image area we can improve the accuracy by factor of \( (100)^{1/2} \approx 10 \). An extra gain (proportional to \( N \)) can be achieved with the use of multiple \( (N) \) bounces of the wave packets, because the change in travel times increases linearly with the number of bounces (Chou and Serebryanskiy, 2002). With the waves traveling through the tachocline the gain is about 8. In this way we may achieve an accuracy of about \( \delta \tau = 0.0025 \) s. [Selecting from MDI and GONG data the wave packets that return to the same point after integer number of bounces Chou and Serebryanskiy estimated the travel time change for waves traversing the tachocline over a solar cycle as 0.05±0.02 s.]

Let us see now what amplitudes of variation produced by the magnetic field and flows at the tachocline can be measured with this accuracy. A maximal contribution to the time delay from magnetic perturbations can be estimated as \( \delta \tau_b = (1 / \nu)(B/8 \pi \rho c^2) d\sigma / R \). Taking \( k = 2 \pi / R \), where \( R \) is the solar radius, \( \nu = 1/150 s \), and the size of magnetic region affecting the integral \( d\sigma = 0.1 R \), we estimate the effect of magnetic perturbations as \( \delta \tau_b \approx 2.510^{-4}(B/10^5 G)^2 \). Hence only field strengths about 1 MG are directly detectable with the above accuracy in delay times. An improvement in accuracy is needed to detect \( 10^4 G \) field by this method.

The magnetic effect is weak because it is a second-order in \( (\nu_0 / \nu) \). Detection of a flow of speed \( \nu \) is more promising because its effect is linear, i.e. \( \delta \tau_\nu / \tau \propto \nu / \nu \). For example, with the travel time accuracy 0.01 s we can detect \( \nu = 0.1 m/s (R / d\sigma) \approx 1 m/s \) near the tachocline. The accuracy of the detection depends on the path-length \( d\sigma \).

5. ADVANTAGE OF MULTI-ANGLE OBSERVATIONS

The determination of dynamo flows requires deep probing of the solar interior. The best way to achieve this objective is the use of Doppler imaging of the Sun from two widely separated positions (Figure 1). The Earth-side imaging can be effectively provided by the ground-based GONG and space-based SOHO/MDI followed by SDO. The second multiple-angle imaging can be provided by a spacecraft orbiting in the circular (Venus) orbit at 0.7 AU (Ruzmaikin et al., 2002).

This will allow the measurements of signals from p-wave packets traveling through all depths of the Sun. When imagers are separated by 45° to 60° the traveling waves traverse the tachocline and close regions above and below it allowing the test for the penetrating meridional flows, gradients of rotation, and turbulent diffusivity. When the separation angles are wider the multiple bounced waves can be measured. At 180° of separation the acoustic signal goes straight through the solar core.

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Figure 1: Imaging from two positions allows the probing of the solar dynamo flows at all depths.

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TIME-DISTANCE ANALYSIS OF GONG+ DATA

S.P. Rajaguru, S.J. Hughes, and M.J. Thompson
Space & Atmospheric Physics Group, The Blackett Laboratory, Imperial College, London, SW7 2BW, UK

ABSTRACT

We show our first time-distance analyses of data from GONG+. We present details of our method of analysing the data and travel-time anomalies measured over an active region when compared to that over a quiet region. The dataset used for studying the active region is from a merged sequence of GONG+ Doppler images.

1. INTRODUCTION

We present time-distance analyses of data from the new GONG+ high-resolution network. In particular we measure p-mode travel-time anomalies in an active region, using the methods of time-distance helioseismology (Duvall et al. 1993). We analyse an 8.5 hour sequence of Doppler images from a merged GONG+ dataset for 1 November 2001 when an active region passed the central meridian at a latitude of 7.5° north. For our study we selected a 256 x 256 pixel area around disk centre, containing the active region, from the time sequence of GONG+ full-disk Dopplergrams.

2. METHOD OF ANALYSIS

(I) Tracking: The rotation-corrected sequence of images were tracked at the latitude dependent solar rotation rate.

(II) Filtering: The 3-D Fourier transform (FT) of the sequence of images were filtered by applying the following filters.

(i) A temporal frequency filter with full transmission starting at 1.7 mHz and ending at 5.3 mHz with Gaussian roll-offs of FWHM 0.4 mHz at either end.

(ii) Filter to remove the f mode by approximating the frequencies of the f and \( p_1 \) modes as polynomials in horizontal wavenumber (Giles 1999).

(iii) A phase velocity filter in the \((k_x, k_y, \nu)\) space to select p modes that travel horizontal distances in the range of 11 - 120 Mm. Here again the filter surfaces in the Fourier space were constructed to have Gaussian roll-offs.

(III) Cross-Correlations: The filtered 3-D FT of the data cube was inverted back in the 2 spatial dimensions, and cross-correlations were then computed in the temporal frequency domain. We have computed time-distance correlations between signals at pairs of points as well as between that at a central point and the average over an annulus. The centre-annulus cross-correlations for this paper were computed for an annulus of radius 52 Mm about a centre point of 9 neighbouring pixels. We scan an area of 180 Mm x 180 Mm (100x100 pixels) covering the chosen active region, which contains a single large spot (See Fig. 4.), by moving the centre point in steps of 11 Mm in N-S and E-W directions, resulting in 256 (a 16 by 16 points map) measurements of cross-correlations. Each measurement is an average of centre-annulus correlations over an area of 36 Mm².

(IV) Fitting the Cross-correlations: We fit the Gabor wavelet (see, e.g., Hughes & Thompson, these proceedings) to the cross-correlations by choosing a peak in the envelope of the first bounce and varying the parameters of the wavelet to recover the phase time, which we use as our estimate of the travel time.

3. RESULTS

The upper panel of Fig. 1 shows a time-distance diagram for an early GONG+ dataset derived from observations made at the Tucson site on 8 May 2000. The different bounces are clearly visible; the straight rays visible at small time-lags are probably artifacts. The lower panel shows similar results for a quiet part of the region of our study. Apart from the different data used, the results in the two panels also derive from different methods of analysis: the upper results use point-to-point cross-correlations, whereas the results in the lower panel use centre-annulus cross-correlations.

Figure 2 compares the first-bounce cross-correlation
Figure 1. The top image shows a t-d diagram obtained using point-to-point correlations in a region of quiet Sun for 8 May 2000 (data from the Tucson site). A point here is the average signal at four neighbouring pixels. The cross-correlation signal at each distance is an average of 40 x 40 pairs of points, half of them in E-W direction and the other half in N-S direction. The lower image is a t-d diagram obtained using centre-annulus correlations, using the merged GONG+ dataset for a region of quiet Sun on 1 November 2001. The signal at the central point is an average of 9 neighbouring pixels, and annuli are 2 pixels thick with cosine tapering. The cross-correlation measurement at each distance is an average of centre-annulus correlations over an area of 36 Mm$^2$.

Figure 2. A comparison of cross-correlation signals in a quiet and an active region for a first bounce distance of 52 Mm. The positive time-lag correlations are shown. The faster arrival of the wave-packet in the active region is clear.

Signals at a distance of 52 Mm in the chosen active region and in a quiet region offset from the active region. The traces measured in the active region are shifted discernably to smaller times, indicating that the wave packets in the active region travel faster than those in the quiet sun.

Figure 3 shows a map of the travel time anomalies measured over the active region. The travel times averaged between inward- and outward-propagating waves (lower panel) are substantially reduced in the active region. We have avoided placing the centre point of annuli in the area covered by the spot. The maximum difference in mean travel times in the active and quiet regions is about 18 s, which corresponds to about 2% change in sound speed close to the surface layers. In this preliminary study we have used only one distance interval of 52 Mm for measuring travel times and so the above value is just a rough estimate assuming that the perturbations sensed are near surface. The difference in travel times between inward- and outward-propagating waves within the region covered by the annulus, which is sensitive to vertical flows and to the divergence of the flow, is substantially greater in the active region than in the quiet sun. Shown in the upper panel of Figure 3 are the travel times from the centre to annulus minus that from the annulus to centre, which indicate a general converging flow of about 200 m/s around the spot region in the near surface layers. Again, these estimates from the travel times are only approximate values, and the acoustic rays travelling the chosen horizontal distance of 52 Mm travel up to a depth of about 16 Mm.

4. CONCLUSIONS

We have measured p-mode travel-time anomalies in an active region using the GONG+ merged data for the first time. In the active region the mean travel times are reduced by about 2%, which reflect the faster wave speed beneath the active region and pos-
sibly also shorter path lengths of acoustic waves due to a shift in the location of the upper turning point of the waves. The difference in travel time between inward- and outward-propagating waves is sensitive to the downflows and flow divergences, and we find that this difference is much larger in the active region than outside it, by about a factor of 100, indicating converging flow magnitudes of about 200 m/s.

We have not yet fully examined the resolution achievable for the full tomographic study of a sunspot using the GONG+ data, and the inversion of full time-distance data from GONG+ is currently underway. But our results presented here demonstrate that GONG+ can be of much use in studying the dynamics of active regions.

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Temporal Variations in the Solar Interior

Chair: M. Thompson
THE INTERNAL ROTATION OF THE SUN

R. Howe
School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
National Solar Observatory P.O. Box 26732, Tucson, Arizona, 85726-6732, USA
e-mail: rhowe@noao.edu

ABSTRACT

Over the past several years, helioseismic data from the Michelson Doppler Imager aboard the SOHO spacecraft, and from the Global Oscillation Network Group, have allowed us to study the changing dynamics of the solar convection zone in greater detail than ever before. We now know that the zonal flows of the so-called torsional oscillation extend well into the convection zone though apparently not to its base, and there seem to be rotation variations of a shorter period around the tachocline region which is crucial to theories of the solar cycle. At higher latitudes, the rotation rate varies strongly during the solar cycle. Modeling and simulation studies attempt to reproduce this behavior with varying degrees of success.

1. INTRODUCTION

2. THE DATA

The MDI instrument has been collecting medium-l data since May 1996 and the GONG project since May 1995. The 'original' results (mainly updates of previous work) presented in this paper are based on 68 overlapping 108-day periods of GONG data, including p-modes up to l = 150, and 30 non-overlapping 72-day periods of MDI data with p-modes up to l = 170 and f-modes up to l = 300. This represents approximately 50% more data than was available to Howe et al. (2000a, 2000b) – enough for substantial improvements in the precision of the measurements. The GONG instruments were upgraded to 1024 × 1024-pixel cameras over the Northern Hemisphere Summer of 2001. The transition period resulted in some degradation of the duty cycle and hence the quality of the inversion results in mid-2001, but no serious discontinuities were found in the data.

The GONG data have recently been re-analysed using Multitaper Spectral Analysis to improve the estimation of the peak parameters. This has resulted in 10-15% more modes being successfully fitted, with the most benefit seen in the lower-frequency modes.

The rotation results were obtained using the same 2d-RLS and 2d-OLA techniques and regularization choices used in the above-cited work.

3. THE GLOBAL PICTURE

Our picture of the mean solar rotation has become much clearer since the early work of Woodard & Libbrecht (1988) on BBSO data, and even since the work of Schou et al. (1998), which was based on the first 144 days of MDI medium-l data. The earlier work established the existence of the shear layer, now commonly known as the 'tachocline', at the base of the convection zone and the subsurface shear layer, and indicated that the rotation rate in the convection zone was approximately constant along radial lines rather than (as some models had predicted) along cylinders parallel to the rotation axis. The tachocline region features prominently in dynamo models of the solar cycle.

Figure 1 shows the mean rotation profiles derived from RLS inversions of the GONG and MDI data. The near-surface shear layer and the tachocline are clearly visible in both data sets. It should be noted, however, that neither feature is fully resolved in these inversions. It is also apparent that the description of the rotation as constant along radial lines is no longer adequate. In fact, the angle of the isorotation contours is almost constant over a wide range of latitudes. This issue is discussed further by Gilman & Howe (2003).

3.1. Comparing MDI and GONG

Figure 1 shows that the results from MDI and GONG agree well in the lower-latitudes in the convection zone and show increasing discrepancies at greater depths and higher latitudes. Schou et al. (2002) carried out a detailed comparison exercise in which three 108-day periods of GONG and MDI data were each analysed using both the GONG ('AZ') and MDI ('CA') algorithms. The results indicated that the discrepancies come almost entirely from the analysis rather than from the underlying data. The excess rotation rate seen in the CA data


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at higher latitudes deep in the convection zone is associated with an anomaly in the CA coefficients at around 3.2 mHz, the cause of which remains obscure. Excluding coefficients at 3mHz and above, and reducing the data set to that common to CA-analysed MDI and AZ-analysed GONG results, produces almost complete reconciliation of the corresponding rotation profiles in the convection zone. The lower rotation rate and apparent oblateness seen in the GONG rotation rate below the convection zone, on the other hand, is believed to be associated with systematic errors in the AZ fitting caused by neglect of the leakage from spectra of the same \( l \) and neighbouring \( m \) values. These issues, however, appear to be more or less independent of the epoch when the data were taken and so do not affect the study of temporal variations.

### 3.2. Near-Surface Shear

The subsurface shear layer has recently been studied in detail by Corbard & Thompson (2002) using the MDI \( l \)-modes. They found that the radial gradient of angular velocity was approximately constant up to 30 degrees latitude and then decreased, reversing sign at the highest latitudes. However, the high-latitude results rely heavily on the modes with \( l \geq 250 \), the reliability of which is somewhat questionable. No evidence of temporal variation of the rotation gradient was found, even though the migrating zonal flows (discussed in Section 4.1 below) passed through the region of interest during the period of the observations considered. Dikpati, Corbard, Thompson, & Gilman (2002), in a related theoretical study, concluded that the near-surface shear alone is not sufficient to account for the whole of the dynamo effect required to drive for the solar cycle; the tachocline-based dynamo effect is still required.

### 4. TEMPORAL VARIATION

In order to study the temporal variation of the rotation, we subtract from each inversion a ‘baseline’ rotation profile. In most of the work illustrated here we use a temporal mean over all the data sets; other possible choices are to use the profile at a specified epoch such as the solar minimum, as was done by Vorontsov et al. (2002), or a smooth profile fit to the temporal mean, as used for example by Schou (1999). Each method has advantages and disadvantages, and results in slightly different pictures of the variations. The broad features of the variation, however, are seen regardless of the method used.

### 4.1. Penetrating Zonal Flows

The pattern of zonal flows, migrating over the course of a solar cycle from middle to low latitudes, was first noted by Snodgrass, Howard, & Webster (1985), and has been studied in surface Doppler measurements by Ulrich (2001). The flows were detected in MDI \( f \)-mode data by Schou (1999), and in \( p \)-mode data from GONG and
MDI by Howe et al. (2000b). The latter work, in conjunction with artificial-data simulations to test the resolution of the inversion results, allowed the flows to be followed to a depth of around 0.092R⊙. More recently, Vorontsov et al. (2002) have claimed to find indications that the flows may penetrate deeper still, perhaps as far as the base of the convection zone. Our own results (Fig. 2) now show hints of organized patterns deeper than was originally thought. This improvement is mostly due to the longer integration time, resulting in a cleaner mean profile to subtract. However, care is needed in interpreting such plots, due to the finite resolution of the inversions, and considerable work remains to be done before a firm conclusion can be reached.

4.2. High-Latitude Changes

At latitudes poleward of around 50 degrees, the rotation shows a strong temporal variation, as seen in Figure 3 and also in Figure 2. In Figure 3, the variation has been referred to a two-term fit to the latitudinal variation of the temporal mean at each depth, in order to emphasize that the high-latitude rotation, at least at the surface seems to lie below this smooth profile during the period of the observations so far. However, the trend of the most recent points suggests that this may not be true throughout the declining phase of the cycle. The work of Vorontsov et al. (2002) suggests that these variations penetrate very deep into the convection zone, and this conclusion seems to be borne out by the results illustrated here. However, the latitudinal resolution of the inversions degrades more quickly with depth at higher latitudes, meaning that the structure of the flows here is not readily apparent; there may be a ‘poleward moving branch’ of the zonal flows, and, for example, Antia & Basu (2001) have interpreted their results as showing this, but the inversions do not appear to unambiguously resolve such a feature.

4.3. Variations Near The Tachocline

Below the region where the zonal flows can be clearly detected, in the vicinity of the tachocline, Howe et al. (2000a) found variations with a period of approximately 1.3 years, with the peak amplitude at 0.72R⊙ and an anticorrelated variation at 0.63R⊙. This result has remained controversial, and was not reproduced in the work of Antia & Basu (2000) or Basu & Antia (2001), or by Vorontsov et al. (2002). The results are also somewhat difficult to interpret due to the poor latitudinal resolution of the inversions at this depth, and a radial resolution insufficient to properly resolve the tachocline. In any case, the amplitude of the signal has been greatly reduced in the more recent observations; it may or may not be interesting that this reduction coincides with higher solar activity. The current results are shown in Figure 4.

The work of Covas, Tavakol, & Moss (2001) on meanfield dynamo modelling suggests that it may indeed be possible under some conditions to have different periods of variation at the top and bottom of the convection zone, described as ‘spatio-temporal fragmentation’.

5. DISCUSSION

The past few years have sharpened our picture of the solar interior rotation and its variations. It is becoming possible to detect subtle variations deep within the convection zone, but further work, both from observation and theory will be needed to confirm the interpretation of these data. Some aspects may become clearer once we have a full solar cycle of data to study.

ACKNOWLEDGMENTS

This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Interamerican Observatory. The author was supported in part by NASA contract S-92698-F, and also acknowledges support by the UK Particle Physics and Astronomy Council through a grant to the University of Birmingham.

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Figure 2. Deviations from mean rotation profile as a function of date and latitude at selected depths, for MDI OLA (left) and GONG RLS (right) inversion results. The greyscale is in nanohertz.
Figure 3. Rotation residuals after subtraction of a smooth constant profile, at selected high-latitude locations. Filled circles represent GONG RLS inferences, filled triangles MDI RLS, and open triangles MDI OLA.

Figure 4. Variations in inferred rotation rate at selected locations near the base of the convection zone for GONG (circles) and MDI (triangles), with filled symbols representing RLS inversion results and open ones OLA.


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LARGE SCALE FLOWS THROUGH THE SOLAR CYCLE

David H. Hathaway

National Space Science and Technology Center
Mail Code SD50, NASA/MSFC, Huntsville, AL 35812 USA
tel: 256-961-7610/ fax: 256-961-7216
e-mail: david.hathaway@msfc.nasa.gov

ABSTRACT

Large scale flows within the solar convection zone are thought to be the primary drivers of the Sun’s magnetic activity cycle. Differential rotation amplifies the magnetic field and converts poloidal fields into toroidal fields. Poleward meridional flow near the surface carries magnetic flux that reverses the polar magnetic polarities. A deeper, equatorward meridional flow may carry magnetic flux toward the equator. These axisymmetric flows are themselves driven by large scale convective motions. Given these intimate connections between the large scale flows and solar activity, it would be surprising if there were not solar cycle variations in the flow characteristics. Some variations, namely the torsional oscillations, are well established. Other variations, namely changes in the meridional flow and in the shear at the base of the convection zone, are more controversial. In this paper I describe the observed characteristics of the solar cycle and the large scale flows and discuss the nature of the solar cycle variations.

INTRODUCTION

Understanding the nature of the solar activity cycle has been a major problem for solar astronomy in spite of nearly 400 years of observations. The 11-year period of the cycle was discovered over 150 years ago (Schwabe, 1844). The equatorward drift of the active latitudes was discovered shortly thereafter (Carrington, 1858). Even the magnetic characteristics of the cycle and the tilt of active regions have been known for over 80 years (Hale et al., 1919). Yet, an understanding of the nature of the cycle seemed beyond our reach until the advent of helioseismology.

With helioseismology came measurements of the Sun’s internal structure and rotation profile that greatly restricted possible dynamo models. Kinematic dynamo models from the 1960’s and 1970’s could reproduce some of the major characteristics of the cycle but these models relied on large scale flow characteristics that helioseismology has shown to be incorrect. Developments in dynamo theory during the 1980’s and 1990’s placed the seat of the dynamo at the interface between the convection and radiation zones where magnetic flux can be stored long enough for dynamo action to be effective. The discovery of the tachocline, the layer of strong radial shear in the zonal flow, at this interface added further evidence in support for the interface dynamo models.

Within the last decade the meridional circulation has been recognized for the role it may play in the solar dynamo. Earlier dynamo models relied on propagating dynamo waves to give the equatorward migration of the active latitudes. This process is augmented with, or replaced by, a deep equatorward meridional flow in some recent dynamo models. Observing the characteristics of the meridional circulation deep within the Sun should discriminate between these dynamo possibilities.

If, as expected, these large scale flows drive the solar cycle we should expect to see variations in the behavior of the solar cycle associated with variations in the large scale flows themselves. In the next section I present some of the characteristics of the solar cycle and their variability. In the following sections I describe the variability in the large scale flows and indicate where significant progress should be made before we can achieve a more complete understanding of the solar cycle.

SOLAR CYCLE CHARACTERISTICS

The most significant characteristics of a solar cycle are its period and amplitude. These quantities are usually determined from smoothed sunspot numbers but similar results are obtained from other indicators such as sunspot area and 10.7 cm radio flux. Fig. 1 shows the last 100 years of smoothed sunspot numbers and sunspot area along with 50 years of smoothed 10.7 cm flux. This illustrates how close the agreement is between these various indicators of solar activity. Using monthly averaged sunspot numbers since 1749 smoothed with a 24-month FWHM Gaussian gives an average cycle period of 10.9 years but with a range from 8.8 to 14.0 years. The average amplitude is 103 sunspots but the range of amplitudes is from 43 to 187.

![Figure 1. Sunspot numbers (solid line), total sunspot area (dashed line), and 10.7-cm radio flux (dotted line) smoothed with a 24-month Gaussian filter over the last century. All three indicators give similar results for the amplitudes and periods of the solar cycle.](imageurl)
Comparing cycle periods with their amplitudes reveals no significant correlation between these two characteristics (in spite of reports to the contrary). However, there is a significant correlation between the period of a cycle and the amplitude of the following cycle (Hathaway, Wilson, Reichmann, 1999). This is shown in Fig. 2. The left hand panel shows the period vs. the amplitude of the same cycle. The correlation coefficient is only 0.3 and the probability of obtaining this distribution from uncorrelated quantities is 0.33. The right hand panel shows the period vs. the amplitude of the following cycle. The correlation coefficient is 0.7 and the probability of obtaining this distribution from uncorrelated quantities is only 0.04. The median values for the last 22 complete cycles are shown with dotted lines in this figure while the best linear relationships between the two quantities are shown with solid lines.

Figure 2. Current cycle amplitudes vs. cycle periods (left panel) compared with following cycle amplitudes vs. periods (right panel). The cycle amplitude is significantly anti-correlated with the period of the preceding cycle.

Two other significant relationships seen in sunspot number characteristics should also be noted. The first is a correlation between the amplitude of a cycle and the rise time – the time it takes to go from minimum to maximum (the Waldmeier effect, Waldmeier, 1935). The second is a correlation between the amplitude of a cycle and the level of activity at the minimum at the start of the cycle (the Amplitude-minimum effect, Wilson, Hathaway, Reichmann, 1998). These two relationships are shown in Fig. 3. The correlation between amplitude and rise time is shown in the left hand panel while that between amplitude and minimum is shown in the right hand panel. The correlation coefficients are 0.7 in both cases. The probability of obtaining the first distribution from uncorrelated quantities is 0.07 while that of the second is a mere 0.001. As in Fig. 2, the median values are shown with dotted lines in this figure while the best linear relationships are shown with solid lines.

These correlations in sunspot number characteristics are consistent with a simple rule for the solar cycle: big cycles start early and rise rapidly to maximum (Hathaway, Wilson, Reichmann, 2002). By starting early they cut short the preceding cycle thus giving it a short period. By starting early they increase the overlap between cycles thus raising the sunspot numbers at minimum. The rapid rise directly reproduces the Waldmeier effect.

Figure 3. Amplitude vs. rise time (left panel) and amplitude vs. sunspot number at minimum (right panel). Big cycles take less time to reach maximum and have more sunspots at minimum.

A vast array of periodicities has been attributed to the solar cycle. Most of these periodicities are found to have little significance upon closer inspection. Three inter-cycle periodicities have been noted previously: a seven to nine cycle periodicity (Gleissberg, 1939); a two cycle periodicity (Gnevyshev and Ohl, 1948); and recently a three cycle periodicity (Ahlawalia, 1998). After removing a linear secular increase in cycle amplitude since the Maunder Minimum (1715), there is little evidence favoring any of these inter-cycle periodicities. Finding the best fit to the inter-cycle amplitude variations using any one of these three periodicities does not significantly reduce the variance in cycle amplitudes (Hathaway, Wilson, Reichmann, 2002).

The number of reported short-term periodicities is even larger. Finding these periodicities is complicated by the Sun’s rotation. We only see one hemisphere at a time so any true periodicities are aliased by our observing window. Amongst these short-term periodicities two have received considerable attention: one with a period of about 155 days (Bai and Sturrock, 1987; Lean, 1990); another with a period of about 2-years (Benevolenskaya, 1995).

An average power spectrum from the last 13 full solar cycles is shown in Fig. 4. It shows sharp 2+ sigma peaks at periods of about 10-years, 2-years, and 150 days as well as broader peaks representing periods near 1-year and near 8-months. The peak at about 150-days was particularly prominent (a factor of three higher) during cycle 21 when it was first discovered in solar flare data (Reiger et al., 1984).

Fig. 4 was constructed from daily International Sunspot Numbers since 1849. The daily numbers were filtered with a 54-day FWHM Gaussian filter to remove any aliasing due to solar rotation and then sampled at 27-day intervals. Each cycle was then fit with the two-parameter function of Hathaway, Wilson, and Reichmann (1994) and this fit was subtracted to give residual sunspot numbers. These residual sunspot numbers display a prominent modulation by the solar cycle itself that is almost certainly the source of the spectral peak at about 0.1 cycles per year. They also exhibit some fairly regular oscillations from time to time including the 2-year “quasi-biennial” variation represented by the peak at 0.5 cycles per year. This quasi-
biennial feature is the likely source of the double peaked maxima seen in cycles 22 and 23. None of these short-term periodicities are reliable enough to provide any predictive power. Neither their strength nor their persistence can be relied upon. The do, however, suggest that some underlying process is at work within the Sun to produce these variations.

![Graph of power spectrum with labels: 13 Cycle Average, 2 Sigma Range, Gaussian Noise Level. X-axis: Frequency (cycles/year), Y-axis: Power.]

Figure 4. Average power spectrum of sunspot number residuals from 13 solar cycles. Sharp peaks at periods of 2-years and 150-days are more than 2-sigma above the noise level. The large peak at 1/10 cycles/year is due to the modulation of the amplitude of the residuals with the cycle period itself.

The locations, orientations, and polarities of sunspot groups provide added information about the solar cycle. The equatorward drift of the active latitudes where spot groups appear was first noted by Carrington (1858) but was referred to as “Sporer’s law” by Maunder (1904) and later authors. This drift is readily evident in “butterfly diagrams” like that presented in Fig. 5. Sunspots appear at the start of a cycle at latitudes near 25° while old cycle spots are still visible near the equator. As the cycle progresses the latitude bands widen and drift toward the equator.

![Butterfly diagram showing sunspot positions (latitude) as a function of time. The equatorward drift of the active latitudes is readily apparent.]

Figure 5. Butterfly diagram showing sunspot positions (latitude) as a function of time. The equatorward drift of the active latitudes is readily apparent.

The cause of this equatorward drift is a significant point of difference between various dynamo models. In some models the drift is due to a dynamo wave that propagates along surfaces of constant angular velocity (Yoshimura, 1975). These models usually give a constant drift velocity as well as a prominent poleward moving branch at higher latitudes (Rüdiger and Brandenburg, 1995). In other models this drift is due to transport by an equatorward meridional flow at the base of the convection zone (Choudhuri, Schüssler, and Dikpati, 1995; Durney, 1997; Dikpati and Charbonneau, 1999; Nandy and Choudhuri, 2002). These models give a drift velocity that mirrors the meridional flow velocity and thus must slow as the equator is approached.

The drift rate of the sunspot latitudes can be measured by following the centroid of the sunspot area in each hemisphere. Care must be taken to separate the cycles around the time of minimum where the cycles overlap. Fig. 6 shows the centroid positions of the sunspots in each hemisphere for each solar rotation since May 1874. The time is taken relative to the time of maximum for each cycle. A second-order polynomial fit through these data confirms what the points themselves show – the drift velocity slows as the latitude bands approach the equator. In fact, the equatorward drift stops at a latitude of about 8°. This general behavior is seen for virtually all hemispheres and cycles. In addition, there is no indication of a poleward moving branch for sunspots. The halt at 8° can be attributed to the effects of the Sun’s rotation on the motions of the rising magnetic loops. As the magnetic loops that form the sunspots rise from the base of the convection zone they tend to move toward the poles and break through the surface at higher latitudes than their original latitudes at the base of the convection zone (Choudhuri and Gilman, 1987; Fan, Fisher, and DeLuca, 1993).

![Graph showing latitudinal drift of the centroid position of sunspot areas for each hemisphere and each solar cycle since 1874. The drift rate is fast at the start of the cycle and slows to a halt at the end.]

Figure 6. Latitudinal drift of the centroid position of sunspot areas for each hemisphere and each solar cycle since 1874. The drift rate is fast at the start of the cycle and slows to a halt at the end.

The latitudinal drift of the sunspot area centroid can be converted into a drift velocity as a function of latitude. Fig. 7 shows the drift velocities as functions of latitude for each hemisphere and solar cycle since 1874. These functions are derived from the second-order polynomial fits to the centroid positions for each hemisphere/cycle.
With one exception (the northern hemisphere of cycle 21, shown with a dashed line in Fig. 7) all hemispheres/cycles show a slowing of the drift rate in lower latitudes. These measurements tend to support the dynamo models that rely on a deep meridional flow to transport magnetic activity. Care should be taken, however, in directly relating the meridional velocities in Fig. 7 to meridional velocities at the base of the convection zone. These velocities must first be corrected for the different radial positions (a 70% reduction in the flow velocity at the base of the convection zone) and then corrected for the change in latitude of the magnetic flux tubes as they rise through the convection zone.

A more surprising correlation is found between the amplitude of the activity in a hemisphere and the drift velocity. This is shown in Fig. 9 where the amplitudes are given by sunspot areas. This distribution about the median values, shown by the dashed lines, is the same as in Fig. 8 and indicates a less than 0.05 probability that this relationship is due to random associations. This result is surprising because correlations between amplitude and velocity and between period and velocity imply a correlation between amplitude and period that is not seen in the sunspot number record (Fig. 2). However, a positive correlation between amplitude and drift velocity is expected from both types of dynamo models.

![Figure 7. Equatorward drift velocity of the sunspot area centroid as functions of latitude for each hemisphere and each solar cycle since 1874. The velocities slow to a halt at about 8° latitude.](image)

As expected, there is a correlation between drift velocity and period of a cycle. This is shown in Fig. 8. The distribution about the median values, shown by the dashed lines, indicates a less than 0.05 probability that this relationship is due to random associations. Hemispheres/cycles with large drift velocities at cycle maximum tend to have short periods. This behavior is expected for both types of dynamos.

![Figure 8. Hemisphere/cycle period vs. drift velocity at cycle maximum. Hemispheres/cycles with fast drift rates have short periods.](image)

Another aspect of the distribution of sunspots and solar activity is the asymmetry between the two hemispheres. North-south asymmetries have been reported for a number of solar activity indicators (Waldmeier, 1971; Carbonell, Oliver, and Ballester, 1993; Ataç and Özgül, 1996). Fig. 10 shows how four different indicators display similar asymmetries. It would be surprising if there weren’t asymmetries—the key for dynamo models is to find out how the asymmetry is associated with the large scale flow characteristics.

![Figure 10. North-south asymmetry seen in four different indicators of solar activity—sunspot area, solar flares, sunspot groups, and absolute magnetic flux. All four indicators show similar asymmetries.](image)
The magnetic behavior of the solar cycle provides some of the most important characteristics for dynamo models. Hale et al. (1919) noted several key characteristics (Hale’s Law): 1) the leading spots in a sunspot group have opposite magnetic polarities to the following spots, 2) the corresponding (leading or following) spots have opposite polarities in the northern and southern hemispheres and, 3) the polarities reverse from one solar cycle to the next. In this same paper the authors note Joy’s determination (Joy’s Law) that: 1) sunspots in a group are spread out along an axis that makes a small angle with the equator, 2) this angle is such that the following spots are at higher latitudes and, 3) that this angle increases from 0° near the equator to about 11° at 30° latitude in each hemisphere. In addition, Babcock (1959) found that the magnetic polarity of the Sun’s polar regions reverse at about the time of cycle maximum.

All of these magnetic characteristics are evident in the magnetic butterfly diagram shown in Fig. 11. This diagram was constructed from NSO synoptic magnetic maps by averaging the magnetic flux in longitude at each latitude position for each solar rotation. Hale’s Law and Joy’s Law produce the shading on the upper and lower sides of the “butterfly wings.” The poleward spots are of one polarity (bright or dark in Fig. 11) in one hemisphere and of the opposite polarity in the other hemisphere and in adjacent cycles for the same hemisphere. The polar fields reverse from cycle to cycle and the reversal takes place at about the time of sunspot maximum.

![Figure 11. Magnetic butterfly diagram from longitudinal averages of NSO synoptic magnetic maps. The trajectories of two fluid parcels in a simple meridional circulation with a peak poleward velocity of 10 m/s are shown by the black curves.](image)

While the equatorward drift of the active latitudes is prominent in Fig. 11, a poleward meridional flow is also evident. The following polarity magnetic elements seen on the poleward edges of the butterfly wings move toward the poles with typical velocities near 10 m/s. The black lines superimposed on Fig. 11 show the trajectories of two fluid parcels on either side of the equator would make if they were transported by a simple meridional flow with a peak poleward velocity of 10 m/s. This diagram makes it very clear that the polar field reversals are produced by the poleward transport of magnetic flux from the active latitudes. Since the following spots are poleward of the lead spots, it is their polarity that dominates and ultimately reverses the polar fields.

**LARGE SCALE FLOWS**

The most significant large scale flow is the differential rotation. The latitudinal variation in the Sun’s rotation rate was first noted in sunspot motions by Carrington (1859). The amplitude of this flow is considerably larger than that of any other large scale flow. Measured relative to the solid body rotation frame of reference (corresponding to the surface rotation at about 30° latitude) the velocity at the equator is some 60 m/s prograde while the velocity at 60° latitude is about 120 m/s retrograde (Fig. 12). The actual value of the equatorial rotation rate is known to depend upon what is being measured. Sunspots and other magnetic features tend to rotate more rapidly than the photospheric plasma and the supergranulation appears to rotate even faster yet (Beck, 1999).

![Figure 12. Zonal velocity (differential rotation) in the photosphere from GONG observations in 1999. The average differential rotation profile is shown by the dotted line while a polynomial fit is shown with the solid line. The velocity range is nearly 200 m/s and the velocity shear is larger in the northern hemisphere at this time.](image)

With the advent of helioseismology, measurements of the Sun’s differential rotation were extended into the Sun to levels well below the base of the convection zone (Rhodes et al., 1990). Such observations have repeatedly shown that the latitudinal differential rotation seen on the surface (Fig. 12) extends through the convection zone and then disappears across the tachocline. An example of the internal rotation profile from Schou et al. (1998) is shown in Fig. 13. A significant feature of the internal rotation profile is the fact that the surfaces of constant angular velocity are oriented nearly radially throughout much of the convection zone. This is in direct conflict with early kinematic dynamos that required these surfaces to be oblate spheroids (Yoshimura, 1975) and with hydrodynamical models of the convection zone that gave cylindrical surfaces (Gilman, 1983, Glatzmeier, 1985). Another notable feature of the internal rotation profile is the shear layer near the surface.
This shallow shear layer was expected from both theoretical arguments (Foukal and Jokipii, 1975) and from hydrodynamical models of solar rotation effects on supergranule convection (Hathaway, 1982). The presence of this near surface shear layer helps to explain the different rotation rates found for different solar features that may be "rooted" at different depths.

Local helioseismology can be used to measure the meridional flow within the convection zone. These measurements give similar results for the direction and amplitude of the meridional flow (poleward at 10-30 m/s) and indicate that the velocity remains nearly constant with depth through the outer parts (as much as the outer half) of the convection zone. An example of the meridional flow determined from rig diagram analysis by Haber et al. (2002) is shown in Fig. 15. This particular example is from 1996, close in time to the surface measurement shown in Fig. 14. In both cases the southern hemisphere shows a larger flow velocity.

The large scale flows include the convective motions as well. The photosphere exhibits at least two types of cellular motions - granules with typical sizes of about 1 Mm and supergranules with typical sizes of about 30 Mm. While arguments are made for the existence of a distinct intermediate sized mesogranule pattern (Nemec et al., 1982), spectra show no significant features at the relevant cell sizes. Fig. 16 shows the velocity spectrum from photospheric Doppler measurements obtained in 1996. In this log-log plot the spectrum shows a nearly linear rise to a peak at wavenumbers of about 120 (30 Mm cells). The rapid fall-off at wavenumbers above 500 is attributed to instrumental resolution and projection effects. This spectrum is accurately modeled by a two component model containing just supergranules and granules (Hathaway et al., 2000).
These convective motions also play significant roles in the Sun's magnetic dynamo. In particular, the flows in supergranules shuffle magnetic elements across the surface in a diffusive, random walk, process. This leads to the decay of active region magnetic structures and the production of the chromospheric network.

Convection cells much larger than typical supergranules are also included in the spectrum shown in Fig. 16. Unfortunately, the relative weakness of these flows in the photosphere makes it difficult to determine their characteristics. However, hydrodynamical models for the solar convection zone (Gilman, 1983; Glatzmeier, 1984; Miesch et al., 2000) clearly show that giant convection cells are needed to maintain both the differential rotation and the meridional circulation. A determination of the properties of such flows within the Sun awaits further developments in helioseismology.

LARGE SCALE FLOW VARIATIONS

Given the intimate relationships between these large scale flows and the solar cycle we should expect to find variations in the flows to be associated with the solar cycle. Unfortunately, good helioseismic measurements (the nearly continuous measurements of GONG and MDI) are only available for the rising phase of a single cycle. We must look to sunspot records and direct Doppler measurements of the photospheric flows to look for inter-cycle variations and systematic variations over the solar cycle.

Howard, Gilman and Gilman (1984) examined the Mount Wilson white light plate collection and measured positions and areas of sunspots from 1921 to 1982. From these data Hathaway and Wilson (1990) found that the solar rotation rate is diminished in the presence of sunspots - the rotation rate is slower at maximum than it is at minimum, slower in big cycles than in small cycles, and slower in the hemisphere with the most spots.

Figure 17. The torsional oscillation signal during cycle 22 (adapted from Ulrich, 2001). The fast streams (dark) start at mid latitudes at the beginning of the cycle and move toward the equator. The poles spin faster at the time of maximum when the polar fields are weak and reversing.

Doppler measurements of the photospheric flows have revealed the presence of torsional oscillations - small variations (~5 m/s) in the rotation profile that propagate toward the equator with the active latitudes (Howard and LaBonte, 1980). Snodgrass (1985) found that these features appear to begin at high latitudes prior to the beginning of the cycle and then move toward the equator with the faster component of the flow on the equatorward side of the active latitudes. Ulrich (2001) has extended these observations through cycle 22 and finds that the fast streams first appear at about 30° at the start of the cycle (Fig. 17). The apparent extension to higher latitudes at earlier times is primarily due to a polar spin up at the time of maximum when the polar fields are weak and reversing.

Helioseismology has shown that these fast and slow streams extend below the surface Kosovichev and Schou, (1997) traced them to depths of 2-9 Mm using f-mode frequency splittings. Howe et al. (2000b) and Antia and Basu (2001) have traced them to depths of at least 60 Mm using p-mode frequency splittings (Fig. 18). Vorontsov et al. (2002) find similar results for the lower latitudes but find a strong poleward migrating branch at high latitudes that extend to the base of the convection zone. If this most recent result is confirmed it would seem to support dynamo models that have poleward propagating dynamo waves at high latitudes.

Helioseismology has also revealed additional variations in the Sun's rotation that may provide important clues to the nature of the Sun's dynamo. Howe et al. (2000a) found evidence for a quasi-periodic variation in the zonal flow in the tachocline. These zonal flow variations produce periodic increases and decreases in the strength of the shear across the tachocline with a period of about 1.3 years. Since the shear in the tachocline is the primary driver for the Sun's dynamo in most dynamo models, we might expect to find variations in dynamo activity that are related to these changes. The top panel in Fig. 19 shows the variation seen by Howe et al. (2000a) at the equator near the base of the convection zone. Comparing these variations with the residual variations seen in sunspot number and 10.7 cm flux (lower panel of Fig. 19) shows little correlation between increases in the tachocline shear and increases in solar activity. Since the Sun did exhibit a resurgence of activi-
ity in late 2001 it will be interesting to see how the tachocline shear was behaving at or around that time.

Figure 19. Variations over the rising phase of Cycle 23 in tachocline shear (top panel – adapted from Howe et al., 2002a) and in solar activity indicators (bottom panel – sunspot number as solid line, 10.7 cm flux as dashed line).

The meridional flow also exhibits variations. Hathaway (1996) attributed some of the disparities in reported meridional flow velocities to the existence of this variability and showed changes in the surface meridional flow seen with GONG prototype data over the course of three years during the decline of cycle 22 (Fig. 20).

Figure 20. Surface meridional flow variations during the declining phase of cycle 22.

Local helioseismology also reveals variations in the meridional flow. Beck, Gizon, and Duvall (2002) find systematic variations in the meridional flow using time-distance helioseismology. They find that the time varying component is represented by zones of divergent flow that are centered on the active latitudes and drift toward the equator with the active latitudes. Haber et al. (2002) also find changes in the meridional flow measured using ring-diagram analysis but not the systematic behavior found by Beck, Gizon, and Duvall (2002). The ring-diagram analysis shows significant asymmetries between hemispheres with converging flows in the active latitudes and a reversal of the flow direction below the surface in the northern hemisphere during the rising phase of cycle 23. A resolution of these apparent conflicts awaits further analyses and additional data.

Determinations of solar cycle variations in the large scale convective motions also await further analyses and additional data. While solar cycle variations in the characteristics of granules have been noted (Muller, 1985), similar studies of supergranules and the small wavenumber end of the convection spectrum have not yet been completed.

CONCLUSIONS

Systematic variations in the large scale flows over the solar cycle are expected from the very nature of the cycle itself. The most robust of these variations – the torsional oscillations – are now known to extend at least 60-70 Mm below the surface. The equatorward propagation of these velocity features from the mid-latitudes is also well established but the behavior at higher latitudes is controversial. The early photospheric observations indicated a simple extension to higher latitudes further back in time. More recent photospheric observations show a somewhat disconnected polar spin up at cycle maximum. On the other hand, the most recent helioseismic results indicate a deep and strong poleward propagating branch.

This has important consequences for the solar dynamo. The zonal velocity decreases inward across the tachocline in the equatorial regions but increases inward across the tachocline poleward of about 30° in each hemisphere. If, as expected, the α-effect (the lifting and twisting helicity imparted to the magnetic flux tubes) is of one sign at all latitudes in each hemisphere, then dynamo waves will propagate one direction in the lower latitudes and the opposite direction in the higher latitudes. However, this poleward migration is not seen in sunspots and dynamo models that include the meridional flow tend to avoid any poleward propagation as well. Confirmation of the poleward migrating zonal flow streams may weigh in strongly on the side of dynamo waves over meridional flow for the mechanism that sets the solar cycle period and the latitudinal drift of the active latitudes.

Another aspect that needs confirmation and further study is the nature and the role of the short-term variations in the tachocline shear. These tachocline oscillations could also have important consequences for the solar cycle.

Variations in the meridional flow over the solar cycle are also somewhat confusing at this time. Two sets of co-temporaneous observations give different results. One set gives diverging flows in the active latitudes – a
result consistent with the production of these meridional flows by the Coriolis force on the zonal flows of the torsional oscillations. The other gives a less systematic variation but with converging flows in active regions—a result consistent with the production of the zonal flows in the torsional oscillations by the Coriolis force on the meridional flows. A resolution of these differences will aid our understanding of the torsional oscillations and their connection to the solar cycle.

Regardless of the resolution of this meridional flow variation problem, it should be noted that an axisymmetric meridional flow cannot maintain the Sun’s rapidly rotating equator. It matters not whether the meridional flow is poleward or equatorward—any such axisymmetric flow will carry angular momentum from the equator toward the poles and from the surface inward. Nonaxisymmetric flows—e.g., the cellular flows produced by thermal convection—are required to maintain the Sun’s differential rotation. A positive detection of these cellular flows, as well as the resolution of the controversies surrounding the solar cycle variations in the other large-scale flows, await new analyses of existing and future data from the instruments of GONG, MDI, and the Helioseismic and Magnetic Imager on the Solar Dynamics Observatory.

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TEMPORAL VARIATION OF ANGULAR MOMENTUM IN THE CONVECTION ZONE

R. Komm\textsuperscript{1}, R. Howe\textsuperscript{1}, B.R. Durney\textsuperscript{2,1}, and F. Hill\textsuperscript{1}

\textsuperscript{1}National Solar Observatory, 950 N. Cherry Ave., Tucson, AZ 85719
\textsuperscript{2}University of Arizona, Physics Department, Tucson, AZ 85721

ABSTRACT

We derive the angular momentum as a function of radius and time with the help of the rotation rates resulting from inversions of helioseismic data obtained from the Global Oscillation Network Group (GONG) and the Michelson Doppler Imager (MDI) and the density distribution from a model of the Sun. The angular momentum as a function of radius shows the strongest temporal variation near the base of the convection zone. This variation extends into the lower convection zone and into the radiative interior and is related to the 1.3-yr periodicity found in the equatorial rotation rate of the tachocline. In the upper convection zone, we find a small systematic variation of the angular momentum that is related to torsional oscillations. The angular momentum integrated from the surface to a lower limit in the upper convection zone provides a hint that the torsional oscillation pattern extends deep into the convection zone. With the lower limit of integration placed in the lower half of the convection zone, the angular momentum fluctuates about the mean without apparent trend, i.e. the angular momentum is conserved within the measurement errors. However, when integrated over the layers slightly below the convection zone, the angular momentum shows the 1.3-yr period and hints at a long-term trend which might be related to the solar activity cycle.

1. INTRODUCTION

In the last decade, great progress has been made in the understanding of the dynamics of the solar interior. The rotation rate in the upper convection zone varies with the solar cycle; the so-called torsional oscillations or zonal flows are detectable in the upper third of the convection zone (Howe et al., 2000a; Antia & Basu, 2000, for example). Recent work suggests that this pattern extends even deeper into the convection zone at least at high latitudes (Vorontsov et al., 2002). At the base of the convection zone, the rotation rate varies with a period of about 1.3 years (Howe et al., 2000b). The cause of this variation is not yet understood, but it might be related to an exchange of angular momentum between the radiative interior and the convection zone (Thompson, 2001). The turbulent motions of the convection zone will redistribute angular momentum on a convective timescale of about a month. Meridional flow and magnetic activity are other mechanisms for angular momentum redistribution. It is thus important to know whether any exchange of angular momentum exists on time scales longer than a few months or years. Such systematic variations might be generated by variations of magnetic fields and could thus provide insights into the operation of the solar dynamo. Organized flows such as torsional oscillations which vary on a time scale of 11 years are clearly related to the magnetic activity cycle. Gough & McIntyre (1998) assumed a large-scale field in the radiative interior to explain the thickness of the tachocline and Gough (2000) suggested that such a field might sustain its own coherent torsional oscillation over at least some extended region of the interior. Also, the transport of angular momentum by Reynolds stresses is essential for the understanding of the meridional motions and equatorial acceleration of the Sun (Durney, 2000a).

In this study, we calculate the angular momentum in the solar interior as a function of time and radius by integrating in latitude the rotation rates derived from rotation inversions. The total solar angular momentum has been calculated before from an inversion of helioseismic data (Pijpers, 1998; Antia, Chitre, & Thompson, 2000). Here, we focus on the temporal variation of the angular momentum in the convection zone and especially on the layers near its base and near its upper shear layer.

2. DATA AND METHOD

We define the angular momentum, $A(h,t)$, of the Sun between spherical surfaces at radius $h$ and the solar
radius, $R_\odot$, as

$$A(h, t) = \int_{R_\odot}^{h} J(r, t) \, dr$$

with

$$J(r, t) = 2\pi \rho r^4 \int_0^\pi \Omega(r, \theta, t) \sin^3 \theta \, d\theta ,$$

with radial distance $r$, colatitude $\theta$, longitude $\phi$, density $\rho$, and angular velocity $\Omega(r, \theta, t)$, which is assumed to be independent of $\phi$. The quantity $J(r, t)$ is the integral over $\theta$ and $\phi$ of $r^4 \rho \sin^3 \theta U_{r\phi}$ ($U_{r\phi} = r \sin \theta \Omega(r, \theta, t)$ is the azimuthal velocity) and from this definition it follows that $J(r, t)$ is the angular momentum of a thin spherical shell with radius $r$ and thickness $dr$. In the calculations of $J(r, t)$ and $A(h, t)$ we assume that the density is a function of $r$ only. The right-hand side of Equation 2 shows that the angular momentum is heavily weighted toward the equatorial regions.

Another quantity of interest is the $\theta$-independent angular velocity, $\bar{\Omega}(r, t)$, that leads to the same angular momentum as $\Omega(r, \theta, t)$ for a thin spherical shell with radius $r$ and thickness $dr$. This quantity is easily derived from Equation 2:

$$\bar{\Omega}(r, t) = \frac{3}{4} \int_0^\pi \Omega(r, \theta, t) \sin^3 \theta \, d\theta .$$

The angular momentum, $A(h, t)$, can then be written as:

$$A(h, t) = \frac{8\pi}{3} \int_{R_\odot}^{h} \rho r^4 \bar{\Omega}(r, t) \, dr .$$

To derive the temporal variation of the angular momentum, we start with the axially-symmetric azimuthal momentum equation as in Equation (13) of Durney (2001) but adding the contribution of the $\phi$-component of the Lorentz force. We further assume that magnetic fields are the main source for the variations of the angular momentum studied in this paper, which simplifies the angular momentum equation to

$$\frac{\partial}{\partial t} (r^4 \rho \sin^3 \theta \Omega(r, \theta, t)) = \frac{1}{4\pi} \frac{\partial}{\partial r} (r^3 \sin^2 \theta B_r B_\phi)$$

$$+ \frac{1}{4\pi} \frac{\partial}{\partial \theta} (r^2 \sin \theta B_\theta B_\phi) ,$$

where $B$ is the magnetic field. The equation for $A(h, t)$ follows immediately from Equation 5,

$$\frac{\partial A(h, t)}{\partial t} = -\frac{1}{2} \int_0^\pi h^3 \sin^2 \theta B_r(h, t) B_\phi(h, t) \, d\theta ,$$

where we have neglected the surface-term. At the surface, and due to magnetic buoyancy, we expect $B_\phi$ to be small. Furthermore there is little doubt that the main source for the solar angular momentum loss (on an evolutionary time scale) is the coupling of the solar wind with the magnetic field.

To calculate the angular momentum, we use 64 GONG data sets processed through the GONG pipeline (Hill et al., 1996) covering the rising phase of the current solar cycle from mid-1995 to late-2001. This time period coincides with the operation of GONG in ‘classic’ mode before the cameras were upgraded in March 2001 and ‘blended’ mode before the upgrade was completed. Each time series has a length of 108 days to achieve good frequency resolution and consecutive data sets are shifted by 36 days to improve the temporal resolution. Mode frequencies were estimated from the 108-day multaper power spectra using the standard GONG analysis (Anderson, Duvall, & Jeffries, 1990; Hill et al., 1998; Komm, et al., 1998). The frequencies within each $(l, m)$ multiplet, characterized by radial order $n$ and spherical harmonic degree $l$, were then fitted to a polynomial series to obtain the so-called $a$ coefficients (Howe, Komm, & Hill, 1999, for example). The coefficient sets were inverted for rotation using the two-dimensional regularized least-squares (RLS) method (Schou et al., 1998, and references therein). In addition, we used 28 MDI data sets analyzed with the MDI pipeline (Schou, 1992). The data sets cover the current solar cycle from mid-1996 to early-2002. The MDI time series do not overlap and have a length of 72 days. The fit is done directly for the mean multiplet frequency and the $a$-coefficients of each multiplet. The resulting coefficient sets are then inverted for rotation. The similarities and differences between GONG and MDI data sets and analysis pipelines are discussed in detail in Schou et al. (2002).

The rotation rates are estimated on a finite grid of 49 positions equidistant in colatitude from pole to equator and of 51 positions in radius from center to surface. We use the density as a function of radius from a solar model by Christensen-Dalsgaard et al. (1996), which is defined on the same radial grid as the inversions and was used to generate the eigenfunctions used for the inversions. We implicitly assume that density fluctuations are small compared to velocity fluctuations and can be neglected.

3. RESULTS

The depth dependence of the average angular momentum reflects the underlying solar model and essentially follows that of the term $r^4$ in Equation (2), since the change in rotation rate is small by comparison. Figure 1 shows the corresponding $\theta$-independent angular velocity $\bar{\Omega}(r)$ averaged over all data sets as a function of radius. The top panel is scaled to emphasize its radial variation, while the bottom panel includes the rotation rates, $\Omega(r, \theta)$, at selected latitudes for comparison. In the convection zone, $\bar{\Omega}(r)$ derived from GONG data is in excellent agreement with that derived from MDI data, which is of course due to the agreement of the derived rotation rates at low and mid latitudes. The good agreement in $\bar{\Omega}(r)$ shows that the relatively large differences between GONG and MDI rotation rates at high lati-
tudes have only a negligible influence on $\Omega(r)$ and thus on the calculated angular momentum. In the radiative interior, the GONG analysis leads to underestimated low-degree splittings, as discussed by Schou et al. (2002), and thus to a smaller value of $\Omega(r)$ compared to MDI.

Figure 2 shows the residual of $J$ (in %), the difference between $J$ and its temporal average divided by the average, as a function of time at three different radial distances. The GONG and MDI results agree quite well within the error bars and show a similar long-term behavior. The fluctuations near the upper shear layer ($r = 0.96R_\odot$, top panel) are small but show a hint of a systematic trend with less-than-average values before 1999 and greater-than-average values after 1999. This trend is related to the torsional oscillations present in the upper third of the convection zone. The residual of $J$ near the base of the convection zone ($r = 0.71R_\odot$, bottom panel) shows a large quasi-periodic variation similar to the 1.3-yr periodicity discovered in the equatorial rotation rate (Howe et al., 2000b). This is not too surprising since the angular momentum of a spherical shell is heavily weighted toward the equator. The middle panel shows the residual of $J$ near the middle of the convection zone ($r = 0.83R_\odot$), which fluctuates around the average with a hint of a slight decrease with time.

Figure 3 shows the difference between $J$ and its temporal average (in $10^{35}$ g cm s$^{-1}$) as a function of time and radius. The largest temporal variations are present near the base of the convection zone and below in the radiative interior. The variation near $R \approx 0.71R_\odot$ shows a significant anticorrelation of $-0.7$ with the variation near $R \approx 0.63R_\odot$, which reflects a similar behavior noticed in the equatorial rotation rate at these depths (Howe et al., 2000b; Gough, 2000), and it also shows an anticorrelation of $-0.6$ with the variation near $r \approx 0.77R_\odot$. This leads to the impression that the fluctuations near the tachocline extend into the radiative interior and also into the lower half of the convection zone. This behavior is common to GONG and MDI results. In the following we focus on long-term variations common to both data sets, since it is unlikely that the two different analysis methods with their different strengths and weaknesses, as discussed by Schou et al. (2002), would lead to similar artifacts.

Figure 4 shows the residual angular momentum, that is the difference between the angular momentum integrated from the solar surface and its temporal average divided by the average, as a function of time and lower integration limit. The residuals show a systematic long-term behavior in the upper third of the convection zone. In these layers, the resid-
Figure 3. The difference between $J$ and its temporal average (in $10^{16}$ g cm s$^{-1}$) as a function of time and radius for GONG (top) and MDI data (bottom). We include the zero contour. The horizontal dashed lines indicate $r = 0.71 R_\odot$, $r = 0.83 R_\odot$, and $r = 0.96 R_\odot$.

Figure 4. Top: The residual angular momentum (in %) as a function of time and the lower integration limit for GONG data. Bottom: The same for MDI data. We include the zero contour. The horizontal dashed lines indicate $h = 0.71 R_\odot$, $h = 0.83 R_\odot$, and $h = 0.96 R_\odot$.

The data show a small trend with greater-than-average values after mid-1999 and less-than-average values before mid-1999 similar to the one seen in Figure 2. This pattern related to torsional oscillations seems to extend below $0.9 R_\odot$ to almost the middle of the convection zone. With the lower integration limit placed in the lower half of the convection zone, there is a hint of a long-term variation in the GONG data, which is anticorrelated to that with the lower integration limit placed in the upper part of the convection zone. Since the MDI data don’t show this trend, we cannot establish its existence unambiguously.

Since the solar dynamo is expected to reside in the upper part of the radiative zone slightly below the tachocline, we calculate the angular momentum of the layers between a lower limit of $h = 0.60 R_\odot$ and an upper limit of $h = 0.71 R_\odot$. In Figure 5, the resulting angular momentum shows a short-term periodicity of about 1.3 years anticorrelated to the one visible in $J(r = 0.71 R_\odot)$ shown in Figure 2. But also shows a small trend with greater-than-average values during cycle minimum and less-than-average values at cycle maximum. This trend can be used as an estimate of the average temporal derivative of the angular momentum (see Equation 6). Since the short-term fluctuations make it difficult to reliably determine the temporal derivative, we perform a simple linear regression to estimate its average value. The value of $h^{-3} \partial J/\partial t$ is $-23 \pm 12 \times 10^5$ g cm$^{-1}$ s$^{-2}$ derived from the GONG data. The regression slope is almost 95% significant. This trend is both physically meaningful and in agreement with numerical calculations (Durney, 2000b). In that model, the stretching of the poloidal field by differential rotation generates the toroidal field and the resulting Lorentz force is negative. Both GONG and MDI data sets are in excellent agreement during epochs where they overlap. But, since the observed trend hinges on the first few GONG data points obtained during 1995, this trend is less noticeable in the MDI data which are compatible with a zero slope.

4. SUMMARY AND DISCUSSION

We study the radial exchange of angular momentum by calculating the quantity $J_{dr}$ where $J_{dr}$ is the angular momentum of a spherical shell. We find a variation near the tachocline with a period of about 1.3 years which is of course related to the periodic variation in the near-equatorial rotation rates (Howe et al., 2000b). We use the same rotation rate data sets as Howe et al. (2000b) but we have a longer time series since two years have passed. The variation near the tachocline is anticorrelated to the variations of the angular momentum at $0.63 R_\odot$ and $0.77 R_\odot$ below and above the tachocline. This gives the visual impression that some fluctuations move across the tachocline into the lower convection zone and into...
Figure 5. The angular momentum integrated from $h = 0.60R_\odot$ to $u = 0.71R_\odot$ for GONG (star symbols) and MDI data (open squares). The MDI data were shifted by $-0.8 \times 10^{-5}$ g cm$^2$ s$^{-1}$, the difference of the means of both data sets. The solid line indicates the linear regression of the GONG data.

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INTERACTION OF SOLAR SUBSURFACE FLOWS WITH MAJOR ACTIVE REGIONS

Deborah A. Haber¹, Bradley W. Hindman¹, and Jurí Toomre¹
¹JILA, University of Colorado, Boulder, CO 80309-0440

ABSTRACT

Solar Subsurface Weather (SSW), which consists of complex and meandering large-scale horizontal flows below the solar surface, has been studied in detail with ring analyses of SOI–MDI data from SOHO. SSW flows are of particular significance since they appear to interact and influence the magnetic fields visible at the surface, with active regions appearing as zones of convergent flow and possible downflows. Such subsurface flows over a range of depths can mechanically twist and displace field lines, possibly leading to unstable magnetic configurations that may flare or erupt as coronal mass ejections. It is highly likely that such flows and magnetic fields are broadly linked in their evolution. We have studied in detail horizontal flow fields in the vicinity of several major active regions observed during 2001 and 2002, finding that at shallow depths there is general flow convergence toward these regions and noticeable flow deflections in the large-scale zonal and meridional circulations. We have detected strengthening of jet-like features in the converging flows occurring over the course of several days. These features are accompanied by prominent diverging outflows at greater depths. It appears that the large-scale flow fields surrounding active complexes have a distinct cellular structure that may contribute to both the overall cohesion of active regions as well as to the movement of magnetic flux within those regions that can lead to flares and other eruptive phenomena.

Key words: Helioseismology; Ring Diagrams.

1. INTRODUCTION

The average properties of large-scale flows beneath the solar surface have now been well studied using a number of global and local helioseismic techniques (e.g. Kosovichev & Schou 1997; Schou et al. 1998; Duvall & Gizon 2000; Howe et al. 2000; Chou & Dai 2001). These flows have been studied over the course of the solar cycle, but the interactions of large-scale flows with individual magnetic active regions are less well defined. Doppler measurements have been used to study directly observable flows around sunspots such as Evershed flows (e.g. Rimmele 1995; Lites et al. 2002), while time-distance helioseismology has reported on supergranular size flows and their interaction with sunspots (e.g. Gizon, Duvall & Larsen 2000; Zhao, Kosovichev & Duvall 2001). However, the influence of active regions on the longer lasting, larger-scale flows (or vice versa) reported here, has not yet been studied in detail. Both time-distance f-mode analysis and ring analysis have been used to make and compare synoptic maps of flows whose average meridional and zonal signals have been removed (Hindman et al. 2003). Those maps show flows averaged over a number of days flowing into active regions at relatively shallow depths beneath the solar surface (about 1 to 7 Mm down). Here we discuss what happens on shorter time scales and at deeper depths.

2. MAPPING OF LARGE-SCALE SUBSURFACE FLOWS WITH DENSE-PACK RING DIAGRAMS

In the presence of flows, acoustic waves with the same horizontal wavenumber propagating in opposite directions have their frequencies split by the Doppler effect. Such frequency splittings are used in local domain helioseismology to measure the flow velocities averaged over the depths where the modes have significant amplitude. In the local helioseismic technique of ring analysis, a spectrum is obtained of the wave field in a localized domain by Fourier transforms (two in space, one in time) of a sequence of tracked and remapped Doppler images (e.g., Haber et al. 2000, 2002). The mode power in the spectrum is distributed along curved surfaces, which when cut at constant frequency appear as a set of nested rings, each corresponding to a mode of the solar waveguide with a different radial order. Ring analyses carefully fit such power spectra with Lorentzian profiles to obtain the frequency splittings due to the presence of flows (which appear as shifts in ring centers).

Through mathematical inversion of the integral equation that relates the frequency splittings to the flow properties, the horizontal velocity can be computed as a function of depth below the photosphere. We use a 1–D RLS inversion technique (Thompson et al. 1996; Haber et al. 2002) locally within a small patch or tile on the solar surface. By repeating the analysis over many such tiles at different locations on the solar disk, a 3–D map of the velocity field within the near-surface layers may be generated. We typically perform analyses on an overlapping
mosaic of regions, 15° in diameter, with centers equally spaced 7.5° apart, roughly filling the solar disk with 189 circular tiles out to 60° from disk center. By repeating this set of dense-pack analyses on a daily interval, we are able to follow the evolution of the flows with time. Solar rotation brings new sectors into view as it removes others, so an individual site only remains visible on the disk for about seven days. We show here the results of such analyses performed on the uninterrupted full-disk Doppler velocity data from the SOI–MDI Dynamics Programs (Scherrer et al. 1995) for the years 2001 and 2002. The trends shown here have also been seen in data from previous years of SOI Dynamics Programs (Haber et al. 2000, 2002).

3. SSW NEAR ACTIVE REGIONS

We have carried out preliminary assessments of subsurface flows in the vicinity of active region complexes using our ring-diagram mappings. The last few years of observations with SOI–MDI have been very interesting in this regard since the sun has been near the peak of its magnetic activity cycle. Figures 1a, b exhibit the increased complexity of SSW in the very active sun in April 2001, as seen in synoptic flow maps for the two depths of 7 Mm and 16 Mm. Shown here are the residual horizontal velocities, where the time averaged mean components at each position on the solar disk have been removed from each of the daily dense packs before generating a synoptic map by averaging in time at a given latitude and longitude. This type of synoptic map accentuates the flows that may be a signature of giant cells, meandering jet streams, and particularly the interaction of the active region complexes with the flows. Outside the regions of magnetic activity in Figure 1a, b, the flows appear to be structured with a predominance of size of roughly 25° to 45° in diameter. The flows near the magnetic complexes are generally more vigorous. This is especially true for active region NOAA 9393 which is centered near latitude 15° north and longitude 150°. This region exhibits strong and largely converging flows at the depth of 7 Mm, and even stronger diverging outflows at the greater depth of 16 Mm. In contrast, in the vicinity of a number of other active complexes (such as at latitude 20° north, longitude 345°) the flows at both depths are converging toward the major axis of the active region. Yet in some active regions in other rotations, we have found that such converging flows and a strong shear line only extend to depths of about 4 Mm from the surface, below which the flows appear to meander with little regard for the active regions evident at the surface, suggesting that the magnetism there may be confined close to the surface. Clearly there is a wide range of subsurface flow behavior associated with active complexes, and we have so far only briefly sampled a few of them.

Figure 1c also shows the diverging behavior of SSW flows from large active regions at a depth of 16 Mm for the beginning of the Dynamics Program in January 2002. In most regions the outflows appear to be perpendicular to the neutral line of the activity but in more complicated regions (such as the one at longitude 135°, latitude 5° south), the convergence axis is not clearly defined. In other regions, (e.g. the one at longitude 270°, latitude 15° south), the flows are convergent at all depths, including the 16 Mm depth shown here.

We are particularly interested in NOAA 9393, for it is a large active region responsible for a sequence of major flares and CMEs. Further, its emergence, maximum activity and decay could be sampled reasonably well in the three solar rotations studied with the SOI–MDI Dynamics Program in the March to May 2001 interval last year. This active complex has also been recently examined with time-distance probing (Kosovichev, Duvall & Zhao 2002). Of course, a given site remains visible to our ring-diagram analyses only for about 7 days, but these coincided with some of the major flaring events during two of the rotations. Figure 2 provides close-up views of the subsurface flows at three depths for the four consecutive days 23–26 April 2001, during which numerous M-class flares (including an M8) and CMEs were detected. The upper two rows, sampling 2 Mm and 7 Mm in depth, reveal that amidst the general converging flows approaching the neutral line from roughly the north and the south, a compact but strong jet forms which streams into the complex from the north. Over the period of jet formation, the coronal magnetic field as viewed by EIT exhibited major restructuring. In sharp contrast, the lower row of images of flows at a depth of 16 Mm reveals striking outflows from the active complex in both northerly and southerly directions. Figure 3 reveals that very similar subsurface flow behavior was seen earlier during 27–30 March 2001 when this complex was first visible in our observing matrix. It also shows both strong converging flows at the 7 Mm depth and divergence at the 16 Mm depth. This interval likewise possessed numerous M flares, and on 29 March an X1 flare that may be this cycle’s champion, accompanied by a prominent CME. Possibly NOAA 9393 was especially active since it is thought to have been formed by fragmented flux tubes that emerged over an extended period in time, rather than being formed by a single large Ω-loop broken into smaller tubes near the surface (Kosovichev et al. 2002).

4. DISCUSSION

It is clear that SSW flows show somewhat different properties near active regions. In some cases it looks like giant cell outflows (e.g. at longitude 160°, latitude 10° south, fig. 1a) may be contributing to the convergent flows seen at the active regions, in the same way that supergranular flows sweep magnetic network to the supergranular boundaries. In some cases, (for example: longitude 108°, latitude 45° south, or longitude 195°, latitude 45° south in Figures 1a, b, the swirling giant-cell size flows appear to have little to do with the magnetic field. In other cases (such as at longitude 345° from latitudes 10° south to about 20° north in Fig. 1b) it looks like a major flow is completely deflected by the magnetic field, as if it had run into a wall.

The outflows from larger active regions at greater depths are harder to understand. If the inflows near the surface
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Figure 1. Synoptic maps of fluctuating horizontal flows (with temporal and longitudinal means removed) during an interval of vigorous magnetic activity. The magnetic field intensity and polarity is indicated by the underlying half tones. (A) and (B) show Carrington rotation 1975 (April 2001) for the two depths of 7 Mm and 16 Mm (respectively). (C) shows Carrington rotation 1985 from the following year (Jan 2002) at a depth of 16 Mm. Twofold interpolation is used to accentuate the flow patterns. Major activity complexes, such as centered in latitude near 15° (north) and in longitude near 150°, exhibit strong, complex converging flows at the depth of 7 Mm, in contrast to the prominently diverging outflow at the greater depth of 16 Mm. Other large-scale outflow patterns are also present, such as those centered in longitude at 210° in the 7 Mm map, that may be associated with giant cells.
Figure 3. Close-up views at three depths of flow evolution (with twofold interpolation) near the active complex NOAA 9393 for the four consecutive days of 27–30 March 2001, obtained with dense-pack ring-diagram analyses. During this interval a large X1 class flare took place on 29 March. Numerous other major flares and CMEs were associated with this magnetic complex. The panels in the upper row (at depth 2 Mm) exhibit an asymmetric convergence of flows toward the main axis of the active region during the first few days. The flows are larger from the south on the left (east) side of the region and from the north on the right (west) side of the region. By 30 March, however, the speeds are more balanced. In the middle row, at a depth 7 Mm, the flows are much larger north of the active region, increasing dramatically by the 30th as a jet forms that streams through the region. However, the lower row (depth 16 Mm) reveals striking outflows from the central part of the active complex, with these flows displaying more asymmetry than those seen in Figure 2. In the right hand extension of this active complex (longitudes 165°–180°), there are still convergent flows towards the region at this depth.
Figure 2. Close-up views at three depths of flow evolution (with twofold interpolation) near the active complex NOAA 9393 for the four consecutive days of 23–26 April 2001, obtained with dense-pack ring-diagram analyses. The reference scale arrow for the flows is indicated to the left of each row. During this interval numerous major flares and CMEs were associated with this magnetic complex. The panels in the upper row (at depth 2 Mm) and middle row (depth 7 Mm) exhibit general convergence of flows toward the main axis of the active region, accompanied by the formation and strengthening of a distinctive southerly jet that impinges upon the complex from the north. In sharp contrast, the lower row (depth 16 Mm) reveals striking outflows from the active complex, oriented nearly perpendicular to the neutral line.
PATTERNS OF VORTICITY ON THE SOLAR SURFACE

Benjamin P. Brown, Harvey Mudd College, Claremont CA, 91711, USA
Herschel B. Snodgrass, Lewis & Clark College, Portland OR, 97219, USA

Abstract

Local Correlation tracking of Hydrogen-alpha images taken at one minute intervals at Big Bear Solar Observatory is used to make flow maps that reveal large-scale, high-velocity patterns that appear to be associated with the Sun's magnetic activity. We discuss the possible connections of these patterns to the azimuthally averaged meridional flow and torsional oscillations. We then use the flow maps to compute global maps of vorticity at the solar surface. The vorticity maps contain plume-like patterns of alternation resembling the patterns seen in the maps of the Sun's background magnetic field. The vorticity plumes may account for the disparity in diffusion constants determined for the dispersal of the field and the polar field reversals.

1) The question of a longitudinal dependence in the torsional oscillations.

The so-called torsional oscillations, discovered by Howard and LaBonte (1980) are familiar to solar physicists as a pattern of azimuthal wind bands that first appear at high latitudes a few years before solar minimum, migrate toward lower latitudes to meet the zone of emerging activity of the next cycle, and continue along this zone until the next minimum. The way in which this pattern bridges successive cycles suggests that the cycles are overlapping, or 'extended.' The analysis of helioseismic global modes has shown that this torsional pattern is present, and remarkably similar, deep within and perhaps clear through the convection zone. This evidence in particular supports the idea that the torsional oscillations play a significant role in the activity cycle, and it may be that some aspect of the early manifestations of this pattern can be used to predict the strength of the magnetic activity to follow.

A major question in the study of the torsional oscillations is whether it is, as usually presented, azimuthally symmetric, or whether it has longitudinal structure. The problem is that, although it covers a wide area, the small (~5 m/s) wind velocity has to be extracted from the more localized but far stronger velocity signals, including ~300 m/s horizontal flows from the supergranulation. The surface pattern becomes apparent only after sequences of observations are averaged, which smears the longitudinal structure. Moreover, the global mode analysis cannot reveal such an asymmetry. The pattern, therefore, has come to be regarded simply as a function of latitude and time, as seen in Figure 1.

Fig. 1. The positive-velocity wind band of the torsional pattern, plotted in standard form as a function of latitude and time. The contour interval is 1 m/s.

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To date we have no widely accepted model for the mechanism of the torsional oscillation and its connection with the cycle, although several have been proposed. Yoshimura (1981) and Schussler (1981) argued that it is a Lorentz-force back reaction, Snodgrass and Wilson (1987), and, independently, Gilman (1992), argued for an azimuthal-roll model involving the thermal shadow of the developing azimuthal field band. More recently, Ulrich (2002) and others have argued that it is akin to the phenomenon known to meterologists as Rossby waves.

Each of these mechanisms could work to produce a pattern like that seen in Figure 1, and there are other possibilities as well. It is essential, therefore, to refine the observational picture, and particularly to determine the longitudinal dependence, before we try to pin down where the torsional oscillations pattern comes from and how it connects with the activity of the cycle.

Some work has been done on this already. Ulrich (2002), using clever sampling methods, finds what appears to be an irregular variation in the signal on scales of ~60 degrees; but the noise level is still too high to be certain of this result. A project is presently underway to systematically “clean” the Mount Wilson data of a number of now-understood instrumental effects, and to re-reduce all of these data from 1967 to the present to a common format, which may lead to a cleaner and more detailed picture. But in the meantime, another avenue has opened up, in the form of flow maps produced by applying local correlation tracking of H-alpha images taken at BBSO.

2) The BBSO H-alpha flow maps.

As part of the present observational program at BBSO, a sequence of full-disk H-alpha images of the Sun are taken at one-minute intervals. The result is a rapidly growing data set of about 300 – 400 such images per day. One of the uses that has been made of these data is to pass them through a local correlation tracking program to create flow maps, an example of which is seen in Figure 2.

Fig. 2  Left: The flow map of H-alpha features for 1 Aug., 1998, produced by local correlation tracking of pairs of H-alpha images spaced at 1-minute intervals, then averaging the results over ~400 adjacent image pairs taken on that date. The direction and length of each arrow represents the local velocity for the pixel located at the tail of the arrow. Center: Average velocities in each strip of latitude. Right: The same day’s Flow map with average velocities subtracted.
There are several advantages to making surface velocity maps by correlating surface features, rather than by measuring Doppler shifts. One is the absence of “limb-redshift” and other non-velocity signals that are present in the Doppler data, and another is that there is less high-velocity contamination from the supergranular cells. Although a flow map thus produced for any pair of images is still very noisy, when all 300 – 400 flow maps for a given day are averaged together, the result appears to be quite reasonable. In Figure 2 one sees both the differential rotation, which dominates the left-hand panel, and also a remarkably organized pattern of local variation. As seen in the right-hand panel, the map becomes more intriguing when one subtracts a longitudinal average. In the map presented here, this subtracted ‘differential rotation’ (center panel) is simply the average flow at each latitude in the left panel. A better technique would be to subtract a mean differential rotation determined over a long time period, but at this stage, we have insufficient data to do this.

Figure 2 shows that very large, well-organized, horizontal high velocity (~1 km/s) patterns, extending over substantial regions, are present at the solar surface. The patterns resemble the flow patterns for deeper layers in the convection zone obtained through local-mode helioseismology studies, seen elsewhere in this volume. We note strong equatorward and poleward components in these flows, which may, if the poleward flows have a slight tendency to be dominant, account for of the overall ~10 m/s poleward meridional flow (e.g. LaBonte and Howard 1982). The East-West components of these flows are similarly strong, and may contribute to the torsional oscillations pattern, or account for it altogether. These flows may even contribute to the overall time-averaged differential rotation. A question arises as to what the features in the H-alpha maps represent. We assume that these features are atmospheric structures at various heights in the chromosphere, and that the mean height will vary from center to limb. As this range of heights and its center-to-limb variation are greater than the range and variation of the depths of formation of the spectral lines observed in the Mount Wilson magnetograms, absolute rates of surface flow are less well determined. We are interested, however, in the character of the residual maps, and uncertainties of a few percent are not of concern. Such large and organized patterns are not readily apparent in the Mount Wilson Doppler arrays, in which only the line-of-sight component of the velocity is determined. But it is obvious from Figure 2 that when one fits the Mount Wilson Doppler data to obtain either the solar rotation or the meridional flow, these large-scale high velocity patterns and the supergranular flows are both being averaged over. This guarantees that the analysis of the longitudinal structure of the torsional pattern is more problematic than what would be suggested by looking at the Mount Wilson residual Dopplergrams alone. For while it is possible that the torsional pattern is created through a slight, latitude-dependent imbalance in the East-West components of these flows, it is also possible that the torsional pattern is a weak, deep-set flow pattern upon which the more random-appearing high velocity flows seen in Figure 2 are superimposed. A similar point can be made about meridional flow: It may be a steady, more or less azimuthally symmetric flow, upon which the high-velocity flows seen in Figure 2 are superimposed, or it may arise from a slight but systematic imbalance in these flows.

3. The vorticity maps.

Comparing the residual map in Figure 2 to the Mount Wilson magnetogram taken the same day (Figure 3a), which shows both the active regions and the background field, one finds that the local patterns of irregular flow tend to be associated with the regions of magnetic activity. A very interesting study of this has already been done (Yurchyshyn and Wang, 2001) which shows the pattern of motions connected with a bipolar spot region. Here it appears that the leading spot, which is plowing through the ambient plasma at a rate of ~100 m/s, produces a turbulent wake, which the follower spot encounters. The apparent effect of this wake is to cause the follower spot to break up and scatter, which may provide an explanation for the well-known phenomenon in which the follower spot is usually more diffuse than the leader.

The patterns seen in Figure 2 suggest large-scale vortical flows in and around the regions of activity. As noted above, our initial interest in these maps was to see whether, when we averaged a long sequence together, the big vortex-like structures would average to yield the torsional pattern. For this to be so, there would have to be a slight preference for the vorticity, assuming it centered in the active latitude zone, to be counter-clockwise in the Northern hemisphere, and clockwise in the Southern
hemisphere. This would cause the zones of shear enhancement to run through the centers of the sunspot 'butterfly wings', as is the case for the torsional pattern (Snodgrass 1987). This is of course not evident in the few day's flow maps, paper by Sheeley, Nash and Wang (1987), wherein it was concluded that they are fraying active region remnants, drifting under the combined influence of supergranular diffusion and meridional flow.

Fig.3. (a) Mount Wilson magnetogram for 1 Aug., 1998. Black indicates positive magnetic field, and white indicates negative. At this time the field at the North Pole had a positive sign, as did the leader spots in the Northern Hemisphere. (b) A map of the vorticity in the solar surface for 1 Aug., 1998, after the quiescent differential rotation has been subtracted. Black indicates vorticity in the clockwise sense, and white represents vorticity in the counter-clockwise sense.

which are all that we have produced to date, and the project awaits our acquisition of a sufficient amount of data. In the meantime we have been studying the (residual) vorticity itself. We define the vorticity to be the line integral of the velocity around a closed loop, and compute this, for a given pixel, by looking at the circulation in the four pixels that surround it. A map of this vorticity, computed for the same day as in Figure 2, is seen in Figure 3b.

The most striking feature in Figure 3b is the pattern of "plumes," which look remarkably like the unipolar plumes often seen in the background magnetic field. Close inspection of the same-day's magnetogram (Figure 3a) shows that the background-field plumes, though not very pronounced on this date, have similar appearance and location. Background-field plumes seen in the magnetograms were discussed in the famous

There are many problems with this explanation, which are discussed by Snodgrass and Smith (2001). In particular, the field structures are too large to be buffeted about by the supergranular cells, and minimum value of the diffusion constant required to fit the data, D ~ 600 km²/s, is more than 3 times larger than the constant measured directly in the fine scans studied by Hagenaa et al. (1999).

It is very tempting to speculate that the plumes seen in our vorticity map (Figure 3b) are long-lived eddies in the active-region wakes, and that they combine to give rise to turbulent streams of poleward and equatorward flow, which could account for the apparent large diffusion constant inferred by SNW. Examination of them at fixed latitude shows that the vorticity varies smoothly in an almost sinusoidal manner, and therefore the derivative with respect to longitude indicates the
boundaries of poleward and equatorward moving flow streams. The cross-correlation of the sign of these inferred streams with the sign of the magnetic field yields a marginally significant correlation between poleward-moving streams and the regions of follower-spot polarity in the background magnetic field. Detailed comparison of Figure 3b with Figure 3a shows that some but not all the poleward moving streams transport following polarity — for in some cases it is the leading polarity that is transported poleward. The score on this date (1 Aug. 1998) is roughly 4 to 2 — that is, of the 6 clearly defined plumes, 4 transport following polarity poleward and 2 transport leading polarity poleward. This lack of perfect correspondence would be expected for a chaotic phenomenon such as turbulence.

It is also tempting to speculate that we are seeing, in the comparison of Figure 3a with Figure 3b, the alpha-dynamo in operation. As we have noted, on this particular day, the plumes in the magnetogram (Figure 3a) are not particularly striking. The plumes seen in the vorticity map (Figure 3b) are far more evident. It will be interesting to see what a vorticity map looks like when the plumes in the magnetogram itself are more striking.

4). Conclusion

All of the results reported here are based on a very small sample of the data. Thus far, we have examined vorticity maps for only a few days, and have presented what may be our strongest case in the data that we present in this paper. An interesting check on our project will be to see whether we can reproduce the Mount Wilson arrays by binning the line-of-sight components of the flows seen in Figure 2. We also will calculate and map the divergence of these flows. Although, as we have noted, the presence of these high-velocity flows that were not anticipated in the analysis of the Mount Wilson data complicates the determination of the longitudinal structure of the torsional pattern, it is possible that this will be offset by the greater amount of detailed information these maps provide. We anticipate much will be learned in comparing the divergence and vorticity maps.

We are presently embarking on a long-term study of these flow maps, in cooperation with Carsten Denker and other members of the staff at BBSO. We wish to thank BBSO Director Phil Goode, who first showed one of us (HBS) a BBSO flow map, and the rest of the staff at BBSO who have been very helpful on this project. We also thank the staff at Mount Wilson Observatory, particularly John Boyden, for supplying us with the Mount Wilson data. Finally, we wish to thank the other members of our Summer research group at Lewis & Clark College, Brenna Gillman of Lewis & Clark, and Tess Williams of Stanford University for their help on various tasks associated with this project.

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References

Irradiance and Helioseismology

Chair: D. Hathaway
ORIGIN OF IRRADIANCE VARIATIONS FROM DISK PHOTOMETER DATA

Martin F. Woodard and Ken G. Libbrecht

1NorthWest Research Associates/Colorado Research Associates Division, 3380 Mitchell Lane, Boulder, CO 80301-5410 USA
2California Institute of Technology and Big Bear Solar Observatory, 264-33 Caltech, Pasadena, CA 91125, USA

ABSTRACT

Eight years of brightness data from the Solar Disk Photometer (SDP) at Big Bear Solar Observatory (BBSO) have been analyzed to investigate the origin of total solar irradiance (TSI) variations. Our analysis has focused on whether non-sunspot variations in the measured TSI are better accounted for by a simple model of emission characterized by a facular contrast function or by a 'hot-band' model.

Previous studies (e.g., Foukal & Lean, 1988) indicated that roughly half of the 'non-sunspot' contribution to the TSI variation comes from bright facular elements in the vicinity of active regions. Speculation about the remaining non-sunspot variation, which alone accounts for the bulk of the observed cycle-related TSI variability, centered on the role of diffuse facular elements versus 'hot bands', the latter inspired by work on buoyantly rising magnetic flux tubes in the convection zone (Parker, 1995). The precision limb photometry from the Mt. Wilson solar distortion telescope (Kuhn, Libbrecht, & Dicke, 1988; Kuhn & Libbrecht, 1991), carried out during the rising phase of solar cycle 22, provided indirect evidence of hot bands. The simultaneously observed TSI variation could be accounted for if most of the excess brightness, seen in the active latitudes at the limb, was produced by 'non-facular' emission, with constant contrast across the solar disk.

To study sources of TSI variability more closely, a Solar Disk Photometer (SDP) was built by Libbrecht and operated at BBSO during the 1990s (Taylor et al., 1998). The most significant new feature of the instrument was the ability to scan the solar disk at various distances from the limb, as well as at the limb. In this way a direct determination of the contrast function of bright features could be made over a range of limb distance in which the facular contrast is known to vary significantly.

The contrast of the bright emission is seen to vary considerably over the range (0.05 < $\mu$ < 0.55) of limb distance sampled by SDP, in a way that is qualitatively consistent with facular emission. To make a more quantitative comparison we extrapolated the observed brightness contrast to the unobserved inner portion of the disk and unobserved wavelengths assuming a facular form for the $\mu$ dependence and a Planck form for the wavelength dependence of the non-sunspot brightness excess, where these quantities were unavailable. The extrapolated contrast function was then converted to a non-sunspot TSI perturbation for each season of data. When corrected for the effect of sunspots, the inferred TSI variation over the rising phase of solar cycle 23 (1996–2000) agrees fairly well with the trend in the observed TSI for the same period. Our results thus substantiate previous findings that most of the TSI variations on solar-cycle time scales, after correcting for the effect of sunspots, are of facular origin, leaving little room for a contribution from hot bands in the current cycle (Woodard & Libbrecht, 2002).

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DEGREE DEPENDENCE OF MODE PARAMETERS WITH SOLAR ACTIVITY IN BISON DATA

W. J. Chaplin1, Y. Elsworth1, G.R. Isaak1, R. New2

1 School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
2 School of Science and Mathematics, Sheffield Hallam University, Sheffield S1 1WB, UK
*E-mail: yelsworth@blam.ac.uk

ABSTRACT
The line-width, velocity power and rate of supply of energy to the low-degree p modes of the Sun are investigated over falling phase of cycle 22 and the rising phase of cycle 23 by fitting in the Fourier transform domain. We see that for the first time for low-degree modes we are able to parameterise the solar activity dependence as a function of the angular degree of the mode.

Key words: Sun: activity – Sun: oscillations

1. INTRODUCTION
The study of the aspects of helioseismology that relate to the excitation and damping of the oscillations has attracted considerable interest. Analysis has shown that the strength of the oscillations can be located within the outer few hundred kilometres of the convection zone (see, e.g., Chaplin & Appourchaux 1999, Chaplin et al. 1999, Kumar & Basu 1999 and Nigam & Kosovichev 1998, 1999). Furthermore, the evanescent tails of these oscillations are contaminated with a correlated component of the convective granulation. The phase properties of the resonant signal are modified to such an extent that the observed profiles in the frequency domain are noticeably altered. The relative size disparity between the imprinted signal that is present when either Doppler velocity or intensity observations are made is believed to explain the opposite sign of asymmetry that is usually seen in the two types of data. Theories of the excitation of the p modes by stochastic turbulence indicate that the forcing is peaked in the outer layers of the convection zone. Evidence that supports those theories comes from the observed asymmetries, observations of individual ‘acoustic’ events and the distribution of the observed mode powers.

Not withstanding the observed asymmetry of p-mode structures, the excitation and damping can be modelled in terms of a simple harmonic oscillator that is both forced and damped. The oscillator is characterised by two independent parameters which are the damping time and the forcing of the oscillations.

The power in the mode and the line width of the mode are extracted by fitting the spectrum of the oscillations. The power is the height times the width and to obtain the energy supply rate one multiplies the height by the width$^2$. It has been shown that these are variable over the solar activity cycle at the level of just over 20 per cent of their mean value. The variation in the power of the modes and their width have been shown to be consistent with the rate of supply of energy to the modes being unchanged over the solar cycle (Chaplin et al. 2000, Komm et al. 2000).

2. EXCITATION & DAMPING PHENOMENOLOGY
A useful analogy for the p modes is a forced, damped harmonic oscillator of the form:

$$\frac{d^2}{dt^2} x(t) + 2\eta \frac{dx}{dt} + \omega_0^2 x(t) = f(t)$$

Where, $x(t)$ is the displacement of the oscillator, $\omega_0$ is its natural angular frequency, $\eta$ is the damping constant, and $f(t)$ is the forcing function. The Fourier transform of the oscillator equation gives the shape of the expected power spectrum in the vicinity of the resonance ($\omega = \omega_0$).

If $F(\omega)$, the power spectrum of $f(t)$ is a slowly varying function of frequency and the Q of the oscillator is high, the profile in the power spectrum is essentially Lorentzian with a radian FWHM of $2\eta$.

The peak height, $H$, is

$$H = \frac{F(\omega_0)}{4 \omega_0^2 \eta^2}$$

The total velocity power of a given mode is proportional to the product of the width and the height, i.e.,

$$V^2 = \pi \frac{F(\omega)}{4 \omega_0^2 \eta}$$

To a good approximation, the solar acoustic spectrum should be represented by an ensemble of such oscillators. The energy (kinetic plus potential) of a mode with associated mass M is given by

$$E = MV^2$$

The rate at which energy is supplied to the modes, $dE/dt$, is the product of the full line width and the energy in the mode.

Thus

$$\frac{dE}{dt} = -\pi \frac{F(\omega)M}{2 \omega_0^2}$$

Note that the negative sign here is conventional. For the oscillations to be sustained, energy has to be supplied to the modes.

3. WHAT DO CHANGES IN THE MODE PARAMETERS TELL US?
We see that the various equations above provide us with different information about the oscillators. Measurements of the modal line widths, $\Delta v$, are


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expected to give a direct measure of the damping rate, \( \eta \); the velocity power, \( V \), reflects the balance between the excitation – as expressed by \( F(\omega) \) – and the damping. The energy supply rate is dependent only on the forcing and is independent of the damping rate. Although the natural frequency of a mode is known to vary through the solar cycle, the fractional change is small and can be neglected in this analysis. Similarly, there is no reason to expect the mode mass to change significantly.

We can therefore derive the following expressions:

\[
\frac{\delta(\Delta v)}{\Delta v} = \frac{\delta \eta}{\eta} \quad \text{and} \quad \frac{\delta (\frac{dF}{dt})}{\frac{dF}{dt}} = \frac{df}{f}
\]

These expressions indicate that changes to the forcing will change the energy supply rate but not the width of the mode structure and the reverse is true for the damping. We can therefore use these parameters to study the solar-cycle-related changes in the in the forcing and the damping of the oscillator. The observed fractional changes in the width, power and energy supply rate for the low-degree modes observed with BiSON are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fractional Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>24 ± 3%</td>
</tr>
<tr>
<td>Power</td>
<td>-22 ± 3%</td>
</tr>
<tr>
<td>Energy supply rate</td>
<td>0 ± 4%</td>
</tr>
</tbody>
</table>

The sense of the variation is that a positive sign indicates that the change is an increase at solar maximum and a negative sign indicates that the parameter decreases with increasing solar activity (Chaplin et al. 2000). Similar results were obtained at higher degree by the GONG network, except that they saw a change in the energy supply rate of approximately -3 ± 4% dependent on the degree of the mode (Komm et al. 2000).

5 DERIVATION OF THE PARAMETERS USING LONG SPECTRA

The method, used to derive the solar-cycle trends quoted above, is based on the coherent analysis of long data sets. The data are fitted using maximum likelihood methods and chi-squared 2 degrees of freedom statistics to a model that is a Lorentzian with the addition of a small degree of asymmetry. Each data point is derived from 108 days of data. This interval, a multiple of the solar rotation period, is chosen to minimize effects dependent on solar rotation.

The data span the period from the Spring of 1991 to the end of 2001. For the most part, the fill of the data is above 70%. Two spectra with fills below 50% have been deleted from the analysis. Incomplete fill can have an impact on the derived parameters for the mode and we have used extensive simulations to make any necessary corrections to the results.

The quality and extent of the data, together with the increased confidence in the derived mode parameters have allowed us to explore the activity variation with individual 1.

There are several choices that can be made in the error with which a result is quoted. The fitting procedure itself will give a formal error determined by the fit. This may however, underestimate the true reliability of the result. Another approach is to use the error derived from the scatter between independent data sets. There is a third option that combines the advantages of the previous two methods. In this situation we take the scatter error and give it a weight that depends on the formal error. This we call the external error. It is usually the most reliable estimate of the error.

6 WHY ONE MIGHT EXPECT 1 DEPENDENCE

Over many years now it has been observed that the oscillations are influenced by the presence of strong magnetic fields on the Sun. Observations have convincingly shown that solar p-mode frequencies are increased by magnetic activity (Woodard & Noyes 1985; Libbrecht & Woodard 1990; Elsworth et al. 1990, 1994; Bachmann & Brown 1993; Rhodes et al. 1993; Jimenez-Reyes et al. 1998). Libbrecht & Woodard (1990) and Woodard et al. (1991) have argued on the basis of observations of intermediate-degree modes that the source of the perturbations must lie near the solar surface. This conclusion is supported by Gough & Thompson (1988a,b), Vorontsov (1988) and Paterno (1990).

The early investigations all assumed the solar magnetic field to be homogeneously distributed over the solar surface (at least on large scales). However, full-disc magnetograms and synoptic maps reveal, that this is not the case. Magnetic activity changes both its strength and its location on the disc with position with the solar activity cycle. There are also clear differences between the rising and the falling phases of individual cycles.

A theoretical framework for the discussion of the effect on the mode parameters of the localisation of the surface solar magnetic activity has been provided by two groups of workers (Dziembowski & Goode et al. (1997, 2000) and Moreno-Insertis and Solanki (2000)) who both use the concentration of the magnetic activity at low latitudes as the cycle approaches its maximum to predict the expected frequency changes. However, Moreno-Insertis and Solanki extend the analysis to include the polar magnetic fields. Thus, although we do not explore it here, there is the potential to distinguish between the predictions of the two theories.

The modes of different degree have differing sensitivities to the conditions near the surface of the Sun. This is true even for the low degree modes studied here. The \( l=0 \) mode is, in principle, uniformly sensitive across the Sun. The observation has some variation in sensitivity across the visible disc because of Doppler imaging in the spectrometer used to make the observation. The sensitivity to the sectoral mode with \( l=|m| \) is higher for other modes in the multiplet.
The higher degree modes ($\ell=|m|=1,2,3$) are all more concentrated toward the equator of the Sun. Hence we expect to see a degree dependence in the mode variation with activity. This variation has been reported by several groups for the frequencies but, until now, no-one has reported the individual $l$-dependence for the other mode parameters at low angular degree.

The low degree modes differ from the higher-degree modes in that they directly probe the energy generating core of the Sun. Furthermore, they are the modes which will be detected on stars other than the Sun. In order to draw proper inferences about the conditions in the interior of the Sun (star), one must be able to remove the surface contamination imposed by the activity. Only then will be able to use the full accuracy that the low degree modes offer.

7 CORRELATIONS BETWEEN THE MODE PARAMETERS AND WITH SOLAR ACTIVITY

To compute the sensitivity of any given mode parameter to the solar activity we first have to select an activity proxy. There are many proxies that one could chose, but we find better correlations with radiative rather than magnetic field indices. One of the best proxies for activity is the 10.7 cm radio flux and it is this variable that we use here.

The individual mode parameters are regressed against the chosen activity to give a slope and an intercept. For the fit, we can choose whether to use the errors as weights and which of the two different errors available to use.

The slope of the fit in fraction change of the mode parameter per unit activity is transformed into a percentage change in the mode parameter from solar minimum to solar maximum by multiplying the slope of the fit by the range of activity. The reference time for the change is taken to be at solar minimum.

Figure 2 shows the degree dependence in the mode power (derived as the height multiplied by the width) and the energy supply rate which is the power times the width. The errors in the width and the height of the modes are strongly anticorrelated, however, the determination of the mode power is robust as it evidenced by the relatively low errors even for $l=3$.

8 OTHER LOW-\(l\) OBSERVATIONS

Although no other group has published the $l$ dependence of the width, power and energy supply rate for these low-degree modes reported here there are some relevant publications. Komm et al. (2002) used both GONG and MDI and medium $l$ (40 to 80) to produce latitudinal information on the distribution of the mode parameters. Their maps show very beautiful correlations between the changes in the mode parameters and the strength and location of the magnetic field on the Sun. The low degree modes from MDI were looked at by Toutain & Kosovichev (2002) who discovered that the modes with $1+m$ even $l$ are more sensitive to solar activity than those with odd $1+m$.

Figure 1 shows the percentage change in the mode width computed for each $l$ value with different ways of weighting the fit. The individuals fits are shown as an indication of the sensitivity of the result to the fitting method and we emphasize that in no way are the four values shown at each $l$ independent realisations. With the exception of the one outlier at $l=0$ (weighted fit with external errors), it is obvious from the graph that the result is not particularly dependent on the type of regression employed. We believe that no inferences should be drawn from the value of the cycle sensitivity for $l=3$ because the low signal-to-noise for these data and the anti-correlated errors between the derived height and width of the mode give rise to significant systematic errors.

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the l= 0,2 and 1, 3 given width the same and so they were not able to study the effect that we see here. 
Finally, the intensity data from VIRGO-SPM, again on SOHO, were studied by Jimenez et al. (2002). The showed the low l changes in the intensity data but did not derive any sensitivity gradients for individual 1 components.

9 DISCUSSION
We have used the long, high quality, data from the BiSON network to show that one can extract cycle dependence for the individual 1 components of low-degree modes. The values that we get are consistent with the averages derived by use previously over all the modes. The sensitivity of the fractional change of the width is such that l=0 is least affected and l=1 and l=2 are more strongly affected. This is consistent with the predictions of Dziembowski & Goode and Moreno-Insertis & Solanki and with the known localization of the magnetic activity. The results for the power in the modes and the energy supply rate are much less clearcut. The data appear to imply that the energy supply rate to the modes is higher for the higher l values. However, we must stress that, that the trend is marginal and may be consistent with no solar cycle related changes. The data for the power in the modes are consistent with no degree dependence in the solar-cycle related changes.

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REVIEWING SOLAR MAGNETIC FIELD GENERATION IN THE LIGHT OF HELIOSEISMOLOGY

Dibyendu Nandy

Department of Physics, Montana State University, Bozeman, MT 59717
email: nandi@mithra.physics.montana.edu

ABSTRACT

Helioseismic observations, in recent years, have vastly improved our understanding of solar interior dynamics. This has challenged theorists to come up with up-to-date models for the origin and evolution of magnetic fields in the Sun – which on the one hand has to be consistent with these new results from the solar interior and on the other hand, has to correctly reproduce magnetic features that have been observed on the Sun since the advent of telescopes and magnetograms. We report here results from such dynamo simulations and address the following questions: Where in the Sun are the strong “sunspot-forming” toroidal magnetic field and the much weaker poloidal field generated? What is the role and nature of the unobserved meridional counterflow? We also discuss possible future helioseismic inputs that may usefully constrain the exact nature of the solar dynamo.

Key words: Sun; Dynamo; Magnetic field; Alpha effect; Meridional circulation; Helioseismology.

1. INTRODUCTION

It is believed that a hydromagnetic dynamo mechanism, sustained by small and large scale plasma motions in the interior of the Sun, generates the solar magnetic cycle. Visible manifestations of the underlying dynamo mechanism on the surface of the Sun have been studied spanning centuries, starting with the telescopic observations of sunspots by Galileo Galilei and Christoph Scheiner in the early 17th Century. Years of solar magnetic field observations have now firmly established some of the notable features of the solar cycle. The two main components of this cycle being the appearance and equatorward migration of sunspots (identified with the toroidal field) at low latitudes and the poleward propagation of the weak magnetic features (concentrated near the poles) on the solar surface (identified with the poloidal field). The relative polarity of the bipolar sunspot pairs reverses every 11 years. A reversal in the polar field occurs at the time of sunspot maximum, again at an interval of 11 years. This shows that the cycle of the weak and diffuse poloidal field is intimately related to that of the sunspot cycle.

Working in spherical coordinates and assuming axial symmetry, the magnetic field in the Sun can be expressed as

\[ \mathbf{B} = B_\phi \mathbf{e}_\phi + \nabla \times (A \mathbf{e}_\phi). \] (1)

The first term on the right hand side of the above equation is known as the toroidal component and the second term as the poloidal component of the magnetic field. The toroidal magnetic field is generated in the solar interior by stretching of the poloidal component by the differential rotation (Parker, 1955a). Before the advent of helioseismology, the internal rotation of the Sun was virtually a free parameter. Consequently, the rotational profile in pre-helioseismology dynamo models were prescribed to ensure solar-like Butterfly diagrams. Helioseismology has now mapped the internal rotation of the Sun (Schou et al., 1998; Charbonneau et al., 1999) and with the discovery of the tachocline - a region of substantial radial shear in the rotation, it is now fairly certain that the strong toroidal fields are produced in this tachocline region at the base of the solar convection zone (SCZ).

Due to its buoyancy, the strong toroidal flux tubes rise up radially from the base of the SCZ to the surface, forming sunspots (Parker, 1955b). Simulations of this buoyant rise (Choudhuri & Gilman, 1987; Fan, Fisher & DeLuca, 1993; D'Silva & Choudhuri, 1993) and flux storage (Moreno-Insertis, Schüssler & Ferriz-Mas, 1992), have established that the strong “sunspot-forming” toroidal field at the base of the SCZ must be of the order of $10^5$ Gauss (G). The strength of the equipartition magnetic field (assuming equipartition between the magnetic and turbulent energies) in the SCZ is only of the order of $10^4$ G. The classical \( \alpha \)-effect, which involves the twisting of the rising toroidal field by helical turbulence to regenerate the poloidal field (Parker, 1955a), cannot work on such strong super-equipartition field. Therefore, alternative mechanisms for the regeneration of
the poloidal field are necessary. One of these alternative scenarios which has received considerable attention in the recent past is the Babcock and Leighton mechanism for the dynamo $\alpha$-effect.

In Section 2 we outline the Babcock-Leighton dynamo, with special emphasis on the model of Nandy & Choudhuri (2002). In Section 3, we review some alternate models of the solar dynamo that use the helioseismically deduced rotation profile. In Section 4 we briefly summarize our present state as far as solar dynamo theory is concerned, and conclude in Section 5 with a discussion on how future helioseismic inputs may help unravel some of the important unanswered questions regarding the solar magnetic field generation process.

2. THE BABCOCK-LEIGHTON FLUX TRANSPORT DYNAMO

The Babcock and Leighton (hereafter BL) idea recognizes that the decay of tilted bipolar sunspot pairs, on the solar surface (resulting from the eruptions of buoyant flux tubes), can regenerate the poloidal field (Babcock, 1961; Leighton, 1969). This provides an alternative mechanism for the $\alpha$-effect, which in this case, is essentially a surface process (in contrast to helical turbulence which takes place throughout the SCZ). Numerous solar dynamo models have been built in recent years based on the BL idea (Choudhuri, Schüssler & Dikpati, 1995; Durney, 1995, 1997; Dikpati & Charbonneau, 1999, Nandy & Choudhuri, 2001; Küker, Rüdiger & Schultz 2001, Nandy & Choudhuri 2002). The tilts of the bipolar sunspot pairs are such that they correspond to a positive value of $\alpha$ in the northern hemisphere of the Sun. From the Parker-Yoshimura sign rule (Parker 1955a; Yoshimura 1975), it follows that at low latitudes, where the radial shear in the tachocline is positive (and hence $\alpha \theta / \theta r > 0$), there would be a poleward propagating dynamo wave. It turns out that a strong enough meridional circulation can “force” the dynamo wave equatorward at low latitudes, thus generating solar-like Butterfly diagrams (Choudhuri, Schüssler & Dikpati, 1995; Durney, 1995). We have constructed a solar dynamo model, with helioseismically determined rotation pattern, based on the BL idea by invoking an $\alpha$-effect that is concentrated in a thin layer near the solar surface. Motivated by results from simulations of flux storage and the buoyant rise of toroidal flux tubes we have implemented a buoyancy algorithm where toroidal fields exceeding $10^9$ G are made to erupt (from the base of the SCZ) to the surface layers (Nandy & Choudhuri, 2001; Nandy & Choudhuri, 2002). The $\alpha$-effect acts on this erupted toroidal field to produce the poloidal field. The poloidal field thus produced in the surface layers is first transported polewards by a combined action of the poleward meridional flow and diffusivity and then downwards to the high latitude tachocline by the meridional down-flow near the poles.

In the outer half of the SCZ, the meridional flow is observed to be poleward (Giles et al., 1997; Braun & Fan, 1998). It follows from mass conservation that there must be an equatorward counterflow somewhere in the solar interior. The exact location of this counterflow, however, remains undetected. Using a flow profile that is confined mainly in the SCZ and the tachocline (as depicted in Fig. 1, top-left) and our model for the BL dynamo, we generate a Butterfly diagram for the toroidal field at the base of the SCZ (Fig. 1, bottom-left). It is clear from the Butterfly diagram that strong toroidal fields form at high latitudes. The shaded regions, denoting the latitudes at which buoyant eruptions occur with the progress of the cycle, show that these simulated sunspot eruptions are confined to high latitudes, in sharp contrast to the observation of sunspots at low latitudes.

The poloidal field, according to the BL idea, is generated only in a thin layer near the solar surface. The only way this poloidal field can be brought down to the tachocline for the re-generation of the toroidal field is by the meridional down-flow near the poles. Thus, the sinking poloidal field hits the high latitude tachocline first. Helioseismic inversions show that the radial shear in the tachocline, which is negative at high latitudes and positive at low latitudes, is stronger at high latitudes. This results in the production of very strong toroidal fields at high latitudes, which subsequently diffuse out to the SCZ giving rise to sunspot eruptions there. It may be noted that this difficulty has been faced by many modelers working with BL type dynamo models (Durney, 1997; Dikpati & Charbonneau, 1999, Kuker, Rüdiger & Schultz, 2001).

Nandy and Choudhuri (2002) recently proposed that
a deep meridional flow, penetrating beneath the tachocline, can solve the discrepancy between the observations and the BL dynamo predictions. In this scenario, the toroidal fields created at high latitudes are dragged down immediately by the penetrating flow (profile shown in Fig. 1, top-right) into the stable layers beneath the tachocline. A toroidal magnetic field becomes buoyant if it is above the base of the SCZ, but its buoyancy can be suppressed if it is put in the sub-adiabatically stratified upper radiative layer just beneath the tachocline. This belt of toroidal field cannot then diffuse out into the SCZ and can be subsequently brought towards low latitudes through this stable layer by the equatorward meridional counterflow. At low latitudes, the upflow in the meridional circulation will bring up this toroidal field belt to the unstable SCZ, from where it can buoyantly erupt to form sunspots. Such a flow profile successfully reproduces the latitudinal distribution of sunspots as shown in the Butterfly diagram in Fig. 1 (bottom-right).

Fig. 2 shows a snapshot of the toroidal field configuration in the solar interior at a particular instant in time. It is seen that a belt of negative toroidal field, created at the high latitude tachocline, is being pushed down into the upper radiative layers. Simultaneously, at low latitudes, a belt of positive toroidal field (which was created at the high latitude tachocline but has now been transported to low latitudes by the equatorward meridional counterflow) is being pushed out into the unstable SCZ - from where it can erupt radially outwards to form sunspots.

One of the striking conclusions from this model is that the toroidal magnetic field belt that ultimately results in sunspot activity at low latitudes, is produced in the high latitude tachocline. In fact, evidence of this is found in other BL dynamo models (Durney, 1997; Dikpati & Charbonneau, 1999, Kuker, Rüdiger & Schultz, 2001), and we argue that this is a generic feature of such BL dynamos, where the sinking poloidal field (that creates the toroidal field of the next cycle) encounters the strong shear in the high latitude tachocline first.

Evidently, meridional circulation plays a very important role in such BL dynamo models, primarily as a transporter of magnetic flux (which is why these class of models are often called flux transport dynamos). It may be expected therefore, that variations in the meridional flow (arising from its inherent turbulent nature) can also account for observed variations in the solar cycle period and amplitude (Hathaway, 2003). BL dynamo models, employing fluctuating components of meridional circulation, have in fact successfully reproduced many of the observed solar cycle variations (Charbonneau & Dikpati, 2000) and such theoretical dynamo models predict that the solar cycle period scales inversely with the meridional flow speed (Dikpati & Charbonneau, 1999; Nandy, 2002)

3. ALTERNATE DYNAMO MODELS: TACHOCLINE AND MEAN FIELD ALPHA EFFECT

Various instabilities associated with the toroidal magnetic field and rotation at the base of the SCZ may also act as the source of an $\alpha$-effect there - such that the poloidal field re-regeneration process can also take place in and around the tachocline. Some of the suggested physical mechanisms which can produce such an $\alpha$-effect are interaction of rising magnetic flux tubes with rotation around the base of the SCZ (Brandenburg & Schmitt, 1998) and global hydrodynamic instability of differential rotation in the tachocline (Dikpati & Gilman, 2001).

Dikpati & Gilman, (2001) raised concerns about solar-like parity violation by BL models and demonstrated that an additional $\alpha$-effect in the tachocline (due to an instability in the differential rotation) can solve this problem of parity violation. At this stage it is not clear whether the parity violation by BL models is a robust phenomenon or rather a feature of the specific model and parameters (such as diffusivity and meridional flow profile within the SCZ) that Dikpati & Gilman (2001) used. Simulations are currently underway to explore this in greater detail. It is also not certain whether such an $\alpha$-effect can really exist in the tachocline, where strong magnetic fields reside and can, in principle, suppress any $\alpha$-effect there. Nevertheless, the tachocline $\alpha$-effect as worked out by Dikpati & Gilman (2001) has certain attractive features, notable amongst them being the production of strong toroidal fields only at low latitudes (due to the profile of this $\alpha$-effect which peaks at middle to low latitudes thus confining the dynamo activity there). The Dikpati & Gilman (2001) model therefore does not need a deeply penetrating meridional flow to ensure eruptions at low latitudes. However, they do need an equatorward counterflow in the meridional circulation to force the dynamo waves equatorward (just like the BL dynamo models) and hence this particular model can also be classified as a flux transport dynamo model. Interested readers may refer to Dikpati & Gilman (2001) for more details. We just point out here that a notable difference between their and our model (Nandy & Choudhuri, 2002) is that in their model, the toroidal field that

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produces sunspots is produced at the low latitudes itself, while in our model, this toroidal field is created at the high latitude tachocline and then transported to low latitudes by a deeply penetrating meridional flow.

In spite of observations of a meridional circulation in (the upper half of) the SCZ, and various physical mechanisms suggesting a positive $\alpha$-effect (in the northern hemisphere of the Sun), many modelers still build models with no circulation, prescribing a negative $\alpha$-effect to ensure the proper migratory behavior of the dynamo wave at low latitudes (Rüdiger & Brandenburg, 1995; Moss & Brooke 2000; Covas et al., 2000). These models, usually referred to as the mean-field models, work with an $\alpha$-effect distributed either throughout the SCZ or near the base of the SCZ. There is also a particular class of model, known as the interface dynamo, in which the source regions for the toroidal field and the poloidal field are separated by an interface layer across which the diffusivity changes discontinuously (Parker, 1993; Charbonneau & MacGregor, 1997; Markiel & Thomas, 1999). The toroidal field in the interface dynamo models is produced and stored in the low diffusivity region beneath the SCZ, while the poloidal field is produced in the high diffusivity regime of the SCZ (the highly turbulent SCZ reduces the toroidal field strength such that the $\alpha$-effect does not get quenched in the main body of the SCZ). All interface dynamo models published till date do not include a meridional circulation. It is important to understand then that the migratory behavior of the toroidal field that we see in mean-field or interface dynamo models is the result of a pure dynamo wave, propagating along iso-rotation contours following the Parker-Yoshimura sign rule, and not a wave driven or “carried” by the meridional circulation.

Since the radial shear in the rotation is negative at high latitudes and positive at low latitudes, a negative $\alpha$ (in these mean-field models) would imply two branches of the butterfly diagram (in the absence of meridional circulation) - a poleward propagating branch at high latitudes (where $\alpha d\Omega/dr > 0$) and an equatorward propagating branch at low latitudes (where $\alpha d\Omega/dr < 0$). The mean-field dynamo models often generate butterfly diagrams with stronger poleward propagating branches at high latitudes (Rüdiger & Brandenburg, 1995; Moss & Brooke 2000; Covas et al., 2000). The reason behind this is not difficult to understand - the amplitude of the radial shear is stronger in the high latitude tachocline! The interface dynamo models on the other hand, rely mainly on the latitudinal shear in the rotation to produce the toroidal field. Consequently, the migratory behavior of the toroidal field in this class of models is not so strongly constrained by the sign combination of the $\alpha$-effect and the radial shear within the tachocline. Recently, however, some questions have been raised regarding the ability of interface dynamos to function with a solar-like rotation profile (Markiel & Thomas, 1999). This result casts some doubt on the workability of such models, although, the final say would probably depend on more simulations, possibly from models including meridional circulation.

4. WHERE DO WE STAND?

It is clear from the preceding discussion that the single most important factor differentiating the various solar dynamo models proposed till date is the physical nature and location of the $\alpha$-effect. Unfortunately, all these different models also explain the main features of the solar cycle to some degree of success, with a particular class of model explaining some particular features more elegantly than others. This means that it is not possible to discriminate between these models easily (if one leaves out personal biases). It is also not unlikely that the Sun works with more than one kind of $\alpha$-effect operating at different locations in its interior. For example, even though a popular argument against the traditional mean-field $\alpha$-effect (due to helical turbulent convection) is that it will be quenched due to the strong super-equipartition fields in the solar interior, it is very likely that this $\alpha$-effect could still work on the more diffuse toroidal field at the base of the SCZ which has not collapsed to form strong flux tubes (or which is left behind after a flux tubes has formed and erupted out to the surface layers), while a BL type $\alpha$-effect works on the surface (due to the decay of tilted sunspot pairs resulting from the erupted flux tubes). The problem of course is that there is no way of knowing whether any of these speculative ideas are true, or to directly determine whether a single kind of $\alpha$-effect, or a certain combination of different $\alpha$-effects, drives the solar dynamo. Undoubtedly then, the single most important uncertainty in solar dynamo theory at present, is the $\alpha$-effect (see also, Bigazzi & Ruzmaikin, 2003).

The question that naturally follows is whether some kind of observational constraint can be put on the dynamo $\alpha$-effect? We explore this issue in the next section, where we discuss helioseismic inputs of the future that may constrain the exact nature of the $\alpha$-effect.

5. POSSIBLE HELIOSEISMIC CONSTRAINTS ON THE NATURE OF THE SOLAR DYNAMO

The main contenders for the $\alpha$-effect and models for the solar cycle, at present, are; the BL $\alpha$ (located near the surface - resulting from the decay of tilted bipolar sunspot pairs) and the corresponding BL dynamo (Durney, 1997; Dikpati & Charbonneau, 1999; Nandy & Choudhuri, 2002); the tachocline $\alpha$ (located in the tachocline - resulting from the global hydrodynamic instability of the differential rotation) and the tachocline dynamo (Dikpati & Gilman, 2001); the mean-field $\alpha$ (located at the base or throughout the SCZ - due to helical
turbulent convection) and corresponding mean-field dynamos without meridional circulation (Rüdiger & Brandenburg, 1995; Moss & Brooke, 2000; Covas et al., 2000).

Let us now refer to a meridional cut depicting the geometry in the interior of the Sun (Fig. 3). In this figure, the iso-rotation contours of an analytic fit to the helioseismically deduced solar rotation are shown with solid lines. The dashed line depicts the base of the SCZ at 0.71\(R_\odot\). \(H_{CZ}\) denotes the upper layers of the SCZ at high latitudes, \(H_T\) denotes the high latitude tachocline, \(L_{CZ}\) denotes upper layers of the SCZ at low latitudes and \(L_T\) denotes the low latitude tachocline.

The BL dynamo (corresponding to a BL \(\alpha\)-effect) will produce the strong toroidal fields at the high latitude tachocline, which is subsequently transported to low latitudes by a deeply penetrating meridional flow as discussed in Section 2. Referring to Fig. 3, it is clear that the toroidal field belt will originate around the region marked \(H_T\) and after a time corresponding to about 11 years it would re-surface to the SCZ through the region marked \(L_T\). Thus, if improved helioseismic techniques could measure the location of toroidal fields at the base of the SCZ, signatures of this strong toroidal field belt should be detected at both the regions \(H_T\) and \(L_T\). Such studies to detect possible signatures of magnetic fields have only just begun and are still at a preliminary state (Antia et al., 2001; Chou & Serebryanskiy, 2002). The toroidal field belt, which in the model of Nandy & Choudhuri (2002) is pushed down by a penetrating flow beneath the tachocline at high latitudes and out into the SCZ at low latitudes, would couple the SCZ to the radiative interior. Therefore angular momentum transport across the tachocline region or variations in the rotation (due to the feedback of the toroidal field belt on the rotation) is expected to occur. This should potentially give rise to temporal variations in the rotation across the tachocline, with the maximum amplitude of these variations peaking every 11 years. The variations should also be such that above and below the tachocline, they would be in anti-phase (because the toroidal field belt would be moving into the lower layers and out of the upper layers of the tachocline). For BL dynamos, since the toroidal field is produced at high latitude and then transported by a meridional flow to low latitudes, such variations (of similar amplitude) are expected at both the regions \(H_T\) and \(L_T\). Howe et al. (2000a) have detected temporal variations around the tachocline, but at much shorter timescales of 1.3 years. Subsequent studies, however, have failed to detect similar variations (Basu & Antia, 2001). In such BL models, the toroidal field belt produced at high latitudes come out into SCZ and collapse to form flux tubes (due to the up-welling in the meridional flow) only at low latitudes. These flux tubes would then buoyantly rise to form sunspots, thus threading the whole of the SCZ from regions \(H_T\) to \(L_{CZ}\) and migrate equatorward. At high latitudes, on the other hand, the amplitude of these oscillations is expected to be much lower - due to the (lack of strong flux tubes and) presence of disconnected magnetic features that are transported towards the poles by the meridional flow near the surface (Makarov & Sivaraman, 1989). The pattern then would migrate polewards near \(H_{CZ}\) and appear to move down with the sinking meridional flow there. Studies of the torsional oscillations patterns are underway and many of them do reflect some of the features discussed here, however, a consensus view regarding the high latitude activity (near about \(H_{CZ}\)) has still not emerged (Howe et al., 2000b; Vorontsov et al., 2002; Basu & Antia, 2003).

The tachocline dynamo, corresponding to an \(\alpha\)-effect located in the low latitude tachocline (Dikpati & Gilman, 2001), would generate the strong toroidal fields only at mid to low latitudes. Since the source regions for the toroidal field and the poloidal field in such models are close together, they can communicate diffusively, and the role of meridional circulation is largely reduced. In this case, most of the activity associated with the dynamo will be confined to the region \(L_T\). Therefore temporal variations around the tachocline are expected to occur only at \(L_T\), strong toroidal fields will also be confined to this region. The torsional oscillation pattern at low latitudes would be similar to that of the BL models, with rising and equatorward migrating bands of faster or slower rotation threading between \(L_T\) and \(L_{CZ}\). At high latitudes (\(H_{CZ}\) and \(H_T\)), dynamic variations are not expected. However, if, as suggested by Dikpati & Gilman (2001), the solar dynamo is working with a combination of the BL and Tachocline \(\alpha\)-effects, then dynamic variations are also expected at high latitudes. But, as opposed to the purely BL case, in such a scenario, the amplitude of the temporal variations around the tachocline is expected to be larger at \(L_T\) as compared \(H_T\) (because there is an additional dynamo source term at low latitudes).
which can amplify the magnetic field there).

Finally, we come to the mean-field dynamo models (usually working with a negative \( \alpha \)-effect) which do not employ meridional circulation. Due to the lack of circulation, the toroidal dynamo waves in this case would follow the Parker-Yoshimura sign rule and there would be an equatorward propagating branch at low latitudes and a poleward propagating branch at high latitudes. Since the amplitude and extent of the high latitude tachocline is larger, these models usually generate a stronger polar branch upwards of 36° latitude and in the region \( H_T \), unless the alpha-effect is strongly constrained to low latitudes (Rüdiger & Brandenburg, 1995; Moss & Brooke, 2000; Covas et al., 2000). This should result in strong temporal variations in the rotation in the high latitude tachocline (\( H_T \)) and a much smaller amplitude variation around \( L_T \). Torsional oscillation patterns would also reflect the Butterfly diagram of these mean-field models, with stronger amplitude (poleward propagating) oscillations rising between \( H_T \) and \( H_CZ \) and lower amplitude (equatorward propagating) patterns rising between \( L_T \) and \( L_CZ \), with the patterns diverging from around 36° latitude (where the shear in the tachocline changes sign).

With that we come to the end of our discussion. We have pointed out that helioseismic studies of dynamic variations in the interior of the Sun can potentially discriminate between different \( \alpha \)-effects and models of the solar dynamo - thus pointing out the right direction in which to proceed. Detection of the exact location of the equatorward counterflow in the meridional circulation is also of vital importance. With the mapping out of the solar internal rotation, one would have hoped that we were close to a non-controversial and well accepted model for the solar cycle. Evidently, that hope has not been realized. However, on the brighter side and as pointed out here, there is much more that could be done, both at the observational and theoretical level. That is indeed a happy situation to be in, and one hopes to learn more about the complex dynamics in the interior of the Sun, along the way.

ACKNOWLEDGEMENTS

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From Solar to Stellar Seismology

Chair: J. Kuhn
THE SEARCH FOR G MODES

T. Appourchaux*
Research and Science Support Department of ESA, P.O.Box 299, NL-2200AG Noordwijk

ABSTRACT

The Phoebus group was set up about 5 years ago with the expressed purpose to detect the g modes predicted by helioseismology to occur in the sun. The current status will be reviewed including new approaches proposed by other groups in the field. Over the years, the upper limit to g-mode amplitude set by Appourchaux et al (2000) (10 mm/s at 10 σ) has been lowered due to a longer time series as well as new detection techniques. Notwithstanding these efforts the chance of a positive detection still appears remote with the current upper limit still way above that predicted by helioseismology. New techniques in particular observations involving limb intensity and/or velocity cross-correlations against various observables, or even the detection of gravitational waves may offer the best hope for a future positive detection.

Key words: intensity - p modes - SOHO - Sun.

1. INTRODUCTION

Since the beginning of helioseismology, the detection of g modes has been the most challenging quest in our field. There were several claims of g-mode detection (e.g. Delache & Scherrer 1983; Thomson et al. 1995), none of which has ever been confirmed. Since the conception of the SOHO mission, one of the goals of this mission was to detect g modes. In 1997, following the lack of g-mode detection by SOHO experimenters, the Phoebus group was formed, with the aim of detecting g modes. The group set an upper limit to the g-mode amplitude of 10 mm/s at 200 μHz (Appourchaux et al. 2000). The Phoebus group also reported on its activity at the Tenerife (Appourchaux et al. 2001) and Boston meeting (Appourchaux 1998; Appourchaux et al. 1998a; Fröhlich et al. 1998). Since the last meeting in Tenerife, there has been new developments in detection techniques, either related to the Phoebus group or a collaboration with this group.

*on behalf of the Phoebus group

In the first section, I review the current techniques deployed for finding the modes. I will especially outline the new techniques developed over the past 2 years. In the second section, I state the current upper limit to g-mode amplitudes. In the third section, I explore new ways of lowering the limit that may lead to a positive detection. In the conclusion, I project myself in the future.

2. G-MODE DETECTION TECHNIQUES: AN AIDE MÉMOIRE

There is now a wealth of techniques that are available for detecting g modes. Any technique aiming at detecting g modes should be able to detect also low-frequency p modes. These modes can indeed mathematically mimic the g modes provided their lifetime is longer than the observing time.

The g-mode detection techniques can be categorized as follows:

- Spectrum estimators
- Mode masking
- Statistical testing
- Patterns
- Data combination

Each of these categories can be combined for providing different methodology for detecting g modes. For instance Statistical testing is required on any Spectrum Estimators derived from any Mode masking. This is an example of a possible combination and other combination are always possible. Nevertheless a must-be-included category is certainly Statistical testing for it provides a safeguard against over-interpreting the data.

2.1. Spectrum estimators

The estimation of the spectrum of the time series is of prime importance when one wants to detect long
lived oscillations like the g modes. The following estimators are at our disposal:

- Fourier spectrum (power spectra)
- Multitapered spectrum
- Random Lag Singular (Cross) Spectrum analysis
- Frequency matching (oversampling and bin shifting)
- Time-frequency spectrum
- Monte-Carlo filtering

The use of Fourier spectrum estimation is widely developed in helioseismology. It has been used by almost any scholar in the field. Its properties are well known and quite often well understood (Bracewell 2000), and so is its statistics (Davenport & Root 1958).

Fourier spectrum estimation is slowly being replaced by multitapered spectra that are widely used in geophysics (For a review, see Thomson 1982); in helioseismology the use of these slepian tapers has been replaced by the more practical (but less accurate) sine tapers (Komm et al. 1999). The statistics of the resulting spectrum is also well known (Thomson 1982). Unfortunately, for long-lived modes, they tend to broaden too much the peaks as shown by Thomson (1982). Tapers should be used for what they were designed in the first place: mean spectrum estimation; therefore they are more useful for short-lived p modes.

Random Lag Singular (Cross) Spectrum analysis is an elaborate technique based on Singular Value Decomposition (RLSSA, RLSCSA: Varadi et al. 1999, 2000, respectively). The technique claimed successful detection of low-frequency p modes (Bertello et al. 2000) but still lacks a proper assessment of its statistical property, i.e. it cannot be excluded that the technique produces large peaks solely due to noise.

Frequency matching has also recently been developed by Gabriel et al. (2002) and by Chaplin et al. (2002) using zero padding and bin shifting, respectively. When one wants to detect signals from oscillators having a lifetime longer than the observing time, there is a fair chance that the frequency bin will not match the frequency of the oscillator. As a result, the peak could be reduced by up to 60% as shown by Gabriel et al. (2002). In order to alleviate this problem one can either oversample the data by using zero padding (Gabriel et al. 2002) or try to tune for the frequency by using bin shifting (Chaplin et al. 2002). This latter technique involves creating many different time series of similar but different length, but without zero padding (Chaplin et al. 2002). In either case, the statistics of the observation is indeed affected. For oversampling, the analytical calculation is somewhat difficult. It has been replaced by Monte-Carlo simulation showing that there are typically 3 independent frequency bins when the time series is oversampled by a factor 5 (Gabriel et al. 2002). The bin-shifting method produces also 3 independent power spectra from the many spectra generated (Chaplin et al. 2002). This factor 3 is certainly not a coincidence and we await theoretical calculations to confirm the numerical observations. An other way of reducing, the frequency mismatch would be to use multitapers (Rafael García, private communication). Given, the possible phase change produced by say 5 tappers, it might be quite likely that the signal would be less missed than it is with Fourier spectra, even at the expense of broadening the peak.

Time-frequency spectrum or wavelet analysis has been used by Gabriel et al. (1998) and by Finsterle & Fröhlich (2001) for looking for the lifetime of candidate g modes. Its statistical properties can be derived from those of the Fourier spectrum.

Last but not least, a new technique for spectrum estimation has been presented during the conference (Grec and Renaud, these proceedings). This is a technique that uses randomly generated window function for mode detection. The generated power spectrum has a well known statistics. This promising technique has been applied to the GOLF time series but should be tried to other time series such as those of GONG.

2.2. Mode masking

Mode masking is required by instruments making images of the Sun. There are several types of masks:

- spherical harmonics masks
- g-mode masks
- optimal masks

Spherical harmonics masks are widely used in helioseismology. They correspond to the displacement or temperature perturbations (velocity or intensity observations). These masks have been used for detecting g modes as in Appourchaux et al. (2000); in which their visibilities are given.

Unfortunately, these masks are rather well adapted for most p modes but not for g modes. For velocity observations, the horizontal displacement must be taken into account. For intensity observations, the oscillations perturb the surface of the Sun and the light it emits (Berthomieu & Provost 1990; Toutain et al. 1999). For either observations, the additional contribution is a non-spherical harmonics function (Unno et al. 1989; Appourchaux & Andersen 1990). In either case, specific g-mode masks can be devised (for intensity observations see Appourchaux & Andersen (1990)).
Masks that are optimal for specific purpose has also been developed. For instance, Appourchaux & Andersen (1990) developed masks that minimize both the leakage from other degree and from other $m$. Similar masks have been derived by Appourchaux et al. (1999b) and by Toutain & Kosovichev (2000). Recently, Wachtet al. (2002) has produced masks optimizing (i.e., minimizing) the noise contribution from the supergranulation noise varying across the solar disks, producing lower detection limits than those of Appourchaux et al. (2000). There are other interesting masking techniques that could make use of the properties of supergranulation. The solar rotation causes the supergranulation to enter and leave the observing window. A way to reduce the resulting noise would be to have a window following the rotation in a region close to the centre of the solar disk where the supergranulation noise is weaker than at the limb.

2.3. Statistical testing

Statistical testing is related to hypothesis testing. This branch of mathematics require proper expertise that the present author may lack. In short, we have basically tried two type of hypothesis:

- **H$_0$ hypothesis:** what is the probability of detecting pure noise?
- **H$_1$ hypothesis:** what is the probability of detecting a signal?

This is a simplification of what is performed, and I would like to apologize to the experts for doing that. The use of hypothesis is related to a decision process (Papoulis 1991); should we accept or reject the hypothesis? It should not be used for implying that say g modes are being detected: i.e., reject H$_0$.

The H$_0$ hypothesis was used by Appourchaux et al. (2000) for providing upper limit to g-mode amplitudes. The statistical method is based on the knowledge of the statistical distribution of the power spectra of full-disk integrated instruments; namely this is a $\chi^2$ distribution with 2 degrees of freedom (d.o.f.). Appourchaux et al. (2000) provide a simple formula for the relative level $\sigma_{\text{det}}$ for which a peak due to noise has a 10% probability to appear in a 70-$\mu$Hz bandwidth. This relative level will depend upon the observing time ($T_Y$) because the number of frequency bins in the bandwidth increases with time. And we have:

$$\sigma_{\text{det}} = 10 + \log(T_Y)$$  \hspace{1cm} (1)

where $T_Y$ is the observing time in years. A similar calculation leading to Eq. (1) can be carried out using spectra obtained by making an $m - \nu$ averaged spectra: the so-called collapsogramme (Appourchaux et al. 2000). The advantage of the collapsogramme is that it enhances mode multiplets while reducing at the same time artifacts due to instrumental effects (Appourchaux et al. 2000). For these 2 cases, the H$_0$ hypothesis can only be used if the statistical properties of the spectrum estimator is known. That is why I emphasized in the corresponding section whether the statistics of the estimator is defined. For instance, the RLSSA and RLSCA of Varadi is an interesting technique lacking a proper knowledge of its statistical properties. This renders the low-frequency p-mode detection of Bertello et al. (2000) of limited value. I must outline that similar results could be obtained by say raising a power spectrum to some unknown and random power; large peaks due to noise might be enhanced by this procedure but given the random nature of the exponent their significance would be doubtful. That is the reason why for any spectrum estimator a proper derivation of the statistics should be performed either through analytical calculation or Monte-Carlo simulations as in Gabriel et al. (2002) or in Chaplin et al. (2002).

The H$_1$ hypothesis has also been used in Gabriel et al. (2002) for setting a probability of detecting a sine wave given the noise in the GOLF data (Gabriel et al. 2002). In this latter case, the statistics is a non-centered $\chi^2$ with 2 degrees of freedom.

Composite hypothesis testing is also of interest but is currently not being developed in our field yet.

2.4. Patterns

The asymptotic behaviour of g-mode frequencies (or periods) was used for easing detection. This was pioneered by Delache in 1983 leading to a claimed detection of g modes (Delache & Scherrer 1983); leading other observers to try the method on their own data (Memorie della Società Astronomica Italiana, vol 55 and references therein). This approach is only of relevance to high order g modes (or very low frequency below 100 $\mu$Hz) for which the asymptotic behaviour applies. Unfortunately, the solar noise increases towards lower frequencies, and the mode spacing dramatically decreases; in addition the situation is even complicated by the introduction of rotational splitting (Fröhlich & Andersen 1985). The mode degeneracy being lifted by the internal rotation of the Sun, it also contributes to the overall pattern.

The collapsogramme technique pioneered by Appourchaux et al. (2000) makes use of the pattern created by the rotational splitting for detecting the modes. It can also be used for full-disk instruments producing a single power spectrum: in this case it is called an overlapogramme. Chaplin et al. (2002) has devised a statistical technique based on the detection of an ordered multiplet (due to rotational splitting) in power spectrum of full-disk integrated data. It has allowed to lower the detection level depending on the number of peaks searched for. The limit (under the H$_0$ hypothesis) they derived can be translated into the equivalent sigma level in a power spectrum for a one-year observing time: 5.9r, 4.5r, 3.8r for a doublet, triplet and a quadruplet, respectively. A similar approach has been used by García et al. (2001) for
the GOLF data. They derived levels for detecting modes of various degree that can be translated for 1-year time series to 6.4σ for an \( l = 1 \) doublet, 5.9σ for an \( l = 2 \) doublet and 4.3σ for an \( l = 2 \) triplet.

An artificial way of reducing the detection limit is to reduce the window over which we want to detect the mode, e.g., by looking in a window centred around theoretical g-mode frequencies. Denison & Walden (1999) provided a simple formula to derive the number of peaks that one can find in a power spectra given a list of given frequencies and a window around these frequencies. Under the \( H_0 \) hypothesis, it is written as follows:

\[
N = N_l(1 - (1 - p_{det})^{2N_w+1})
\]  

(2)

where \( N_l \) is the number of frequencies guiding the search, \( p_{det} \) is the probability level needed for identifying a peak and \( N_w \) is half the window size in units of bins. When \( 2N_w + 1 \)\( p_{det} \) is much smaller than 1, we can rewrite Eq (2) as:

\[
N = N_l(2N_w + 1)p_{det}
\]

(3)

This simple formula is quite useful to realize that the number of identified peaks will increase with the size of the window. This is the drawback of such a method: spurious peaks will be detected in this manner that are likely to be wrongly identified as g modes. Here we should remind the reader, that theoretical p mode frequencies had been in error of a few tens of \( \mu \)Hz until it was realized that the error came from our inability to model properly the surface of the Sun (Christensen-Dalsgaard 1990). Therefore care should be exerted when using theoretical frequencies as a guideline for searching for g modes.

2.5. Data combination

The use of different data sets related to different observables and/or wavelengths could very well be the solution to the g-mode detection. There is no doubt that the combination of more than 2 signals could considerably lower our detection limit. Observables such as radial velocity, intensity fluctuation, limb displacement and/or brightening are polluted by different source of noise such as supergranulation and active regions that produce different signatures. We can list the possible combinations in 2 categories:

- use of one instrument
- use of more than one instrument

Autocorrelation falls into the first category. The collapsogramme technique used by Appourchaux et al. (2000) belongs to the first category. An interesting technique developed by García et al. (1999) use a longer sampling time (twice as long) to create two independent time series of the GOLF data.

The Multivariate Spectral Regression Analysis (MSRA) belongs to the second category (Koopmans 1974; Appourchaux et al. 2000). It assumes that the modes are predominantly coherent over the observing time. The basic assumption is that low-order, low-degree p modes and g modes will have lifetimes that are significantly longer than the observing time. The MSRA has been used on VIRGO by Finsterle & Fröhlich (2001) allowing to claim detection of low-frequency low-degree p modes.

The RLSCSA described above is also a technique making use of different data sets for mode detection.

![Figure 1. Current upper limit for GOLF (diamond), BiSON (square) and MDI (black and grey curve) derived under the 10-% probability limit as in Appourchaux et al. (2000). The GOLF limit is obtained by Gabriel et al. (2002) using nearly 6 years of data. This latter limit is close to that derived by Wachter et al. (2002) for sectoral modes and for 2 years of MDI data. The BiSON limit is derived for a quadruplet from Chaplin et al. (2002) using 9 years of data. The MDI limit is derived from Appourchaux et al. (2001); Kuhn et al. (1997) for 1 year of data; the black curve corresponds to the radial displacement and the grey curve to the total displacement (radial and horizontal). The black thick continuous and dashed curves are the simplified version of the theoretical g-mode amplitude for \( l=1 \) predicted by Kumar et al. (1996) and Gough (1985), respectively.]

3. ON LOWERING THE LIMIT: THE PRESENT

At the time of writing, the canonical figure of 10 mm/s given by the Phoebus group in Appourchaux et al. (2000) has already been lowered by time and by newer analysis technique. Figure 1 shows the current upper limit obtained by various instrument. The limit is now closer to 3.5 mm/s for BiSON due to the use of a nine-year time series and of the multiplet ordered technique described above. For GOLF the limit is about 6.5 mm/s due to a data set about 6 years long; similar to actual limit of the Phoebus group.

As mentioned earlier by Appourchaux (1998) the
limit does not decrease like $T^{-\frac{1}{2}}$ but like $T^{-\frac{1}{3}} \log(T)$. That is a slower decrease than for instance shows that the limit of 10 mm/s would be 4.6 mm/s for a 100-year time series instead of the 1 mm/s commonly thought. This is due to the probability limit which needs to be kept constant (Appourchaux 1998). If we were to do otherwise it would mean that we would lower our probability level and accept more peaks that would likely be due to noise. Therefore, the decrease is bound to be not as fast as expected. So it is quite clear that a large amount of cleverness will be required to detect the $g$ modes. The hope for detecting the $g$ modes in our lifetime remains quite weak. That is why we need to explore other ways and means of lowering the limit.

4. ON LOWERING THE LIMIT: THE FUTURE

There are other possibilities for detecting the $g$ modes. We know that we have still a long way to go (at least a factor 20). Hereafter I summarize the possible orientation that our search will take:

- New observable: Limb measurement
- New instrumentation for measuring solar radial velocities
- New techniques: general-relativity effects

Hereafter, we describe each direction in more detail.

4.1. Limb measurement

The limb provides two types of observable: a physical displacement of the surface of the Sun, and a relative intensity fluctuation. Both types of measurement have been used by Kuhn et al. (1997) for detecting solar modes using MDI limb data. Unfortunately, if we were to detect the $g$ modes with the limb displacement only, we would not be able to go to frequencies lower than 100 $\mu$Hz because the $g$ modes becomes more and more horizontal, e.g., there is no limb displacement detectable (See Fig. 1). Using intensity fluctuations, Appourchaux & Toutain (1998) and Toner et al. (1999) showed that $p$ modes could be detected in the guiding signals of VIRGO/LOI instrument and of the MDI limb data, respectively. This observed amplification at the limb, predicted by Toutain et al. (1999) will be used for detecting $g$ modes with the PICARD instrument (Thullier & Meissner 2002). PICARD is an approved CNES mission to be launched in 2006. An amplification factor of 3 to 5 in amplitude might not be enough to detect the $g$ modes, but we hope that by combining the PICARD data with other observables (intensity, velocity) we may be able to detect the $g$ modes. The predicted upper limit for PICARD is 1 mm/s in 2 years (Dämé et al. 1999). We may hope that a similar limit be reached by the recently approved Helioseismic and Magnetic Imager (HMI) of NASA's Solar Dynamic Observatory (SDO) (for more information on HMI look at hmi.stanford.edu).

4.2. New instrumentation for measuring solar radial velocities

There are now projects being developed for making better measurements of the solar radial velocities. The instrument called MR15 or NCGOLF proposed to derive solar radial velocities from 15 measurements of the profile of the Sodium line (Turck-Chièze et al. 2001). Although no mention of the capability of the instrument is described, it is hope that the improvement will range between 0.1 and 1 mm/s, with a goal of going down to 0.1 mm/s (Turck-Chièze et al. 2001) provided that the solar noise can be beaten. An instrument of a very novel design is being developed by the Université de Nice (Jacob 2002, and their web site www-astro.unice.fr). It is based on a Mach-Zender interferometer making use of the whole spectrum for deriving radial velocities for planets, stars or the Sun. It has great potential for space use given its compact nature, its lack of moving parts and the heritage of MDI regarding solid Michelson interferometers. The performance of the instrument

\[1\] Michelson Doppler Imager aboard SOHO
Earth orbit, borrowing some of the technologies being developed for LISA. The one-year detection limit they provided makes the mission extremely appealing to helioseismologists; the limit is below that of the theoretical amplitude predicted by Kumar et al. (1996). Figure 3 summarizes the limits of LISA of ASTROD compared with the theoretical amplitudes for $l = 2$. It must be pointed out that Christensen-Dalsgaard (2002) has independently made a similar analysis leading to conclusions of similar tone; for once it seems that the present paper seems to be slightly more optimistic than that of Christensen-Dalsgaard (2002). Unfortunately, we agreed that the rest of the universe$^4$ could be the limiting factor for g-mode detection and not the Sun itself.

5. CONCLUSION

There are now numerous techniques that have been developed for g-mode detection, allowing to lower somewhat the upper limit given by Appourchaux et al. (2000). The upper limit to g-mode amplitudes has now been lowered to about 3 mm/s at 200 $\mu$Hz. Unfortunately, I believe that the techniques currently available will not provide the minimal factor 10 needed for reaching the theoretical amplitudes of Kumar et al. (1996) or of Gough (1985).

It is timely to start planning for other space missions that could help us to reach for the goal. This could be achieved with the PICARD mission or even with HMI of SDO that are bound to be launched in 2007. In the more distant future, the LISA mission is going to be launched in 2012 (or so), it will not only provide g-mode detection possibilities but also enabling technologies for a future mission based on spacecraft laser ranging such as ASTROD.

Finally, I truly hope that I will not have to repeat these statements for the next 18 years$^5$, and that the detection of g modes would have been achieved in this time frame.

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$^2$Laser Interferometry Space Antenna
$^3$Astrodynamical Space Test of Relativity using Optical Devices
$^4$Christensen-Dalsgaard (2002) explicitly mentions the galactic binary stars
$^5$Until my retirement...
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ELEVEN YEARS OF IRIS FREQUENCIES AND SPLITTINGS

LUAN, UMR 6525, Universite de Nice Sophia Antipolis, Nice, France.

ABSTRACT

Having acquired since July, 1989, a complete 11-year solar cycle of full disk data, the IRIS++ network has now made available to anyone the longest helioseismic data base to-date. A few results obtained from this very long time series are briefly presented here, with some emphasis on the low degree p-mode frequencies themselves, and their rotational splittings that have been estimated with unprecedented accuracy.

1. INTRODUCTION

The IRIS (International Research on the Interior of the Sun) network has deployed around the world 6 instruments that measure the line of sight velocity integrated across the complete surface of the visible Sun, by means of a sodium vapor resonant cell. It has acquired its first data point on the early morning of July 1st on the remote mountain site of Kumbel, Uzbekistan. Thanks to the addition of a Cacciani's MOF operated during the same summer at JPL, California, and the beginning of operation of the second IRIS site at Oukamden, Morocco, during the fall of the same year, the network data bank can usefully be exploited from the very beginning, so that at the turn of the new millenium, a complete solar cycle of 11 years has been achieved. However, despite real human efforts from the team who initiated the network project, and then from the heroic observers and their local teams in the various network sites, it has never been possible to approach closely enough the ideal duty cycle of 100 percent. Two different strategies have been developed in parallel to make the data bank suitable for good science. First several cooperations have been developed with scientific groups operating different but not inconsistent instruments. Several other summer seasons of the JPL MOF have thus been included in the data bank, with the later addition of the complete data set from the Tenerife Mark-1 instrument (through an official agreement with the BiSON network) and from the Hawaiian LOWL instrument (Gelly et al, 1998; Salabert et al, 2002). The second strategy consisted in using a partial gap filling method, that takes advantage of a peculiarity of the signal obtained in full disk helioseismology: the existence of a regular and almost equidistant set of peaks in the Fourier domain implies that the signal is nearly periodic in the time domain, being repeated with a more than 80 percent correlation every 4 hours or so. Data gaps not longer than 8 hours can then efficiently be filled by the data collected before and after the gap (Fossat et al, 1999). Fig. 1 shows the result of this method applied to the summer season of 1991. Its reliability becomes better and better with sharper peak profiles (implying much longer than 8 hours lifetime of the modes) and more precisely equidistant distribution of these p-modes peaks. In practice, the periodicity of nearly 4 hours is efficient for the study of all powerful modes in the five minute range, while it can be used twice, or once at 8 hours in the low frequency domain (below 2 mHz or so), thus increasing by one more step the final duty cycle. The annual final annual duty cycles are mostly above 70 percent in the five-minute range, and above 90 percent in the low frequency range. It must be noted that merging the various mentioned data sets with the sodium line full disk data is not an exactly straightforward task, as first the spectral line is not the same in all data sets (Potassium versus Sodium), and moreover, the MOF data sets from JPL (sodium) and from Hawaii (Potassium) consist of full disk images, that must first be integrated for simulating as well as possible the optical integration made by the Iris sodium cell. This has been studied in some detail by the various partners involved, and described by Salabert et al (2002).

2. FREQUENCIES

P-mode have been regarded as one dimensional damped and randomly excited oscillators. Their corresponding peaks in the Fourier power spectrum are then asymptotically described by Lorentz profiles. No asymmetry has been taken into account in the line profiles, so that a small bias cannot be excluded. In the frequency range comprised between 2.2 and 4.5 mHz or so, individual power spectra have been


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computed using time series of 4 months, two consecutive time series being overlapping by two months. The pairs of odd and even degrees have been fitted together. It has been checked many times (see for instance Fiery-Fraillon et al., 1998) that once the rotational splitting of the doublets and triplets is known within a few percent, the estimation of the central frequency is significantly more robust if the splitting is entered as a constant and not as a free parameter in the fitting process, as no individual splitting can really be estimated with an accuracy of a few percent. In the same spirit, the doublets and triplets are assumed to be symmetrical in amplitudes, as this too has been checked to make the fit more robust, and is part of the asymptotic model. The p-mode parameters are then extracted, for each four-month time series, by means of a maximum likelihood line fitting technique, using the standard Chi square distribution. The fit itself, however, is a non standard fit. It had to be specially adapted for the peculiar case of the repetitive music gap filling method, as this gap filling produces a modulation of the background noise. Fiery-Fraillon and Appourchaux (2001) have noted that the modulation is different for the noise and for the pics, and they have studied a fit that provides unbiased recovery of the line shapes when the gap filling is efficient at 100 percent. The frequency uncertainties are estimated by the formulation of Toutain and Appourchaux (1994).

The individual mode frequencies have then been simply averaged across all individual 4-month estimations, from mid 1989 to 2000, thus across a complete solar cycle of 11 years. No attempt for any solar cycle correction have been made, the frequencies are directly averaging all the solar activity levels.

3. THE HIGH FREQUENCY RANGE

Beyond 3.8 mHz or so, the pairs of even or odd modes can no longer be efficiently resolved since their separation decrease with frequency as their linewidths increase and cross the separation between 3.7 and 3.8 mHz. The rotational splitting is becoming really negligible and can simply be ignored without significantly damaging the frequency accuracy. A fitting process can still find the two components of a given pair, but with an suddenly increased error bar, that makes them less interesting in comparison with the more precise lower frequencies. Our frequency table still show, with large error bars, individual frequencies till around 4.5 mHz, except for the l=3 modes where no effort has been made to extract them beyond 3.4 mHz as the large error bars would have made this effort meaningless. It must also be noticed that for the study of the time dependence of p-mode parameters along the 11 years of this data set, only cross correlations between different spectra have been used beyond 3.8 mHz, this providing better shift measurements than the comparison of uncertain individual frequencies (Salabert et al., 2002).

Beyond the acoustic cutoff frequency of about 5.5 mHz, only the original full disk sodium data has been used, as it is the most sensitive to solar oscillations in this range because of the higher altitude probed in the solar atmosphere. Individual daily time series have been selected as being at least 9 hours long, thus providing just enough resolution for clearly showing the pseudo mode fringes. Many individual daily spectra have been averaged. The visibility of the fringes is quite clear (Fig. 2). On the other hand, the frequency shift along the solar cycle has been detected till the cutoff frequency as abruptly changing its sign around 4 mHz (Fig. 3), but this drift is still unclear in the pseudo mode range, where the signal to noise ratio does not permit to obtain a significant evidence for a frequency shift in one or the other direction.
4. THE LOW FREQUENCY RANGE

The solar core rotation remains relatively poorly constrained by the p-mode splittings, as these modes spend an unfortunately short fraction of their travel time in crossing the core itself. Moreover, the rotational splitting of the low degree p-modes is extremely small, in most cases smaller than the peak linewidths, so that its accurate measurement has been very difficult. Indeed, when there is a significant peak overlap, the splitting results depend on the peak fitting strategies, and none of the various attempted strategies can claim to be definitely better than others. Only in the very low frequency range, where the linewidth becomes very narrow, can the splitting be estimated without any risk of a bias produced by the line wings overlap. The mode frequencies themselves can then also be estimated with a much better accuracy, so that even if these modes do not penetrate quite as deep as the higher frequencies inside the solar core, their much smaller frequency error bar will eventually make them more useful in the inversion of the solar core models (Couvidat, 2002). But in this range, the amplitudes become so small that very long times of integration are necessary, providing both higher frequency resolution and/or more statistical confidence.

A special care has been given to the measurement of the low frequencies and their rotational splittings, as the unusually long duration of the Iris++ data base makes presumably possible to reach an unprecedented accuracy. Below 2 mHz, the linewidth have been shown (Libbrecht, 1988) to decrease very rapidly with decreasing frequency. That means that the frequency and phase lifetimes of the p-modes becomes very long, so that the repetitive music gap filling method can be extended without damage to twice the 4-hour standard value. The resulting annual duty cycles are then superior to 90 percent in most cases. On the other hand, the frequency shifts due to magnetic activity also decrease very rapidly with decreasing frequency. We have then started by using power spectra with one-year resolution, checking if the variation from year to year remains visible or not. The answer is yes. Even becoming smaller and smaller, the frequency shift from year to year is still clearly visible, so that a time of integration longer than one year will spread the power and thus will not improve the situation of the splitting measurement. However, it has been checked that if the mode frequencies are still not stable at their linewidth scale and at the one-year bin resolution, the components of the l=1 doublets and l=2 triplets tend to partly move together year after year, so that the apparent splitting is little affected by the mode frequency shift. A little, but not as much as the model of independent components would imply.
Our strategy has then been to measure the position of the doublets and triplets year by year, assuming a known splitting of 805 nHz for the double synodic splitting present in the data. For each mode, the frequency has been computed as the average of all individual annual estimations, and all the annual profiles have been slightly shifted relatively to each other to make their central frequency coincide before being averaged for the final splitting measurement. With this 8-hour gap filling method, it happens that taking separately the sodium and the potassium data still results in reasonably good final duty cycles, generally between 60 and 70 percent. Then these independent data sets have also been used by means of their cross spectra, in which case there is no approximation of relative calibration of sodium versus potassium. Everything has been averaged, cross spectra and full network power spectra. Fig. 4 shows the results of this 10-year average for the same doublet of \(l=1, n=12\), and Fig. 5 shows the same without repositioning them year after year. The rms amplitude of each component is the same in both cases, of the order of 6 mm/s, but the linewidth in visibly sharper in the average power spectrum computed after repositioning, so that the fine measurement of the splitting is made more precise. Fig. 6 shows that with decreasing frequency, this non independence between the components of the multiplets seems to improve, so that they can still be tracked accurately down to \(l=7\) despite the relatively poor signal to noise ratio of the IRIS data in this low frequency range.

5. FREQUENCIES AND SPLITTINGS
RESULTS

Table 1 to Table 4 give the complete list of Iris ++ frequencies and rotational splittings, with their error bars. The splitting measurements have not been attempted on the \(l=3\) modes, because the accuracy could not be satisfactorily compared to other values. In the same spirit, only the low frequency splittings of \(l=2\) have been given a special care. In this case (\(l=2\)) the frequency asymmetry of the triplet has also been looked for, with a null result (the average asymmetry of the measured triplets is 6 \(\pm\) 10 nHz)

The weighted averages of the \(l=1\) and \(l=2\) splittings are respectively 431.7 1.2 and 433.5 1.6 nHz, giving an overall average of 432.5 \(\pm\) 1 nHz, if such an overall average makes some sense! These splittings, consistent with other recent measurements (like Gelly et al, 2002) but significantly more accurate, tend to confirm a slight decrease of the rotation rate when going down from the radiative envelope towards the core itself (Corbard, 2002). However of course, only the future hoped detection of g-modes, with their deeply rooted kernels, will be able to confirm this inference.

6. COMPARISON WITH GOLF RESULTS

The results of Gelly et al (2002) use almost 2000 days of Golf data, about half shorter than the IRIS++ data bank, but of course with a significantly lower noise level and mostly a nearly ideal duty cycle, be-
Table 2. IRIS frequencies and splittings of l=1 p-modes averaged from 1989 to 1999. Splittings and their error bars are in nHz

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Table 3. IRIS frequencies and splittings of l=2 p-modes averaged from 1989 to 1999. Splittings and error bars in nHz

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sides the interruption of a few months in 1998. The time series before (blue channel) and after the interruption (red channel) have been treated separately, so that two different frequency tables are provided, one covering the minimum of activity, and the second one the rising phase and the maximum of the next cycle. We have compared our tables with the average of these two Golf tables.

First the global agreement is excellent between the two frequency tables, the mean difference being extremely small, about 0.06 μHz.

A comparison of error bars has been made very simply, by the mean value of their ratio σ(iris)/σ(Golf). It shows that Iris frequencies are estimated slightly less accurate than the corresponding Golf frequencies, this ratio being of the order of 1.1. However, this is mostly due to the high frequency range, beyond 3.7 or 3.8 mHz, where the poorer signal to noise ratio of Iris combined with the poorer filling factor is really damaging the precision. In all the low and medium frequency range, and up to 3.7 mHz or so, the two data sets have a comparable quality, the longer time series of Iris just compensating the better quality of Golf.

Another interesting point is the rms comparison of individual frequency differences d with the expected differences D given by the square of the sum of estimated error bars. The rms value of d/D is found to be of the order of 2 to 2.5 (depending on taking into account or not the two largest individual differences, far too large to just be random uncertainties). It shows that at least one set of error bars, or more probably both are optimistically underestimated.

Now the comparison of the splitting results is also extremely interesting and informative. If the mean values are perfectly consistent within their error bars, the mean ratio σ(iris)/σ(golf) of these error bars is 0.45, more than a factor 2 in favor of Iris. On the other hand, the same comparison of the rms individual differences d with the expected differences D shows a very small ratio of the order of 0.3, showing that this time, one or both sets of error bars are, quite a lot in fact, pessimistically overestimated. It is possible to go one step further by assuming that all splittings should be the same and then checking their internal consistency. The rms departure around the weighted average, relative to the individual error bars, is found to be 0.8 for Iris and 0.5 for Golf. It means that both sets of error bars are overestimated, mostly Golf, while they are computed by means of
Table 4. IRIS frequencies and error bars of l=3 p-modes averaged from 1989 to 1999 Hz

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the same assumption of independence of individual components than the frequencies error bars, that are underestimated! It seems then that this assumption cannot be regarded as valid. The error bars themselves, in this case, could well contain a real physical information, the amount of interdependence of individual components inside the split multiplets. Further investigation of this point is worth being considered, by taking care that both data sets are treated identically, specially for the error bar estimations.

Now, taking into account the fact that the Golf error bars are at least twice better than claimed, and even the Iris error bars, even much smaller, are still overestimated, it means that the overall mean splitting is finally measured within less than 1 nHz, an impressive result indeed that, unfortunately, is still not able to give a definitive answer to the rotation rate of the deepest solar core!

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Corbard, T., 2002, these proceedings.
ON THE CHARACTERIZATION OF HIGH-DEGREE MODES: A LESSON FROM MDI

Sylvain G. Korzennik\textsuperscript{1}, Cristina Rabello-Soares\textsuperscript{2,1}, and Jesper Schou\textsuperscript{2}

\textsuperscript{1}Harvard-Smithsonian Center for Astrophysics, 60 Garden St, Cambridge, MA 02138, USA
\textsuperscript{2}Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA

\vspace{1cm}

ABSTRACT

High degree power ridges (up to $\ell = 900$) were computed and fitted for several two to three-month-long time-series of full-disk observations taken with the Michelson Doppler Imager (MDI) on-board the Solar and Heliospheric Observatory between 1996 and 1999. A detailed discussion of the modeling of the ridge power distribution, and the contribution of the various observational and instrumental effects on the spatial leakage, in the context of the MDI instrument, are presented.

The result of this work is a better understanding of the problems associated with the characterization of high degree modes. We present the instrumental and observational requirements needed to achieve a determination of high degree mode frequencies whose residual systematic errors associated with the accuracy of the ridge to mode correction scheme are smaller than the uncertainty of the fitting itself.

Key words: Helioseismology; high-degree ridge fitting.

1. INTRODUCTION

The characteristics of the solar oscillation power spectrum is such that above some spherical harmonic degree – whose specific value is frequency dependent – individual modes and their spatial leaks overlap as to blend into ridges. Moreover, it has been shown (Korzennik, 1999) that the horizontal component of the observed line-of-sight velocity signal causes the amplitude of these leaks to be asymmetric. The resulting power distribution of the ridge is therefore not centered around the target mode frequency, resulting in a substantial offset between ridge and mode frequency.

On the other hand, an accurate determination of these high-degree mode frequencies would allow us to further constrain the solar structure and dynamics in the outer 2 to 3\% in radius. Such new constrain can in turn improve our knowledge of the equation of state, chemical composition, etc... (see for instance Rabello-Soares et al., 2000, and references therein).

We present here some key elements of an extensive work on the characterization of high-degree modes using full-disk observations taken with the Michelson Doppler Imager (MDI) on board SOHO (additional details can be found in Korzennik et al., 2002).

2. DATA USED AND FITTING METHODOLOGY

Four consecutive epochs of continuous full-disk (2\" / pixel) velocity images, each lasting between two and three months and taken on nearly yearly intervals from 1996 to 1999, were analyzed.

Estimates of the limit spectra were computed by subdividing the time-series of spherical harmonic coefficients in 4096-minute-long intervals. Each interval was Fourier transformed – using an oversampling factor of 2 – and the resulting spectra for a given epoch were averaged. This procedure lead to power spectra with a frequency resolution of 4.1 $\mu$Hz, a relatively low resolution. With such resolution ridges at high degrees are well resolved while individual modes that are otherwise resolved at intermediate degrees blend into ridges.

A single asymmetric profile was fitted in the least-squares sense to these low-resolution spectrum according to:

$$ P_{\ell,m}(\nu) = \sum_n \tilde{A}_{n,\ell,m} P(\tilde{x}_{n,\ell,m}, \gamma_{n,\ell,m}) + B_{\ell,m}(\nu) $$

(1)

where

$$ \tilde{x}_{n,\ell,m} = \frac{\nu - \tilde{\nu}_{n,\ell,m}}{\gamma_{n,\ell,m}} $$

(2)

$$ P(x, \alpha) = \frac{1 + \alpha(x - \alpha/2)}{1 + x^2} $$

(3)
and
\[ \log B_{\ell,m}(\nu) = \sum_{j=0}^{2} b_j(\ell, m) \nu^j \]  

where \( \hat{A}_{n,\ell,m}, \hat{\gamma}_{n,\ell,m}, \hat{\nu}_{n,\ell,m} \) and \( \hat{\alpha}_{n,\ell,m} \) are the ridge amplitude, width, frequency and asymmetry parameters for a given \((n, \ell, m)\) ridge fitting. For practical reasons, this fitting was carried out only every 5th \( \ell \) for 100 \( \leq \ell \leq 250 \) and every 10th \( \ell \) for 250 \( < \ell \leq 900 \), and only for some 50 equally spaced \( m \) values at each \( \ell \). From the resulting ridge frequencies, the ridge frequency splittings were parameterized in term of Clebsch-Gordon coefficients (\( \tilde{a}_i \), for \( i = 1, 6 \), see Ritzwoller & Lavelle, 1991).

3. STRATEGY: RIDGE MODELING AND SYNTHETIC SPECTRA GENERATION

To properly model the ridge power density we have to include the complete leakage matrix. For line-of-sight velocity observations, \( i.e. \):
\[ \tilde{\nu}_{n,\ell,m} = (V_r Y_{\ell}^m, V_h \partial_\phi Y_{\ell}^m, V_h \frac{1}{\sin \theta} \partial_\theta Y_{\ell}^m). \]  

the leakage matrix is:
\[ C_{\ell',m',\ell,m} = \int Y_{\ell'}^m*Y_{\ell}^m \sin \theta \cos \phi \, d\Omega \]  

\[ C_{\ell',m',\ell,m}^\theta = -\frac{1}{L} \int Y_{\ell'}^m*\partial_\theta Y_{\ell}^m \cos \theta \cos \phi \, d\Omega \]  

\[ C_{\ell',m',\ell,m}^\phi = \frac{1}{L} \int Y_{\ell'}^m*\partial_\phi Y_{\ell}^m \sin \phi \, d\Omega \]

where \( \ast \) represents the complex conjugate operator, \( V_r \) and \( V_h \) the radial and horizontal velocity amplitudes, and \( W \) the spatial window function. The complete leakage matrix is the sum of the radial component \( C_r \) and the horizontal components \( C^\theta + C^\phi \). Thus a mode with a given \((\ell', m')\) will leak in the \((\ell, m)\) power spectrum with an amplitude attenuated by a factor \( C_{\ell',m',\ell,m} \) where:
\[ C_{\ell,m,\ell',m'} = C_{\ell,m,\ell',m'}^r + \beta_{n,\ell} (C_{\ell,m,\ell',m'}^\theta + C_{\ell,m,\ell',m'}^\phi), \]

and where \( \beta_{n,\ell} \) is the horizontal-to-vertical displacement ratio.

Rather than evaluating these integrals, we have computed these coefficients numerically by producing simulated images of single spherical harmonics that were decomposed using the same code used to decompose the observations. This calculation was carried out on an subset of \( \ell \) and \( m \) from which all other values were interpolated. By using this approach we can include additional instrumental effects - like the effect of image distortion and/or the instrumental point spread function - to better mimic the actual observations.

4. EFFECT OF THE DIFFERENTIAL ROTATION

Woodard (1989) predicted that the power distribution between spatial leaks is further altered since the solar differential rotation distorts the angular part of the eigenfunctions. The rotation being axisymmetric, this distortion results in additional mode mixing over the degree \( \ell \) only, namely:
\[ \tilde{C}_{\ell,m,\ell',m'} = \sum_{\ell''} G_{\ell',\ell''} C_{\ell,m,\ell'',m'} \]

and where \( G_{\ell',\ell''} \) is zero unless \( \ell' = \ell'' \) is even. An analytic expression for the coefficients \( G_{\ell',\ell''} \) was derived in that paper using a canonical expression of the solar differential rotation. Figure 1 illustrates the value of these coefficients as a function of \( m \), for two values of \( \ell \). This variation with \( m \) results in an additional redistribution of the power density of the ridge that mostly affects the ridge splitting coefficients.
Figure 2 illustrates the frequency and the splitting coefficients offsets between mode and ridge fitting resulting from the power distribution by the leakage matrix discussed above. These offsets are compared to the observed offsets estimated from intermediate degree modes that are resolved when using high frequency resolution spectra but that blend into ridges in the low frequency resolution spectra we used to fit ridges.

This figure shows that the frequency offsets are barely affected by the mode mixing, and that our theoretical estimates agree overall with the observed offsets (qualitatively and quantitatively), but not yet in the details. It also clearly shows that the offsets of the odd splitting coefficients are mainly due to the mode mixing caused by the distortion of the eigenfunctions by the differential rotation.

The value of the rotation profile at the surface is needed to correct high-degree splitting coefficients resulting from ridge fitting, but these splitting coefficients should be the ones that give us additional information on the rotation profile in the outer 2 to 3%. While we anticipate that a bootstrapping and/or iterative method can be devised to refine our knowledge of the rotation profile just below the surface from high-degree splittings, any estimate of corrected rotation splittings at high degrees is de facto coupled to the rotation profile used to compute the corrections.

5. INSTRUMENTAL EFFECTS

We have computed the effect on the complete leakage matrix of several instrumental effects, namely:

- Plate scale error
- Image distortion
- Instrumental point spread function (PSF)
- Image orientation, i.e.: angles $P$ and $B_0$

In each case synthetic power spectra were generated, using corresponding leakage matrix values, and fitted to estimate frequency and splitting offsets.
2003ESASP.517D...9G

\[ \varepsilon_r \equiv \frac{\Delta r}{r} = a_r \left( \frac{r}{r_m} \right)^2 - 1 \]  \hfill (11)

where \( a_r = 1.1 \times 10^{-3} \), \( r \) is the distance from the detector center, \( \Delta r \) the image distortion, and \( r_m \) is the observed image mean radius.

However, this expression does not account for the elliptical shape of the MDI image, an additional distortion that is also on the order of 0.1%. The most likely cause is that the CCD detector is tilted with respect to the focal plane. The shape of the solar image resulting from such a tilt is indeed elliptical, and a tilt of 2.6° can account for the observed 0.1% effect.

The effect of this image distortion on the ridge to mode correction is shown in Fig. 5. This figure shows that by including such image distortion, or model of these corrections matches rather well the observed offsets (except for the very low frequencies).

5.3. Instrumental Point Spread Function

The PSF of MDI instrument in full-disk mode was estimated using the HGEOM procedure on MDI full-disk images. HGEOM, a procedure part of the GONG reduction and analysis software package (GRASP), returns an estimate of the azimuthally averaged modulation transfer function (MTF, i.e., the Fourier transform of the PSF), following the methodology described in Toner & Jeffries (1993). This method reduces the 2D solar image to a radial profile (by computing the mean over concentric annuli) and computes the Hankel transform of this profile. By exploiting the zero-crossing properties of the Hankel transform one recovers the true image dimensions and estimates the MTF.

The estimates of the PSF for the four different epochs are consistent and their widths correlate with the amount of defocus. Unfortunately, there is strong evidence that the PSF of the MDI instrument is not azimuthally symmetric. The power distribution of the images show a marked variation with azimuth – on the order of 20% near focus.

Tarbell et al. (1997), using phase-diversity analysis, estimated the PSF of the MDI instrument in focus and in its high resolution configuration (i.e., when the image is magnified by a factor 3.2). This PSF shows a clear angular dependence, with a secondary peak with an amplitude of 15%, attributed to the residual optical aberrations of the instrument components.

Finally, we should also point out that the best focus changes across the CCD by nearly one focus step. This is caused by some residual field curvature and will produce a radial variation of the PSF. The optical characteristics of the MDI instrument are such that the instrumental PSF has to be a function of both the radius and the azimuth.
Figure 4. Effect of plate scale error on the frequency and the frequency splitting coefficient \( a_1 \). Top panels: results from models. Bottom panel: observed values, based on intermediates degrees, for all four epochs.

5.4. Image Orientation

The MDI instrument is in principle oriented to maintain the column direction of the CCD aligned with the position of the solar rotation axis. This was checked using an inter-comparison of MDI and GONG images. These tests lead to estimates of a residual error in the position angle of the MDI images of 0.20 ± 0.05° and 0.19 ± 0.04°, based on observations of the transit of Mercury and drift scans used, respectively, to determine the GONG image orientation.

The residual error on \( B_r \), the Carrington latitude at disk center, results from the precision of the Carrington elements. Giles (1999) has shown that for MDI this error is around 0.1°.

6. RESULTS FROM MDI

6.1. Uncertainties versus Corrections and the Associated Error Budget

Table 1 compares the range of uncertainties resulting from ridge fitting to the magnitude of the ridge to mode corrections, for both the frequency and the rotational splitting coefficients. While the uncertainties resulting from the fitting scale with the ridge width, Table 1 shows that the required corrections are about an order of magnitude larger than the uncertainties. On the other hand, to produce useful high-degree mode parameters we need to be able to compute these corrections with a precision substantially better than the uncertainties.

6.2. Changes with Epoch

To look for changes with epoch, that would be the result of changes in activity, we first corrected the frequency and frequency splittings for the known plate scale error. We then assumed that all other systematic errors are constant in time, and look at changes with respect to 1996. As expected, the frequencies change with activity; these changes are comparable to a simple linear extrapolation at high \( \ell \) of GONG or BBSO observed changes for comparable changes in activity.

<p>| Table 1. Comparison between uncertainty and ridge to mode correction |
|----------------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>( \nu ) ([\mu\text{Hz}])</th>
<th>Fitting Uncertainty (^{(a)})</th>
<th>Largest Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 ) ([\text{mHz}])</td>
<td>0.3 – 1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>( a_2 ) ([\text{mHz}])</td>
<td>0.2 – 0.8</td>
<td>5.0</td>
</tr>
<tr>
<td>( a_5 ) ([\text{mHz}])</td>
<td>0.15 – 0.7</td>
<td>2.0</td>
</tr>
</tbody>
</table>

\(^{(a)}\) given range corresponds to a ridge width range of 10 – 60 \( \mu\text{Hz} \).
Figure 5. Effect of image distortion: frequency offset between mode and ridge fitting when including or not image distortion. Crosses correspond to a model that include the complete image distortion, dots correspond to the model without any image distortion – as shown in Fig. 2; circles correspond to the 1996 observational values.

Table 2. Optimal error budget for the ridge to mode correction

<table>
<thead>
<tr>
<th></th>
<th>Expected precision</th>
<th>Change in $\nu$ [\mu Hz]</th>
<th>$a_1$ [nHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>plate scale</td>
<td>0.01 %</td>
<td>0.220</td>
<td>0.06</td>
</tr>
<tr>
<td>diff. rotation</td>
<td>0.03 %</td>
<td>0.054</td>
<td>0.045</td>
</tr>
<tr>
<td>image distortion</td>
<td>0.01 %</td>
<td>0.150</td>
<td>0.04</td>
</tr>
<tr>
<td>instrument PSF</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>orientation, $P_{eff}$</td>
<td>0.05°</td>
<td>0.006</td>
<td>0.40</td>
</tr>
<tr>
<td>orientation, $B_0$</td>
<td>0.02°</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>correction precision</td>
<td>0.272</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fitting uncertainties</td>
<td>0.1 – 0.6</td>
<td>0.3 – 1.0</td>
<td></td>
</tr>
</tbody>
</table>

Unfortunately, changes in splittings or linewidths remain dominated by systematic errors. The asymmetry does not seem to have changed with epoch, except, maybe, for the $f$-mode.

6.3. Optimal Error Budget

Table 2 presents an error budget projected for the expected precision of the elements included in our model. The sobering conclusion of this exercise is that our estimate of the achievable precision of the frequency and splitting coefficients corrections are still comparable to the fitting uncertainties.

For the frequency corrections, the dominant contributions come from the plate scale error and the image distortion. If both could be known to an accuracy of $10^{-5}$, the precision of the correction would be further reduced to 0.06 $\mu$Hz, a useful value since it become substantially smaller than the uncertainties.

7. CONCLUSIONS

Now that the elements that contribute to the power distribution of high-degree ridges are better understood and that we better understand the MDI instrument imaging imperfections, we must – and will in the near future – reprocess the Dynamics data. This reprocessing shall include the correct image scale, the known image distortion, and an improved estimate of the image orientation in the spatial decomposition. We anticipate that the precision of the ridge to mode conversion of the reprocessed data will be better than at least half of the fitting uncertainties.

The same observational and instrumental considerations must be addressed now that the GONG instruments have been upgraded, but multiplied by 6 for the instrumental ones. Similarly, the higher spatial resolution planned for HMI on SDO will impose tighter constraints on the optical design and characterization of that instrument.

ACKNOWLEDGMENTS

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AMPLITUDE MODULATION OF LOW DEGREE P-MODES – COMPARISON OF BISON AND VIRGO

Bo Andersen¹, Torben Leifsen², William J. Chaplin³, Yvonne Elsworth³

¹Norwegian Space Centre, N-0212 Oslo, Norway, +4722511800, bo@spacecentre.no
²Institute of Theoretical Astrophysics, University of Oslo, N-0315 Oslo, Norway
³School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

ABSTRACT

Using both VIRGO and MDI data we have previously studied the amplitude variation of the \( l=0 \) p-modes for radial orders 12 to 32. In this study we extend the investigation backward in time to 1992 by including data from the BISON network. For the large amplitude modes there is a strong correlation between the space based radiance measurements from VIRGO and the ground based Doppler shift measurements from BISON. The extreme rotational modulation of \( l=0, n=22 \) is confirmed to be a phenomenon confined to the period of minimum solar activity. Also with neighbouring \( l=1, n=21 \) a clear modulation is seen at slightly lower frequency. Some persistent frequencies occur in other \( l=0, l \) modes, but not to the same level in time and amplitude.

1. INTRODUCTION

The solar p-modes are excited just below the visible solar surface by millions of independent and relatively short lived events. The observed time variation of the amplitudes of global low degree modes is the accumulated effect over time of these events. The overall variation on time scales from a few days and upward is connected to the interaction of the modes with the solar atmosphere as well as the possible influence from the presence of active regions.

The low degree solar p-modes show amplitude modulation at all observable timescales. For some modes a large fraction of this modulation occurs at the solar rotation frequency (Leifsen et al 1998, and references therein). For other modes there seems to be little or no deterministic component in the modulation. Only intermittent correlation between the modulation of different modes have been observed (Foglizzo et al. 1998). This is to be expected if the excitation of modes is completely stochastic (Chang 1996).

In this work we extend the previous studies in time to get a clearer picture on possible connections between the observed parameters and the solar cycle. The current work utilizes data that cover more than ten years.

2. DATA AND DATA REDUCTION

The VIRGO investigation (Fröhlich et al., 1997) provides near continuous time strings of solar irradiance and low resolution radiance. In this study we have used data from the blue channel (405 nm) of the VIRGO sunphotometers (SPM). The time string used spans from 29 January 1996 to 31 May 2002 with very few interruptions. The total data coverage is more than 98%. The BISON data series consist of Doppler shift measurements carried out by resonance spectroscopy of the Potassium 770 nm line and covers the period from 1. January 1992 to 20. December 2001 with an average fill of nearly 74%. The details of the BISON data collection and reduction can be found in Chaplin et al. (1998) and references therein.

The time variation of the individual \( l=0 \) p-modes have been developed by means of a Hilbert transform as described by Leifsen et al (1998). The Hilbert transform was calculated for a bands of 2, 3, 5 and 10 \( \mu \text{Hz} \) centred on the individual \( l=0-1-2 \) p-modes. The range of radial orders studied is from 12 to 32. The highest effective time resolution of the calculated amplitude variation of each mode is about two days and the data are sampled twice per day.


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3. RESULTS

This study has utilized the time series generated of the mode amplitude variation for the individual modes both from the BISON and SPM data. If nothing else is stated, the studies have been carried out for all the bandwidths isolating the individual modes and they give similar results.

For the overlapping time period (29-01-1996 to 20-12-2001) the BISON and SPM data have been extensively compared, both in the time and frequency domain. The most striking feature is the close similarity between the space/irradiance and the ground-based/Doppler shift data. A typical example of this is shown in Figure 1.

This clearly confirms the previous conclusions from comparisons between MDI and SPM data (Leifsen et al 1998) that all major features in the generated time series are related to the individual modes.

Figure 1. Upper panel: Amplitude variation with time for l=0, n=22 as observed with BISON (solid), VIRGO/SPM (dash-dot) and SOI/MDI (dashed). Lower panel: Power spectrum of 3/4 years of amplitude data from 1. May 1996 for BISON (solid), VIRGO/SPM (dash-dot) and SOI/MDI (dashed).

Figure 2. The greyscale image shows the coherence between the individual mode amplitude time series generated from BISON and SPM data for the one year period starting 29. January 1996.

Figure 3. The upper panel show an image created of one year power spectra of the l=0, n=22 mode amplitude time series generated from the BISON data. The spectra are shifted by 20 days. The bottom panel show the time distribution of the power in the range 0.38-0.5 \muHz.
Figure 4. The autocorrelation functions of the $l=0$, $n=22$ mode calculated for approximately one year periods. The BISON data are shown as solid lines and the VIRGO SPM data as dot-dashed lines. The strong rotational modulation is seen mainly in the 1996 data set.

Figure 2 shows the coherence of the BISON and SPM amplitude modulations for the observation of the $l=0$ and $n=22-32$ modes. The coherence drops off for high and low $n$-values. The drop-off for low $n$-values can easily be explained by the fact that the lines become narrower and the signal to noise/ratio is lower causing the uncorrelated solar background signal to dominate. At the high end the causes are not clear and unambiguous. The lifetimes of the modes are decreased such that the width of each mode covers the whole bandwidth used for isolating the modes. If the intrinsic similarity of the modes, independent of the observing technique is maintained, the correlation should not decrease. The reduced signal to noise level may contribute, but not at the level observed. From our observations we believe that the observed reduced correlation is due to the fact that for high $n$-values there is a strong correlation between the structure of the modes and both the underlying solar background signal. This signal is not correlated in radiance and Doppler shift. Also for these $n$-values the difference in observation height in the solar atmosphere between BISON and SPM ($\approx500$ km) will contribute to the de-correlation.

The exceptional rotational modulation of the $l=0$ and $n=22$ mode described in Leifsen et al. (1998) has been further investigated. The BISON data show the same modulation as the SPM data. The time variation of the low frequency end of the power spectra of BISON mode variation is shown in Figure 3. We see here the large increase in power at the solar minimum, but have to recollect that in this period there was a recurring “active longitude”. For other modes there are longer periods of semi-persistent power near the rotational frequency as well as other frequencies. However no other mode show the same amount of relative change at isolated frequencies as shown in Figure 3. Figure 4 show the autocorrelation function for 11 yearly time series of the mode amplitude of the $l=0$ and $n=22$ mode. We see the strong similarity between the BISON and SPM data except for the 1998 period due to the SOHO vacation.
The strong rotational modulation is seen only for 1996 and slightly so for 1992. Looking at other modes for \( l=0 \) and for \( n=18-22 \) we see an over-abundance of persistent amplitude modulation around the solar rotation frequency and twice that. For \( l=1 \) the periods are more typically 20 and 10 days as compared to 25-30 and half that for \( l=0 \). We can speculate if this effect is an artifact or contains information on how the internal geometry of the non-radial modes may affect the accumulation of all the small individual excitations. In addition the first harmonic component is generally more visible in the BISON data. The latter effect may be connected to the effect of the spatial sampling on the solar surface that BISON does.

We have studied the autocorrelation and cross-correlation of the different time series of mode amplitude. We find a significant cross-correlation between the different modes observed with BISON. This, however, is caused by the effect of gaps due to the lower fill factor than in the SPM data. No consistent cross-correlation is found between the different \( n \)-values from the SPM data. We see that the width of the autocorrelation functions vary systematically with \( n \). This is shown in Figure 5. We see that at towards the low end the width drops of toward the resolution limit. This is caused by the increased contribution from the background solar signal that has little or no time correlation at these frequencies (Appourchaux et al 2002). The steeper reduction of the SPM data also shows that this is a signal to noise effect. Towards high \( n \) the width drops of due to the shorter lifetime of the modes. The decrease is, taking into account the time resolution limit, consistent with other measurements of high order mode lifetimes.

4. CONCLUSIONS

Using a combined data set of SPM and BISON covering nearly a solar cycle we claim to see:

- A consistent and correlated variation of the low degree mode amplitudes observed in BISON and SPM up to \( n=23 \). Above this the correlation is reduced with increasing \( n \).
- There are large variations in all the mode amplitudes at all timescales from a few days to several months. Semi-persistent frequencies lasting months to years occur. There is an overabundance of these frequencies at or slightly above the rotational frequency.
- The extreme rotational modulation of \( l=0, n=22 \) in 1996 is confirmed with the BISON data. Also the \( l=1, n=21 \) mode show some of this at about 2/3 of the rotation period. Both these modes are situated at the peak of the envelope of the p-mode spectrum where the dominating damping effects change.
- The rotational modulation of some modes may be an effect of a special active region configuration near the solar minimum in 1996 or it may be coupled directly to the minimum. Only a continuation of the observations into the upcoming minimum may resolve this question.
- The lifetimes of the modes can be seen directly in the width of autocorrelation function of the mode amplitude time variation.

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A REVISITED DESCRIPTION OF ACOUSTIC MODES

P. A. P. Nghiem
CEA/DSM/DAPNIA/SAp, CE de Saclay, 91191 Gif sur Yvette Cedex, France
tel: +33 (0)1 69 08 92 64 / fax: +33 (0)1 69 08 65 77
e-mail: papnghiem@cea.fr

ABSTRACT

In the framework of asteroseismology, it would be useful to assess the situation in theoretical understanding and prediction of acoustic modes of the Sun, to better estimate the reliability we can expect when studying other stars. Available precise seismology calculations have given many useful informations concerning the solar interior. One can nevertheless note that at large frequencies, analytical and numerical calculations are in very good agreement while being different to observational data up to tens or hundreds of microHz. At low frequencies in contrast, numerical results remarkably agree with observations while analytical results are substantially different, especially for low degree modes. I try to address these problems by proposing a slightly revisited understanding of acoustic eigenfrequencies, based on the concept of acoustic waves in a locally homogeneous environment. The deduced local wave formalism can better conciliate the numerical-analytical-observational results in the major range of frequencies, while proposing a diagnostic for the Sun surface. These results can be useful for seismology of Sun-like or intermediate-mass stars where acoustic- and gravity-mode regions are well separated in frequency.

INTRODUCTION

Since several decades, intensive theoretical works have been devoted to the study of solar acoustic oscillations, which combined to many observational data, have led to decisive progresses in the Sun interior description. In what extent those theoretical results will help to interpret the data from other stars that begin to be collected and that will be massively available in the very near future? It is worth to assess the situation of theoretical predictions in helioseismology, to estimate their reliability for asteroseismology.

Let us examine the acoustic eigenfrequencies as seen by the three actors: semi-analytical results (Vorontsov 1991, Lopes & Turck-Chièze 1994, Roxburgh & Vorontsov 2000), numerical results (Christensen Dalsgaard 1982, 1997, Unno et al. 1989), and observational data (Rhodes et al. 1997, Bertello et al. 2000, García et al. 2001). In Fig. 1 it can be seen that only two of the three actors agree at each of the two distinct frequency ranges, with a very good precision, while being noticeably different to the third one. This situation records a great advance as compared to the beginning of helioseismology. But it also means that a global progress is still to be performed. The reliability of the semi-analytical calculation, whose role is to give the physical understanding, becomes questionable when its results do not predict the good trend. The numerical calculation fulfills its role of giving precise predictions, but presents an important shift at high frequency.

A further examination of the physical mechanisms invoked should help to clarify the problems. Unfortunately, the most advanced semi-analytical methods giving the best results until now employ very complex formalisms. The existence of trapped waves and cavity is even not clear. The use of an horizontal eigenvalue proportional to either $l(l+1)$ or $(l+0.5)^2$, and the related passage or not of the radial mode through the solar center, are still debated. That point, relatively secondary for the Sun, will become more crucial for other stars, for which only the lowest-degree ($l$) modes will be detected.

It is possible that the problems lay in the physical understanding of the mechanisms leading to acoustic eigenmodes. In the following, we will try to revisit that point, and propose a formalism constructed on basical physics of trapped wave and cavity.

![Fig. 1. Eigenfrequency comparison between observational data (---), numerical results (+ -), and envelopes of semi-analytical results (+,0 roughly estimated from resp. Lopes & Turck-Chièze 1994, Roxburgh & Vorontsov 2000), for the degrees $l$ = 0 to 10.](image)

A REVISITED DESCRIPTION

Basically, eigenfrequencies are the signature of waves trapped in a cavity, a wave is the propagation of an oscillation, an oscillation is a continuous exchange between kinetic and potential energy around an equilibrium state. Where this last one ends, there is no more oscillation, thus no possible propagation, the wave
is reflected, that is a turning point, determining the cavity limit. In a huge gaseous sphere under gravity like a star, two kinds of equilibrium can be considered: the equilibrium between strata of different pressure, and the pressure equilibrium inside each stratum, that is the local homogeneity. Every pressure perturbation generates two oscillations: a gravity one around an equilibrium stratum, and an acoustic one around the local equilibrium pressure. The two induced waves have wavelengths at two different scales, respectively large and small, and, unless their frequencies are close to each other, do not interact between them. For the Sun, except around 500 MHz where coupling has to be considered, the acoustic wave can be studied completely separately from the gravity wave.

In the present approach, in addition to the classical hypothesis of adiabaticity, spherical symmetry, linear oscillation, the one of purely acoustic wave is also considered (but the asymptotic approximation is not used). By definition, such a wave can only propagate in a local homogeneous environment, or more precisely homogeneous as regard to its wavelength \( \lambda \). The propagation condition can be expressed as:

\[
\lambda = \frac{2\pi}{k} < a_E H_{\rho(k)}
\]

where \( k \) is the wavenumber, \( H_{\rho(k)} \) the pressure scale height projected on the propagation direction, and \( a_E \) a constant to be determined. If the condition (1) is no longer satisfied, the wave can no longer propagate, and is reflected. If it happens when the pressure is increasing, a 'pressure wall' is encountered, and on the contrary case, a 'pressure vacuum' is encountered. The wave propagation and reflection are so far completely defined in their principle.

More precisely, the wave can be described as the propagation of small eulerian perturbation quantities \( x' \) of equilibrium quantities \( x_0 \), \( x \) standing for either the pressure \( p \), or the density \( \rho \), or the gravitational potential \( \phi \). In terms of lagrangian perturbations \( \delta x(\vec{r}) \) at the location \( \vec{r} = \vec{r}_0 + \vec{\phi}' \), where

\[
\delta x(\vec{r}) = x'(\vec{r}_0) + \vec{\phi}' \cdot \nabla x_0,
\]

the classical equations of conservation can be written as (see e.g. Christensen-Dalsgaard 1998):

\[
\begin{align*}
\rho_0 \frac{\partial^2 (\vec{\phi}')}{\partial t^2} + \nabla p - \rho_0 \nabla \phi_0 \cdot \rho' \nabla \phi_0 &= 0 \\
\delta \rho + \rho_0 \nabla (\vec{\phi}') &= 0 \\
\delta \rho &= \frac{\Gamma_1 p_0}{\rho_0} \delta p \\
\Delta \phi' &= -4\pi G \rho_0'
\end{align*}
\]

where \( t \) stands for the time, \( G \) the gravitational constant and \( \Gamma_1 \) the equilibrium adiabatic exponent. Keeping in mind that here all the spatial derivatives of \( x_0 \) vanish due to a locally homogeneous structure, the divergence of (3) and the second-time derivative of (4) combined with (5) and (6) give

\[
\frac{\partial^2 \rho'}{\partial t^2} - \frac{c_0^2 \phi'}{\rho_0} = 0
\]

where

\[
c_0^2 = \frac{\Gamma_1 p_0}{\rho_0}.
\]

By searching solutions with a locally plane wave form

\[
x' = A \cos(k \cdot \vec{r} - \alpha + \psi)
\]

the dispersion relation can be derived from (7) giving the wave number \( k \)

\[
k^2 = \frac{\omega^2 + 4\pi G \rho_0}{c_0^2}.
\]

Until now, only cartesian coordinates are used. In the presence of spherical symmetry, the differential equations can also be derived in spherical coordinates \((r, \theta, \phi)\), and solutions in locally spherical wave with harmonics \( Y(\theta, \phi) \) can be tried

\[
x' = \frac{A'}{r} Y(\theta, \phi) \cos(k r - \alpha + \psi)
\]

where \( k_r \) clearly the radial component of the former wave vector \( \vec{k} \). It can then be shown with the Legendre polynomials that its horizontal component is given by (Christensen-Dalsgaard 1998)

\[
k_h = \sqrt{\frac{\ell(\ell + 1)}{r^2}},
\]

and (10) gives

\[
k_r = \sqrt{\frac{\omega^2 + 4\pi G \rho_0 - \ell(\ell + 1)}{c_0^2 r^2}}
\]

where \( \ell \) is the wave degree. It must be noticed that the classical eigenvalue \( E = \ell(\ell + 1) \) is used here, and will be maintained all along this work.

To reach the eigenfrequencies, it is sufficient to concentrate on the motion projected on the radial axis, and the waves we consider can be represented by

\[
x' = \frac{A'}{r} \cos(k r - \alpha + \psi)
\]

where \( x' \) stands for \( \rho', \rho', \phi', \) or \( \vec{\phi}' \), this last one being the radial component of the displacement \( \vec{\phi}' \). When the acoustic wave is reflected between the external and internal limits of a star, there is formation of a standing wave in a cavity, resulting from the superimposition of forward and backward waves

\[
x'(r) = \frac{A'}{r} \cos(k_r r - \alpha + \psi) + \frac{A'}{r} \cos(-k_r r + \alpha + \psi')
\]

\[
= \frac{2A'}{r} \cos(-\alpha + \frac{\psi + \psi'}{2}) \cos(k_r r + \frac{\psi - \psi'}{2})
\]

which is a typical trapped wave expression, presenting nodes and antinodes separated in phase by \( \pi \). Following Fig.2, it is clear that the phase advance between the turning points \( r_l \) and \( r_s \) should verify
\[ \int_{r_1}^{r_2} k \, dr = (n + \alpha) \pi \]  
(16)

where \( n \) is the number of nodes along \( r \), called the mode order, and \( \alpha \) is a parameter to adjust following the phase conditions at \( r_1 \) and \( r_2 \). For example, if there are two antinodes at the two limits, \( \alpha = 0 \), if there are one node and one antinode, \( \alpha = -0.5 \).

**Fig. 2.** Typical standing wave with nodes and antinodes separated by \( \pi \) in phase.

Let us remark that the last relation that appears naturally above is called the Duvall relation. It is already widely used in different approaches and was introduced first by Duvall (1982). Even so, attention must be paid that \( r_2 \) is usually confused with \( R \), the star radius, while it is not the case here. This apparently small subtlety is in fact fundamental. Indeed, assuming \( r_2 \equiv R \) is an extremely crude approximation because it leads to frequency errors of tens of \( \mu \text{Hz} \). Any attempt to compensate for it, is equivalent to make a variable change while integrating (16), therefore to work with a wavenumber \( k_2 \) that is not the real one. So \( \alpha \) is not the same as the one employed here. Moreover, the phase conditions at external and internal turning points are completely mixed. The overall consequence is that one is working in a kind of phase space that is at odds with the real world.

Here, by working with \( r_2 \) and not \( R \), we stay in the real physical space and can take advantage of the classical rules of physics. The eigenmode determination becomes similar to what is done for an acoustic cavity on Earth, just like for any musical instrument. The eigenfunctions projected on the radial direction are given by (15), and the eigenfrequencies are determined by the Duvall expression (16). Thus, to obtain the full solution, it remains to determine the internal and external turning points together with the phase conditions at these points, namely \( r_1 \), \( r_2 \) and \( \alpha \).

**THE TURNING POINTS**

The external reflection conditions are the simplest ones, for the location as well as for the phase. This question is discussed in Nghiem (2002), only the results are presented here. As indicated by (1), the acoustic wave is reflected near the surface, where a 'pressure vacuum' is encountered. The external turning point is given by \( r_2 \) verifying

\[ \frac{1}{k_r} \frac{\omega^2}{c_0^2} = \frac{2\pi}{11.3 H_p} \]
(17)

and the phase at \( r_2 \) is so that there is an antinode there, like for a standing wave along a rope with a free end.

For the internal turning point, let us consider first the non radial case (\( l \neq 0 \)). In the depth of a star, where the structure can be considered as homogeneous, there is no problem of boundary conditions and thus no genuine reflection point. Nevertheless, when only the radial projected propagation is considered, one can speak about an internal turning point \( r_l \), the location where the radial component of the wave vector vanishes:

\[ k_r = 0 \iff \frac{c_0^2}{l^2} \frac{4\pi G \rho_0}{l(l+1)} = \frac{\omega^2}{l(l+1)}. \]
(18)

**Fig. 3.** Right or left member of (18) for the Sun, so that \( r_l \) can be immediately determined once the frequency and the degree of the modes are known. For the degrees \( l \geq 5 \), the curves begin to merge with the one, independent of \( l \), where the contribution of gravity is neglected like in the asymptotic approximation. For the smaller degrees, the difference with this last approximation is appreciable, even far from the centre, because although \( \rho_0 \) decreases quickly with radius, \( c_0^2 / l^2 \) decreases also quickly. A gap has even to be noticed for the mode \( l = 1 \). Globally, gravity pulls the low-\( l \) modes toward the centre.

To derive \( \alpha \), we need to determine the phase at \( r_l \). Let us remark that the standing wave formation must also imply geometric conditions. Since so far \( r_l \), \( r_2 \) and the phase at \( r_2 \) are exclusively determined by physical conditions, one could predict that \( \alpha \) depends mainly on geometrical ones. We make here a step further by
assuming that the expression of $\alpha$ depends solely on geometrical conditions. One can then look more precisely at the internal turning point: the wave trajectory is tangential to the sphere of radius $r_i$. On this sphere, or the one just above, the standing waves create a pattern of bumps and hollows whose characteristic areas are proportional to $r_i^2/l$. When this latter area tends toward zero, that means that the wave goes almost straightforward, without being deviated: there is almost no turning point. So, for the radial projected motion, the phase shift between forward and backward wave tends to zero, and following (15) with $k_r = 0$, one can conclude that there is an antinode at $r_i$. As there is already an antinode at $r_s$, this implies that $\alpha$ tends to zero. Let us try to look for an expression of the form

$$\alpha = b \frac{r_i^2}{l}$$

(19)

where $b$ is a constant to be determined.

On the one hand, the case where $r_i^2/l$ tends to zero clearly corresponds to the extreme limit where the frequency tends toward infinity. On the other hand, the lowest frequency can be used to determine $b$. In the absence of gravity, think about e. g. any cavity with one free end on Earth surface, it is normal to consider that the fundamental mode is the one with only one antinode at one end, and one node at the other, namely $l = 1$, $n = 1$. That implies that $\alpha = 0.5$. Every element necessary for the calculation of this mode are thus available. For the Sun, we obtain:

$$v = 331.3 \ \mu\text{Hz}, \ \alpha = 0.5, \ \frac{r_i}{R} = 0.353, \ \frac{r_i}{R} = 0.730.$$ 

Thus

$$\alpha = -\frac{4(r_i/R)^2}{l}$$

(20)

As the expression of $\alpha$ does not depend on environment physical conditions, the introduction of gravity does not change it, only its value will change via the change of $r_i$. Everything is now ready for the complete calculation of the $l \neq 0$ modes.

For the radial modes

$$l = 0, \ k_h = 0, \ k_r = k > 0$$

(21)

there is a priori no turning point like when $l \neq 0$. It is very tempting to conclude that these modes pass through the star centre which is a node, since due to symmetrical reasons it must stay immobile. In this case: $r_i = 0$, and $\alpha = 0.5$. But one realizes that these values induce eigenfrequencies that are lower than numerical or observed results by several tens of $\mu\text{Hz}$. This suggests that the cavity so defined is too large, and these modes do not touch the centre. In fact, the linear solution of local wave cannot be applied at $r_i = 0$. For the local spherical wave, it is obvious that the center is a singular point according to its expression (15).

Instead of leaving the linear solution, we can invoke the spherical symmetry to introduce a new concept, the ‘wave internal pressure’. When going toward the center, Eq. (15) indicates that each point of the wave surface will experience an indefinitely increasing pressure which comes from the neighbouring points on this even surface, not from the background. When this internal pressure becomes too strong, the effect is the same than for a ‘pressure wall’ and the wave rebounds on it. From the plane wave formalism, this indefinitely increasing internal pressure can also be seen by looking at the pressure due to the whole set of trajectories (straight line) on one trajectory.

Let $p_i$ be that internal pressure, it is inversely proportional to the radius, and the corresponding pressure scale height is

$$H_{pl} = r_i.$$ 

(22)

Following (1), there is reflection when

$$\lambda = a_i H_{pl} \Rightarrow k_r = \frac{2\pi}{a_i r_i}$$

(23)

where $a_i$ is a constant to be determined. Like at the surface, that is a problem of boundary condition which is not contained in the physics of wave propagation. By merely searching for a continuity with the $l = 1$ modes, one can deduce $a_i$, $\alpha$, then the expression giving $r_i$ (also represented in Fig. 3):

$$\alpha = -0.2$$ 

(24)

$$\frac{k_r}{r_i} = \frac{1.265}{r_i} \Rightarrow \frac{c}{r_i^2} + \frac{4\pi G p_0}{c r_i} = \frac{\omega^2}{1.6}.$$ 

(25)

This is merely an empirical way of doing which is not really rigorous. Anyhow, like $a_i$ for the external boundary condition, the coefficient $a_i$, once determined, should be universal, i. e. independent of any star, because it simply reflects the ‘pressure wall’ condition for a wave.

RESULTS

We now have every ingredient to calculate the eigenfrequencies. It is worth gathering here the whole set of equations used. Let us remind that, given the 2 parameters $l$, $n$ which are respectively the mode degree and order, a system of 2 equations with 2 unknowns $\omega$ and $\alpha$ has to be solved:

$$\int_{r_i}^r k_r \, dr = (n + \alpha)\pi$$

(26)

$$\alpha = -\frac{4(r_i/R)^2}{l} \text{ for } l \neq 0, \quad \alpha = 0.2 \text{ for } l = 0$$

(27)

with

$$k_r = \sqrt{\frac{\omega^2 + 4\pi G p_0}{c_0^2} - \frac{l(l + 1)}{r^2}}$$

(28)

$r_i$ given by
$k_r = 0$ for $l \neq 0$, \quad $k_r = 1.265 r_1$ for $l = 0$ \quad (29)

and $r_2$ given by

$$\frac{1}{k_r} \frac{\omega^2}{c_0} = \frac{2\pi}{11.3H_p}.$$ \quad (30)

We would like to stress that Eq. (26) to (30) are all the equations we need to determine the eigenfrequencies.

That allows to appreciate the extreme simplicity of the present approach. One can also notice that only 3 functions of the equilibrium structure are involved, $c_0(r)$, $\rho(r)$ and $H_p(r)$. $H_p(r)$ is in fact only important at the surface in the determination of $r_2$, $\rho_0(r)$ acts through the gravitational potential, which plays a crucial role toward the center, being more important for lower degree modes.

We also remark that those equations are obtained without using asymptotic hypothesis. The results induced in the following sections will thus present the same quality at low as well as at large degree or order, at the exclusion of very low frequencies where the coupling with gravity modes is not taken into account.

The results obtained with the above equations are compared to the ones coming from numerical calculations in Nghiem (2002). An inversion method of $H_p$ is also validated, allowing to find out the $H_p$ surface profile using by the Isothermal Atmosphere Approximation.

In Fig. 4, 5, 6, 7, the internal, external turning points, the phase $\alpha$, and the eigenfrequencies compared now to observational data, are given for the modes $l = 0$ to 10.

The solar model called Btz (Brun, Turck-Chièze & Zahn 1999), is employed, together with observational data coming from the SOHO spacecraft (MDI data from Rhodes et al. 1997; GOLF data from Bertello et al. 2000, and from García et al. 2001). The frequency discrepancy varies strongly with the frequency range, but is mostly independent of the degree $l$. As this behaviour is very specific to the external limit, it is straightforward to conclude that it comes from a discrepancy on $r_2$. If, like with numerical data, it can be assumed that this is due to a model-Sun discrepancy of the $H_p$ and $\Gamma_1$ surface profiles, while the $c_0$ profile is not to be questioned, the inversion above noted can be applied. Those inverse profiles are given in Fig. 8, 9. The resulting frequency discrepancy is shown in Fig.10: it is contained in $\pm2.5$ $\mu$Hz for the range 1000-5000 $\mu$Hz. It indicates the limit of the present semi-analytical approach, which is mainly due to the approximative treatment of the phase shift at the internal turning point. The numerical calculation should not face this problem, so if it uses the external boundary condition proposed here, it would arrive to an agreement with observation better than the $\mu$Hz in the whole frequency range. Theoretical predictions and observations become now coherent: semi-analytical results give the general trend together with the physical understanding, and numerical results give the precision.

The above type of inversion of surface parameters could be used as guidelines for a more sophisticated...
description of this region. A first attempt to infer the surface magnetic field from that, is performed in Nghiem et al. (2002). But the physical mechanisms involved at the surface remain to be identified, and the change or not of the sound speed to be clarified.

CONCLUSIONS

A description very close to the basical physics of standing wave leads to a very simple formalism. It also avoids the use of two approximations that are already known to be the weak points: the asymptotic approximation for semi-analytical calculations, and the Isothermal Atmosphere Approximation for both semi-analytical and numerical calculations. The quality of the theoretical predictions is improved, and the better coherence with observational data should ameliorate their reliability when studying other stars.

The approach proposed here can be applied to Sun-like stars where gravity and acoustic modes are well separated. For more massive stars, the coupling between them becomes stronger, and the simple analytical formalism given here has to be improved. But numerical calculations using the same stopping condition than here at the surface would continue to be pertinent.

The clarification of the internal turning points for the lowest-\(l\) modes could also help to better characterize the very near-center region of the stars.

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Fig. 8, 9. Profiles of the pressure scale height (plus zoom at the external limit), and of \(\Gamma_j\) at the surface. Continuous line: initial profiles coming from the Sun model. Dashed line: inversed profiles from observational frequencies.

Fig. 10. Resulting frequency discrepancies with observational data for the modes \(l = 0\) to 10, in the presence of the inversed Hp profile of Fig. 8. The symbols are defined in Fig. 4.
Structure of the Solar Interior

Chair: T. Brown
SUPERGRANULATION: NEW OBSERVATION, POSSIBLE EXPLANATION

Mark Peter Rast
High Altitude Observatory, National Center for Atmospheric Research, Boulder, Colorado, USA

ABSTRACT

We briefly review the main observational properties of the solar supergranulation: divergent horizontal flow, weak thermal signature, oscillatory power, and super-rotation. We present new photometric measurements which attempt to disentangle the magnetic network and convective contributions to the supergranular continuum intensity contrast, and suggest that the convective signal has now been detected with some confidence. We propose and examine a purely advective model for the supergranular flow and show that large-spatial and long-temporal supergranular (and mesogranular) scales naturally arise through the collective interaction of many small-scale and short-lived granular downflow plumes. We show that dynamically unsteady behavior in such a model can yield oscillatory power and speculate that superrotation may be achieved given an underlying size dependent rotation rate. Finally, we discuss the uncertainties and observational predictions of this highly simplified model.

Key words: convection; granulation; supergranulation.

1. OBSERVATIONAL PROPERTIES

The solar supergranulation is characterized by large-scale (32 Mm) long-lived (20hr) horizontal flow of amplitude 0.3 – 0.5 km/s (Hart, 1954, 1956; Leighton, Noyes, & Simon, 1962). The flows diverge from cell centers outward, terminating at boundaries outlined by strong photospheric magnetic fields and chromospheric network (Simon & Leighton, 1964). The motion suggests a convective origin, with warmer fluid presumably upwelling near the cell centers and cooler fluid descending at their boundaries. However, measurements of the supergranular continuum intensity fluctuations have yielded precisely the opposite signature: positive continuum intensity enhancement at the cell boundaries (Beckers, 1968; Frazier, 1970; Skumanich, Smythe, & Frazier, 1975; Foukal & Fowler, 1984; Lin & Kuhn, 1992). Since this enhancement is significantly limb-brightened, it is likely of magnetic origin (Beckers, 1968). In Section 2 below, we describe new work that attempts to disentangle the magnetic contribution to the supergranular continuum contrast from the assumed underlying convective signal.

Measurement of the vertical component of the supergranular flow is likewise confounded by the presence of magnetic fields. While upflows are elusive and complete lanes of downflowing material (analogous to those seen for granulation) are not observed, isolated downdrafts occur at the vertices of supergranular cells. Frazier (1970) measured flows of ~ 0.1 km/s amplitude, highly correlated in both position and amplitude with magnetic flux density. Since that time, spatially intermittent downflows at supergranular boundaries have been reported with amplitudes ranging from tens to hundreds of meters per second (Musman & Rust, 1970; Deubner, 1971; Skumanich, Smythe, & Frazier, 1975; Worden & Simon, 1976; Giovanelli & Slaughter, 1978; Küveler, 1983; Wang & Zirin, 1989; Hathaway et al., 2002). While some of the discrepancy in these measurements may be associated with spectral line formation heights, much of it also probably reflects spectral line distortions due to the presence of magnetic fields (Miller, Foukal, & Keil, 1984; Stenflo et al., 1984; Stenflo & Harvey, 1985) which are always closely associated with the flows. Unfortunately, disentangling the magnetic and flow contributions, even in polarimetric observations, is difficult and model dependent, with the magnitude of inferred downflows sensitive to the assumed thermodynamic and geometric relations between the field and the flow (Solanki, 1986; Solanki, 1989; Bellet Rubio, Ruiz Cobo, & Collados, 1997; Briand & Solanki, 1998; Frutiger & Solanki, 1998).

The horizontal supergranular flows can be obtained either from Doppler measurements away from disk center or, near disk center, by tracking the advection of magnetic elements or smaller scale flows (Simon, 1967; November & Simon, 1988; Simon et al., 1988; Wang & Zirin, 1988; Muller et al., 1992; Wang et al., 1996; Zhang et al., 1998; Roudier et al., 1999; De Rosa, Duval, & Toomre, 2000; Shine, Simon, &...
Hurlburt, 2000; Lisle, De Rosa, & Toomre, 2000). This latter approach largely confirms that the primary supergranular flow pattern is radially outward from the cell interior, but it also uncovers some interesting details. After nearing the cell boundaries the flows carry the tracers toward the supergranular vertices. The network boundaries are incomplete, with spatially intermittent sites of high magnetic flux density well correlated with sites of strong convergence. Additionally, significant horizontal flow structure exists within the cell interiors. Sites of both convergence and divergence are embedded within individual supergranules. The supergranular flow thus appears to be dominated by a collection of isolated magnetized downflows which induce broadly divergent but structured horizontal flows within the cell interiors and strongly converge horizontal flows in their vicinity at the supergranular boundaries. This observational picture fits well with the highly simplified model presented in §3 below.

With the basic thermal and flow properties of the solar supergranulation under continued investigation, other curious properties have been reported. Longitudinal cross correlation of the supergranular flow pattern suggests that the pattern rotates faster than the surface plasma (Duvall, 1980; Snodgrass & Ulrich, 1990), as determined from direct Doppler measurements, and faster than the outer 5% of the solar convection zone (Beck & Schou, 2000), as determined by helioseismology. Even more curiously the flow pattern apparently rotates at a rate faster than the magnetic network (Snodgrass & Ulrich, 1990). The rate is size dependent with larger scale features rotating faster than smaller ones (Duvall, 1980; Beck & Schou, 2000). This ‘super-rotation’ has been recently confirmed for flow fields obtained from $f$-mode time-distance helioseismology (Duval & Gizon, 2000; Gizon, Duvall, & Schou, 2003). Fourier transformation of those fields yields excess power in prograde vs. retrograde modes. Moreover, the spectrum shows maximum-power offset from zero temporal frequency.

In light of these unusual properties and the lack of conclusive proof of a convective origin, other speculations have arisen to account for supergranulation: gravity wave modulation of the convective motions (Lindzen & Tung, 1976), a transition with depth in the magnetic field filling factor (Foukal, 1977), granular pumping of a solar rip-tide (Cloutman, 1979), $r$-mode–convective coupling (Wolff, 1995), spatial correlation between exploding granules (Rust, 2000), and the superposition of traveling waves of unknown origin (Gizon, Duvall, & Schou, 2003). Convective theories themselves abound (Simon & Weiss, 1968; Vickers, 1971; Roxburgh & Tavakol, 1979, Van der Borgh, 1979; Bogart, Gerasch, & Macauslan, 1980; Gerasch, 1985) in attempts to explain the observed intermediate (much greater than a scale height, much less than the depth of the layer) supergranular length scale and the coexistence of at least two (possibly three or four) somewhat discrete convective scales in the Sun. Many of these convection theories invoke the mean depths of hydrogen and helium ionization (two such depths for helium) for scale selection (Leighton, Noyes, & Simon, 1962; Simon & Leighton, 1965). It should be noted in this regard: first, the total depth over which ionization actually occurs (as opposed to its mean) is quite broad due to significant ionization state perturbations associated with cool downdrafts; second, the thermodynamic perturbation (to the adiabatic exponent for example) caused by helium ionization is small (even for HeI and yet smaller for HeII) and not well separated in depth from that caused by hydrogen ionization; and third, all ionization effects on convective flow dynamics are destabilizing and contribute to a reduction in convective scales (Raust, 1991; Raust & Toomre, 1993; Raust, Clune, & Toomre, 2002). If ionization effects are important in solar convective scale selection, it is through the introduction of small scales into what would otherwise be a larger scale flow, not by the organization of the flow on scales corresponding to the mean depth of their occurrence. As we will discuss in §3 below, a more important process in scale selection on the Sun is likely the extreme sensitivity to radiative losses in the photosphere, the consequent formation of vigorous downflowing plumes, and the collective interaction of these.

2. THE INTENSITY SIGNATURE OF THE SUPERGRANULAR FLOW

The solar supergranulation is characterized by large scale divergent horizontal flow and is outlined by magnetic network. There are thus two competing contributions to the associated continuum intensity fluctuations (Figure 1). On one hand, the continuum intensity is enhanced in the network regions because of the reduced opacity of the gas there, due the presence of magnetic fields and consequent gas pressure reduction. On the other hand, the underlying convective motions, which are the presumed cause of the
observed divergent flows, should bring higher temperature fluid upward in the cell centers and thus contribute to enhanced continuum intensity there. We attempt to disentangle these competing contributions using data from the Precision Solar Photometric Telescope (PSPT).

The PSPT regularly produces 2048 × 2048 full disk solar images at three wavelengths: red continuum (607.1 ± 0.11nm), blue continuum (409.4 ± 0.22nm), and CaII K (393.5 ± 0.12nm). Detector variability is determined using an implementation of the Kuhn, Lin, and Loranz (1991) algorithm, and the solar center-to-limb variation is carefully measured using an orthogonal-function decomposition (Rast & Meisner 2002). Removal of the detector variability and solar limb darkening yields contrast images \( \delta I/I_0 \), where \( \delta I = I - I_0 \), \( I \) is the image intensity and \( I_0 \) is the limb fit, of high photometric precision (≈ 0.1%). For this study, simultaneous high quality images at all three wavelengths are selected based on the disk-center root-mean-square contrast, the disk-center spectral power, and the limb-profile width. Continuum images are resized and aligned with the co-temporal CaII K images using bilinear interpolation, and a central 512 × 512 pixel subarray is extracted at each wavelength, yielding an image triplet. Visual inspection verifies feature (sunspot, pore, faculae) alignment in these triplets to about one pixel accuracy.

The CaII K image from each of these aligned triplets is used to identify network cells, their centers and radii. Two different methods were tried. A subjective technique involves initial identification of the cell centers by hand, maximization of the cross correlation between the network CaII K intensity and a doughnut shaped object whose radius and center are adjusted within a limited range, and visual approval of the resultant fit. This technique has the advantage of selecting only nearly circular objects which are readily identifiable as network cells. It has the disadvantages of being subjective and tedious. A more objective technique involves thresholding and skeletonization (Berilli, Florio, & Ermolli, 1998; Berilli et al., 1999), with barycenters and radii (in the four grid directions) measured for each network cell of a continuously tiled pattern. The results obtained for a limited supergranular sample are only weakly dependent on the identification technique, and since a very large number of supergranules are required for this study, the skeletonization technique is employed exclusively to obtain the results reported here.

Photospheric continuum intensity fluctuations, in nonactive regions, are dominated by the solar granulation. In order to measure the smaller underlying network and supergranular contributions it is necessary to reduce the granular noise by either spatial or temporal averaging. If multiple images taken during a single day are carefully aligned and averaged, the granular signal which evolves on short time scales is reduced while the longer lived supergranular pattern remains. In this way it is possible to directly visualize the network contribution to the continuum intensity (Liu, 1974; Figure 2). To quantitatively measure the continuum contrast of the supergranulation, we determine, at each wavelength and independently for each network cell, the azimuthally averaged radial intensity profile as a function of distance from the cell center. These individual profiles are then averaged over all cells after size normalization. This procedure effectively suppresses the granular intensity fluctuations and reveals any underlying azimuthally symmetric network and supergranular contributions, assuming that the cells are nearly circular and that their intensity profiles are similar (independent of size). The azimuthal average performed uses an equal number of equal area annuli for each supergranule, and the size normalization and averaging of the individual radial profiles uses only the data present in the images, binning it rather than interpolating to form the averages. Small supergranules thus individually contribute a noisier signal but
Figure 3. Correlation between Kitt Peak magnetic flux density and PSPT CaII K intensity for the central 512×2 image pixels from two separated dates. Binning and averaging the non-sunspot pixels (those values above the diagonal line over-plotted in (a)) yields the curves shown in (b). A strong correlation between CaII K intensity and magnetic flux exists which is relatively insensitive to the chosen independent variable and observation date. This holds even for very low values of the magnetic flux density (inset). Kitt Peak magnetogram de-rotation (courtesy K. Harvey) was required to align the images from which these plots were made.

The number of them is greater. The spatial resolution of the final average radial profile is coarsest near the center, but the noise properties are uniform with radius and accurately reflect the fluctuations (primarily granular) in the images.

In order to separate magnetic from convective contributions to supergranular intensity contrast, we apply a sequence of increasingly severe masks to the images before measuring the radial intensity profiles as described above. These masks are constructed from the CaII K images, blocking all pixels with CaII K emission above given thresholds, and are applied to all three aligned images. The efficacy of such an approach depends on the empirical correlation between intensity and magnetic flux density in the CaII K images. We illustrate this relationship in Figure 3 using 512×512 disk center data from two pairs of carefully aligned PSPT and Kitt Peak magnetogram images. The images were aligned (unwarping the magnetogram) using small compact high-flux-density objects, but the correlation holds, on average, even for very low values of the magnetic flux density. This is important, because recovery of the convective supergranular signal requires very severe masking of the magnetic field. Since very few pixels remain in the severely masked images, granular noise is more weakly suppressed by the supergranular averaging described. This necessitates the measurement of many supergranular cells, with the results presented here obtained by averaging the signal from 7300.

The uppermost curves at all wavelengths in Figure 4 plot the radial profile of the supergranular contrast from the cell centers outward towards the edge (r/r⊙ = 1) without magnetic field masking. Network is on average ~ 4% brighter in CaII K and ~ 0.7% and ~ 0.5% brighter in blue and red continuum than are network cell centers. These values are higher than but consistent with previous studies (Beckers, 1968; Skumanich et al., 1975). Each successive curve below those for the unmasked images in Figure 4 plots the radial contrast profiles obtained after applying increasingly severe masks to the images. Not only is the mean continuum intensity of the remaining pixels reduced, but, for the second and more severe masks applied (third curve down and lower), the continuum intensity within the supergranular cells is on average higher than the mean.

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2This reduction suggests a significant quiet network contribution to solar irradiance variations.
value of the unmasked pixels in the image (shown by the plot asymptote at large distance, \( r/r_0 = 2.5 \)). This is consistent with a convective signal. The error bars, over-plotted with dashed curves in Figure 4, indicate that, for the fourth curve down the detection is three sigma (standard deviation of the mean) significant. Using identical processing on shifted images (so that the supergranular identification and network masking are no longer appropriate) yields a null result of the same significance. The curves plotted in Figure 4 for the most severe image masking hint at an intensity profile within the supergranular cell that, while still above the mean value of the image throughout, increases toward the cell boundary. This is consistent with the flows expected from the model presented in §3, but requires additional data to verify its statistical significance.

3. A PURE ADVECTION MODEL FOR SUPERGRANULAR AND MESOGRAINULAR SCALE SELECTION

The dynamics of solar granulation is dominated by a delicate balance between the upward advection of heat and its radiative loss in the photosphere. A simple balance between these yields an estimated minimum ascent velocity of 2 km/s to support losses\(^3\) (Nordlund, 1985). Local fluctuations in the photospheric flows that reduce upward advection of heat lead to heat loss rates in excess of advective supply, local cooling, and consequent new downflow plume formation.

High resolution observations of continuum intensity (de Boer, Kneer, & Nesin, 1992; Hirzberger et al., 1997; Hirzberger, 2002) and Doppler velocity (Nesin et al., 1992; 1993; 1997; Hirzberger, 2002) indicate that maximum granule upflow occurs close to the center of small granules but immediately adjacent to intergranule downflow lanes in larger granules. Moreover, many granules (31.2%; Hirzberger et al., 1999) evolve by expansion, central darkening, and fragmentation through the formation of dark radially-directed lanes (Rösch, 1960; Carlier et al., 1968; Namba & Diemel, 1969; Mehlheter, 1978; Kitai & Kawaguchi, 1979; Kawaguchi, 1980; Bray, Loughhead, & Durrant, 1984; Namba, 1986). This fragmentation process occurs rapidly and vigorously (expansion to 3-5 arc-seconds within ~ 8 minutes) during the phenomenon known as an ‘exploding’ granule, but the basic scenario is common to less vigorous fragmentations.

Both the observed granular flow topology and the process of granule fragmentation may be understood in terms of the properties of compressible downflow plumes (Rast, 1995; 1999; 2000). A compressible thermal downflow plume in a stratified domain initiates upflows which, in the upper scale heights of the layer, are confined to its near proximity. This is because the upflows are driven by local buoyancy and pressure gradient forces induced by the downflow plume (Rast, 1998). During granule expansion, the upflows move with the downflows causing the velocity in the granule center to decrease, ultimately falling below that necessary to sustain radiative losses (Figure 5). Fragmentation then occurs, with the formation of a new central downflow plume, followed by the initiation of new downflow lanes as a result of flow reversal, stagnation, and excess cooling (Rast et al., 1993). The lanes form in those directions predisposed to weak flow as determined by the distance to and strength of the neighboring downflow boundaries, which set the upflow magnitude in the granule interior (Rast, 1995). While the mean state of the solar envelope must adjust to ensure total flux transport, the local granular dynamics of the radiatively cooled photospheric boundary layer is best described as an advection-fragmentation process, with the horizontal advection of granular boundaries leading to local cooling and frequent new downflow plume formation. The fundamental length scale of granulation is associated with the horizontal distance over which downflow induced pressure fluctuations drive upward convective heat transport at a level sufficient to sustain radiative losses.

\(^3\)We note, that is only through the release of latent heat of hydrogen ionization by recombination in the photosphere that radiative losses can be sustained with upflows of this magnitude. If only thermal energy were available, as in an ideal gas, much greater upflow velocities (\( \sim 15 \text{ km/s} \)) or much greater temperature fluctuations (\( \sim 10x \)) would be required.
It has been suggested, based on simple advection models (Rast, 1999) and three-dimensional convection models (Cattaneo, Lenz, & Weiss, 2001) that mesogranulation results from the collective interaction of granular flows. We investigate here whether both mesogranular and supergranular scales can result from the collective advective-interaction of granular downflow plumes. More specifically we examine the question, can large-scale long-lived patterns be produced by the advective interaction of small-scale short-lived objects? Remarkably, by taking the horizontal-advection profile from simple thermal plume simulations and the number density of new granular-plumes initiated directly from granulation observations, pure advective interaction can recover the gross spatial and temporal scales associated with both mesogranulation and supergranulation.

Current computational resources are unable to adequately resolve both granular and supergranular scales in a direct numerical simulation of compressible convection. To explore the issues of scale selection, we simplify the convection problem, casting it as an n-body simulation. We heuristically reduce the Navier-Stokes equations to the mutual advection of downflow plumes on a horizontal plane. The unit element of the n-body simulation is the radial advective flow profile associated with each of the individual plumes. This profile is taken from fully compressible direct-numerical-simulations of single thermal downflow plumes in a stratified domain (Rast, 1998). The horizontal flow into such a plume decays exponentially with distance from the downflow site. The decay length depends on both the depth at which the flow measurement is made and the geometry of the plume (Figure 6). This length scale is one of two principle parameters governing the model. Implementing the advection fragmentation process described above for granular dynamics to determine the sites of new plume formation requires discretizing and calculating the vertical flow field over the entire horizontal plane in order to pinpoint where and when the upflow velocity drops below some critical amplitude. That type of advection model has been successfully computed in one-dimension and shows clustering on large scales (Rast, 1999), but it is prohibitively expensive in two dimensions especially when supergranular lifetimes are of interest. For the model we present here, we take the new granular plume sites to be randomly distributed over the two-dimensional plane, with the number density per granular lifetime the second principle parameter. This means that only the horizontal flow must be calculated and that only at the n-body sites. Starting from an initially random distribution of plume sites, we apply the following set of rules:

1. Individual plumes are created at a fixed rate at random positions in the $x-y$ plane. The amplitudes of new plumes are normally distributed, with a full-width at half-maximum of 0.5, about a mean of 1.0.

2. The domain is one-time periodic in all directions, with the advection profile of individual plumes truncated at a distance of one domain width.

3. Plume amplitudes decay exponentially in time, with a time constant equal to one.

4. All plumes are advected horizontally by the vector sum of the flows at their position due to all other plumes.

5. Plumes merge when they lie within one unit of distance of each other. The new plume position is given by the amplitude weighted average position, and the new amplitude is taken as the sum.

6. Plumes are removed when their amplitude decays to less than 0.001 (this rarely happens when advection and merger dominate the solution).

7. The solution is advanced with a time step equal to one-half the minimum distance between any two plumes divided by the maximum horizontal velocity at any of their positions.

Many such n-body simulations have been performed, varying both the width of the advective flow profile and the number of new granular plumes generated per unit area and granular lifetime. These are summarized by Figure 7. When the plume creation rate is low, and as a consequence the plumes on average decay away before undergoing significant merger, no larger scale organization occurs (Figure 7j). However, when sufficient numbers are initiated, clustering produces large amplitude objects with long lifetimes, sustained by the advective influx of those of smaller amplitude. The temporal stability of the large-scale pattern increases as more granular plumes are made (less time dependence in the solutions as one moves to the right in Figure 7).
and the separation of the large amplitude objects increases with increased width of the advective profile (going from bottom to top in Figure 7). Figure 7c is a single snapshot sampling the quasi-steady pattern produced when the advective profile is taken directly from an axisymmetric compressible plume simulation (Figure 6) and the number of new plumes made corresponds to 31.25% of all granules in the 64 × 64 granule area during each granule lifetime. This corresponds to the fraction of solar granules observed to fragment (Hirzberger et al., 1999) and thus presumably produce a new downflow plume. The spatial separation between large-amplitude long-lived plumes in this solution is very similar to the mean size scale of the solar supergranulation, and we would interpret those to be the sites of supergranular vertices.

A more quantitative assessment of the separation length scale associated with plumes of differing amplitudes can be made using the amplitude probability distribution function. Expectation values (averages over many time steps in the solution) of such histograms are shown in Figure 8, each curve plotting the number of objects (normalized to sum to the total) as a function of amplitude for various plume creation rates. For low creation rates, merger is not important, and most plumes have very small amplitudes (they all decay, so the peak in the amplitude distribution is at the low amplitude cutoff). For higher creation rates, two distinct peaks in the distribution occur. (Note that Figure 8b is a continuation of Figure 8a with a reduced vertical scale.) The first peak results because, at these rates, new plume creation often leads to local merger within a granule lifetime. The second peak results because some plumes grow large enough so that their advective domain of influence allows them to ‘live forever’. Increased creation rates yield fewer of these large objects, each with greater amplitude, distributed more sparsely in the domain.

Typical separation length-scales associated with these distribution peaks can be obtained by dividing the domain area by the integrated number of objects of a given amplitude or greater, and taking the square root of this quantity. For the n-body simulation illustrated by Figure 7c, such length-scales have values of 4.0 (with the shoulder on the distribution peak corresponding to objects of typical separation 7.3) and 30. While there is some uncertainty in the scaling, because unit length in the n-body simulation corresponds to the plume merger distance rather than the granular length scale, the results are sur-
prisingly close to the observed scales of mesogranulation and supergranulation, particularly given the simplicity of the model and the uncertainty in the advection parameters chosen. More important than the precise numbers obtained, may be the fact that this highly simplified model can easily produce a large scale long-lived pattern from the advective interaction of many small scale short lived granular objects and that in doing so it produces two such distinct secondary scales.

A quantitative estimate of lifetime is more difficult to obtain. The reason for this is that the n-body simulation produces a time series of plume positions, points on the two dimensional plane. While the human eye and brain are very good at recognizing persistent patterns in such a collection of points, and in fact the positions of the large amplitude plumes are quite steady, any slight changes in their positions results in zero temporal cross correlation. One possible solution is to discretize the two dimensional domain and calculate the total (sum over the contributions from all the objects) horizontal flow-field, or its divergence, everywhere. Unfortunately, this is computationally expensive. One needs to compute the flow field at high spatial and temporal resolution over many realizations of a time series extending many supergranular lifetimes. We test the utility of such an approach using a one-dimensional version of the mutual advective-interaction simulation. The behavior and amplitude distribution function of this one dimensional problem are remarkably similar to those of the two dimensional solution (one caveat being that, because of the geometry, the plume production rate must be much higher in one dimension than two to sustain long lived flows). An example of a time history of the flow divergence in one-dimension is shown in Figure 9, with space extending horizontally and time running bottom to top. Sites of strong convergence (shown dark), separated by many granular length scales, last many granular lifetimes.

The two-dimensional power spectrum of Figure 9 shows surprising structure. As illustrated by Figure 10, it shows both background ridges and isolated peaks. Preliminary study suggests that the source of the peaks is the quasi-regular spacing of the sharp convergence sites in Figure 9, which yield essentially the Fourier transform of a comb of delta functions. Since these sites are spaced fairly regularly and merge at somewhat regular intervals, the power spectrum shows a maximum away from zero. Moreover, the statistics of the granular plume formation events imposes a relationship between the amplitude of and the time interval between perturbations to the convergence site positions. Small amplitude perturbations to their spacing occur with high frequency, because small fluctuations in the random distribution of new plumes occur often. This is directly visible in Figure 9 as the small amplitude jitter in the positions of the dark features. Large amplitude perturbations to this spacing, however, require statistically less likely perturbations to large scale averages of the new plume positions. For example, for
one of the quasi-steady plume sites to merge with another there must be a net imbalance in small plume occurrence averaged over the large-amplitude plume spacing. Without an average growth or decay of a large-scale convergence site relative its to neighbors, which requires a statistically less likely, and thus less frequent, excess or deficit of new plume initiation in its domain of influence, the large scale pattern would be stable. More work is required to fully assess the impact of these perturbations in convergence site position on the spectral power, but it is likely, and apparent in Figure 10, that oscillatory power can result from a purely advective (not wave, as suggested by Gizon, Duvall, and Schou (2003)) phenomenon.

4. PREDICTIONS, SPECULATIONS, AND UNCERTAINTIES

We have proposed a highly simplified model of the solar supergranulation suggesting that this long-lived large-scale patterning results from the collective advective interaction of many small-scale short-lived granular downflow plumes. The model is successful at approximately producing the observed spatial scales of both mesogranulation and supergranulation when the two critical governing parameters are taken from single plume simulations and granulation observations. Lifetimes for these scales are more difficult to quantitatively measure, but preliminary results suggest that they are qualitatively correct, mesogranular and supergranular scales displaying intermediate and long lifetimes respectively. Moreover, the spectral power distribution of a reduced one-dimensional solution suggests that a purely advective phenomenon can produce excess oscillatory power without recourse to an underlying wave phenomenon. Significantly more work needs to be done to quantify and understand these temporal and spectral properties.

One can speculate that this purely advective model of supergranular scale selection may also lead to super-rotation of the supergranular pattern. If large amplitude plumes rotate faster than their small amplitude granular counterparts, then it may be that, because they temporarily clear a path in the granular plume field, they feel a net forward advective force. This would imply that if the magnitude of super-rotation were latitudinally dependent then so would be the vigor and scale of the supergranulation, with somewhat more vigorous somewhat larger length scale flows being associated with more super-rotating regions. To date this is just speculation.

Other model predictions are on a more firm basis. The most obvious it that the downflows associated with supergranulation should be greatest or clustered near the vertices. While the model predicts large amplitudes for these flows, those amplitudes are likely exaggerated by the constraints imposed; no areal increase in size was accounted for during plume merger, and complete merger rather than clustering was assumed. In this regard, it is important to consider the behavior of magnetized plumes (as they are likely to be in the solar environment) and the relationship between the field and the flow. The model also predicts a particular distribution for the horizontal flow divergence within a supergranular cell. It suggests (Figure 9) relatively uniform flow divergence, perhaps structured on the mesogranular scale, within the supergranular cell interiors, with peaks, due to fluid acceleration, near downflow sites at the boundaries. Preliminary PSPT measurements (§2) hint at intensity fluctuations consistent with this, but high spatial resolution flow measurements are required to fully test the model.

Finally, it is important to remember that the advective model proposed reduces a complex physical system to a single interaction. It is far from a complete description, but may offer some measure of insight into the process of scale selection in a highly nonlinear system.

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HIGH-DEGREE P-MODES AND THE SUN’S EVOLVING SURFACE

E.J. Rhodes, Jr.1,2, J. Reiter3, and J. Schou4

1 Dept. of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089, USA
2 Astrophysics and Space Sciences Section, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109-8099, USA
3 Zentrum Mathematik, Technische Universität München, D-85747 Garching, Germany
4 W.W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085, USA

ABSTRACT

Two of the most glaring problems in contemporary helioseismology are the limited availability of high-degree p-mode frequencies for use in inversions of solar internal structure and the lack of high-degree frequency splitting coefficients for use in inversions of solar internal dynamics. A third major problem is the lack of a consensus regarding the mechanism underlying the temporal shifts of the p-mode frequencies. The lack of high-degree frequencies and the lack of similar high-degree frequency splittings have occurred because of the inherent difficulties in measuring such frequencies and frequency splittings in high-degree power spectra without the inclusion of numerous systematic errors. We will first point out the importance of high-degree p-modes to helioseismic inversions. Next, we will describe recent progress we have made in estimating high-degree frequencies, including the use of both asymmetric and symmetric profiles in our fits. We will then demonstrate that the inclusion of corrections for the eigenfunction distortion due to latitudinal differential rotation removes long-standing discontinuities in the high-degree frequency splittings. We will go on to describe our recent efforts of increasing the sensitivity of the p-mode frequencies to changing levels of solar activity through the use of observing runs which are as short as three days in duration. Finally, we will also describe planned efforts at verifying recent discoveries in solar internal dynamics through the reduction and analysis of full-disk Dopplergrams obtained during Solar Cycle 22 prior to the beginning of the GONG and MDI projects. Taken together, the recent improvements in the estimation of high-degree frequencies, frequency splittings, and the availability of useful data from Solar Cycle 22 indicate that a renaissance in global helioseismology is now at hand.

Key words: solar oscillations; high-degree frequencies; temporal frequency shifts.

1. INTRODUCTION

Within the past five years the field of helioseismology has matured to the point where inversions of solar internal structure and dynamics (the so-called “global” methods) have been joined by several different techniques (among them ring diagram analysis, time-distance analysis, acoustic holography, and acoustic imagery) which attempt to infer the properties of the solar interior beneath individual active regions (the so-called “local” methods). While many exciting new results have indeed been obtained with the local methods, there is still much important solar physics to be studied with global methods. The importance of continued research in both areas of helioseismology is emphasized elsewhere in these Proceedings in the closing lecture (Thompson, 2003).

2. IMPORTANCE OF HIGH-DEGREE P-MODES TO HELIOSEISMIC INVERSIONS

2.1. Distinctions Between Structural, Abundance, and Rotational Inversions

Some of the most exciting results that helioseismology has provided have come from numerical inversions of the solar p-mode oscillations. Specifically, these results have come from inversions of datasets of either the oscillation frequencies themselves or of the so-called frequency splittings of the tesserical and sectoral oscillation modes. The inversions of the various frequency datasets have yielded inferences of the thermodynamic structure of the solar interior, most notably of the sound speed there, while the inversions of the frequency-splitting datasets have provided inferences of the two-dimensional angular velocity of the solar interior.

The inversions of the different frequency datasets have been referred to as “structural” inversions (see e.g. Gough et al., 1996; Basu et al., 1996; Kosovichev et al., 1997), and “abundance” inversions (see e.g. Basu and Antia, 1995; Kosovichev, 1996; Antia and Chitre, 1998), while those of the frequency-splitting datasets have become known as “rotational” inversions (see e.g. Thompson et al., 1996; Schou et al., 1998). With the exception of a small number of structural inversions which have included limited sub-sets of the high-degree (l ≥ 150) p-mode frequencies, such as the pioneering sound speed inversion of
Christensen-Dalsgaard et al. (1985), the vast majority of the structural and abundance inversions have only included frequencies of the low- (ℓ ≤ 4) and the intermediate-degree (5 ≤ ℓ ≤ 150) oscillations.

As we will illustrate below, this limitation has prevented helioseismologists from accurately probing the thermodynamic structure of the outer solar convection zone. In particular, the persistent absence of high-degree frequencies from past inversions has prevented us from inferring the sound speed, density, and adiabatic gradient in the outermost three to four percent (by radius) of the solar interior. In fact, it is these very shallow sub-photospheric layers which are the most sensitive to the details of the formulation of the so-called “equation of state” (EOS) of solar material. The structure of these layers is also affected by the amount of helium that is present in them and hence any inversion for the helium abundance should include them. In spite of the exciting progress which has been provided by the recent helioseismic structural inversions, the nearly-complete exclusion of the high-degree frequencies sorely limits us from taking full advantage of the unique plasma physics laboratory which the outermost layers of the solar interior represent.

2.2. Potential Improvements in Determination of Solar Internal Sound Speed

The importance of extending the frequency datasets which serve as the inputs to solar structural inversions to include large numbers of high-degree frequencies was emphasized by Christensen-Dalsgaard (1998), who argued that there is an urgent need for a better understanding of the potential sources of error in the calculation of solar models and he also pointed out that most of the ℓ-v diagnostic plane has not yet been utilized in probing solar models through structural inversions. More recently, Christensen-Dalsgaard and some of his colleagues have presented detailed calculations which show the improvements which can be made in the mathematical functions known as kernel functions which are utilized in the structural inversions to provide the spatial sampling of the solar structural variables (Rabello-Soares et al., 2000). These kernel functions are important in structural inversions because the inversion results represent localized estimates of the sound speed or other variables which can be interpreted as convolutions of the true solar quantities with the localized averaging kernels. Clearly, a set of narrow averaging kernels is desirable in order to maximize the radial resolution of the inverted sound speed profile.

Rabello-Soares and her co-authors computed these kernel functions for both a set of MDI frequencies which did not include any high-degree frequencies whatsoever (from Schou, 1998; which they referred to as their Intermediate-Degree, or ID, set) and they also re-computed these same kernel functions using a dataset in which they replaced these intermediate-degree frequencies with a sub-set of the 7480 intermediate- and high-degree MDI frequencies which our team presented at the SOHO-6 Workshop (from Rhodes et al., 1998; which they referred to as their High-Degree, or HD, set). These two different frequency datasets are shown together in Figure 1a, which is taken from Rabello-Soares et al. (2000). In

Figure 1. a. (top) Comparison of ℓ-v coverages of ID mode set in gray and the additional points which were combined with the ID mode set to comprise HD mode set in black. Except for the f-ridge all of the ID frequencies corresponded to ℓ < 200. b. (bottom) Four pairs of radial averaging kernels computed using the HD mode set in black and using the ID mode set as the dashed curves.

In Figure 1a the points which are common to both the ID and to the HD sets are shown in gray, while the additional points in the more extensive HD set are shown in black. Some of the results of these calculations are shown in Figure 1b. This Figure, which is also taken from Rabello-Soares et al. (2000), shows the manner in which the inclusion of just these few additional high-degree frequencies causes the kernel functions to become sharper and higher than when only the intermediate-degree modes are employed. The kernels computed from the ID mode set are shown as the dashed curves in Figure 1b, while the corresponding kernels which were derived from the complete HD dataset are shown in black. Not only are the black kernels sharper and taller than are the dashed kernels, but the kernels for the two outermost target radii (e.g., r/R⊙ = 0.99 and 0.9975) are positive everywhere rather than having negative contributions as in the case of the dashed curves.

2.3. Importance of High-Degree p-Modes to Studies of the Equation of State

While the solar convection zone is adiabatically stratified, the solar material is not adiabatic at the very top of that zone. Rather, the gas is undergoing the transition from being largely-neutral to largely-ionized in the hydrogen and helium ionization zones which are located in the outermost few percent (by radius) of the solar interior. It turns out that helioseismology can probe important effects of the equa-
tion of state through the ionization-induced decrease of the adiabatic gradient $\Gamma_1$ that occurs in these ionization zones. For such a study, the most promising region is the second ionization zone of helium, which is sufficiently deep to be largely unaffected by the remaining near-surface uncertainties. A number of early papers (Berthomieu et al., 1980; Lubow, Rhodes and Ulrich, 1980; Ulrich, 1982; Ulrich and Rhodes, 1983; Shibahashi et al., 1983, 1984; Noels et al., 1984) suggested that improvements in the solar equation of state could reduce the discrepancies which then existed between the theoretical and observed solar oscillation frequencies. Later, Christensen-Dalsgaard, Däppen, and Lebreton (1988) showed that the MHD equation of state reduced these discrepancies for a large range of oscillation modes. Still later, in a comparison between solar models based on the "EFF" (Eggleton et al., 1973) and the Mihalas-Hummer-Däppen (MHD) (Hummer and Mihalas, 1988; Mihalas, Däppen, and Hummer, 1988; Däppen et al., 1988) equation of states, helioseismology clearly showed the effect of the leading-order Coulomb corrections (Christensen-Dalsgaard and Däppen, 1992; Däppen et al., 1993; Christensen-Dalsgaard et al., 1996).

An alternative approach to the equation of state was independently developed for the OPAL opacity project at the Lawrence Livermore National Laboratory (Rogers, 1986; Rogers et al., 1996). In the initial set of helioseismological tests of the MHD and OPAL equations of state it initially appeared that the OPAL EOS was providing a closer fit to the oscillation frequencies than was the MHD EOS (Christensen-Dalsgaard et al., 1996). A dramatic demonstration of the improvement which the inclusion of the high-degree frequencies can make in such EOS testing is shown here in Figure 2a, which is from Rabello-Soares et al. (2000). This figure shows the radial profile of the difference in the squared sound speed between two different solar models (in the sense MHD-OPAL) as the solid line and two different inversions which were carried out to infer this profile as the two sets of points. The inversion which employed only the ID mode set is shown as the set of gray triangles, while the inversion which incorporated the entire HD mode set is shown as the set of black circles. It is clear that the inversion which included the high-degree frequencies can measure the sound speed difference between these two solar models much more closely to the surface than can the other inversion. Furthermore, from the agreement between the solid line and the black circles, it would appear that the inclusion of the high-degree frequencies allows the inversion to do an excellent job of measuring this important difference. The addition of a complete set of high-degree frequencies should improve the accuracy of this comparison even more.

Another Figure which also shows the value of the high-degree frequencies in discriminating between the MDI and OPAL equations of state is Figure 2b. This Figure shows the differences in the intrinsic $\Gamma_1$ instead of the sound speed differences for the two models that were used to generate Figure 2a. Here the solid line is the radial profile of the differences in the intrinsic $\Gamma_1$ of the two models, while the gray set of triangles is the result of the inversion which incorporated only the intermediate-degree frequencies, and the black set of circles is again the result of the inversion which incorporated the high-degree frequencies as well. The solid line clearly shows the locations of the two different helium ionization zones and the black set of circles clearly does a better job of matching the solid line at the solar surface. Clearly, high-degree frequencies with even smaller errors will be helpful in better resolving the two ionization zones in future inversions.

2.4. Importance of High-Degree Frequency Splittings for Internal Dynamics Inversions

An area of great difficulty in the studies of the solar internal rotation just below the photosphere is the estimation of accurate rotational frequency-splitting
coefficients for \( \ell \geq 120 \). For example, Korzennik (1990) demonstrated that both the odd- and even-ordered frequency-splitting coefficients computed for such degrees exhibited sizeable discontinuities as functions of degree. Later, Rhodes et al. (1998) demonstrated the existence of similar jumps in the high-degree splitting coefficients computed from 1996 MDI power spectra. The lack of understanding of the true cause of these discontinuities has led to a rejection of ad hoc correction schemes and that rejection has led in turn to the near-absence of high-degree internal dynamic inversions. Instead, our knowledge of the dynamics of the shallow sub-photospheric layers has primarily come from the inversion of \( f \)-mode splittings for \( \ell \leq 300 \) (Kosovichev and Schou, 1997; Schou, 1999), from time-distance analyses such as those described by Kosovichev et al. (2001), and from the inversion of \( p \)-mode frequency splittings determined from ring-diagram analyses (Haber et al., 2002). Since inversions of accurate high-degree rotational splitting coefficients will be necessary for the confirmation of the results of these local helioseismic techniques, our new-found ability of computing rotational splitting coefficients which do not have large discontinuities in them, which we will demonstrate in Section 3.3, will be essential for such comparisons.

3. RECENT IMPROVEMENTS IN THE FITTING OF SOLAR POWER SPECTRA

3.1. Development of Our First \( p \)-Mode Fitting Method

In attempts to generate multiple sets of \( p \)-mode parameters which span widely different solar conditions which are also highly accurate, we have developed a total of three different methods for fitting the peaks in solar oscillation power spectra. We began our efforts by developing the first of our three generations of power spectral fitting techniques in order to fit the power spectra which were computed as part of the MDI experiment’s Medium-\( \ell \) Program. The first-generation technique which we developed has come to be known as the “single-peak, averaged spectrum” method (which we will refer to from now on as Method 1). In this method we employed a single, symmetric Lorentzian profile to fit each peak in each set of \( m \)-averaged spectra. However, the use of only a single Lorentzian profile to represent a set of closely-spaced oscillation peaks and sidelobes introduces errors into all of the \( p \)-mode parameters as is shown here in Figure 3a. In this panel we show that the use of a single Lorentzian profile to represent the \( \ell = 50, n = 10 \) oscillation mode does not adequately fit the lower portions of this peak due to the proximity of the spatial sidelobes to the central peak of interest. The scaled residuals of this fit are also shown in Figure 3b.

Isolated \( p \)-mode peaks can be measured only for degrees less than about 220, with the exception of the fundamental mode, which has been detected as a set of isolated peaks up to \( \ell = 296 \). In addition, even for degrees for which some individual modal peaks can be identified at low frequencies, these peaks become blurred together as the frequency is increased. This overlap of peaks and sidelobes produces a so-called “ridge” of observed power which must be fit. The problems inherent in the fitting of broad ridges rather than sharp, isolated peaks of observed power were first addressed by Libbrecht and Kaufmann (1988), who showed how these problems resulted in a so-called “frequency pulling” of the measured ridge-fit frequencies away from the “true” solar frequencies for the high-degree modes. Due to the severity of such frequency-pulling effects, we attempted to develop a frequency-correction procedure in which we fit the low- and intermediate-degree power spectral peaks with both a set of wide fitting ranges and with a set of narrow fitting ranges for our Lorentzian profiles. An example of the profile which resulted when the wide fitting range was employed to fit the same \( \ell = 50, n = 10 \) mode is shown here as Figure 3c. Even at the scale of this Figure it should be evident that the vertical dashed line which represents the frequency that Method 1 determined when the wide fitting range was employed does not agree with the location of the central peak in the observed spectrum. The scaled residuals between this “wide fit” and the observed profile are shown in Figure 3d. These residuals show the obvious disagreement between the fitted profile
and the individual sidelobes. We stress that while this disagreement is obvious in this case of a set of resolved peaks, the same problems are occurring in the cases of the broad ridges of power at the higher frequencies and higher degrees even though the observed peaks do not show evidence of the individual sidelobes. We tried to correct our "wide fit" frequencies for the effects of the mode-blending using the differences between the narrow- and wide-fit frequencies, but by the fall of 1998 we became convinced that such an approach would never be successful because it relied on an extrapolation of such frequency differences to much higher degrees than those modes for which both fitting methods could be carried out.

3.2. Development of Second Fitting Method: The WMLTP Method

The second-generation fitting method is our so-called "Windowed, Multiple-Peak, Averaged-Spectrum" Method, which we will refer to herein as either Method 2 or our WMLTP method. As its name implies, the WMLTP method is applied to m-averaged power spectra, just as was the case with Method 1. Initially, our WMLTP method incorporated a sum of five symmetric Lorentzian profiles and it included the m-averaged leakage matrix for the instrument which generated the power spectra being fit; however, due to the relatively high duty cycle of the MDI spectra which we developed this method to fit, we did not convolve the theoretical profile with the power spectrum of the so-called temporal window function of each observing run. An example of a fit using this early version of Method 2 is illustrated here in Figures 3e and 3f. These panels indicate that the absence of the temporal convolution caused the fitted profile to miss about one-half of the peaks in our observed MWO power spectrum. Subsequently, we modified the code to increase the number of profiles which were included to as many as nine and we also convolved the theoretical profile with the power spectrum of the actual temporal window function of each observing run. The dramatic improvement in the fitted profile which resulted when we computed the actual window function for our 1996 MWO time series is illustrated here in Figures 3g and 3h. The fitted profile now can be seen to pass through all of the peaks in the observed spectrum and the scaled residuals are relatively small and do not depend upon frequency. We also note that this version of Method 2 provided more accurate amplitudes and widths than did Method 1 or the earlier versions of Method 2.

In our most recent improvement to this WMLTP fitting method we have replaced the symmetric profiles with the asymmetric profile of Nigam and Kosovichev (1998). This change has removed one of the previous limitations to widespread use of the WMLTP method. The details of how the Nigam and Kosovichev profile is implemented in the WMLTP fitting method are given by Reiter et al. (2002a). An example of how well the asymmetric WMLTP Method fits observed power spectral peaks when the individual modal peaks can be separately resolved is given in Figure 1 of Reiter et al. (2002a). Additional comparisons of an observed power spectral ridge with fitted profiles which were computed using both the symmetrical Lorentzian and the asymmetric profiles are shown here in Figure 4. In Figure 4a we show the observed ridge for $\ell = 750$, $n = 2$ with the fit using the symmetric profile superimposed as the solid curve. In Figure 4b we show the residuals between the observed ridge and the symmetric profile. These residuals show obvious, systematic, frequency-dependent trends. In Figure 4c we show the same observational ridge as in Figure 4a, but with the asymmetric profile employed instead of the symmetric Lorentzian profile. In Figure 4d we show the residuals between the observations and the asymmetric profile. In contrast to the residuals of Figure 4b these residuals are much smaller in general and they do not show such large frequency-dependent trends.

In Figure 5a we show the $\ell - \nu$ diagram which resulted when we fit the m-averaged power spectra computed from the 1996 MDI Dynamics Run time series using the WMLTP method and the asymmetric profile of Nigam and Kosovichev (1998). In Figure 5b we present the frequency dependence of the Full-Width-at-Half-Maxima, FWHM, of the fits using the same spectra and the same profile. In Figure 5c we present the frequency dependence of the logarithm of the fitted power densities computed from the same spectra and the same profile. In Figure 6a we show the frequency dependence of the asymmetry parameter, $B$, as defined in equation (2) of Reiter et al. (2002a). In Figure 6b we show the frequency dependence of the differences between the frequencies computed using the symmetric Lorentzian profiles and the asymmetric profiles. Inspection of both panels of Figure 6 shows that the frequency differences have a frequency dependence which is very similar to that of the asymmetry parameter, with a strong oscillation in both quantities occurring for frequencies above 5000 $\mu$Hz. We have also compared the FWHM values and the power densities computed using the symmetric and asymmetric profiles and we have found that the FWHM values show the largest relative differences at the same high frequencies. However, the power densities have large relative differences both at low frequencies where the fitted power densities are low and at frequencies above 5000 $\mu$Hz, where the power densities are also low.
Figure 5. a.(top) E-U diagram of frequencies computed from MDI 1996 Dynamics Run m-averaged power spectra using WMLTP Method with asymmetrical profile. b.(middle) Frequency dependence of full-width-at-half-maximum of the same fits as in a. above. c.(bottom) Frequency dependence of the logarithm of the power density from same fits as in a. above.

3.3. Development of the “Maximum-Likelihood, Multi-Peak, Tesserar-Spectrum” Method

We have already discussed the large discontinuities which have been seen in the odd-ordered, rotational frequency splitting coefficients for degrees running from roughly $\ell = 140$ to $\ell = 220$. In an effort to compute splitting coefficients which did not suffer from such discontinuities, we developed a third-generation method of frequency estimation which employs the zonal, sectoral, and tesseral power spectra rather than using $m$-averaged spectra. This method, which we refer to as Method 3 allows to simultaneously obtain both splitting coefficients and frequencies. Some details of Method 3 are given in Reiter et al. (2002b).

As was the case for Method 2, either the symmetric Lorentzian profile or else the asymmetric profile of Nigam and Kosovichev (1998) can be invoked in Method 3.

Recently, a preprint written by Korzennik, Rabello-Soares, and Schou (2002) called attention to the much-earlier paper by Woodard (1989) in which he described the distortion which solar latitudinal differential rotation introduces into the spherical harmonics which govern the spatial behavior of the solar f- and p-modes. Specifically, Woodard (1989) showed that the distortion of high-degree p-mode eigenfunctions by a slow, axisymmetric differential rotation can be expressed as a superposition of the unperturbed eigenfunctions of the same radial order $n$ if Coriolis forces are neglected. Upon seeing this preprint, we immediately modified our Method 3 to correct the spatial leakage matrices which it employs to include the corrections for this distortion.

Figure 6. a.(top) Frequency dependence of asymmetry parameter, $B$, as defined by Reiter et al. (2002). a.(bottom) Frequency dependence of differences between frequencies computed using symmetric Lorentzian profiles and the asymmetric profile described in the text.

When we employed this modified version of Method 3 to the fitting of our reference set of 1996 MDI Dynamics runs zonal, sectoral, and tesseral power spectra, we discovered immediately that the discontinuities had disappeared from the odd-ordered splitting coefficients. Comparisons of our original, uncorrected odd splitting coefficients with our new, distortion-corrected splitting coefficients for the $n = 1$ $p$-mode ridge are shown here in Figure 7a, which is taken from Reiter et al. (2003). Even a cursory inspection of Figure 7a shows clearly that the corrections which Woodard (1989) described have removed the jumps in the odd coefficients. We are elated to have finally solved one of the most long-standing problems in the use of high-degree splittings in solar dynamics inversions. In addition to removing the jumps in the odd splitting coefficients, the inclusion of the differential rotation correction also improves the multiplet-averaged frequencies which result from Method 3. This is shown here in Figure 7b, which is also taken from Reiter et al. (2003), where we have compared the average multiplet frequencies with the theoretical frequencies of Model S calculated by Christensen-Dalsgaard et al. (1996). As can be seen the combination of radius corrections and optical distortion corrections caused by MDI instrumental problems with the perturbation of the resulting leakage matrix by solar differential rotation smoothes the fitted frequencies significantly.

4. STUDIES OF SHORT-DURATION P-MODE FREQUENCY SHIFTS

While it is now a well-established fact that the frequencies of the low- and intermediate-degree solar $p$-mode oscillations do indeed change with time in
response to changing levels of solar activity, there is currently no consensus as to the solar origin of these changes. This situation was summarized clearly by Kuhn (2001), who described the various changes which have been seen in solar $f$- and $p$-mode frequencies, frequency splittings, horizontal flow velocities, and solar diameter measurements as diagnostics of what he called the "acoustic solar cycle". Consequently, we have been working to improve this situation by employing short-duration time series in the computation and fitting of $p$-mode frequencies and widths. We have adopted this strategy because the use of lengthy time series as in most past studies of the temporal frequency shifts of these modes has averaged out the daily variations in the corresponding activity indices during those time series. Such extensive temporal averaging has likely reduced the sensitivity of the resulting $p$-mode frequencies to the changing levels of activity. It has been our hope that the use of short-duration observing time series, during which solar activity is more constant, may allow us to increase our sensitivity of the frequency changes to the mechanism which causes them.

The $p$-mode frequency datasets which we employed in this study were computed with our WMLTP fitting method. In this study this fitting method was applied to a total of five sets of $m$-averaged MDI power spectra which were computed from observing runs which were either 4320 or 4800 minutes in length. This fitting method was also applied to a total of six MWO sets of $m$-averaged power spectra which were all computed from observing runs which were slightly less than 4320 minutes long due to the time of sunset at the end of the third observing day of each run.

To extend previous studies of $p$-mode frequency shifts so that both the high-frequency and high-degree modes would be included in the comparisons, we compared all of our various MDI datasets and we compared the four of our six MWO datasets which had the highest duty cycles; however due to systematic shifts between the MDI and MWO frequency datasets at frequencies above 5000 $\mu$Hz, we chose not to cross-compare any of the MDI datasets with any of the MWO datasets. These inter-comparisons yielded a total of ten tables of MDI frequency shifts and a total of six tables of MWO frequency shifts. In order to study the dependence of these frequency shifts upon changing levels of solar activity, we also computed the average 10.7-cm radio flux and the average magnetic plage strength index for each observing interval. In the top panel of Figure 8 we show the average 10.7-cm radio flux values as a function of time for all 11 of our different frequency datasets and for other, longer-duration observing runs available to us from MWO, MDI, and GONG+ in order to place the levels of solar activity in the context of the most recent 12 years. This Figure indicates that we have been able to obtain frequency datasets covering more than one entire solar cycle.

In Figure 9a we show the frequency dependence of the unbinned frequency shifts between two datasets.
which correspond to minimum and maximum levels of solar activity in order to illustrate the frequency dependence of frequency differences corresponding to very large differences in levels of activity. Past studies of the temporal behavior of the p-mode frequencies have shown that such frequencies increased along with the rising level of activity between the two runs. Figure 9a shows that such increases were seen, but only up to a frequency of about 5000 μHz. On the other hand, for frequencies between 5000 and about 6200 μHz the frequencies actually went down from the first interval to the second. This negative “dip” in the frequency shifts is very similar to a similar dip in a comparison of unpublished 1990 and 1987 South Pole frequencies which was included in a review talk at the SOHO-4 Workshop by Harvey (1995). It is also similar to a dip in a comparison of 1991 Mees Obs. and 1987 South Pole data which was reported by Ronan, Cadora, and LaBonte (1995). We note that this “dip” in the frequency shifts is roughly centered at the location of the acoustic cut-off frequency as shown by Steffens et al. (1998). On the other hand, the dip is somewhat lower in frequency than the calculated frequency shifts published by Johnston, Roberts, and Wright (1995), and by Jain (1995) all of who found a strong dip in the shifts at a frequency of about 6200 μHz. In Figure 9b we show a similar comparison between two of our 3-day MDI frequency datasets. Once again, solar activity was increasing, but in this case the difference in activity was much smaller since both runs in Figure 9b were obtained near solar minimum; hence, the frequency differences in this panel are considerably smaller than are the comparable differences in Figure 9a.

To better illustrate the anti-correlation of the shifts in the lower and higher frequency regimes, we have plotted the binned frequency shifts as functions of frequency for all 10 of the MDI frequency-difference datasets in Figure 10. In Figure 10a we show the binned differences for the four intervals during which solar activity increased between the two runs by the largest amount, while in Figure 10b we show three of the sets of binned differences for time intervals when the activity was relatively unchanged, and in Figure 10c we show three sets of the differences which corresponded to decreases in the level of activity from the first run to the second run of each pair. The curves in Figure 10a show that in each of these sets of frequency differences the lower frequencies did increase as expected, but they also show that the higher frequencies ranging between 5000 and 6000 μHz dropped as shown in Figure 9. On the other hand, the curves in Figure 10c show clearly that the low-frequency shifts were in fact negative as would be expected from the earlier studies. Figure 10c also shows that above 5200 μHz the frequency shifts changed sign and the “dip” mentioned above was replaced by a “peak” in the frequency differences.

In order to study the response of our MDI and MWO frequency shifts to changing levels of solar activity in more detail, we first selected three different points along the curves of Figure 10. Specifically, we selected the frequencies of 3625, 4875, and 5625 μHz as being representative of the low- and high-frequency regimes of our curves. Next we subtracted the mean 10.7-cm flux values of the different observing runs from one another and we generated a table of differences in the radio flux. Next, we performed linear regression analyses of the different sets of frequency shifts upon the differences in 10.7-cm flux. In addition we also subtracted the frequency shifts located at the minima of the curves in the top panel of Figure 10 (i.e. at 5625 μHz) from the shifts located at the peaks of the curves at 4875 μHz. These differences illustrated the difference in behavior of the two different frequency regimes along the curves shown in Figure 10. The results of regressing the 16 binned shifts at 3625 μHz upon the 16 differences in the 10.7-cm flux are shown in upper-left panel of Figure 11, while the results of regressing the 16 shifts at 4875 μHz upon the same 16 10.7-cm flux differences are shown in the upper-right panel of Figure 11. Furthermore, the results of regressing the 16 frequency shifts in the “dip” at 5625 μHz upon the 16 10.7-cm flux differences are shown in the lower-left panel of Figure 11 and the results of regressing the 16 differences of the
frequency shifts at 4875 and 5625 µHz are shown in the lower-right panel. As with most past studies of the low- and intermediate-degree shifts, the shifts at 3625 and at 4875 µHz show positive slopes of 35.8 and 61.3 nHz/SFU, respectively, where SUF = 1 solar flux unit. These slopes are roughly ten to 20 times larger than the slopes found in most of the long-term studies, which were generally less than 4 nHz/SFU. In the lower panels of Figure 11 the slopes were -137.7 and 199.2 nHz/SFU, respectively. In both of these cases the absolute values of the slopes were substantially greater than the slopes in the upper two panels, suggesting that the higher frequencies have greater sensitivity to differences in solar activity. We also note that the fourth slope is simply equal to the difference in the second and third slopes. In all four panels of Figure 11 the ten mean MDI frequency shifts are shown as the xs, while the six mean MWO frequency shifts are shown as the triangles.

The regression analyses summarized in Figure 11 are merely examples of the relationship between the frequency shifts and changes in solar activity. In Figure 12 we show the frequency dependence of the regression slopes, intercepts, and correlation coefficients which resulted from our correlations of only the ten sets of MDI binned frequency shifts illustrated in Figure 10 with their associated changes in 10.7-cm flux. The upper two panels of Figure 12 show how the linear regression line rotates rapidly with increasing frequency above 5000 µHz, while the lower panel shows that the higher-frequency shifts were definitely anti-correlated with the lower-frequency shifts.

5. EXTENSION OF INTERNAL FLOW MEASUREMENTS TO SOLAR CYCLE 22 USING MWO 60-FOOT TOWER DATA

The availability of extensive time series of nearly-uninterrupted full-disk solar dopplergrams from the MDI experiment since April of 1996 has resulted in two exciting discoveries concerning the dynamics of the outer layers of the solar interior. The first of these two discoveries is the demonstration by Kosovichev and Schou (1997) and by Schou (1999) that the so-called “torsional oscillations” are not merely surface features, but instead extend inwardly throughout at least the outer one-third of the solar convection zone. The second of these two discoveries is the apparent emergence in 1998 of a submerged cell of meridional circulation in which the prevailing flow direction is opposite that at the surface (Haber et al., 2002). Due to the important implications of both of these discoveries, we are now planning to carry out retrospective studies of the torsional oscillations and the meridional circulation during Solar Cycle 22 in an attempt to see if the torsional oscillations extended as far inward during that cycle as during the current cycle and to see if a second cell of meridional circulation was present during that declining phase of Cycle 22. To carry out these studies we are planning to use yearly time series of full-disk Dopplergrams which we have been obtaining at the MWO 60-Foot Solar Tower since the summer of 1987. These observations provide a unique opportunity to extend the discoveries made with MDI data into the pre-MDI era.

6. CONCLUSIONS

We have demonstrated the importance of including the high-degree frequencies in structural inversions and we have demonstrated that we can now compute high-degree frequency splittings without the large discontinuities which were present in all previous studies of the high-degree splittings. We have also addressed the current controversy over the mechanism of the p-mode frequency shifts by our use of shorter time series than have generally been used in such studies. Specifically, we have shown that our different sets of computed frequencies of both the
intermediate- and high-degree modes have such high signal-to-noise ratios that we have been able to compare time series as short as three days in duration. We have also shown that many of the frequencies above 5000 μHz are anti-correlated with changes in solar activity and that they are also more sensitive to such changes in activity than are the frequencies below 4000 μHz. Hence, we have demonstrated the response of the p-modes to short-term changes in solar activity. Both of these results indicate that short-duration, high-frequency shifts are more sensitive to changes in solar activity than are the shifts measured by long-duration studies which use only the lower frequencies. Lastly, we have described plans to study the dynamics of the interior during Solar Cycle 22.

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PROBING THE SOLAR CORE WITH BISON: THE CHALLENGE AT LOW $\ell$ AND LOW FREQUENCY

W. J. Chaplin$^1$, Y. Elsworth$^1$, G. R. Isaak$^1$, B. A. Miller$^1$, R. New$^2$, and B. Pintér$^2$

$^1$School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, U.K.
$^2$School of Science and Mathematics, Sheffield Hallam University, Sheffield, S1 1WB, U.K.

ABSTRACT

In this contribution we touch upon a few issues of relevance to the current status of low-angular-degree (low-$\ell$) p-mode Helioseismology. In particular: the precision in frequency, both historic and current, achievable at low $\ell$; the quest to extend the low-frequency detection threshold nearer to the p-mode fundamental; and the level of agreement between frequencies extracted from different data using various analysis techniques.

Key words: Sun: interior – Sun: oscillations.

1. FREQUENCY PRECISION AT LOW $\ell$

When the first estimates of individual low-$\ell$ frequencies were presented some two decades ago (Claverie et al. 1981; Grec, Fossat & Pomerantz 1983) the values given possessed fractional precisions of the order of a few parts in $10^4$. The accumulation of much greater quantities of data, together with improvements in the quality of the observations, has increased this to modern levels of several parts in $10^6$, i.e., an improvement by two orders of magnitude. Chaplin et al. (2002a) discussed these issues at length, and, furthermore, demonstrated excellent consistency in the magnitudes of uncertainties quoted for different data (e.g., MDI and GOLF) when allowance was made for the observation length, $T$, and any reduction in signal-to-background ratio caused by the imposition of a window function (e.g., for ground-based networks like BISON and IRIS).

If a given mode is well resolved in the frequency domain, so that a prominent signature of the resonance is observed to lie over several bins, the precision with which its frequency can be determined should scale as $T^{-1/2}$ (e.g., Libbrecht 1992). Chaplin et al. divided the BISON database into contiguous pieces of different length, and in so doing were able to demonstrate such a dependence by measuring the precision achievable at each length (having allowed first for solar-cycle variation). Given such behaviour, it is interesting to look back in the historic record to determine whether levels of precision quoted are consistent with those given in modern analyses. Examples of the latter include: Thiery et al. (2000), Garcia et al. (2001) and Gelly et al. (2002) for GOLF; Bertello et al. (2000a, b) for GOLF and MDI; Toutain et al. (1998) for MDI; and Chaplin et al. (1999; 2002b) for BISON.

The first low-\ell frequencies to be presented with associated uncertainties in the refereed literature were those of Elsworth et al. (1991). These came from ten 2-month stretches of observation made by the early 2/3-station BISON over the period 1981 through 1988. While in modern practice it is usual to fit models to peaks in the raw frequency spectrum that maximize a merit function which reflects the underlying negative-exponential statistics (Duvall & Harvey 1986; Anderson, Duvall & Jeffries 1990; Toutain & Appourchaux 1994), Elsworth et al. used a non-distorting smoothing function and then fitted under the assumption that normally distributed errors applied.

Their quoted radial-mode uncertainties (i.e., the sample errors characteristic of a single 2-month set) are rendered as stars in Fig. 1. Shown also are the Chaplin et al. (2002a) uncertainties that come from the analysis of a 10-yr BISON set. The dotted line shows the Elsworth et al. uncertainties extrapolated to $T = 10$ yr, under the assumption that they scale as $T^{-1/2}$. These are used to match quite well the errors given for the genuine 10-yr set. That there remains a suggestion of some residual overestimation is only to be expected. No allowance was made by Elsworth et al. for changes in frequency across the solar cycle when they averaged their ten sets; and their detections at low frequencies may have been more marginal than those achievable in modern data of the same duration. Nevertheless, these results indicate that the levels of precision quoted were consistent with those expected, as based on comparison with the more extensive database since accumulated.
2. THE CHALLENGE AT LOW FREQUENCIES

One of the major observational challenges of modern low-\(\ell\) Helioseismology is the goal of detecting unambiguously modes well below 1000 \(\mu\)Hz, with the ultimate aim of detecting the fundamental and encroaching upon the mixed and \(g\)-mode regime. Several different techniques have been applied in an effort to uncover low-frequency \(p\) modes (e.g., Bertello et al. 2000b; Garcia et al. 2001; Chaplin et al. 2002b) and \(g\) modes (Appourchaux et al. 2000; Gabriel et al. 2002; Wachtet al. 2002).

2.1. Mode lifetimes: impact on detectability

The majority of modes studied to date are those for which the resonant peaks are resolved in the frequency domain, i.e., the linewidth, \(\Delta\nu_{\text{nl}}\), extends over several bins. The transition to the unresolved regime, where all power from a mode resides within one bin, occurs at the threshold (e.g., see Chaplin et al. 2002c):

\[
T = 2/\pi \Delta\nu_{\text{nl}} = 2\tau_{\text{nl}},
\]

where in the above \(T\) is the observing time and \(\tau_{\text{nl}}\) the lifetime (e-folding time) of the mode.

An estimate of the locus of the threshold frequency along which \(T = 2\tau_{\text{nl}}\) for low-\(\ell\) modes is shown in Fig. 2 as a solid line. It separates the 'resolved' regime above, and the 'unresolved' regime below. The dashed lines mark the 1\(\sigma\) uncertainty envelope on the location of the locus. Also shown (crosses) are the locations in frequency of the \(n = 1\) to 9 radial modes (from model 'S' of Christensen-Dalsgaard et al. 1996). The estimate of the location of the locus was derived by extrapolating a power-law fit of the mode widths fitted over 1500 \(\leq \nu \leq 2000\) \(\mu\)Hz (see Chaplin et al. 2002a).

With the current lengths of well-filled data available to observers (e.g., roughly \(T \approx 11\) yr from BiSON; \(T \approx 6\) yr for MDI and GOLF) it is evident from Fig. 2 that the \(T = 2\tau_{\text{nl}}\) threshold lies at approximately \(900\) \(\mu\)Hz if one assumes the complete sets are used. The lowest frequency \(p\) mode for which independent corroboration is available at low \(\ell\) from two different instruments is \(\ell = 0\), \(n = 6\) at \(\approx 972\) \(\mu\)Hz (Bertello et al. 2000b; Garcia et al. 2001 from GOLF; Chaplin et al. 2002b from BiSON data). This suggests that only now are we beginning to encroach on the unresolved regime.

In this regime we will be denied the benefit of power being present across several bins; when this is the case tests can be applied that look for the signature of damping, e.g., those developed by Chaplin et al. (2002b). However, the signal-to-background level (in power) in the modes will improve linearly with \(T\) (assuming a constant noise-power spectral density); in contrast, in the regime where the modes are well resolved the signal-to-background remains constant (Chaplin et al. 2002c).

To show explicitly why this is so, consider a time series of length \(T\), sampled at a cadence of \(\Delta t\). For simplicity, take the example of a simple set that contains two components: first, a stochastically excited \(p\) mode with mean-square power \(V_{\text{nl}}^2\) that is damped, giving linewidth \(\Delta\nu_{\text{nl}}\); and second, a normally distributed ('white') noise source with zero mean and sample standard deviation \(\sigma\). When the power spectrum of the time string is computed this yields a peak of maximum height \(H_{\text{nl}}\) against a background level \(B\). The mean level \(B\) which satisfies Parseval's
equation in the frequency domain is:

$$B = 2\sigma^2 \Delta t,$$

where we take a power spectral density (power per Hertz) scaling.

The way in which $H_{nt}$ relates to the other modal parameters depends upon whether the mode is resolved or unresolved. In the oversampled or resolved regime, the lifetime of the mode, $\tau_{nt}$, is smaller than the observing time $T$. When $T \gg \tau_{nt}$, the time series will then extend over a whole series of independent realizations and the modal peak will be well resolved (i.e., lie across many bins) in the frequency domain. What follows is based upon the explicit assumption that the underlying mode profile can be described as a Lorentzian function. Even though the modal peaks are asymmetric at low $\ell$ (Toutain et al. 1998; Chaplin et al. 1999; Thiery et al. 2000), the magnitude of this is so small as to make a Lorentzian description sufficiently accurate for our purposes here. The maximum height (power density) of the underlying limit form of the mode in the power spectrum (taking a mean square scaling) is then:

$$H_{nt} = \frac{2V_{nt}^2}{\pi \Delta \nu_{nt}}.$$ 

Were $T$ to be reduced to such an extent that $T \ll \tau_{nt}$, the underlying profile would be so narrow as to confine all power within one bin. Here, we assume that the signal is commensurate with the window function. In the resulting unresolved regime we have conditions that therefore tend toward those expected of a simple undamped sine wave,\(^1\) where $H_{nt} \sim V_{nt}^2 T$. With reference to Equation 3, the changeover to this regime occurs when $T = 2/\pi \Delta \nu_{nt}$ (cf. Equation 1).

So, a full description of $H_{nt}$ in terms of the other parameters is:

$$H_{nt} = \begin{cases} \frac{(2V_{nt}^2)/(\pi \Delta \nu_{nt})}{T \gg 2\tau_{nt}} \\ \sim V_{nt}^2 T & T \ll 2\tau_{nt} \end{cases}$$

(4)

The signal-to-background ratio at which the mode appears is therefore:

$$H_{nt}/B = \begin{cases} \frac{(V_{nt}^2)/(\pi \sigma^2 \Delta \nu_{nt} \Delta t)}{T \gg 2\tau_{nt}} \\ \sim (V_{nt}^2 T)/(2\sigma^2 \Delta t) & T \ll 2\tau_{nt} \end{cases}$$

(5)

Equation 5 shows that provided the mode is unresolved ($T \ll 2\tau_{nt}$) the signal-to-background ratio increases linearly with $T$. However, given fixed noise and mode characteristics, and a chosen observational cadence, it is constant and independent of $T$ once the mode is oversampled and resolved (i.e., for $T \gg 2\tau_{nt}$).

\(^1\)Here, we refrain from using an exact equality in the equation since the actual form the peak will take – the sinc-squared function of the envelope of the time series – will lead to some, albeit minimal, power leakage.

2.2. Frequency differences from various observations at low $\ell$ and low $n$

In Fig. 3 we show the difference in reported low-$n$, low-$\ell$ frequencies of four analyses of GOLF or MDI data, and the corresponding frequencies of Chaplin et al. (2002b). All five sets have been subjected to a procedure that corrects for the solar cycle (e.g., see Chaplin et al. 1998). Differences at $\ell = 0$ are plotted as square crosses; those at $\ell = 1$ as triangles; and those at $\ell = 2$ as diagonal crosses. The sources of the four comparison sets are: Toutain et al. (1998) for MDI; Bertello et al. (2000a, b), Garcia et al. (2001) and Gelly et al. (2002) for GOLF.

At first glance the level of agreement would appear to be reasonably satisfactory. However, as pointed out by Chaplin et al. (2002b), one must remember that since we are all observing one and the same star the stochastic signature of the modes, and much of the solar background (which may show some variation depending upon the observational technique), will be correlated strongly in independent datasets that cover the same epoch. While the comparison sets considered here are not strictly contemporaneous, the extent of correlation should nevertheless be borne in mind when the significance of differences between sets that overlap is considered. (Note that the errors shown here were derived from simply adding the separate uncertainties in quadrature, i.e., under the erroneous assumption of complete statistical independence.)

To illustrate the importance of correlation, consider the simple example of taking the difference of two observables, both of which possess 1σ uncertainties of similar magnitude $\delta \nu$. When the two are uncorrelated the error in the difference will be given by:

$$\sigma_{no-corr} = \sqrt{2}\delta \nu.$$ 

However, if they are correlated – such that the coefficient describing this is $\rho$ – the above must be modified to (e.g., Barlow 1995):

$$\sigma_{corr} = \sqrt{2}\delta \nu \cdot \sqrt{1 - \rho},$$

so that:

$$\sigma_{corr} = \sqrt{1 - \rho} \cdot \sigma_{no-corr}.$$ 

Chaplin et al. (1998) recognized this point when they considered the statistical distribution of the differences in frequency of BISON and GOLF, MDI and LOI data available from the first few months of the ESA/NASA SOHO mission. They found a clear reduction in the absolute median value of any given set of differences (which should be $\sim 0.67$ for uncorrelated data) when contemporaneous comparisons were made. For example, the absolute median difference between contemporaneous 8-month BISON and GOLF sets was found to be $\sim 0.36$; if one seeks recourse (naively) to Equation 6, this suggests an average degree of correlation between these data of $\sim 70$ per cent. Since the extent of correlation will be frequency dependent (i.e., it will be affected
Figure 3. Difference in reported low-\(\ell\) frequencies of four analyses of GOLF or MDI data, and the claimed BiSON detections of Chaplin et al. (2002b). All five sets have been subjected to a procedure that corrects for the solar cycle (e.g., see Chaplin et al. 1998). Differences at \(\ell = 0\) are plotted as square crosses; those at \(\ell = 1\) as triangles; and those at \(\ell = 2\) at diagonal crosses. The sources of the four comparison sets are: Toutain et al. (1998) for MDI; Bertello et al. (2000a, b), Garcia et al. (2001) and Gelly et al. (2002) for GOLF.
by the signal-to-background ratio, contributions to the background, and the influence from variations in analysis) a thorough comparison demands that the coherency (e.g., Koopmans 1974) be computed across the spectrum.

With reference again to Fig. 3, the \( \ell = 0 \), \( n = 8 \) difference is somewhat out of line (combined uncertainty) in both the Bertello et al. and Garcia et al. plots (the Bertello et al. and Garcia et al. measures agree well); furthermore, the Gelly et al. difference is actually outside the plotted range (difference 0.16 \( \mu \)Hz) suggesting some disagreement concerning claims for this mode. That there are not a greater number of such disparate comparisons is, perhaps, surprising. This is because under conditions where the S/N is low a detection may be made on account of the presence of prominent power in one bin only, whilst the underlying mode peak may be several bins wide (which, given Fig. 2, is probably the case for all modes bar \( \ell = 0 \), \( n = 6 \) in Fig. 3). Under these circumstances there is a tendency to underestimate the true uncertainty associated with the claimed detection, since a value for the centroid corresponding to – or heavily dependent upon – the location of the lone, ‘prominent’ bin may be biased. This indicates that a great deal of care must be taken in arriving at suitable estimates of the errors.

ACKNOWLEDGMENTS

We would like to thank all those who are – or have been – associated with BiSON, in particular: H. K. Williams, J. Litherland and J. Allison in Birmingham and P. Fourie at SAAO: and our hosts at the South African Astronomical Observatory (SAAO); the Carnegie Institution of Washington; the Australia Telescope National Facility (CSIRO); E. J. Rhodes (Mt. Wilson, California); and members (past and present) of the IAC, Tenerife. BiSON is funded by the UK Particle Physics and Astronomy Research Council.

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NEWS FROM THE IRIS PROJECT

G. Grec\textsuperscript{1} and F. X. Schmider\textsuperscript{2}

\textsuperscript{1}D\textsuperscript{é}partement Cassini, UMR 6529 du CNRS, Observatoire de la C\textsuperscript{ô}te d'Azur, BP 4229 06304 Nice, France.
\textsuperscript{2}D\textsuperscript{é}partement d'\textsuperscript{A}stronomie, UMR 6525 du CNRS, Universit\textsuperscript{é} de Nice Sophia-Antipolis, 06108 Nice Cedex 2, France.

ABSTRACT

The IRIS network is presently redefined, including 2 original instruments and 2 collaborations. The main objective is to survey the variation of the p-mode spectrum with the solar activity.

An new instrumental design is under development, using a static Fourier spectrometer for the Doppler shift monitoring. It is now used for the jovian seismology. This compact design can be used for helioseismology or bright stars and is likely suitable for a space experiment. Used together with a classical instrumentation, this alternative design can cancel the "solar noise" and improve the detection of the low-frequency modes.

1. STATUS OF THE IRIS NETWORK

The network is now the result of the collaboration of 3 formerly independent networks and the merging of their data, coming from instruments using different designs (Salabert \textit{et al.} 2002):

- The 2 original IRIS instruments in Ouzbекistan and Morocco, the first being now in Parkent and the second waiting for the replacement of parts. Iris uses a sodium atomic resonance filter.
- The LOWL instruments located on Mauna Loa. That is an imaging device using a potassium filter. The velocity is averaged over the whole disk.
- The MK 1 instrument in the Teide Observatory. It uses a potassium filter.

There is a wide collaboration in data analysis devoted to p modes and the variation of their frequency during the solar cycle. A data base of the Doppler shift observed during the years 1989 to 1999 is available at http://www-astro.unice.fr/iris/Irisdata/Distri_1.1/page.html.

The future of this collaboration will include jovian seismology (see Sec. 2) and stellar seismology: photometric observation of Alpha Cen A and B from Concordia, the permanent station presently built in the Dôme C area, at an elevation over 3000 m on the antarctic plateau.

2. A STATIC INTERFEROMETER FOR ASTROSEISMOLOGY

2.1. Fourier Transform Interferometer used for Doppler imaging

The line of sight velocity for the seismology of astronomical source (mainly the Sun, so far) is usually deduced from the measurement of the Doppler shift of a spectral line. That is made using an optical filter like do IRIS (Grec \textit{et al.} 1991), GOLF (Gabriel \textit{et al.} 1995) and the other devices using atomic resonance, or using the Michelson interferometer introduced for GONG (Leibacher \textit{et al.} 1996) and in spatial version for MDI (Scherrer \textit{et al.} 1995).

Alternatively, an interferometric device tuned with a high difference of the optical path of the 2 optical beams can produce the Fourier Transform of the optical spectrum. Instead of the optical wavelength, the variable is there a geometrical position. That is the so-called FTS, or Fourier Transform Spectroscopy. In the usual implementations, a tuned interferometer is used to scan a domain of this space (Mosser \textit{et al.} 1998). The Doppler shift is deduced from the global drift of the optical spectrum (Fig.1).

In the ideal case of an optical bandpass containing a periodic feature of several lines, its Fourier transform will consist of a single fringe in the interferometric figure. Monitoring such a single fringe can be done with a fully static interferometer such as a Mach-Zender one. In this case a different optical measurement provides the phase shift of the fringes and then the Doppler signal (Fig.2).

Such a device presents several specific properties:

- For a selected bandpass, the sensitivity to the velocity signal is proportional to the optical path difference (OPD).
- A proper selection of the optical bandpass ensures a good contrast of fringes at a high OPD. The OPD of the interferometer is then adjusted to the same value.

- At opposite to instruments monitoring one line (GOLF, MDI), it is not very sensitive to line profile variations.

- It can easily be designed as an imaging device, to be used for helioseismology with angular resolution. It can also be used for stellar seismology.

- Several polarizations of the entrance beam can be monitored simultaneously for polarimetry.

2.2. The device

The device is designed as a focal equipment for any telescope. Starting from the entrance, the successive parts are:

- Field diaphragm, lens and prefilter.
- Interferometric prism.
- Wollaston.
- Data acquisition camera

The main optical parts are shown on the Fig. 3. The interferometer is made from 2 parallelepipeds, glued on a beam splitter. This assembly produces 2 output beams with different optical path: for one arm of the interferometer the reflexion on the external face is due to a metallic coating, for the second arm that is a total dielectric reflexion on the glass to air dielectric. That produces a \( \pi/2 \) phase difference for the 2 polarisations. A metallic/dielectric/metallic coating is used for the beam splitter. Due to the existing symmetries in optical paths, this coating introduces no phase shift.

The 2 parts of the interferometer are made in different glasses. The materials are selected for their dilatation coefficient and the dependency of their optical index on the temperature changes. The objective is to reduce as low as possible the sensitivity of the optical path difference to the thermal variations.

The Wollaston separates the 2 polarisations of each beam. This scheme gives in same time 4 output images with different phases. The optical throughput is firstly related to the size of the Wollaston, it corresponds now to a 60 arcsec field for a 2m telescope.

No moving part is needed. The measurement is a differential over the simultaneous images and suppresses the camera-shutter noise.

2.3. Principle of measurement

The bandwidth \( \Delta \sigma \) is selected around the central wavelength \( \sigma_0 \), the interferometer produces the Fourier transform of this optical spectrum (OFT).

In the OFT the fringes have a period \( P = 1/\sigma \) and vanish within a length \( L = 1/\Delta \sigma \). The pattern of the spectral lines in the optical spectrum produces fringes at higher optical path difference (OPD).

When the optical spectrum is moved due to Doppler effect, the phase of the fringes observed for a fixed OPD will change (Fig. 2). The actual relative phase \( \phi \) is calculated from the intensity measured on the corresponding points of each 4 images, this calculation involving a calibration table, as usual for any field-dependent device. The variation \( V - V_0 \) of the velocity is deduced from the Eq. 1.

\[
\phi = 2\pi \int f(x, y) \Delta \sigma \frac{V - V_0}{c}
\]

where \( f(x, y) \) depends on the position \( x, y \) in the focal plane, \( \Delta \sigma \) is the optical passband selected by the prefilter, \( c \) the velocity of the light. The function \( f(x, y) \) is rather complex but can be calculated. It comes also from the instrumental calibration. The dependency of the optical path difference on the position \( x, y \) is included in \( f(x, y) \).

The better sensitivity for the velocity measurement depends on the selected bandpass together with the corresponding adjustment of the OPD. The signal to noise ratio (i.e. the sensitivity) depends on the product of the fringes contrast measured in the Fourier transform of the observed optical spectrum by the optical path difference (Fig. 4).

As a first analysis, identical changes of the profile of several lines correspond to a variable convolution in the OFT. Then they don't impact on \( \phi \), i.e. the measured velocity, but only on the fringe contrast. That is the case for similar lines coming from the same optical depth in the Sun, the sensitivity to the convective motions is then reduced.

2.4. Present status of the instrument

A prototype of this device has been developed by F. X. Schmider and J. Gay for the jovian seismology, a network project called SYMPA (Gay & Schmider 2000). Preliminary observations have been performed, showing performances in agreement with theoretical ones (Fig. 5 and 6).

The following performances are expected from the preliminary analysis:

- Photon noise on Jupiter: \( 4 \text{ cm s}^{-1} \) in 8 h on a 1.5 m telescope.
• Residual drift: 16 m s$^{-1}$ °K$^{-1}$, requiring 1 mK temperature accuracy, rms value integrated corresponding to the temporal frequency resolution of an expected jovian mode.

• Detection of modes of degree $l \leq 25$ when the seing is better than 1.5 arcsec.

2.5. Conclusion

This device is now at a low level of integration, so far designed to be used with the horizontal beam of a Nasmyth focus. Nevertheless, this compact design, using a static optical device and a CCD camera) can make a good competitor for a space detector. It requires a small 5 cm diameter collector in the case of solar measurements.

The major limitation in the low-frequency solar observations is the so-called solar noise. The understanding and the reduction of this noise is one of the challenges of the emerging projects.

The detection of the Doppler shift on a single absorption line results from the measurement of the relative variations of the photometric level in a few spectral windows along the wings, like it is done for GONG (Leibacher et al. 1996) and on SoHO the instruments GOLF (Gabriel et al. 1995) and MDI (Scherrer et al. 1995). The line shape is a complex function (Roddier 1965) and may change along the time due to the convection, the local magnetic effects, the varying pattern of the active zones, etc... The measurement is sensitive to the changes of the line shape, as a difference of the behaviour of the "red" and the "blue" wing can be misinterpreted as a Doppler shift.

The measurement of the Doppler shift using the Fourier transform of the optical spectrum is sensitive to the spacing of several lines. The changes in the optical spectrum have a specific effect in this case, dependingly on the correlations existing between the shape of the selected lines.

The 2 measurements are then complementary and their association can improve the detectability of the low-frequency solar modes.

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Figure 1. The use of an interferometric device for the measurement of a Doppler shift. An FTS monitor the variations of the fringes pattern related to the Fourier transform of the optical spectrum.

Figure 2. Example of a set of interferometric images. A linear combination of 4 simultaneous images gives the velocity field. The image analysis depends on the optical path difference. For a given point of the image, we determine the phase $\phi$ of the observed fringe from the signal level $A, B, C, D$ for the 4 images and from the calibration (including the position in the focal field). The velocity is calculated from the Eq. 1.
Figure 3. Functional scheme.

Figure 4. The solar spectrum is limited to a narrow band centered at the wavelength $\sigma = 6170\text{Å}$. The optical prefilter select a bandpass $\Delta\sigma \approx 50\text{Å}$ (upper panel). OFT is the Fourier transform of this function. The sensitivity $S$ of the interferometer (lower panel) is the product of the fringes contrast measured in OFT by the optical path difference. The maximum sensitivity corresponds to the sharp line at ODP $\approx 1.24$ cm. This line contains several thousands of fringes, the interferometer is tuned to select one of those. The vertical scale for $S$ is the raw value in cm. The s/n ratio of the velocity measurement depends on $S$. 

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Figure 5. Mechanical and optical scheme of the prototype used for Jupiter observations. The frame is a 40 cm square. No moving part is required for the Doppler measurement. As the choice of glasses compensates the thermal drifts, no thermal control is implemented. The acquisition camera is a commercial product.

Figure 6. The FTS used at the Nasmyth focus of the 1.5m telescope at Calern Observatory. The image on top is the result of a single exposure during the Jupiter observations, showing the 4 interference patterns.
UNDERSTANDING THE CONVECTIVE SUN

Regner Trampedach\textsuperscript{1}, Dali Georgobiani\textsuperscript{1}, Robert F. Stein\textsuperscript{\textdagger}, and Åke Nordlund\textsuperscript{2}

\textsuperscript{1}Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA, tel: +1 517-355-9200 x2444/fax +1 517-353-4500, E-mail: trampedach@pa.msu.edu

\textsuperscript{2}Theoretical Astrophysics Center and Astronomical Observatory, Juliane Maries Vej 30, DK-2100 København Ø, Denmark

\section*{ABSTRACT}

Hydrodynamical simulations of the surface layers of the Sun, has greatly improved our understanding and interpretation of solar observations.

I review some past successes in matching spectral lines, improving the agreement with high-degree p-mode frequencies and matching the depth of the solar convection zone without adjustable convection-parameters.

Our solar simulations contain p-modes, and are used for studying the asymmetry of p-mode peaks and to calibrate the conversion between the observed velocity proxies and the actual velocities.

Key words: Convection; Solar atmosphere; Solar structure; Helioseismology.

\section{1. INTRODUCTION}

One of the main lessons from the convection simulations, is that the interplay between the radiative field and the hydro-dynamics plays a decisive role in determining the structure of the Solar atmosphere (Stein & Nordlund, 1989).

The convective driving force is buoyancy produced by the radiative cooling in the photosphere: The hot plasma, rising from the interior, does not loose entropy until very close to optical depth, $\tau = 1$. Here the plasma cools precipitously and is recycled back into the convection zone, in narrow and turbulent down-drafts. The super-adiabatic gradient at the top of the convection zone is dominated by the entropy deficient down-drafts. A good overview of the properties of the simulations was written by Stein & Nordlund (1998).

These asymmetries between the smooth up-flows and the turbulent down-drafts, have an effect on both the stratification of the Solar atmosphere and on the emergent light from the photosphere. And except for neutrinos, this light is all we can observe of the Sun. These differences with respect to 1D stellar atmospheres affect helio-seismology by changing the resonant cavity of the $p$-modes and by improving the interpretation of, e.g., dopplergrams. Furthermore, since the simulations hide nothing from us, they can be used as ideal test-beds for theories about wave-propagation, -excitation (Nordlund & Stein, 2001; Stein & Nordlund, 2001; Skartlien et al., 2000), -damping and other interactions.

The convection simulations provide a self-consistent and parameter-free stellar atmosphere model. There are no free convection parameters (mixing-length has two) and there are no micro- and macro-turbulence parameters. The numerical resolution is a parameter in the simulations, but it is not entirely free. Both the stratification and spectral line-profiles converge at finite resolution, as found by Asplund et al. (2000a).

Validation of the convection simulations by all Solar observations will therefore give us more confidence in simulations of other stars, than can conventional 1D stellar atmospheres. The emerging field of astero-seismology is a potential benefactor from this.

\section{2. PAST SUCCESSES}

Improved physics, e.g., going from one to three dimensions, is not reason enough for embracing a new model. It also has to improve the agreement between models and observations before we can have confidence in predictions by the new model. Therefore the present, short review of our past successes

\subsection{2.1. Depth of the Solar Convection Zone}

Rosenthal et al. (1999) used a horizontal and temporal average of a 3D Solar simulation as an atmosphere for a conventional 1D Solar structure
model (Christensen-Dalsgaard & Frandsen, 1983; M_0=1.00 T_\text{eff}=5770.21 g_\text{surf}=2.74e+04 \alpha=1.8473

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Comparison of 1D solar model with horizontally- and temporally averaged convection simulation.}
\end{figure}

Christensen-dalsgaard, 1982). The 1D structure model was based on the same composition, equation of state and opacities as the 3D simulation, and in the atmosphere Rosenthal et al. (1999) used the temporally and T-\tau averaged relation from the simulation (Trampedach et al., 2002).

This consistency between the 3D- and the 1D-model, allowed for a matching of the adiabat of the Solar convection zone. The entropy in the bulk of the Solar convection zone is determined by the entropy jump at the surface and depends strongly on the detailed interplay between convection and radiation. This region is much better described by the 3D simulations, than by 1D mixing-length-type models. Further down in the convection zone, all convective perturbations become small and can be linearized, and convection is governed by overturning of the rising plasma caused by simple mass conservation on the back-drop of the radial density gradient. This regime is adequately described by a mixing-length model, but its two parameters are undetermined by the theory. Matching density and temperature at a common pressure point, deep in the simulation, allowed a calibration of the main mixing-length parameter, \alpha_0 = 1.8473 \pm 0.002.

The 3D- and 1D-models, matched and combined in this way, has a convection zone depth of d_{\text{CZ}} = 28.61 \pm 0.02\%, in good agreement with that inferred from inversion of helioseismic observations, d_{\text{CZ}} = 28.7\% \pm 0.3\% (Christensen-Dalsgaard et al., 1991) and d_{\text{CZ}} = 28.7 \pm 0.1\% (Basu & Antia, 1997).

The uncertainties for both the \alpha-calibration and the convection zone depth, are based on the scatter obtained when performing the matching on individual time-steps of the simulation. Systematic effects are not included.

Fig. 1 compares the averaged simulation with the matched 1D-model on the common depth-scale. The zero-point of the \(z\)-scale is where the averaged simulation has \(T(z) = T_{\text{eff}}\).

From the first three panels of Fig. 1 we notice how the atmosphere is expanded with respect to the 1D model. This expansion has two approximately equal contributions: 1) An opacity effect. Since convective temperature perturbations in the photosphere become rather large (order of 10^3 K), and the opacity is highly non-linear because of the very temperature sensitive H^\(-\) ion, perturbations to the radiation field are asymmetric between the up-flows and the down-drafts. The high opacity in granules hides the warm rising plasma from our view and is not compensated by equally warmer down-drafts. This leads to a sort of convective back-warming, increasing the pressure-scale-height and expanding the atmosphere. 2) A dynamical effect, commonly called turbulent pressure:

\[ P_{\text{turb}} = \langle \langle u^2 \rangle \rangle, \]

where \langle \langle \rangle \rangle denote horizontal and temporal averaging. The turbulent pressure is displayed in the lower right panel of Fig. 1, and it contributes more than 13\% of the total pressure in the photosphere. The turbulent pressure is not well described and normally not included in mixing-length models, but to perform the matching to the simulation, it was necessary to include it at the matching point and it was truncated above (dashed line).

From the lower left panel of Fig. 1 we see that the super-adiabatic part of the convection zone is confined closer to the photosphere than compared to the 1D model. This gives a larger super-adiabatic peak (since the area is the same, describing the same entropy jump) leading to an overall larger temperature gradient in the photosphere.

2.2. p-mode frequencies

Christensen-Dalsgaard et al. (1996) used the matched and combined 1D- and 3D-model of the Sun described in Sect. 2.1 for calculating adiabatic p-mode frequencies.

Fig. 2 compares the frequencies from a 1D Solar model, in this case model S by Christensen-Dalsgaard et al. (1996), with the observed. The frequency differences are weighted by the mode-mass, and we see that the differences are predominantly a function of frequency. This is the signature of errors in the surface layers of the model.

Fig. 3 provides a similar comparison, but this time using the matched and combined 1D- and 3D-model from Sect. 2.1. We see a dramatic change, with the
high frequency differences more than halved. This big difference is due to the expansion of the resonant cavity with respect to a 1D mixing-length model, as described in Sect. 2.1.

For periods of 5 minutes and longer, the case is less clear-cut. This points to some remaining problems that fall in two main categories. It can either be problems with the stratification of the simulation (from, e.g., inadequate opacity, radiative transport, equation of state, numerical resolution, etc.), or it can be our poor understanding of mode-physics. Near the top of the convection zone the p-modes are clearly non-adiabatic, propagate in a very in-homogeneous medium and interact with convection—none of which is accounted for yet.

2.3. Spectral Lines

All observations of the Sun (except for neutrino observations) detect photons originating somewhere in the photosphere. Since this region is also the top of the convection zone, with large temperature fluctuations, a significant turbulent pressure and a complicated velocity-field, it is rather fortunate that we nevertheless have been able to make meaningful interpretations with 1D models.

The complications introduced by the interactions between convection and radiation, is illustrated in Fig. 4. It is a plot of the profile of the disk-center intensity of the 6767 Å NiI-line from one snap-shot of the Solar simulation. The thick line is the surface averaged NiI-line and the thin lines are the line-profiles at 36 random points on the surface of the simulation.

We clearly see that none of the contributions from individual points look like the surface averaged line.

2.4. Abundance Analysis

The simulations can also be used as atmosphere models for abundance analysis. Classical abundance analysis is based on 1D atmosphere models with no velocity fields, and with micro- and macro-turbulent
velocities as well as the abundance as free parameters in the fitting process. Often some additional width of the damping (Lorentz-) component of the profile is needed in order to reproduce the observations, provided by a fourth free parameter: the damping enhancement factor.

From the simulations we have self-consistent density-temperature- and velocity-fields and we can abandon the use of micro- and macro-turbulence. Also, with a real velocity-field the line-profiles are not symmetric any longer, but exhibit bisectors and convectional shifts agreeing with observations, as shown by Asplund et al. (2000b).

This detailed agreement with the shape of lines, revealed that the Ni blend with the 6300Å O-line, shown in Fig. 5, had previously been underestimated. Lambert (1978) argued that the observed line's high degree of symmetry and proximity to the predicted wavelength, were strong indicators of the Ni-line contributing less than 5% to the equivalent width. With the asymmetrical line-profiles and convectional blue-shifts from the simulations, we find that the observed line is significantly shifted from the predicted wavelength and that the observed line could not be that symmetric unless the Ni-line contributes about 25% to the equivalent width.

Accounting correctly for the blend, lowers the oxygen abundance from the 6300Å-line by -0.13 dex. Additional 3D-effects from the simulation lowers the abundance by an additional -0.08 dex (Allende Prieto et al., 2001).

The abundance of other key elements has been reanalyzed in a similar fashion: carbon by Asplund et al. (2003a), nitrogen by Asplund et al. (2003b) and iron by Asplund et al. (2000c), as shown in Tab. 1. The results of 1D abundance-analysis listed in Tab. 1 are those compiled by Anders & Grevesse (1989).

Finally, progress in the treatment of atomic physics and line-broadening (Barklem et al., 1998, and references therein) has made it possible to abandon the last free parameter, the damping enhancement factor.

### Table 1. Solar abundances from 1D- and 3d-models.

<table>
<thead>
<tr>
<th>Element</th>
<th>1D</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>8.55 ± 0.06</td>
<td>8.44 ± 0.05</td>
</tr>
<tr>
<td>N</td>
<td>7.97 ± 0.04</td>
<td>7.80 ± 0.05</td>
</tr>
<tr>
<td>O</td>
<td>8.78 ± 0.07</td>
<td>8.66 ± 0.03</td>
</tr>
<tr>
<td>Fe</td>
<td>7.64 ± 0.03</td>
<td>7.45 ± 0.07</td>
</tr>
</tbody>
</table>

3. CALIBRATING MDI

We are working on calibrating the “Michelson Doppler Imager” (MDI, Scherrer et al., 1995) on board the “Solar and Heliospheric Observatory” (SOHO, Domingo et al., 1995) against the simulations. MDI observes the 6767.79Å Ni-line through a set of filters and a combination of the intensity in these filters, α, is used as a velocity-proxy, as detailed by Scherrer et al. (1995).

![Figure 6. Line-shape and bisectors for the MDI-line during a few p-modes.](image)

Fig. 6 shows the behavior of the Ni-line in the simulation, through 2.5 periods of the fundamental mode, as shown in the insert. The line-profiles are...
plotted on the lower, absolute wavelength-scale, and the bisectors on the upper velocity-scale, exaggerated by a factor of three compared to the wavelength-scale. The different line-styles correspond to different phases in the mode, as shown with vertical lines in the inset. From the profiles, and from the bisectors in particular, it is obvious that the p-modes do not merely shift the line back and forth. The line also changes asymmetry and sways with phase of the mode.

Fig. 7 shows (dashed line) the currently employed translation between α and the actual line-of-sight velocity. This curve is derived from shifting a Gaussian line-profile of same depth and width as the Nil-line, and convolving with the filter profiles (Scherrer et al., 1995). The large range is needed in order to cover the range of velocities introduced by the rotation of the Sun, SOHO’s orbit around the L1 Sun-Earth Lagrange-point, as well as the radial movement of the L1-point. Notice that all these three sources of velocity are simple translational movements of the photosphere relative to MDI, producing simple shifts of the Nil-line. The p-mode range is about 0.2 in α and less than 0.5 km s\(^{-1}\) in line-of-sight velocity.

On top of this I have plotted the equivalent curves (solid) derived from the p-modes excited in the simulations. I have plotted a series of curves corresponding to nine position-angles on the solar disk, as indicated in the plot. It is clear that there is a limb-effect although it is rather moderate for translational velocities between -2 and +3 km s\(^{-1}\), and \(\mu = \cos \theta > 0.5\). Note that the high-resolution MDI-images reach out to \(\mu = 0.4\). We also notice how both the limb-effects and the deviations from the 1D calibration, grow with larger translational velocities.

Despite the rather simplistic model, the pure translation of a Gaussian (dashed line) reproduce the disk-center \(\nu(\alpha)\)-transformation rather well, save an offset. The offset will of course have no effect on the derived p-modes, but the change of \(u_2(\alpha)\) with \(\mu\) has the potential to redistribute power in the low-\(l\) region.

Barely discernible in the plot, we also see deviations between the p-mode response and the translational response, in particular for \(u_2,\text{trans} < -4\) km s\(^{-1}\). The p-modes from the simulations are seen as small line-segments centered on the integer values of \(u_2,\text{trans}\). The deviation from the translational response is caused by the swaying of the Nil-line during the period of a mode, as illustrated in Fig. 6. This fortunately increases MDI’s sensitivity to p-modes as compared to translational velocities.

4. SOLAR P-MODE LINE-ASYMMETRIES

We have also used the 43 hour long Solar simulation to investigate the spectrum of p-modes excited in the simulation (Georgobiani et al., 2002). In particular, we recover the same asymmetries as observed (e.g. Duvall et al., 1993).

On of the most intriguing results from the p-modes in the simulations, is the reversal of symmetry in temperature when measured in two different ways. When measured at local \(\tau(x,y) = 1\) (observable) we recover the observed intensity spectrum and the asymmetry reversal with respect to velocity. If instead measured at the geometrical height where the average optical depth, \(\langle \tau \rangle\), is unity (not observable) the mode-peaks in the temperature spectrum has the same shape as in the velocity spectrum.

The \(\tau(x,y) = 1\) is a rather corrugated surface, reflecting the temperature differences between the high-entropy up-flows and the cool down-drafts, as well as the high temperature sensitivity of the atmospheric opacity (from the H\(^-\) ion). During an oscillation the increase in temperature leads to increased opacity moving outward the point actually observed. This suppresses the observed amplitude in temperature, because of the decrease of temperature with height.

Radiation effects therefore dampens the modes in temperature or intensity, as evident from the figures by Georgobiani et al. (2002). The “un-dampened” mode power in temperature, at \(\langle \tau \rangle = 1\), is a full order of magnitude larger, but the granulation-noise is only larger by a factor of five. The key to the mystery of the asymmetry-reversal seems to lie somewhere in the transition from \(\langle \tau \rangle = 1\) to \(\tau(x,y) = 1\), as investigated by Georgobiani et al. (2003)
5. CONCLUSION

Our solar convection simulations agree well with solar observations and have explained a number of problems: 1) Joining the simulations continuously with a 1D envelope models, determines the mixing-length parameter $\alpha$, and leads to the helioseismically inferred depth of the Solar convection zone. 2) A large part of the discrepancy with observed $p$-mode frequencies is explained by an expansion of the atmosphere (acoustic cavity) caused by 3D dynamic and radiative effects. 3) The detailed shapes of spectral lines is recovered with the simulations without any adjustable parameters. 4) This gives us more confidence in abundance-analysis based on the convection simulations, as mentioned for the C, N, O and Fe abundances. The internal scatter in abundances from different lines is smaller, and agreement with meteoritic abundances is improved.

We have also used the simulations for improving our understanding of $p$-modes in the Sun and their interactions with convection. $p$-modes excited in the simulation turn out to display the observed reversal of asymmetry between intensity and velocity. Since the simulations cannot hide anything from us (as does nature) the clues necessary for resolving the puzzle are certainly to be found somewhere in the simulations (Georgobiani et al., 2003).

We are in the process of starting a large-scale simulation with resolution $512 \times 512 \times 163$ covering a region $36 \times 36 \times 10 \text{ Mm}$ for, e.g., tomography and studies of super-granulation and effects from magnetic fields. Efforts to further improve the treatment of microphysics and radiation are also in progress.

Acknowledgments

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MAGNETOCONVECTION AND MICROPOROSITY

D. J. Bercik\textsuperscript{1}, A. Nordlund\textsuperscript{2}, and R. F. Stein\textsuperscript{3}

\textsuperscript{1}Space Sciences Laboratory, University of California, Berkeley, CA 94720-7450, U.S.A.
\textsuperscript{2}Astronomical Observatory / NBII/AFG and Theoretical Astrophysics Center, Juliane Maries Vej 30, DK-2100 Copenhagen \O, Denmark
\textsuperscript{3}Physics and Astronomy Department, Michigan State University, East Lansing, MI, 48824, U.S.A.

ABSTRACT

We report on results from a series of radiative magnetoconvection simulations in a 12 Mm $\times$ 12 Mm $\times$ 3 Mm near-surface solar layer. Initially unipolar, vertical magnetic field at average field strengths of 0 G, 200 G and 400 G is imposed on a fully relaxed hydrodynamic convective state. Magnetic field is swept to the intergranular boundaries by the convective flows, where it is compressed to kilogauss field strengths. The shapes and intensities of magnetic features typically evolve on the same time scale as the granulation pattern; however, the underlying magnetic structure evolves on a much longer time scale.

Occasionally, dark, high field strength features form that have properties consistent with observed micropores. The micropores primarily form when a small granule submerges and the surrounding magnetic field moves into the resulting dark 'hole'. The fluid flow inside micropores is suppressed by the strong magnetic field. The surrounding walls of a micropore heat its edges by lateral radiation, but the micropore experiences a net cooling through vertical radiation. The resulting thermodynamic structure of micropores stabilize them against destruction, allowing some micropores to exist for many granulation time scales.

Key words: Convection; Magnetic fields.

1. INTRODUCTION

We have conducted a series of magnetoconvection simulations to investigate the interaction between magnetic field and convection on the scale of granulation. While much progress has been made observationally in the study of small-scale magnetic features (such as bright points), it is difficult to trace the evolution of features that do not have well correlated proxies (such as micropores) (e.g., Berger & Title 2001). We use the simulations to investigate if micropores can form through the merger of existing magnetic flux.

The numerical approach is based on the hydrodynamic code of Nordlund and Stein (Stein & Nordlund 1998; Nordlund & Stein 1990), modified to include magnetic fields. The physical region under investigation is a thin solar surface layer. The dimensions are 12 Mm $\times$ 12 Mm $\times$ 3 Mm, extending 500 km above the surface to the temperature minimum and extending 2.5 Mm below the surface into the convection zone. The simulations are run on a three-dimensional non-staggered Cartesian grid. We solve the equations for mass, momentum and internal energy conservation and the induction equation for the vector potential. Pressure and temperature are found from a tabular equation of state, which includes ionization. The simulations include non-gray, LTE radiative transfer using a 4-bin opacity distribution function. The boundaries are periodic in the horizontal directions, and open in the vertical direction, with the magnetic field tending toward a potential field at the top boundary.

The initial state for the simulations was created from a snapshot taken from an existing lower resolution, hydrodynamic simulation. The snapshot was then interpolated to the desired resolution and physical size and allowed to relax thermally. The resulting initial state was used to start three simulation runs, each differing by the magnitude of the initial vertical field strength. All imposed fields were unipolar. Each of the scenarios was run for approximately three hours of solar time.

1. $B_{x,0} = B_{y,0} = B_{z,0} = 0$ at each grid point. This is a control run representing the purely hydrodynamical state.

2. $B_{x,0} = B_{y,0} = 0, B_{z,0} = 200$ G at each grid point. This case represents a moderate field strength plage region.

3. $B_{x,0} = B_{y,0} = 0, B_{z,0} = 400$ G at each grid point. This case represents a high field strength plage region.


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2. STRUCTURE OF MICROPORES

Magnetic field is swept to intergranular boundaries by the divergent upflows inside granules. The magnetic is compressed into "flux sheets" in the intergranular lanes and "flux tubes" at the intergranular vertices. At least near the surface, these tubes are not symmetric, isolated structures, but form star-shaped patterns and often are connected to nearby "flux sheets".

A typical simulation micropore has an area given by its intensity signature ($I < 0.8 < I >$) in the range $1 - 2 \text{ Mm}^2$. The magnetic size is slightly larger, as the edges of magnetic structures are brighter where material flows down along the current sheet. The micropores have fluxes of $1.5 - 3.0 \times 10^{10} \text{ Mx}$. They evolve on the time scale of granulation, with only a few lasting for several granule turnover times. The sizes, intensities and fluxes are consistent with the smallest observed pores (Keil et al. 1999; Leka & Skumanich 1998; Stüttelin 1998). Micropores are strong magnetic field concentrations where the unit optical depth surface is at a lower temperature that the surrounding medium, causing the micropore to appear dark (Figure 1). The temperature structure inside is superadiabatic, causing the micropore to cool and become partially evacuated. Weaker field concentrations show downflow throughout their interiors, but the velocities inside micropores are suppressed due to higher field strengths; downflows still exist through a sheath at the outer boundary, however (Figure 2). The suppression of velocity in the micropore interior leads to a reduction in the convective energy transport. Radiative transport becomes a more important factor in the balance of energy. The micropores are cooled by radiation in the vertical direction and heated by radiation from their hot sidewalls (Figure 3). The vertical cooling is nearly balanced by the horizontal heating and the micropore structure is fairly stable.

3. FORMATION OF MICROPORES

The formation process for a micropore is shown in Figure 4. The left panel shows a mask where high field strength points ($B > 1.5$) and low emergent intensity points ($I < 0.8 < I >$) are dark. A high-field, dark ring forms first, with the interior subsequently decreasing in intensity and rising in field strength. The middle panel shows the emergent intensity for the same times. A small granule near position (1.5, 2.5) is in the process of disappearing, leaving behind a dark hole. The magnetic field is shown in the right panel. The disappearing granule is surrounded by magnetic field that gets pushed into the space the granule leaves behind. There is a gradient in the magnetic field strength at the interface between a granule and intergranular lane, with weaker field closest to the granule and increasing into the lane. Therefore, weaker field enters the dark region first. Once the magnetic field fills the region left behind by the sinking granule, it is compressed to high field strength by surrounding granules. The merger process described here is an alternative method of formation to the emergence of a monolithic flux structure. This type of formation process is similar to that seen in network bright point observations, which has been named formation by granule compression (Muller & Roudier 1992). The network bright points appear to form in large, dark spaces between granules as the magnetic field gets compressed by converging granules. The formation time was found to be rapid, occurring in approximately four minutes. Further observations have confirmed the process in the network (Roudier et al. 1994) and also in plage regions (Berger & Title 2001).

4. CONCLUSIONS

A series of numerical simulations was used to investigate magnetic fields in a convectively unstable atmosphere in order to get a better understanding of the interactions and features seen in solar observations. The general properties of the magnetic field agree well with observations. Magnetic field is transported from granules to intergranular lanes by convective flow. The field gathers in the intergranular lanes and is compressed. Magnetic features form preferentially at the vertices of intergranular lanes.

The simulations have shown a possible formation mechanism for micropores. Existing field merges at the site of a disappearing granule. The formation process must be relatively fast (less than a granular turnover time). Some micropores are large enough to survive for more than a few convective turnover time scales. Even though these structures survive, they are constantly being deformed and squeezed into the intergranular lanes by changes in the granulation pattern. Over the course of their lifetimes, larger magnetic structures undergo many splittings and mergers. This dynamic evolution can cause the magnetic features to lose their intensity signature while still being a coherent magnetic structure, suggesting that their evolution cannot be traced (at least in its entirety) by these signatures. A longer time span is necessary to make a statistical study of micropores. The scope of the investigation here is limited to plage-like conditions, and it would be useful to study other field topologies. However, even with a limited number of samples, we now have data to get radiation diagnostics, such as Stokes profiles, for three-dimensional magnetic features.

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Figure 1. Background: filled contours of the magnetic field in 250 G intervals. Overlay: contours of the temperature from 4000 K to 16000 K in 1000 K intervals. The $\tau = 1$ depth is shown as the thick line around $z = 0$ Mm. The “flux tubes” are significantly cooler than their surroundings at a given geometrical depth.

Figure 2. Background: filled contours of the magnetic field in 250 G intervals. Overlay: velocity field. The $\tau = 1$ depth is shown as the thick line around $z = 0$ Mm. The flow is suppressed in the interior of the strong established flux tube in the center, but not the evacuating “flux tubes” on either side.
Figure 3. Radiative heating and cooling. Units are $10^{10}$ erg g$^{-1}$ s$^{-1}$. The top three panels show the net radiative heating/cooling and its relation to the temperature and magnetic fields. The bottom three panels show the contribution of vertical, inclined and nearly horizontal rays. The interior of the strong established "flux tube" is nearly in radiative equilibrium, with cooling by vertical rays nearly balanced by horizontal heating from the side walls.
Figure 4. Images of magnetic field (right panels), emergent intensity (center panels) and a mask showing only low intensity, strong field locations (left panels). Micropores form when magnetic flux from surrounding intergranular lanes moves into locations where a small granule disappears.
Prospects for the Future

Chair: M. Rast
HELOISEISMOLOGY PRESENT AND FUTURE

Michael J. Thompson
Space & Atmospheric Physics Group, The Blackett Laboratory, Imperial College, London, SW7 2BW, UK

ABSTRACT

The subject of helioseismology is in a exciting phase of expansion. In terms of the methods being developed and applied, the new and challenging results coming out from the high-quality data that present observational campaigns, networks and missions are providing, the arrival of GONG*, the prospects for new missions including Solar Dynamics Observatory, and the recent ground-based results and exciting prospects for asteroseismology of other solar-like stars. I pick up on some themes of the conference, and expand on the above developments and activities.

Key words: global-mode helioseismology; local helioseismology; temporal variations; asteroseismology.

1. INTRODUCTION

A striking feature of this conference and of the current state of our subject is the wealth of methods that are being developed and applied in helioseismology. Global-mode helioseismology is the most mature of the methods, particularly in its application to inversions for the spherically symmetric hydrostatic structure (using the mean multiplet frequencies) and the rotation (using the odd component of the m-splitting) of the solar interior, e.g. Gough et al. (1996) and Thompson et al. (1996). Some of the local helioseismic methods have also been around for a few years, but they are now being used more extensively. 

Ring-diagram analysis has been applied to mapping the subsurface flows in the outer 10 Mm or so – e.g. González Hernández et al. (1998); Haber et al. (2002) – and we have seen at this meeting its application to structure-like inversions under active and quiet areas (Simmons, Basu & Bogart, these proceedings). Time-distance helioseismology has developed rapidly from its early beginnings (Duvall et al., 1993) to be an impressively powerful technique producing some of the most enticing results in the field (Kosovichev et al., 2000). Acoustic holography is less widely used but has produced interesting results under and around sunspots, as well as the completely novel imaging of active regions on the far side of the Sun (Lindsey & Braun, 2000a,b). Other such work of note has been developed and applied by members of the Taiwanese Oscillation Network (TON) (Chang et al., 1997). New methods are also being developed, including waveform analysis, which I discuss briefly below.

I shall make no attempt to summarize comprehensively the meeting: the papers in the rest of this volume will give a far better account of the full range and depth of science presented. There were many examples of excellent application of the methods mentioned above. For example, the ring-analysis results Toonre showed intriguing synoptic maps of the evolving flows of so-called Solar Subsurface Weather (SSW) in the upper part of the convection zone, and flows under a particularly strong active region. The results complement time-distance studies of subsurface flows in strongly magnetic regions by Zhao et al. (2001). Among the results of acoustic holography I was particularly struck by the intense, almost point-like concentrations Braun and Lindsay showed in power maps focussed at depths of some 8 Mm beneath penumbral regions: these concentrations, which are in fact 5 Mm across or so, are perhaps interpretable as collapsing flux tubes. Results from global-mode helioseismology included temporal variations in the convection-zone rotation in the form of zonal banded flows known as torsional oscillations, discussed here by Vorontsov, see also Vorontsov et al. (2002). Apparently related structures in the meridional circulation have also been detected with time-distance helioseismology (Beck et al., 2002) The 1.3-year oscillation in the rotation rate near the base of the convection zone and in the outer part of the radiative interior, reported by Howe et al. (2000), was discussed several times. I shall return to this issue later. Shibahashi considered the use of additional constraints as well as global mode frequencies to probe the solar core.

Evidence for the wave-like properties of supergranulation (Gizon et al., 2003) was presented. Irradiance variations were discussed in back-to-back talks by Kuhn and Woodard. In particular, the question of whether irradiance variations are all superficial was addressed, but no consensus appeared to be reached. This is an intriguing and indeed important question if we are to understand how the Sun’s energy output varies, and whether the interpretative modelling of irradiance data variations in terms of the surface coverage of sunspots and faculae (discussed in his talk by Fröhlich) in fact captures all of the essential process. The enormous promise of asteroseismology was presented by Bedding who discussed that there are quite a number of stars now (even without counting the roAp stars) which have been found from
ground-based observations to exhibit solar-like oscillations. I shall expand on some of these themes below.

2. THE COMPLEMENTARITY OF HELIOSEISMIC METHODS

Global-mode helioseismology is based on analysis of the Sun’s oscillations in terms of global modes. As is well known, if the Sun were spherically symmetric, non-rotating, etc., its normal modes of oscillation would take the form

\[ \xi_{nlm}(r, t) = \xi_{nl}(r) Y_l^m(\theta, \phi) e^{-i\omega_{nlm}t} \]

where \( n, l \) and \( m \) are respectively the radial order, the degree and the azimuthal order of the mode, and \( (r, \theta, \phi) \) are spherical polar coordinates with origin at the centre of the Sun. Moreover, the mode frequencies \( \omega_{nlm} \equiv \omega_l \) would be independent of \( m \). The total surface displacement at \( r = R \), in this ideal scenario, is of course a superposition of all such modes:

\[ S(\theta, \phi, t) = \sum_{nlm} a_{nlm} \xi_{nl}(R) Y_l^m(\theta, \phi) e^{-i\omega_{nlm}t} \]

where \( a_{nlm} \) is the physical amplitude of the mode.

Of course we cannot observe \( S \) over the whole Sun at once, and also in the case of Doppler velocity what we measure is a projection onto the line of sight. The properties of an individual mode are estimated in the case of spatially resolved observations by projecting the observed signal onto a mask; commonly that mask is a (possibly apodized) spherical harmonic – see e.g. Howe & Thompson (1998) and references therein.

Departures from spherical symmetry – caused by rotation, magnetic fields or asphericity of the structure of the Sun, for instance – modify this picture. As is well known, slow rotation (at angular rate \( \Omega \) much greater than the frequencies of the modes) raises the degeneracy of the modes within a multiplet of given \( n \) and \( l \). Since the star is still axisymmetric, \( m \) is still a well-defined quantum number.

Rotation and more spatially complex departures from spherical symmetry will also modify the eigenfunctions. This is sometimes referred to as mode coupling, but I think that term is misleading and I prefer to speak of mode distortion. Martin Woodard pointed out some years ago the importance of appreciating the distortion of high-degree modes by differential rotation (Woodard, 1989). A global mode has a particular frequency: the different rotation rate at different latitudes means that the local dispersion relation varies, and so for a fixed frequency the horizontal wavenumber of the mode will vary with location.

Almost the simplest example of such symmetry breaking is slow rotation at a rate \( \Omega(r) \) that depends only on distance from the centre of the Sun. To first order (in \( \Omega/\omega_n \)) the eigenfunction now comprises a term proportional to \( Y_l^m \) plus first-order corrections proportional to \( Y_{l-1}^m \) and \( Y_{l+1}^m \) in general (Gough & Thompson, 1990); i.e. it is distorted, with contributions from three spherical harmonics, not just one. There is thus a “coupling” of spherical harmonics, but not in a meaningful sense a coupling of modes. Nonetheless, if the observations are projected onto spherical harmonics then this distortion of the eigenfunction means that the same mode will be detected at not only its principal \( l \) value but also at \( l \pm 1 \).

The reason that the distortion involves only immediately neighbouring spherical harmonics is that we have chosen the simplest rotation law: one that has no angular dependence. If the rotation law instead were proportional to \( \cos^k \theta \), for instance, it would introduce to the eigenfunction terms proportional to \( Y_l^m \) with \( l' = l - k, l - k + 2, ..., l + k \). A more localized perturbation, such as the perturbation to the wave dispersion relation caused by a sunspot or active region, will require even more spherical harmonics to describe the eigenfunction.

In principle, one might make use of the information in the relative amplitude of the global mode in different spherical harmonics to learn about the form of the symmetry-breaking perturbation. In practice, even for a benign perturbation such as latitudinally independent or slowly varying rotation rate, I am not aware that anyone has applied this idea to solar data. To study localized perturbations, it will often be preferable to use local methods. This is already being done for localized subphotospheric structures, of course. Also, longitudinally varying structures and possible small-scale deviations from sphericity in the deep interior might be detectable with acoustic holography or time-distance helioseismology.

For other problems, global-mode helioseismology will continue to be the method of choice. The message is, we need different methods to address different questions, and we should ensure that we have at our disposal all the tools we have developed. Different analysis techniques will be most useful for studying structure under sunspots and studying axisymmetric structure of the Sun’s core, for example.

3. PRESENT AND FUTURE DEVELOPMENTS IN ANALYSIS METHODS

3.1. Global-mode helioseismogoy

By comparison with local helioseismology, global-mode helioseismology is mature and well-understood. It is certainly not the case, however, that everything is understood. The inferred solar rotation rate from MDI and GONG project data differ in some parts of the solar interior by much more than their quoted uncertainties (Schou et al., 2002). The cause of the discrepancy seems to be in the estimation of mode parameters from the observational data (‘peak-bagging’): if the same peak-bagging is applied to observations from MDI and GONG, the inferred rotation rates are in good agreement with one another. The discrepancies are most apparent at mid to high latitudes in the lower convection zone and in the outer part of the radiative interior – where a systematic error
in the MDI peak-bagging seems to be the cause—and in the deep interior, where the GONG peak-bagging tends to underestimate the splittings. These discrepancies probably do not come from trivial errors in the implementation of the peak-bagging, but rather from the more fundamental assumptions underlying the two different methods. The MDI algorithm assumes a model for the leakage between different modes and uses a parametrisation in \( n \) to fit frequencies within each multiplet, whereas the GONG algorithm essentially fits each mode frequency independently and proceeds without a leakage model.

An area of active development in global-mode helioseismology is the measurement of the parameters of the high-degree modes (Rabello-Soares et al., 2001; Reiter et al., 2002a, also Rhodes, these proceedings). The fundamental problem in this regime is that the modes no longer form discrete peaks in the power spectrum and so the methods to measure their parameters involve fitting in a continuous manner along ridges of constant \( n \) (‘ridge-fitting’). Among the issues currently being addressed is the effect of peak asymmetry on the modes (Reiter et al., 2002a), an effect that must also be relevant for fitting in ring-analysis. Also, inversion of high-degree data introduces additional considerations, since the \( l \)-dependence of the ray paths in the near-surface regions cannot be neglected (Di Mauro et al., 2002).

Comparisons between global mode parameters deduced from ridge-fitting, and local modal properties deduced from ring analysis would be a valuable check on both and in particular on the estimated uncertainties that the practitioners attach to their parameter estimates. A group of us have started such a comparison, but the work is not yet very far advanced.

3.2. Ring-diagram analysis

Of the local methods, ring-diagram analysis is the closest in spirit to the global-mode analysis in that the wavefield is still analysed in terms of wave frequencies. (The oscillations are quantised in \( n \) in the radial direction, though not in the horizontal directions.) There is inevitably a trade-off to be made in terms of the size of patch chosen for the ring analysis. The smaller the patch, the better in principle the localization on the solar surface; but the worse the resolution in (horizontal) wavenumber \( (k) \) space, since that resolution \( \Delta k \sim 1/D \), where \( D \) is the linear dimension of the patch. Typically, tiles of size 15° square in heliocentric coordinates are chosen.

To date, of the implementations of local methods, that of ring analysis is the one best able to process data fast enough for e.g. synoptic maps of subsurface flow, with the promise to produce these as standard products from GONG+ and the Solar Dynamics Observatory.

3.3. Time-distance helioseismology

Time-distance helioseismology has already shown its potential for probing local structures and flows near the surface, and holds great promise also for probing local structures and flows deeper down. Particularly noteworthy are the investigation of flow underneath a sunspot (Zhao et al., 2001) and the structure under sunspots (Kosovichev et al., 2000). Most work to date has been done in the ray approximation, and there are still uncertainties about how to model correctly the sensitivity of the measured travel-times to subsurface conditions. Improvements are based upon modelling more completely the wave propagation, either from single or distributed sources (Gizon & Birch, 2002; Jensen & Pijpers, 2003).

There must also be doubts regarding the interpretation of the cross correlations and travel times in regions of strong magnetic fields, such as when the waves propagate through a sunspot. To my knowledge, the conversion of acoustic modes to other magneto-acoustic modes (in particular the slow mode) have not yet been taken into account in interpreting the correlation of points inside and outside a spot. Probing sunspots using two-bounce travel paths, so that no measurements in the spot region itself are needed, will avoid some concerns about time-distance results in such regions; but it is still necessary to address the complexities of effects of magnetic field on the wave propagation.

3.4. Acoustic holography

I shall say little about acoustic holography since I have not worked with it and know little about its intricacies. The method is flexible and capable of producing interesting results which complement those that have been obtained from the other methods. More forward modelling with simulations might help elucidate the images produced by holography. It would be interesting to see such modelling coupled with inversion techniques in the interpretation of the holographic results.

3.5. Other methods

I am sure that at future GONG meetings we shall see the fruits of more new methods of data interpretation. One such new development is waveform analysis, which term I interpret broadly as using more aspects of the wavefield than just the travel times currently inferred from e.g. two-point correlations. Such methods have been developed in geoseismology (Pratt, 1999) and, as discussed at this meeting, efforts are underway to transfer that technology to helioseismology, cf. Tong et al. (2003). A development that one might call ‘stochastic wavefield analysis’
involving the fitting of the whole cross-correlation has recently been presented (Woodard, 2002). Methods such as these may tell us more about not only the subphotospheric structures and flows but also aspects of the form of the source of the wave excitation.

3.6. Intercomparisons of methods

This meeting saw some initial efforts to compare the inferences from different local methods. Much more extensive testing and validation of the local helioseismic methods must be a high priority in order for us to place confidence in the rapidly expanding body of results from this area of helioseismology. Such tests can take place different levels of sophistication. A splinter-group discussion at this meeting agreed to a coordinated comparison of inferences from different methods applied to the same solar data. I hope we shall see the results of that comparison in 2003. In such a comparison, of course, we do not know the ‘truth’ – the actual solar flow, structure, etc. – but at least we look to find consistency between the inferences of the different methods. All the methods could agree but be wrong, though we have a perhaps naive hope that the methods are sufficiently different that that would not be the case. In the longer term, a more definitive test would be to apply the methods to artificial data for which the true underlying flow or structure would therefore be known. The forward problem solved to create such data must capture enough of the physics of the wave propagation to provide appropriate data for the method under study. In my view the same artificial data do not necessarily have to be appropriate for testing all of the local methods: it may be that it would be prohibitively complicated to capture enough physics to satisfy practitioners of all of the local methods that the data are an appropriate test for their method, since the methods do ostensibly use different aspects of the wavefield.

It is also important to quantify the resolution and error properties of the local methods. This has been done extensively in global-mode helioseismology, with the development of averaging kernels to measure resolution and the quantitative evaluation of the propagation of data errors to the inferences (Christensen-Dalsgaard et al., 1990; Schou et al., 1994; Rabello-Soares et al., 1999) including error correlations (Howe & Thompson, 1996). In ring analysis, credible efforts to evaluate the depth resolution with averaging kernels have been made, e.g. Haber et al. (2002), but the development of fully three-dimensional averaging kernels is still under study. For time-distance helioseismology, some studies of the resolution have been undertaken (Korzennik, 2001) and quantitative measures are now developed (Jensen, 2002). It is desirable that such measures of resolution and statistical uncertainties in the reconstruction become routine when presenting the results of time-distance and other local inversion solutions.

4. A SELECTION OF SCIENCE OPPORTUNITIES

As we look particularly to NASA’s Solar Dynamics Observatory (SDO) mission in about five year’s time, building upon the work that is already being done with MDI and other high-resolution data, I think there are great opportunities for helioseismology to tackle substantial science problems in solar physics. Below, I discuss just three: solar activity and the coupling between the solar interior and exterior; the solar dynamo; and temporal variations of the solar interior. Together these areas can lead to a fundamental understanding of solar activity with important topical corollaries for prediction of space weather and perhaps too of climate change.

4.1. Solar activity

Improving our understanding of the coupling between the solar interior and its exterior is a major goal of SDO and, particularly with the widespread development of local helioseismic techniques, the wider community of solar physicists are recognising the potential for helioseismology to provide the link between what occurs beneath the surface and the activity above the photosphere and in the corona. We have already begun in helioseismology to probe the interactions between flows and magnetic field, and the subsurface structures under sunspots, and the evolution of active regions even before they appear at the photosphere. All the local methods are contributing to these advances.

The question of whether the observed irradiance variations are all superficial is of great interest. Inversions of global-mode even a coefficients have strongly indicated latitudinally dependent thermal structure in the convection zone, and there are hints (Antia et al., 2003) that this may vary over timescales of years, thus perhaps contributing to storage and release of energy in the convection zone on the timescale of the solar cycle. The evidence for temporal variations of the latitudinal structure is however quite tentative, and study over a longer period of time is definitely required. Moreover, the kernels used for inverting for latitudinal structure perturbations should be carefully considered: it is not obvious to me that the present kernels, derived essentially by taking those for spherically symmetric perturbations and weighting with the squared modulus of a spherical harmonic, are appropriate. Modelling too must help clarify the possible latitudinal and longitudinal thermal structures that might exist in the convection zone and modify the local flux at the photosphere.

4.2. Elucidating the solar dynamo

Helioseismology has already substantially changed the view of the solar dynamo by revealing the existence of the tachocline shear layer which is now the favoured site for large-scale dynamo action in the solar interior. It is possible that there are multiple scales of dynamo action taking place in the Sun, with small-scale dynamo activity caused
by the interaction of convection in the upper convection zone with small-scale flux there, and large-scale dynamo involving the ordering and intensification of the field by large-scale action in and near the tachocline. There is a resurgence of interest in flux-transport dynamos, in which meridional circulations transport newly generated small-scale field or old flux (decayed active-region flux, for instance) from the surface layers and eventually down into the tachocline – see, for example, Dikpati et al. (2002). A helioseismic determination of the meridional circulation in the lower convection zone would constrain such models. Near-surface measurements of the circulation, e.g. by ring analysis (Haber et al., 2002), indicate a poleward flow, albeit possibly modified in the northern hemisphere during the recent rising phase of the solar cycle. Of course, mass conservation in a steady state dictates that \( \nabla \cdot \mathbf{\mu} = 0 \). Consequently, any axisymmetric poleward circulation in the upper convection zone must close with an equatorward flow somewhere at greater depth. However, the increase in density with depth means that the flow may be quite feeble and its magnitude hard to measure. Nonetheless, measuring the travel times between points separated by 45° or more on the solar surface may reveal the existence of such flows.

As discussed at this meeting, the direct effect on the waves of magnetic fields in the tachocline region may be even harder to detect. However, an intriguing result by Chou & Serebryanskii (2002) indicates an anomalous perturbation over the rising phase of the solar cycle to the travel times of waves whose ray paths bounce around the Sun exactly eight times before returning to their starting point: such ray paths are distinguished by having their lower turning points just at the location of the tachocline. It is suggested therefore that such an anomalous perturbation could be a signature of the evolving magnetic field at that depth. However, the rather broad Fresnel zone of such waves at the base of the convection zone makes it curious that a similar perturbation is not also at least partially present in the travel times of the waves corresponding to ray paths that close around the Sun in seven or nine bounces.

4.3. Temporal variations

With helioseismic observations now extending over many years, it is possible to probe directly temporal variations of the solar interior on timescales up to a decade. Detecting such variations would give completely new constraints on the dynamics of the interior. As discussed at the meeting, a reported detection that has caused some debate is a quasi-periodic 1.3-year oscillation in the rotation rate in equatorial regions just above and beneath the base of the convection zone (Howe et al., 2000). The evolution of this oscillation in more recent data was presented by Toomre et al. (these proceedings). As discussed in the latter paper, although initially unexpected, this periodicity has previously been found in various heliospheric and geomagnetic measurements, which hints at a physical linkage between the region around the tachocline, presumably through the photospheric and coronal magnetic fields, to the solar wind. There is also theoretical calculations that indicate how such a period of variation might originate (Covas et al., 2000). That the oscillation is not detected in data such as LOWLs, currently only available in one-year averages, is hardly surprising: because the averaging time is a substantial fraction of the putative 1.3-year period, any signal with that period would have its amplitude reduced in one-year averaged data to a quarter of its true amplitude. The amplitude found by Howe et al. (2000) is 6-8 nHz peak-to-peak, and one quarter of this would not be significantly above the noise level. In my oral presentation, I stated that I considered that the 1.3 year signal was definitely in the helioseismic data (though not necessarily originating deep in the interior, although no-one has demonstrated how a 1.3-year period systematic error could enter the data). In support of my assertion that the signal is in the data, I cited not only Howe et al.'s paper but also Basu & Antia (2001). Sambani Basu told me afterwards that in so doing I misrepresented their work – which I certainly do not want to do – and that the signal they see in the data has a period of 1.1 years. (This would then be barely different from one year, a period which might arise from various artifacts in the observing process.) What I had in mind was Fig. 8 of Basu & Antia (2001), which shows a signal in the GONG data peaking at a frequency of 0.8 year\(^{-1}\). The 1.3-year oscillation is potentially the first direct dynamical signature of variations in the region where many believe the Sun's large-scale magnetic dynamo has its seat: it is desirable that others seek to verify it or disprove it with other datasets (with sufficient temporal resolution). I suggest that the archive of Mount Wilson Observatory helioseismic observations would be an excellent source of independent data stretching back quite a few years that could cast further light on the phenomenon. Of course, we should be alert to variations elsewhere on the Sun, and not necessarily with the obvious timescales.

Temporal variations of mode parameters other than the frequencies – e.g. Chaplin et al. (2003) – will help us understand better not only the physics of the modes but also the activity-induced changes of the convection and other properties of the subphotospheric layers.

5. ASTEROSEISMOLOGY

An area that will see great activity in the next few years is the seismology of solar-like stars more distant than the Sun. By solar-like, I mean dwarf and subgiant stars possessing substantial convective envelopes and likely therefore to exhibit solar-like turbulently excited modes. The present, promising state of the subject was well reviewed by Bedding at this meeting. Several groups have now made credible detections of solar-like oscillations from ground-based observations, for example in \( \alpha \) Cen and \( \beta \) Hyi (Bouchy & Carrier, 2002; Bedding et al., 2001). Other stars in which solar-like oscillations have been detected include the G7 giant star \( \xi \) Hya (Frandsen et al., 2002).

The ground-based observations of solar-like oscillations are remarkable, but new satellite missions promise to transform the field with observations that are both qualitatively and quantitatively beyond what can currently be...
achieved. The first data of this new generation of observations will come from one or more of three small-satellite missions which are planned: the Canadian mission MOST, due for launch in 2003; the mostly French mission COROT, due for launch in 2005; and the Danish mission Rømer/MONS, also due for launch in 2005. A prospect on a wholly larger scale is the ESA mission, Ed-dington, due for launch in 2007: this will have a dual role of primarily asteroseismology for two years and primarily extra-solar planet-finding for three years (Favata, 2002). In a similar time-frame, the NASA mission Kepler will also perform a dual planet-finding and asteroseismology role. Seismology of solar-like stars will of course enable us to explore physical conditions not found in the Sun, specifically looking at the structures of stars of different age, metallicity and mass. Observations of stars other than the Sun will not reveal the high-degree modes, but the methods of analysing low-degree solar modes will be applicable. Nonetheless, work is required to develop and refine inversion methods for use when only low-degree data are available, particularly for investigating a star’s structure. We may not in the near future make spatially detailed inferences about an individual star’s internal rotation except for the Sun — but it should be feasible to get some limited information about the internal rotation of other stars.

These missions will also provide a wealth of data on other variable stars. Here a major problem is very often that of identifying which modes are seen, for example, in delta Scuti variables. It may be that the much more sensitive observations from space will pick out low-amplitude modes that fill in the ‘missing’ modes and thus make mode identification easier; but it seems likely that, overall, mode identification will remain one of the greatest obstacles to performing detailed seismology of other types of variable star.

6. FINAL WORDS

The existing and new networks and missions will give helioseismologists great opportunities to make important discoveries about the solar interior and its role in solar activity. GONG+ is now operating, providing continuous, high-resolution mapping of the solar oscillations. SOHO continues its operation for a few more years; and SDO will provide great opportunities for helioseismology to exploit further the techniques discussed above in pursuit of major scientific questions about the working of the Sun and its interaction with its environment. Major issues to be addressed by SDO include the interior-exterior coupling, flux-flow interactions, active-region structure and evolution, and the solar dynamo. Other missions may, in the future, enable us also to view the Sun from more than one vantage point, so that time-distance helioseismology or other techniques can trace rays in a single bounce deep into the radiative interior to probe longitudinal or other aspherical structures there and investigate flows not practically accessible to helioseismology using global modes. It remains important to continue measuring, with BISON, GOLF and other instruments, the low-degree modes which penetrate the core, to measure or put constraints on temporal variations of the deep interior and to push to ever lower-order modes, as Chaplin discussed at this meeting. The g modes that would tell us so much about the core remain elusive, but must still be sought. The suggestion to detect them from their modulation of the acoustic cavity of the p-modes has been investigated before, but perhaps deserves to be looked at once again.

Finally, the ground-based and much anticipated satellite data for asteroseismology will make accessible a new dimension of exploration of the structure of stars. Many of us find that a very exciting prospect and a natural extension of what we have achieved for the Sun.

ACKNOWLEDGMENTS

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Posters
EFFECTS OF FLARES ON SOLAR OSCILLATION CHARACTERISTICS

Ashok Ambastha1, Sarbani Basu2, and H. M. Antia3

1 Udaipur Solar Observatory, Physical Research Laboratory, P. O. Box 198, Udaipur 313001, India
email: ambastha@prl.ernet.in
2 Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.
email: basu@astro.yale.edu
3 Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India
email: antia@tifr.res.in

ABSTRACT

We use ring diagram analysis to study the effects of solar flares on p-mode oscillation characteristics. We study the changes in the amplitude, frequency and width of acoustic modes using data before, during and after a few of the major flares during the current solar cycle. Mode power is found to be enhanced during and after some flares, though the enhancement is not seen in all flares.

Key words: Sun: oscillations; Sun: activity.

1. INTRODUCTION

Solar flares are amongst the most energetic phenomena observed on the solar surface. There have been observations (Kosovichev & Zharkova 1998, 1999) suggesting that some large flares excite waves on the solar surface. Most of the flare energy is emitted in the chromosphere and corona, however, some white light flares suggest that there may still be substantial energy released in the photosphere which can affect the solar oscillations. In order to study the effect of flares on solar oscillation modes we use the ring diagram technique (Hill 1988; Patrón et al. 1997; Basu et al. 1999) applied to 3D spectra from the MDI instrument. We use spectra obtained before, during and after the flares to study if there is any variation in mode characteristics such as frequency, width, power and asymmetry.

2. DATA AND TECHNIQUE

To study the effect of flares, we have chosen 10 active regions that had flares when they were close to the central meridian. Major flares from these regions around the time covered by our study are given in Table 1.

Table 1. Characteristics of solar Active Regions (AR) and Flares

<table>
<thead>
<tr>
<th>NOAA No.</th>
<th>Flare Class</th>
<th>Location</th>
<th>Date</th>
<th>Start ( UT )</th>
<th>End ( UT )</th>
</tr>
</thead>
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<td>S21W03</td>
<td>15MAR98</td>
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<td></td>
</tr>
<tr>
<td>8185</td>
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<td>S26E19</td>
<td>26MAR98</td>
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<td></td>
</tr>
<tr>
<td>8192</td>
<td>M2.4/3N</td>
<td>S24W03</td>
<td>27MAR98</td>
<td>21:48 23:17</td>
<td></td>
</tr>
<tr>
<td>8485</td>
<td>M6.2/2N</td>
<td>N23W39</td>
<td>16MAR99</td>
<td>21:34 21:46</td>
<td></td>
</tr>
<tr>
<td>8525</td>
<td>M4.4/2N*</td>
<td>N15E32</td>
<td>03May99</td>
<td>05:36 06:32</td>
<td></td>
</tr>
<tr>
<td>8539</td>
<td>M5.2/5N</td>
<td>N16E19</td>
<td>10May99</td>
<td>05:22 05:37</td>
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<tr>
<td>9026</td>
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</tr>
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<td>9076</td>
<td>M2.1/1F</td>
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</tr>
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<td></td>
</tr>
<tr>
<td>9081</td>
<td>X1.2/3B*</td>
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<td>9046</td>
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<tr>
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<td>S21E31</td>
<td>06APR01</td>
<td>19:10 19:31</td>
<td></td>
</tr>
<tr>
<td>100</td>
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<td>15:20 16:00</td>
<td></td>
</tr>
<tr>
<td>359</td>
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<td></td>
</tr>
<tr>
<td>358</td>
<td>M2.3/1F</td>
<td>S22W27</td>
<td>11APR01</td>
<td>12:56 13:49</td>
<td></td>
</tr>
<tr>
<td>357</td>
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<td>S22W38</td>
<td>12APR01</td>
<td>02:56 03:29</td>
<td></td>
</tr>
<tr>
<td>356</td>
<td>X2.0/SP</td>
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<td>12APR01</td>
<td>09:39 10:49</td>
<td></td>
</tr>
<tr>
<td>9433</td>
<td>X4.0/2N</td>
<td>N14E23</td>
<td>23APR01</td>
<td>20:15 20:43</td>
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</tr>
</tbody>
</table>

Table 1. We have calculated flare indices for the selected active regions using the quantity \( Q = i \times \tau \) to quantify the daily flare activity summed over the duration of tracked data sets using the method described by Kleczek (1952), and Atac & Oguz (1998). This relationship gives an approximate estimate of the total energy emitted by the flares in the active region. In this relation, \( i \) is the intensity scale of the importance, and \( \tau \) is the duration of the flare in minutes. This flare index (FI), along with other properties of regions studied is given in Table 2. The Table also lists the Magnetic Activity Index (MAI)

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NOAA 9393 was the largest active region observed in the last 20 years. It produced 23 M and 5 X class flares (including the largest X20 flare of Solar Cycle 23) during its disk transit of March 24–April 5, 2001. It possessed a very large MAI value (184.8 G). Out of the two large proton flares, the X1.7/1B event on March 29, 2001/10:15 UT occurred when the AR was close to the disk-centre at N20W19, making it favourable for solar oscillation characteristics studies. NOAA9026 produced two proton flares, an X2.3/3B event on June 06, 2000/15:25 UT, and a M5/3B event on June 10, 2000/17:02 UT, respectively. Both these events were associated with halo CMEs. The event of June 6, 2000 was well covered by MDI data sets, when an additional X-class, and an M-class flare also occurred. These events correspond to a large flare index, $FI = 1783$, which is the largest in our study.

In order to study the effect of flares on solar oscillation modes, we use the ring diagram analysis technique on data sets from MDI. Each of the data sets covers a region of about $16^\circ \times 16^\circ$ on the Sun and a time interval of 1664 minutes. We have chosen the data from available sets when the active region was close to central meridian. The data sets used are listed in Table 2. This table gives the heliographic coordinates of the central point as well as the central meridian longitude at midpoint in time. For convenience each region is identified by the serial number listed in the first column. We take the 3D spectra of oscillation power for each of the regions. Mode characteristics are known to be affected by magnetic fields in active regions (Rajaguru et al. 2001) and hence we need to account for differences in MAI values of different regions. In particular, the power in modes is known to reduce as the MAI increases.

To fit the 3D spectra we have adopted a model with asymmetric peak profiles as used by Basu & Antia (1999)

$$P(k_x, k_y, \nu) = \frac{e_{P1}}{k^4} + \frac{e_{P2}}{k^4} + \exp(A_0 + (k - k_0)A_1 + A_2(k_x^2) + A_3(k_y^2)S_z^2 + 1)$$

where

$$x = \frac{\nu - ck^2 - U_x k_x - U_y k_y}{w_0 + w_1(k - k_0)}$$

$$S_z = S^2 + (1 + Sx)^2$$

and the 13 parameters $A_0, A_1, A_2, A_3, c, p, U_x, U_y, w_0, w_1, S, B_1$ and $B_2$ are determined by fitting the spectra using a maximum likelihood approach (Anderson et al. 1990). The parameter $S$ measures the asymmetry in the peak profile. The form of asymmetry is the same as that used by Nigam & Kosovichev (1998). The mode characteristics we are interested in are the frequency ($ck^2$), peak power ($\exp(A_0)$), half-width ($w_0$) and asymmetry parameter $S$.

### Table 2. Regions studied using ring diagram analysis

<table>
<thead>
<tr>
<th>No.</th>
<th>Lat.</th>
<th>Lon.</th>
<th>CM</th>
<th>Start time</th>
<th>End time</th>
<th>MAI (G)</th>
<th>FI</th>
</tr>
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<tr>
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<td>22.52</td>
<td>30</td>
<td>060</td>
<td>13:00:38</td>
<td>14:34:21</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>R02</td>
<td>22.52</td>
<td>30</td>
<td>045</td>
<td>14:34:56</td>
<td>15:07:39</td>
<td>70.8</td>
<td>66</td>
</tr>
<tr>
<td>R03</td>
<td>22.52</td>
<td>30</td>
<td>030</td>
<td>15:07:13</td>
<td>16:10:56</td>
<td>91.2</td>
<td>306</td>
</tr>
<tr>
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<td>22.52</td>
<td>30</td>
<td>015</td>
<td>16:10:31</td>
<td>17:14:14</td>
<td>109.9</td>
<td>322</td>
</tr>
<tr>
<td>R05</td>
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<td>30</td>
<td>000</td>
<td>17:14:27</td>
<td>18:22:10</td>
<td>129.5</td>
<td>357</td>
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<tr>
<td>R06</td>
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<td>035</td>
<td>20:10:24</td>
<td>21:20:56</td>
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<td>388</td>
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<tr>
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<td>30</td>
<td>050</td>
<td>21:20:56</td>
<td>22:31:27</td>
<td>240.0</td>
<td>420</td>
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<td>060</td>
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<td>23:41:59</td>
<td>293.5</td>
<td>458</td>
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<tr>
<td>R09</td>
<td>22.52</td>
<td>30</td>
<td>060</td>
<td>23:41:59</td>
<td>24:52:31</td>
<td>293.5</td>
<td>458</td>
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<tr>
<td>R10</td>
<td>22.52</td>
<td>30</td>
<td>060</td>
<td>24:52:31</td>
<td>25:53:55</td>
<td>293.5</td>
<td>458</td>
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<tr>
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<td>30</td>
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<td>25:53:55</td>
<td>26:55:19</td>
<td>293.5</td>
<td>458</td>
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<tr>
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<td>27:56:43</td>
<td>293.5</td>
<td>458</td>
</tr>
<tr>
<td>R13</td>
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<td>30</td>
<td>060</td>
<td>27:56:43</td>
<td>28:58:17</td>
<td>293.5</td>
<td>458</td>
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<tr>
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<td>29:59:41</td>
<td>293.5</td>
<td>458</td>
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<td>29:59:41</td>
<td>30:60:15</td>
<td>293.5</td>
<td>458</td>
</tr>
</tbody>
</table>

### 3. RESULTS

Fig. 1 summarises the results for each of the 10 ARs that we have studied. The lowest panel in each figure shows the evolution of MAI and the Flare index as calculated by us. The horizontal axis is the serial number of the regions, which are separated generally at intervals of about 1 day. The upper panels show the evolution of amplitude and width of modes. For this purpose we have taken an average over all modes.
in frequency interval 2.5–3.0 mHz for f-modes \( (n = 0) \) and in interval 3.0–3.5 mHz for p-modes \( (n > 0) \). All differences shown in the figures are with respect to the first region.

For the active region NOAA9393 the largest flare occurred in region R76, when the region was close to the central meridian. The magnetic field did not evolve significantly around that time. The mode-amplitudes are found to be maximum for R76 which also has the highest MAI value. The mode-amplitudes increase from region R74 to R75, even though the MAI also increased. This was unexpected, since as mentioned before, non-flare active regions showed a decrease in mode-amplitudes with increase in MAI (Rajaguru et al. 2001). This increase is likely to be due to flares that occurred during R75. After that the mode-amplitudes remained essentially constant until R78 even though more flares had taken place. During R79 the mode-amplitudes are substantially reduced even though the MAI value had reduced. Thus the effect of MAI appears to be compensated by that of the flares. The width of the modes has also reduced during the flare.

Fig. 2 shows the difference in mode characteristics in the region covered by NOAA9393 at different times. The frequencies are not significantly affected during this period, however, the small difference that is seen is of opposite sign before and after the flare. The frequency differences are positive for post-flare spectra which may be expected from the fact that magnetic field is highest in region R76. But in the preflare spectra although the magnetic field is smaller the frequency differences are negative, which is contrary to expectation. This could be due to complex changes taking place before the flare. Most of the change in frequency occurs for high-frequency modes which suggests that changes are confined to outermost layers close to the photosphere.

In order to check if similar effects are seen in other flares we look at the data from NOAA9026. This region has the largest flare index amongst all region studied and the magnetic field had not changed significantly during the period under consideration. The power in oscillations appears to have increased after region R51 which had the largest flare. The region R52 also had a substantial flare index and shows maximum mode-amplitude, which could be due to flares. The mode-amplitudes for R52 is found to be larger than that for R51 which had a larger flare activity. It is possible that the effect of the large flare in R51 persisted for some time. In this case too, the width of the modes decreased after the flare. The frequency difference is also found to be similar to that for NOAA9393.

The active region NOAA8539 had a minor flare during R41 and very little flare activity during other times. The magnetic field in this region was also rather low. Despite this it shows enhanced power during R41 and R42, which could be due to the flare. The width of modes also reduced, like in other cases.

The active region NOAA8485 had a flare during R24 and also showed a slow increase in the magnetic field during this period. The mode-amplitudes increased substantially between R21 and R20, but started declining after R22, despite the flare. It is not clear what the effect of the flare was in this case.

The active region NOAA9415 had a major flare during R82, but the mode-amplitudes were essentially constant over the interval R80–R83. The magnetic field for this AR decreased substantially after the flare. It is possible that amplitude increase due to reduction in magnetic field was compensated by
amplitude-enhancement due to the flares.

The active region NOAA8179 had a flare during R3 and R4 and also had a significant increase in magnetic field during this time. In this case there was very little variation in the power in the modes. It is possible that the power enhancement due to flares was compensated by the reduction in power expected from the increase in the MAI.

4. CONCLUSIONS

During some flares the power in acoustic modes appears to increase beyond the normal value expected from the influence of magnetic field. The frequencies of high degree modes appear to be reduced before some flares. This could be due to either change in structure of this region or complex magnetic field variation leading to flares. These changes should be confined to outermost layers around the photosphere. These signatures are not seen in all flares and there is considerable variation between different flares. The extent by which the power appears to be enhanced by flares does not have any obvious correlation with the flare index.

The observed variations in the effect of flares may perhaps be attributed to the relative amounts of deposited flare energy at the lower and deeper layers, and particularly in the photosphere. The large X-class of flare alone is not an adequate indicator of this deposition, as this parameter is related mainly to the emission from the higher, coronal level. On the other hand, a lower X-class, but higher $H_\alpha$ class of the flare may be more important when it comes to effects on oscillation modes. Similarly, it may be expected that the rather rare, white-light flares should excite the p-modes significantly. The flare index alone may not be a good indicator of the magnitude of flare, in affecting the photosphere. In fact, a large FI may result due to several long duration, but small magnitude flares integrated over the time-interval of MDI data. On the other hand, it is possible that a very impulsive flare, which would be more important, was of rather short duration, thereby giving a smaller FI. Much more work is needed to understand the flare to flare variations.

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SOLAR OSCILLATION PARAMETERS: SIMULTANEOUS VELOCITY-INTENSITY
SPECTRAL & CROSS-SPECTRAL FITTING

Caroline Barban and Frank Hill
National Solar Observatory, Tucson, AZ, USA

ABSTRACT

We use the Severino et al. (2001) model for simultaneously fitting four spectra: V (velocity) and I (intensity) power, I-V phase difference and I-V coherence to observational data. We show that this model allows us to reproduce well the observed spectra for \( \ell = 15, 20, 25, 30, 35, 40, 45, 50 \) at low and intermediate frequencies. At high frequencies, the contamination of the spectrum by leaks may prevent fitting the data with the model. A study of the fit parameters as a function of frequency shows the well-known behavior of the mode amplitude and width, but additional modes are needed for a physical interpretation of all fit parameters. Comparing the oscillation parameters from the multi-spectral fitting and from using only the V spectra shows that the oscillation frequency differs by at most 0.1 \( \mu \)Hz.

Key words: Sun: oscillations, Velocity and Intensity data.

1. INTRODUCTION

Helioseismology seeks to infer the properties of the solar interior from the oscillation mode parameters, primarily the frequency. Thus, increasing the precision and accuracy of the mode parameter estimates correspondingly improves our knowledge of the solar interior. The parameters are typically determined by fitting a theoretical profile to the observed Doppler velocity (V) power spectrum, neglecting the information contained in the spectrum of total intensity (I). This information content is demonstrated by the qualitative differences, such as substantial frequency shifts and the sense of the line asymmetry, that are unmistakable in simultaneous V and I spectra (e.g. Nigam et al. (1998), Duvall et al. (1993)). Clearly the physics of the oscillation determining the mode parameters cannot depend on the observed quantity, thus the differences are likely to arise from the excitation mechanism. Further, these differences can be used to derive a more complete and accurate model of the solar oscillations improving the mode parameter estimation and providing new measures of the excitation. Using the model of Severino et al. (2001), we simultaneously fit four spectra: V and I power, I-V phase difference and I-V coherence. We present here preliminary estimates of the oscillation parameters determined from this model using GONG data. In Sect. 2 and 3, the data and the model are presented. In Sect. 4, the results obtained from fitting this model to the data and the basic oscillation parameters are compared with those estimated from a single spectrum model. Finally, Sect. 5 is devoted to conclusions and perspectives.

2. DATA

We used 9 GONG months (324 days, from 1996 Oct. 28 to 1997 Sept. 16, duty cycle=85%) of I and V spherical harmonic time series corresponding to \( \ell = 15, 20, 25, 30, 35, 40, 45, 50 \). We corrected the spherical harmonic coefficients for the frequency splitting \( \Delta \nu_{rot} \) due to solar rotation by multiplying each coefficient by \( e^{-\frac{2\pi\ell}{\Omega}\Delta \nu_{rot}(\ell,m)\ell} \). The frequency splitting were computed from the rotation coefficients \( a_\ell \) of Duvall et al. (1986) and the following equation:

\[
\Delta \nu_{rot}(\ell,m) = \sum_{l=1}^{L} a_l P_l(-\frac{m}{L})
\]

where \( L^2 = \ell(\ell + 1) \) and \( P_l \) are the Legendre polynomials of degree \( l \). We computed I power \( (P_I) \) and V power \( (P_V) \), phase difference \( (\Phi_{I-V}) \) and coherence \( (COH) \) with:

\[
P_{I,V}(\ell,\nu) = \sum_{m=-\ell}^{\ell} |T_{I,V}(\ell,m,\nu)|^2
\]

\[
\Phi_{I-V}(\ell,\nu) = \arg \sum_{m=-\ell}^{\ell} T_{I}(\ell,m,\nu)T_{V}^{\ast}(\ell,m,\nu)
\]

\[
COH(\ell,\nu) = \frac{\left| \sum_{m=-\ell}^{\ell} T_{I}(\ell,m,\nu)T_{V}^{\ast}(\ell,m,\nu) \right|}{\sum_{m=-\ell}^{\ell} |T_{I}(\ell,m,\nu)|^2}
\]

---


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where $T_V(\ell, m, \nu)$ and $T_I(\ell, m, \nu)$ are the Fourier transforms of the rotation-corrected $I$ and $V$ coefficients; asterisks denote the complex conjugate quantity.

3. HELIOSEISMIC SPECTRA MODEL

To reproduce the four spectra ($V$ and $I$ power, $I-V$ phase difference and coherence), we used the model of Severino et al. (2001). This model is based on a coherent ($c$) component which includes the $p$-modes ($p$), a fraction of the solar background that is correlated ($cc$) with the $p$-modes and a fraction of the solar background that is uncorrelated ($ccu$) with the $p$-modes; and an uncoherent component or noise $n$. According to this model, the expression of the four helioseismic spectra are:

- for the $V$ power spectrum:

$$ P_V(\nu) = |V_P(\nu) + V_{cc}|^2 + |V_{cu}|^2 + |V_n|^2 $$

with $V_P + V_{cc} = [V_P(\nu)e^{i\Phi_{V_P}(\nu)} + |V_{cc}|e^{i\Phi_{V_{cc}}}$

where $|V_P(\nu)|$ and $\Phi_{V_P}(\nu)$ the solution of the equation of a damped harmonic oscillator:

$$ |V_P(\nu)|^2 = \frac{|V_P(\nu = \nu_0)|^2 \eta^2}{(\nu_0 - \nu)^2 + \eta^2} $$

$$ \Phi_{V_P}(\nu) = -arctan\left(\frac{\eta}{\nu_0 - \nu}\right) $$

with $\nu_0$ the resonance frequency, $\eta = 1/(2\tau)$ where $\tau$ is the mode life time, $\nu$ the frequency.

- for the $I$ power spectrum:

$$ P_I(\nu) = |I_P(\nu) + I_{cc}|^2 + |I_{cu}|^2 + |I_n|^2 $$

with $I_P + I_{cc} = |I_P(\nu)e^{i\Phi_{I_P}(\nu)} + |I_{cc}|e^{i\Phi_{I_{cc}}}$

where $|I_P(\nu)|$ the solution of the equation of damped harmonic oscillator:

$$ |I_P(\nu)|^2 = \frac{|I_P(\nu = \nu_0)|^2 \eta^2}{4\pi^2(\nu_0 - \nu)^2 + \eta^2} $$

and with $\Phi_{I_{cc}} = \Phi_{I_{cc-V_{cc}}} + \Phi_{V_{cc}}$ and $\Phi_{I_P}(\nu) = \Phi_{I_{P-V_P}} + \Phi_{V_P}(\nu)$.

- for the phase difference spectrum:

$$ \Phi_{I-V}(\nu) = arg\{(I_P(\nu) + I_{cc})(V_P(\nu) + V_{cc})^* + I_{cu}V_{cu}^*\} $$

- for the coherence spectrum:

$$ COH(\nu) = \frac{(I_P(\nu) + I_{cc})(V_P(\nu) + V_{cc})^* + I_{cu}V_{cu}^*}{\sqrt{P_I(\nu)P_V(\nu)}} $$

with: $I_{cu}V_{cu} = |I_{cu}|e^{i\Phi_{I_{cu}}} |V_{cu}|e^{-i\Phi_{V_{cu}}}$

$$ = |I_{cu}| |V_{cu}| e^{i(\Phi_{I_{cu}} - \Phi_{V_{cu}})} $$.  

The equations presented here are for a fixed $\ell$ value and for rotation corrected, $m$-averaged data, then the indices $\ell$ and $m$ are no longer present in these equations.

**Figure 1.** $\ell - \nu$ diagram: + corresponds to oscillation modes where the S/N is sufficiently high in $V$ and $I$ for fitting the modes; diamonds indicate modes where the fit is “visually” good and the fitting parameters are “physically” correct; squares show modes where a comparison of fitting parameters obtained from the multi-spectra fitting method and from a single $V$ power spectrum is possible.

**Figure 2.** Modeled (solid line) and observed helioseismic spectra (dots) for $\ell = 20$, $n = 9$.

4. SIMULTANEOUS V AND I FITTING

The $I$ and $V$ power spectra are normalized by fixing to unity the observed power at the frequency corresponding to the left side of the frequency range used for the fit. All the background components are considered constant in the frequency range used for the fit. We then searched for the best fit between the model and the data by minimizing $\chi^2$. 14 free parameters $(|V_P(\nu = \nu_0)|, |I_P(\nu = \nu_0)|, |I_{cc}|, |V_{cc}|, |I_{cu}|, |V_{cu}|, |V_n|, |I_n|, \eta_0, \Phi_{I_{P-V_P}}, \Phi_{V_{cc}}, \Phi_{I_{cc-V_{cc}}, \Phi_{I_{cc-V_{cc}}}})$ are used for the fit procedure.

A search for adequate S/N in V and I data for the fit process was made for $\ell = 15, 20, 25, 30, 35, 40, 45, 50$; 142 modes were found to fulfill this requirement (see + in Fig. 1). For these modes, when necessary, some spectral intervals were excluded from the fit because of contamination by leakage. Among these 142 modes, we obtained a total of 34 “visually” good and “physically” correct fits where the amplitude is

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positive. Some unphysical negative amplitude values were obtained, probably due to incomplete leakage removal. The 34 good fits are at low or intermediate frequencies (see diamonds in Fig. 1). For these modes $\chi^2/N - N_F$ (where $N$ is the number of independent points and $N_F$ the number of the fit parameters) is between 0.99 and 1.77. An example of acceptable fitted and observed spectra for $\ell = 20$, $n = 9$ is shown in Fig. 2.

4.1. Study of model parameters

The fit parameters as a function of frequency are presented in Fig 3 and 4. As expected, we found a “bell” shape for $|V_{p}(\nu = \nu_0)|$ and $|I_{p}(\nu = \nu_0)|$ and increasing FWHM with frequencies with a “plateau” at intermediate frequencies. $|V_{cu}|$ and $|I_{cu}|$ decrease with frequency (except for a few points) as does $|V_{ns}|$; however $|I_{ns}|$, $|V_{cc}|$ and $|I_{cc}|$ seem to be roughly constant with frequency. $\Phi_{I_{p}-V_{p}}$ is between 70° and 100° with the higher values at high frequencies. A value, less than 90°, the theoretical temperature-velocity phase difference value for an adiabatic evanescent wave, can be explained by a physical mixing between the modes and the underlying background (Severino et al. 1998, Nigam & Kosovichev 1999). $\Phi_{V_{cc}}$, $\Phi_{I_{ns}-V_{ns}}$ and $\Phi_{I_{cu}-V_{nc}}$ seem to have an horizontal “V” shape behavior. No particular behavior is found with $\ell$.

4.2. Comparison with single spectrum fit

A comparison between frequency, FWHM and amplitude ($V_{p}$) determined from the multi-spectral fitting method and the same parameters determined using only the $V$ power spectrum was performed. The absolute oscillation frequency difference is less than 0.1 $\mu$Hz; however there is a systematic tendency for the frequency to slightly higher in the multi-spectral fit. For $V_{p}$ the difference is random and the absolute value is less than 3; and for the FWHM the difference is also random and the absolute value is less than 0.35 $\mu$Hz (Fig. 5). No particular behavior of these differences as a function of $\ell$ is found.

5. CONCLUSION AND PERSPECTIVES

To determine the solar oscillation parameters, we simultaneously fit four spectra ($V$ and $I$ power, $I-V$ phase difference and coherence) using the model of Severino et al. (2001). A total of 34 oscillation modes at low and intermediate frequencies and for $\ell = 15, 20, 25, 30, 35, 40, 45, 50$ are successfully fitted. For these modes, the model does a good job of reproducing the observed spectra. The difficulty fitting the model to the observed modes at high frequencies is probably due to leaks which are not yet sufficiently removed. A more careful procedure to remove the

Figure 3. Fitting parameters $V_{p}$, $I_{p}$, $V_{cu}$, $I_{cu}$, $V_{cc}$, $I_{cc}$, $V_{ns}$, $I_{ns}$, $\Phi_{V_{cc}}$, $\Phi_{I_{ns}-V_{ns}}$, $\Phi_{I_{cu}-V_{nc}}$ and $\Phi_{I_{p}-V_{p}}$ as a function of frequency.
leaks in the fitted spectral range needs to be developed.

The oscillation amplitude and width found from the multi-spectral fit shows their well-known behavior. The behavior with frequency of the other background parameters needs to be further studied by fitting additional modes.

A comparison of the oscillation parameters estimated from the multi-spectral and single-spectrum fits shows that the oscillation frequency differs by at most 0.1 \( \mu \)Hz but with a systematically higher value in the multi-spectral fit. The differences in both the amplitude and FWHM are random, with a maximum amplitude difference of 3 and a maximum FWHM difference of 0.35 \( \mu \)Hz.

Future work will include estimating the fitting errors; increasing the number of fitted modes by a better correction of the leakage; fitting individual \( m \) modes; developing a physical interpretation of all fit parameters; and studying the cycle behavior of the noise components.

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NEAR-SURFACE SOUND SPEED FROM RING DIAGRAMS

Sarbanii Basu\(^1\), Brooke Simmons\(^1\), and Richard. S. Bogart\(^2\)

\(^1\)Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.
\(^2\)Stanford University, HEPL Annex A202, Stanford, CA 94305-4085, U. S. A.

ABSTRACT

We determine the near-surface solar sound-speed profiles using frequencies of high-degree modes determined from from ring-diagram analyses of isolated active and nearby quiet areas. We have analyzed MDI full-disc-resolution data for this work. In order to be able to invert high-degree modes obtained from the ring-diagram analysis, we have had to make several improvements in the inversion codes.

Key words: Sun: oscillations; Sun: interior.

1. INTRODUCTION

Ring-diagram analysis can determine the frequencies of high-degree modes (\(l \geq 200\)) for small regions of the Sun (Antia & Basu 1999). These modes can in principle be used to infer the properties of the region under study.

Ring-diagram analyses have shown that frequencies of active regions are much higher than the those of quiet regions (Rajaguru et al. 2000; Bogart et al. 2002). Low and intermediate-degree global-modes obtained from data observed when the Sun was at different activity levels do not show any degree dependence (expect that related to mode inertia) in the frequency differences, and the differences do not translate to any any difference in structure. Differences in frequencies of high-degree modes between active and quiet regions obtained from ring-diagram analysis show a dependence on degree and the lower turning point (Rajaguru et al. 2000). Thus these modes can be used to study differences in structure between the quiet and the active regions.

We present the results of a first attempt at inverting the frequency differences between active and quiet regions to obtain the sound-speed differences between the regions as a function of depth. We use MDI full-disc Dopplergrams for this work.

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2. DATA AND TECHNIQUE

We consider two active regions: (1) Carrington Rotation(CR) 1922, Longitude 15\(^\circ\), Latitude 7.5\(^\circ\)N, (2) CR1963, Longitude 75\(^\circ\), Latitude 22.5\(^\circ\)N. For the first active region, we choose two quiet regions at the same latitude: CR 1923, Lon 120\(^\circ\) and CR 1923 Lon 37.5\(^\circ\). We do the same for the second region and choose two regions: CR1963, Lon 337.5\(^\circ\) and Lon 52.5\(^\circ\). We consider areas of 16\(^\circ\) × 16\(^\circ\) for each region. The Magnetic Activity Indices (MAI) of the different regions are listed in Table 1.

The MAI was calculated by integrating the unsigned field values within the same regions and over the same intervals as those over which the regions were tracked to calculate the power spectra, using available 96-minute magnetograms. The same temporal and spatial apodizations were used. Only the strong field index was calculated. This was done by setting all fields less than 50 Gauss to zero. One of the reasons of calculating just the strong field index, rather than the total field index is that the 96-minute magnetograms are a mix of 5-minute and 1-minute averages. The averages do have slightly different zero levels and noise levels, which can make a difference at low fields, but is not expected to make much difference if the quantity of interest is only the strong field index. In addition, to remove effects of cosmic rays, outliers were removed. These were defined as
pixels with field values differing by a factor of more than six from the average of their neighbors, if that average was more than 400 G.

Each region tracked for 8096 minutes when they were at the central meridian. The 3D power spectrum was calculated and was fitted to determine the frequencies. To fit the 3D spectra we use a model with asymmetric peak profiles as used by Basu & Antia (1999)

\[
P(k_x, k_y, \nu) = \frac{e^{B_1}}{k^3} + \frac{e^{B_2}}{k^{4/3}} + \frac{\exp(A_0 + (k - k_0)A_1 + A_2(k_x)^2 + A_3(k_xk_y))S_x}{x^2 + 1}
\]

where

\[
x = \frac{\nu - ck^p - U_xk_x - U_yk_y}{w_0 + w_1(k - k_0)}
\]

\[
S_x = S^2 + (1 + Sx)^2,
\]

and the 13 parameters \(A_0, A_1, A_2, A_3, c, p, U_x, U_y, w_0, w_1, S, B_1\) and \(B_2\) are determined by fitting the spectra using a maximum likelihood approach (Anderson, Duvall & Jeffries 1990). The parameter \(S\) measures the asymmetry in the peak profile. The form of asymmetry is the same as that used by Nigam & Kosovichev (1998). The mode characteristics we are interested in are the frequency \((ck^p)\), peak power \((\exp(A_0))\), half-width \((w_0)\) and asymmetry parameter \(S\).

The difference in frequencies thus obtained can be inverted. In order to invert the high degree modes obtained we have to improve the inversion code. The changes needed have been described by Simmons & Basu (this volume).

**Figure 1.** The frequency differences between the different regions of CR1922/CR1923. Only \(n = 1\)–\(5\) modes are fitted successfully. Note that in Panels (b) and (c) the frequency differences are not 0 at low frequency.

**Figure 2.** The frequency differences between the different regions of CR1963. Only \(n = 1\)–\(5\) modes are fitted successfully. Note that the differences in Panel (a) are larger than those in Fig. 1(a). That is because although both regions compared are quiet, they have different MAIs.

**Figure 3.** The sound-speed difference between the two quiet regions of CR1922. The vertical error bars are 1σ errors in the inversion results due to the errors in the data. The horizontal error bars are a measure of the resolution of the inversions.

### 3. RESULTS

The frequency differences between the different regions at CR1922 and CR1963 are shown in Figs. 1 and 2 respectively. On average the sets have \(f\) modes of degree 350 to 1200, \(n = 1\) modes of degree 230 to 1200, \(n = 2\) modes of degree 220 to 900, \(n = 3\) modes of degree 210 to 850, \(n = 4\) modes of degree 200 to 700 and \(n = 5\) modes of degree 200 to 530. Only modes with frequency less than 4.5 mHz are used for the inversions.

Fig. 3 shows the sound-speed differences obtained by inverting the differences in Fig 1(a). Note that the differences are zero to within errors. This is not surprising given that the frequency differences are close to zero. The results that we consider reliable.
Figure 4. The sound-speed difference between the active region and the two quiet regions — CR1922 lon. 52.5° and CR1932 lon. 337.5°.

Figure 5. The sound-speed difference between the active region and the two quiet regions of CR1963.

are shown in the figure. The overlapping error bars tell us that the region has been oversampled and the results at different points are therefore correlated. The downturn at large radii is unlikely to be real, but needs further investigation.

Fig. 4 shows the differences in sound-speed between the active and the two quiet regions of CR1922/CR1923. We can see that beyond a certain depth, the active regions have a higher-subsurface sound speed than quiet regions. This is consistent with the inferences of Kosovichev et al. (2000, 2001). We are unable to invert the immediate sub-surface layers reliably and hence cannot see the negative sound-speed differences that they see in that region. There is a sign of a downturn, but we are unable to confirm that at this time.

Fig. 5 shows the sound-speed difference between the active and quiet regions of CR1963. The results are qualitatively similar to those in Fig. 4. However, the sound-speed differences are larger and probably reflect the larger differences in magnetic activity. The sound-speed differences are so large in this case that the break-down of the linear approximation used in the inversion equations is a real concern. The upturn at lower radii is not real and is a reflection of the lack of low degree modes.

3.1. Can one combine global and local modes?

Given that lower degree modes (l = 100–150) may help get better inversion results, the question arises whether one can add global modes to the mode set. There is a possible danger of systematic effects. To
test what we get, we have added frequency differences of \( l = 100 \) to \( l = 150 \) modes from global mode sets. We add 6 such sets (3 GONG and 3 MDI). The list is given below in Table 2. The GONG data were obtained from 108-day time series. The MDI data were obtained from 72-day time series.

The GONG differences were taken with respect to data from GONG months 10–12. Data collection for this set started on Mar 26, 1996. The MDI differences are with respect to set 1216. Data collection for this set started on May 1, 1996. The three sets in each group have different activity levels as measured by the 10.7 cm radio flux. The GONG reference set has a radio flux of 72.7 sfu, the MDI reference set has a radio flux of 72.4 sfu. Although inversions of the global-mode differences do not show any difference in structure (see Basu & Antia this volume), nonetheless adding these to local-helioseismology data may cause different effects and hence we need to check with a number of different sets.

Fig. 6 shows the sound-speed difference obtained when the different sets of global modes are added to the CR1922/CR1923 data. Note that all sets give results similar to that obtained with only the local data at most radii. There are some differences between the results obtained from the different sets, and although the formal errors in the inversions obtained from the enhanced set are smaller, the scatter between them is of the same order as the inversion errors of the un-enhanced set. The difference in the results between the un-enhanced and enhanced sets at \( r > 0.996R_\odot \), and the difference between the GONG and MDI enhanced results in Fig. 6(b) imply that without higher degree modes we are not likely to succeed in inverting the region between 0.996 and 1\( R_\odot \).

Fig. 7 shows the sound-speed difference obtained when the different sets of global modes are added to the CR1963 data. Note that these results too show a difference in the results obtained by the GONG enhanced data and those obtained by the MDI enhanced data. It could be that the sound-speed differences are no longer in the linear regime. All the enhanced results are however, consistent (within errors) with the un-enhanced results. However, the smaller formal errors and the differences between the GONG and MDI enhanced results show that one needs to be careful when adding global modes to local modes.

4. CONCLUSIONS

We can successfully invert the differences in frequencies obtained from ring-diagram analyses. The sound speed below active regions is larger than that below quiet regions, beyond about 0.996\( R_\odot \) (i.e., 2.8 Mm). We are not yet able to invert successfully at smaller depths. The results are similar to those obtained by Kosovichev et al. (2000, 2001). The sound-speed differences between strong active regions and quiet regions can become large enough that the linearization used to obtain the equations that are inverted may not be valid. It is not completely clear yet whether one can enhance the mode sets obtained from ring-diagram analysis with global modes.

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This work was supported by NASA NAG5-10912 to SB and RSB. This work utilizes data from the Solar Oscillations Investigation / Michelson Doppler Imager (SOI/MDI) on the Solar and Heliospheric Observatory (SOHO). SOHO is a mission of international cooperation between ESA and NASA. MDI is supported by NASA grants NAG5-8878 and NAG5-10483 to Stanford University. This work also utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Inter-American Observatory.

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TEMPORAL VARIATIONS OF SOLAR STRUCTURE

Sarbani Basu\textsuperscript{1} and H. M. Antia\textsuperscript{2}

\textsuperscript{1}Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.  
\textsuperscript{2}email: basu@astro.yale.edu  
\textsuperscript{2}Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India  
\textsuperscript{2}email: antia@tifr.res.in

ABSTRACT

We have analysed GONG and MDI data for the past 7 years to determine if there are any changes in solar structure. We fail to find any change in the solar interior. In the process of investigations, we find that there are possible systematic differences between the pre- and post-recovery MDI data for the high degree (\(T \gtrsim 120\)) modes.

Key words: Sun: oscillations; Sun: interior.

1. INTRODUCTION

That solar oscillations frequencies change with time has been known since the last solar cycle (Libbrecht & Woodard 1990). The change in frequencies is clearly seen in the current solar cycle with high-precision data from both the Global Oscillation Network Group (GONG) and the Michelson Doppler Imager (MDI). The accumulation of data by these two projects over the last seven years enables us to make a detailed study of changes that take place within the Sun as the solar cycle progresses.

We use GONG and MDI data to study how the radial as well as latitudinal dependence of structure changes with time. We also study possible temporal variation in solar radius using the F-mode frequencies. Since any study of time variations pre-supposes that the data do not have time-dependent systematic errors. We check for that in the data.

2. DATA AND TECHNIQUES

We have used data obtained from 108-day observations from GONG (i.e., three GONG months) and 72-day observations from MDI. We use 68 temporally overlapping data sets from GONG starting from 1995 May 7 and ending on 2002 February 22, with a spacing of 36 days between consecutive data sets.

\textbf{Table 1. Data sets used}

<table>
<thead>
<tr>
<th>Set #</th>
<th>Start Day</th>
<th>10.7 cm Flux$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GONG sets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1–3</td>
<td>May 7, 1995</td>
<td>75.1</td>
</tr>
<tr>
<td>10–12</td>
<td>Mar 26, 1996</td>
<td>72.7</td>
</tr>
<tr>
<td>24–26</td>
<td>Aug 12, 1997</td>
<td>91.0</td>
</tr>
<tr>
<td>34–36</td>
<td>Aug 7, 1998</td>
<td>131.3</td>
</tr>
<tr>
<td>63–65</td>
<td>Jun 16, 2001</td>
<td>180.8</td>
</tr>
<tr>
<td>66–68</td>
<td>Oct 2, 2001</td>
<td>214.0</td>
</tr>
<tr>
<td>MDI sets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1216</td>
<td>May 1, 1996</td>
<td>72.4</td>
</tr>
<tr>
<td>1432</td>
<td>Dec 3, 1996</td>
<td>73.2</td>
</tr>
<tr>
<td>1936</td>
<td>Apr 21, 1998</td>
<td>108.5</td>
</tr>
<tr>
<td>2224</td>
<td>Feb 2, 1999</td>
<td>130.7</td>
</tr>
<tr>
<td>2800</td>
<td>Sep 1, 2000</td>
<td>175.3</td>
</tr>
<tr>
<td>2944</td>
<td>Jan 23, 2001</td>
<td>164.4</td>
</tr>
<tr>
<td>3160</td>
<td>Aug 27, 2001</td>
<td>219.8</td>
</tr>
</tbody>
</table>

$^a$ Units of \(10^{-22} J \, s^{-1} \, m^{-2} \, Hz^{-1}\)

The MDI data consists of 30 non-overlapping data sets starting from 1996 May 1 and ending on 2002 August 21 (Schou 1999). In order to keep figures from overcrowding we have not shown results of all data sets in all figures. The ones that are shown are listed in Table 1. In the table, the heading “Start Day” is the beginning of the 108 day observational period for the GONG sets and the 72 day period for the MDI set. Also listed in the table is the average 10.7 cm radio frequency flux over the observing period, which is known to be a good solar activity index.

The radial sound-speed profiles were obtained by SOLA inversions (Pijpers & Thompson 1992, 1994) applied to solar structure inversions (see Rabello-Soares et al. 1999 for details). The latitudinal sound-speed distribution was determined by 2d RLS inversions (Antia et al. 2001a). The position of the CZ base was determined by the method described by Basu & Antia (1977).


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Figure 1. The scaled frequency differences (panel a) between the GONG sets listed in Table 1 and GONG set 10–12. The differences have been plotted after averaging in bins of 25 modes. Only one set of errors has been shown for the sake of clarity. The radio frequency flux for each set is in parentheses. Panel (b) shows the relative sound-speed differences in the Sun at different times as obtained by inverting the GONG frequency differences.

3. RESULTS

Fig. 1(a) shows the scaled frequency differences between the GONG sets listed in Table 1 and GONG set 10–12. The quantity $Q_{nl}$ is the ratio of the mode inertia of a mode of degree $\ell$ order $n$ to that of a mode of degree 0 with the same frequency as the mode of degree $\ell$ order $n$ (Christensen-Dalsgaard & Berthomieu 1991). Since the scaled-differences are just a function of frequency, we plot the scaled frequency differences averaged in bins of 25 modes. A similar result is seen for MDI data. From the figure we see that as the activity level increases, the frequencies increase. The frequency shift is known to be well correlated to solar activity (Howe et al. 1999).

Figs. 1(b) and 2 show the sound-speed differences obtained by inverting the frequency differences between the different GONG and MDI sets respectively. The inversion results are reliable till about $0.965 R_\odot$. We do not see any significant, or systematic change in the sound-speed with time in the GONG results. MDI results however show a qualitative difference in the results of sets obtained before SOHO was temporarily out of contact (i.e., sets 1432, 1936) and the other sets that were obtained after contact was resumed. The difference occurs above the CZ base. All the post-recovery sets show a positive $c^2$ difference with respect to the reference set 1216. The fact that this temporal variation is not seen in the GONG results leads us to believe that there are systematic differences in the pre- and post-recovery MDI data, particularly at the high degree end.

In order to test for systematic differences between high-degree frequencies as a function of time, we plot the frequency differences as a function of degree. Since there is a steep frequency dependence of the frequency differences at each degree, we average the differences for modes that have frequencies between 2 and 3 mHz. We use a common mode set for this. The results for GONG and MDI are shown in Figs. 3 and 4. Note that there is no obvi-
ous degree dependence in frequency differences in the GONG data. But for the MDI data the pre-recovery differences show no obvious degree dependence, the post-recovery sets do, particularly above $l = 120$ or so. Since the GONG results for data obtained at the same time do not show any such degree dependence this figure supports our hypothesis that there are non-solar systematic differences between pre- and post-recovery data. The sound-speed differences obtained by inverting the differences between the different MDI sets when the mode-set is restricted to modes with $l < 120$ show an absence of any systematic differences between pre- and post-recovery data. The results are similar to the GONG results shown in Fig. 1(b).

Further evidence of systematic errors is obtained when comparing GONG and MDI results of sound-speed asphericity and the f-mode frequency variations. The fundamental or f-mode is a surface gravity mode and its frequency is not very sensitive to solar structure. These frequencies have been employed to determine the solar radius (Schou et al. 1997; Antia 1998). There have been conflicting claims about variation in solar radius, based on temporal variations of the f-mode frequencies. Dziembowski et al. (2001) have claimed that solar radius is decreasing at a rate of 1.5 km/year during the last few years. They express the f-mode frequency variation as

$$\Delta \nu_l = -\frac{3}{2} \frac{\Delta R_l}{R} \nu_l + \frac{\Delta \gamma_f}{I_l}$$  \hspace{1cm} (1)$$

where $\nu_l$ is the frequency of f-mode for a degree $l$, $\Delta R_l$ is inferred change in solar radius. The second term is believed to arise from variation near the solar surface and $I_l$ is the mode inertia.

Fig. 5 summarises the results obtained using such a fit to MDI data sets, where frequency differences with respect to a standard solar model are considered for each of the 30 available MDI data sets. The resulting temporal variation in $\Delta R$ and $\Delta \gamma_f$ as well as the $\chi^2$ per degree of freedom for each of the fits are shown in Fig. 5. It appears that there is an oscillatory trend with a period of about one year in all three quantities. This oscillatory trend has been studied by Antia et al. (2001b) and is most likely to be an artifact of data analysis. It is clear from $\chi^2$ values that most of the fits are not good and the assumed form given by Eq. 1 does not fit the observed data. Ignoring the oscillatory trend in the inferred radius variation, which is unlikely to be of solar origin, it appears that most of the variation has occurred between 1998.4 and 1999.4 which is exactly the period when contact with SOHO was lost. This becomes more clear if we look only at the filled squares in the figure which are at intervals of one year and correspond to data sets which give relatively good fits. The variation in solar radius before or after the break in SOHO data is negligible. This is again most likely to be due to systematic variations in instrumental characteristics during recovery of SOHO.

Using the even order splitting coefficients it is possible to infer the latitudinal variation in sound speed (Antia et al. 2001a). Fig. 6 shows $\delta c^2/c^2$ at $r = 0.96R_\odot$ obtained using MDI data. To bring out the temporal variations the temporal average at each latitude has been subtracted from all results. The top panel which shows the results obtained using all modes shows a distinct temporal variation. Before
the gap the asphericity is positive at low latitudes, and negative at high latitudes. This is reversed after the gap. This temporal variation is not seen in GONG data as well as in the bottom panel of Fig. 6 which shows the MDI results when only modes with $\ell < 110$ are used.

Since the solar dynamo is generally believed to be located close to the base of the convection zone, it would be interesting to study temporal variations in this region. No significant temporal variation is found in depth of the convection zone (Basu & Antia 2001).

4. CONCLUSIONS

We find no observable change in the solar sound-speed profile as a function of time. There is a systematic difference between the high-degree modes of the pre- and post-recovery data from MDI. This systematic difference affects the sound speed in the convection zone. This systematic difference also affects the estimate of solar radius and in fact, apart from oscillatory trend with a period of one year, almost all temporal variation in solar radius is found to occur during the time when the contact with SOHO was lost. The aspherical component of sound speed in outer convection zone is also affected by this systematic error resulting in a large temporal variation during the gap in data.

There are two alternatives: either the Sun has had some interesting transition during the time when SOHO was not operational or these apparent temporal variations are due to systematic variations in instrumental characteristics during recovery of SOHO satellite. The fact that these variations are not seen in GONG data would tend to suggest that there are indeed some systematic variations in MDI instrument, which should be accounted for before inferring possible temporal variations inside the Sun.

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TEMPORAL VARIATIONS IN THE ROTATION RATE IN THE SOLAR INTERIOR

Sarban Basu$^1$ and H. M. Antia$^2$

$^1$Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.  
$^1$email: basu@astro.yale.edu

$^2$Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India  
$^2$email: antia@tifr.res.in

ABSTRACT

The frequency splittings obtained from GONG and MDI observations over the last 7 years are used to study how the rotation rate of the solar convection zone has evolved with time. The bands of faster and slower than average rotation rate are found to move towards the equator at low latitudes, while at high latitudes they move towards the poles. The low latitude bands also move upwards with time, and they extend almost to the base of the convection zone. We find no significant temporal variation in the rotation rate in the tachocline region.

Key words: Sun: oscillations; Sun: rotation; Sun: interior.

1. INTRODUCTION

With the accumulation of GONG and MDI data over the last seven years it is now possible to study the temporal variation in the rotation rate in the solar interior. The solar rotation rate is known to show temporal variations, with bands of faster and slower rotating regions moving towards the equator with time (Howe et al. 2000a; Antia & Basu 2000). This pattern is found to penetrate to a depth of about 0.1$R_\odot$. Antia & Basu (2001) extended this study to higher latitudes to find that at high latitudes these bands move polewards. In this work we extend these studies to longer time interval and also investigate the radial variations in the zonal flow pattern.

The seat of the solar dynamo is believed to be near the base of the convection zone and one may expect some changes in this region during the solar cycle. Howe et al. (2000b) have reported 1.3 yr oscillations in the rotation rate in equatorial region at $r = 0.72 R_\odot$. However, this periodicity has not been seen in other investigations (e.g., Antia & Basu 2000; Corbard et al. 2001), and hence, needs to be investigated further.

2. DATA SET

We have used data sets from GONG and MDI for this investigation. These sets consist of the mean frequency and the splitting coefficients. We use GONG data for months 1–68, which cover the period from 1995 May 7 to 2002 February 22. We have used the 68 temporally overlapping data sets each covering a period of 108 days with a spacing of 36 days between two consecutive data sets. The MDI data sets (Schou 1999) consist of 30 non-overlapping data sets each covering a period of 72 days, starting from 1996 May 1 and ending on 2002 August 21 with some gaps during 1998–99 when contact with SOHO was temporarily lost.

We use a 2D Regularized Least Squares (RLS) inversion technique (Antia et al. 1998) to infer the rotation rate in solar interior from each of the available data sets.

![Figure 1](image)

**Figure 1.** A contour diagram of the rotation-velocity residuals at $r = 0.98 R_\odot$ obtained using 2D RLS inversion of GONG data. The solid contours are for positive $\delta \nu_{rot}$, while dotted contours denote negative values. The contours are drawn at interval of 1 m/s. The zero contour is not shown.


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3. ROTATION RATE IN THE CONVECTION ZONE

The rotation rate in the outer layers of the Sun shows temporal variations with a pattern similar to the well known torsional oscillations at the surface. To study the temporal variations in the rotation rate we look at the residuals obtained by subtracting the temporal mean of the rotation rate from the rotation rate at any given time. The residuals at a depth of $0.02R_\odot$ below the surface from GONG data are shown in Fig. 1. This figure actually shows the linear velocity corresponding to the residual in rotation rate, i.e.,

$$\delta v_{\text{rot}} = \delta \Omega r \cos \theta,$$

where $\theta$ is the latitude. From this figure it can be seen that there are distinct bands of faster and slower than average rotation rate, and that these move towards the equator with time at low latitudes. At high latitudes, the bands seem to move towards the poles.

![Figure 1.](image)

**Figure 2.** Superposed contour diagram of the rotation-velocity residuals (black contours) at $r = 0.98R_\odot$ and surface term in asphericity inversions (gray contours). The solid contours are for positive values, while dotted contours denote negative values.

Fig. 2 compares the temporal variation in zonal flows with those seen in the surface term of the inversion for asphericity. Antia et al. (2001) have demonstrated that the latter is well correlated with temporal variation in magnetic flux at solar surface. It can be seen that the band of positive zonal flow velocity is narrower than the surface term of asphericity. The equatorward edge of the two bands is roughly the same.

Fig. 3 shows the rotation rate residuals from GONG data plotted as a function of time and radial distance at latitudes of $15^\circ$, $30^\circ$, $45^\circ$ and $60^\circ$. At low latitudes there is a clear trend of contours moving upwards with time. From this we can deduce that the pattern rises upwards with time at a rate of about $0.05R_\odot$ per year or about 1 m/s. At $15^\circ$ latitude, the pattern clearly penetrates to depths greater than $0.1R_\odot$ inferred in earlier works, reaching almost to the base of the convection zone. At other latitudes the depth of penetration cannot be discerned from these figures. At high latitudes the time evolution of pattern with depth is not clear either. Near the tachocline region there are many fluctuations which are probably due to errors in inversion and thus may not have any significance.

**Figure 3.** The rotation-velocity residuals from GONG data as a function of time and radial distance at selected latitudes. The contours of constant residual velocity are shown at intervals of 1 m/s, with solid contours showing positive values and dotted contours showing negative values.

Fig. 4 shows the residual in rotation velocity at different latitudes as a function of time at $r = 0.98R_\odot$ from both GONG and MDI data. It is clear that different latitudes are at different phases of the pattern and the location of minima and maxima have strong latitudinal variations. The amplitude of variation is distinctly smaller around a latitude of $50^\circ$ beyond which the shift in phase is gradual. If we assume that the zonal-flow pattern is periodic with a period of 11 years, the non-sinusoidal nature of variation implies the presence of higher harmonics of the period. We fit an expression of the form

$$\delta v_\phi(r,t,\theta) = \sum_{k=1}^{N} a_k(r,\theta) \sin(k\omega_0 t + \phi_k)$$

where $\theta$ is the latitude and $\omega_0 = 2\pi/P_0$ is the basic solar cycle period, with $P_0 \approx 11$ years. Since the various terms are not orthogonal over the limited time period for which data are available the amplitudes of these terms also depends on the number of terms included. It is found that inclusion of $k = 2$ term tends to suppress the dominant $k = 1$ term. If this term is dropped then we can fit $k = 1$ and 3 terms. Fig. 5 shows the amplitudes of the $k = 1$ and $k = 3$
components as a function of radial distance and latitude. It is clear that the first term dominates and the amplitude of higher harmonic is less than half of the basic frequency. Similar results have been found by Vorontsov et al. (2002). The higher harmonic is found to be significant only near the surface. Furthermore, from the amplitude of $k = 1$ term it is clear that the pattern penetrates to a good fraction of the convection zone depth at low latitudes and possibly at high latitudes too. The pattern is not well defined at intermediate latitudes of around $40^\circ$.

![Figure 5](image5.png)

*Figure 5.* The contours of constant amplitude of $k = 1$ and 3 terms in the fit to zonal flow velocity. The contours are shown at intervals of 1 m/s, with dashed contour showing the 1 m/s level.

To check for possible periodic oscillations near the tachocline region, we calculate the residual in the rotation rate at different depths and latitudes. Some of the results are shown in Fig. 6, which can be compared with Fig. 2 of Howe et al. (2000b). We do not see any significant periodic signal in either the GONG or the MDI results at any latitude or depth. Even in inversion results of Howe et al. (2000b), the periodicity is not clear in MDI data and appears to show up only in GONG data.

![Figure 6](image6.png)

*Figure 6.* The rotation-rate residuals as a function of time at a few selected radii and latitudes. The radii are marked at the top of the figure, while latitudes are marked in each panel. The continuous line shows the results obtained using GONG data, while the points show the results from MDI data using 2D RLS inversion technique.

### 4. THE TACHOCLINE

Since the rotation rate in the tachocline region is steeply varying function with radius, inversions may not give reliable results in this region. To study the properties of the tachocline we use the three techniques described by Antia et al. (1998). These are: (1) a calibration method in which the properties at each latitude are determined by direct comparison with models; (2) a one dimensional (henceforth, 1d) annealing technique in which the parameters defining the tachocline at each latitude are determined by a nonlinear least squares minimisation using simulated annealing method and (3) a two-dimensional (henceforth, 2d) annealing technique, where the entire latitude dependence of the tachocline is fitted simultaneously, again using simulated annealing. To improve the accuracy we take weighted average of these three results.

In all techniques the tachocline is represented by a
model of the form (cf., Antia et al. 1998),

\[
\Omega_{\text{tac}} = \frac{\delta \Omega}{1 + \exp[(r_t - r)/w]},
\]

(2)

where \( \delta \Omega \) is the jump in the rotation rate across the tachocline, \( w \) is the half-width of the transition layer, and \( r_t \) the radial distance of the mid-point of the transition region. Here, \( \delta \Omega, r_t \) and \( w \) are functions of latitude. The properties we are interested in are the position and the thickness of the tachocline, and the change in rotation rate across the tachocline. No significant temporal variation is seen in any of these properties (Basu & Antia 2001).

Table 1. Mean properties of the tachocline at different latitudes

<table>
<thead>
<tr>
<th>Lat. (°)</th>
<th>( \delta \Omega ) (nHz)</th>
<th>( r_t ) ((R_\odot))</th>
<th>( w ) ((0.01R_\odot))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.60 ± 0.42</td>
<td>0.6920 ± 0.0018</td>
<td>0.67 ± 0.13</td>
</tr>
<tr>
<td>15</td>
<td>17.68 ± 0.23</td>
<td>0.6912 ± 0.0018</td>
<td>0.75 ± 0.12</td>
</tr>
<tr>
<td>45</td>
<td>30.32 ± 0.52</td>
<td>0.7096 ± 0.0018</td>
<td>1.16 ± 0.10</td>
</tr>
<tr>
<td>60</td>
<td>67.62 ± 0.72</td>
<td>0.7103 ± 0.0022</td>
<td>1.47 ± 0.19</td>
</tr>
</tbody>
</table>

Since there is no significant temporal variation in any of the properties of the tachocline, we take temporal average over all data sets and techniques to improve the accuracy and the results are shown in Table 1. There is clearly a systematic variation in the position of tachocline with latitude while the variation in thickness is less clear. From Table 1, it appears that there is little difference between depth and thickness of tachocline between latitudes of 0° and 15° and between latitudes of 45° and 60°. It could be hypothesised that latitudes 0° to 30° and > 30° form two parts of tachocline which are distinct with different depths and thicknesses, while there is very little variation with latitude within each part.

5. CONCLUSIONS

The rotation-rate residuals, obtained by subtracting the time-averaged rotation rate from that at each epoch, show a pattern of temporal variation similar to the well known torsional oscillations observed at the surface, with bands of faster and slower rotation moving towards the equator with time. At high latitudes the bands appear to move towards the pole. This is quite similar to what is seen in theoretical results of Covas et al. (2000). At low latitudes the bands of faster and slower rotation appear to move upwards at a rate of about 1 m/s and the pattern penetrates almost to the base of the convection zone. Assuming that the zonal flow pattern has a fundamental period of 11 years we find that in addition to the fundamental period, the third harmonic is also significant in outer layers.

We do not find any evidence for the 1.3 year periodicity in equatorial regions at \( r = 0.72R_\odot \), reported by Howe et al. (2000b). There is no significant temporal variation in the position, width or the extent of jump across the tachocline. The position of tachocline shows a significant variation with latitude, and the tachocline is found to be prolate, with a difference of \((0.018 ± 0.003)R_\odot\) in position between the equator and a latitude of 60°, which is consistent with results obtained by Charbonneau et al. (1999).

ACKNOWLEDGEMENTS

This work utilises data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Inter-American Observatory. This work also utilises data from the Solar Oscillations Investigation/ Michelson Doppler Imager (SOI/MDI) on the Solar and Heliospheric Observatory (SOHO). SOHO is a project of international cooperation between ESA and NASA. M. D. is supported by NASA grants NAG5-8878 and NAG5-10483 to Stanford University.

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THE MAGNETIC FIELD IN THE CONVECTION ZONE.

Alberto Bigazzi and Alexander A. Ruzmaikin
Jet Propulsion Laboratory, California Institute of Technology, CA USA

ABSTRACT

One of the key questions in solar physics that remains to be answered concerns the strength and the distribution of the magnetic fields at the base of the convection zone. The flux tube dynamics requires that toroidal fields of strength as large as 100 kilogauss be present at the base of the convection zone. The kinetic-magnetic equipartition argument leads to smaller field strengths. For possible detection of these relatively small (compared to pressure effects) fields by helioseismic methods it is important to know the range of the field strengths and their distribution.

We estimate a range for the toroidal magnetic field strengths at the base of the convection zone using dynamo simulations in a spherical shell. These simulations involve the distribution of rotation provided by helioseismic inversions of the GONG and MDI data. Combining the simulations with the observed line-of-sight surface poloidal field we extract the spatial pattern and magnitude of the mean toroidal magnetic field at the base of the convection zone.

1. INTRODUCTION

The inner distribution of the solar magnetic field is poorly known. Despite the striking regularity of the solar activity, we still lack an understanding of its causes and the mechanisms by which it operates.

The layer that we are able to directly observe the magnetic fields on is the photosphere. From observations of sunspots we know that magnetic fields with intensities of the order of thousands of Gauss are produced somewhere below the photosphere and can live for months before decaying away. The topological and magnetic structure of sunspots (as expressed by Hale’s and Joy’s laws) suggest that they harbor intense, toroidal magnetic flux tubes. The poloidal component of the solar magnetic field is much weaker compared with the toroidal field.

The intensity and spatio-temporal distribution of magnetic fields inside the Sun represent a challenge for any theory explaining the origin of solar activity. On one hand, the highly localised sunspot field points toward local dynamics. On the other, the much more diffuse poloidal field more naturally fits into a mean-field model. One needs to combine two approaches to explain the solar cycle: the flux tube dynamics and the mean field dynamo. Thin flux tube dynamics has been very successful in explaining features connected to sunspots, see e.g Choudhuri & Gilman 1987, Fan Fisher & De Luca 1993, Caligari, Moreno-insertis, & Schüssler 1995. The periodic migration of sunspots toward the equator and the field reversal cannot be explained by this dynamics alone. The explanation of the reversals of the field and of the equator-ward migration of the activity were in fact one of the great achievements of the mean field theory originated by Parker, Steenbeck, Krause and Rädler (Parker 1955, Steenbeck, Krause and Rädler 1969). Yoshimura (1975a) proved that waves mainly propagate along the isosurfaces of angular velocity. With the present knowledge of solar differential rotation, this would imply that inside the bulk of the convection zone waves travel radially outwards, not giving rise to equatorward migration. Migration may be restored in the shear layer at the base of the convection zone.

While it is possible to generate high magnetic fluxes by means of differential rotation, to store them for enough time to allow for their intensity to build up would not be feasible in the convection zone because such magnetic flux tubes would erupt in a timescale of months (Parker 1975, Moreno-insertis 1986). Helioseismology tells us that around 0.7 \( R_\odot \), a sharp radial change in the solar rotation curve happens in a layer whose thickness can be as small as 0.02 \( R_\odot \) (see Christensen-Dalsgaard et al. 1991, Basu & Antia 1997, Kosovichev 1996, Corbard et al. 1998, Charbonneau et al. 1999).

The same layer may allow for fields up to \( 10^8 \) Gauss to be stored, what would be needed for thin flux tube dynamics to work in the case of the Sun (Moreno-Insertis et al. 1992, Ferriz-Mas & Schüssler 1994).

Following Ivanova & Ruzmaikin (1977), Parker (1993) discussed a model of mean field dynamo that includes a sharp gradient of turbulent diffusivity and two distinct location for the sources of the magnetic field, that is differential rotation and helical turbulent motions. Stronger toroidal fields can in fact be produced in the region where diffusivity is smaller, just below the convection zone. Separating the shear layer and the source of the alpha-effect, moreover, would allow for alpha not be quenched.
by the strong underlying magnetic field. A very thorough investigation of this kind of models has been carried out by Charbonneau and MacGregor, 1997. We will use in our Profile II a similar setup to one of those discussed in the aforementioned paper.

In the following, we are going to study how different profiles of the α-effect may influence the spatial distribution of fields. We shall consider the profile of diffusivity and the rotation curve as given.

Mean field, kinematic dynamo cannot predict the absolute values of the generated magnetic fields, only their ratio. The measured mean radial field at the surface can then be used to infer the value of the toroidal magnetic field in the interior, once the ratio is known.

2. DYNAMO MODEL

We consider axisymmetric solutions of the mean field dynamo. We assume for the solar rotation a simple analytical fit to the profile reconstructed by helioseismic where the surface rotation curve

\[ \Omega_0 = \Omega_0 \left(1 + a_1 \cos^2 \theta + a_2 \cos^4 \theta\right) \]

is made smoothly match the core rotation \( \Omega_C \) in a layer of thickness \( 0.2R_\odot \) at \( 0.692R_\odot \). Equatorial rotation is \( \Omega_0 = 2.865 \times 10^{-6}\,\text{s}^{-1} \) and the core rotation taken as the value of \( \Omega_C \) at \( 30^\circ \) latitude. \( \theta \) here is colatitude. \( a_1 = -0.126 \) and \( a_2 = -0.159 \). The radial profile of \( \partial \Omega_0 / \partial r \) at the equator is shown in solid line in Figure 1. The sign of this gradient is opposite at higher and lower latitudes. Turbulent diffusivity \( \eta \) is constant throughout the Convection Zone and we have it drop a factor 10 to 50, in a layer of thickness \( 0.2R_\odot \) at a location either coincident with that of the rotational shear layer or slightly above it, at \( 0.713R_\odot \). Both these values have been worked out in the context of Helioseismology, see Christensen-Dalsgaard et al. 1991, Basu & Antia 1997, Kosovichev 1996, Corbard et al. 1998, Charbonneau et al. 1999. In reality the drop in turbulent diffusivity is estimated to be of the order of \( 10^6 \). In Figure 1 the diffusivity profile along the radial direction is represented by the solid dashed line, its gradient marking the bottom of the Convection Zone. In solid line, the radial derivative of the rotation which defines the shear layer is plotted.

The mean field dynamo equation, written in terms of scalar potentials (Krause & Rädler, 1980) is solved numerically in a \( r - \theta \) meridional semi-disk. Second order finite differences in space and a third order Runge-Kutta scheme for time advance are used. In most of the runs a grid of \( 60 \times 40 \) is used. Ideal conductor boundary conditions are used at the interface with the core and radial field condition is implemented at the surface. Regularity condition are imposed on the axis.

No model for non-linear α-quenching has been used. This is consistent with our assumption that the flow field is not influenced by the magnetic field.

2.1. The α-effect

Given the lack of constraints on the form of the α-effect, we shall discuss how different alphas influence the spatial distribution of the the toroidal and poloidal fields. Figure 1 shows the distribution in radius of α. In all models, a latitudinal dependence of \( \cos(\theta) \) is included. This is standard in dynamo theory and reflects the property of the alpha-effect that being antisymmetric with respect to the Equator. In case IV, an additional factor of \( \sin^2(\theta) \) is present. This has been discussed by Rüdiger & Brandenburg, 1995, and has the effect of shifting the magnetic field patterns closer to the equator, which better reproduces the observed patterns of sunspot migration.

Profile I, the diamonds in Figure 1, is maximum in the bulk of the Convection Zone, at 0.82 \( R_\odot \) in our case. It is null at the surface. This form of the α-effect takes into account the influence of the rotation on helical turbulence, see Zeldovic, V. Ruzmaikin, Sokoloff 1983. Profile II, squares, is sharply peaked at 0.7 \( R_\odot \), just above the shear layer, and represents an Interface Dynamo model. As already mentioned, this profile was used by Charbonneau and MacGregor (1997). Profile III is, instead, non-vanishing only in the outer shell of the Convection Zone. This kind of “surface dynamo” has been discussed in conjunction with meridional circulation, see e.g. Choudhouri et al. 1995. Profile IV, triangles, has a constant value throughout the whole Convection Zone. It drops to zero close to the surface and at the base of Convection Zone. This profile has often been adopted in the literature. The last case considered, V, α has two contributions: one coming from the bottom of the convection zone where it is negative and proportional to the gradient of η, combining the effects due to the decrease in the intensity of the turbulence and the stratification. The other, coming from the convection zone, is taken to be the same as in I. Yoshimura (1975b) has used this form for the α-effect.

<table>
<thead>
<tr>
<th>( \alpha(r) )</th>
<th>( r_0 )</th>
<th>( \alpha_0 )</th>
<th>( \gamma )</th>
<th>( T )</th>
<th>( r_{\text{max}} )</th>
<th>( B_r / B_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>.75</td>
<td>4.1</td>
<td>8.2</td>
<td>.026</td>
<td>.71</td>
<td>.84</td>
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<tr>
<td>II</td>
<td>.7</td>
<td>27.5</td>
<td>19</td>
<td>.047</td>
<td>.67</td>
<td>.80</td>
</tr>
<tr>
<td>III</td>
<td>&gt; .8</td>
<td>4.1</td>
<td>18</td>
<td>.008</td>
<td>.78</td>
<td>.92</td>
</tr>
<tr>
<td>IV</td>
<td>.7   / .98</td>
<td>4.23</td>
<td>5</td>
<td>.015</td>
<td>.9</td>
<td>73</td>
</tr>
<tr>
<td>V</td>
<td>.75</td>
<td>5</td>
<td>180</td>
<td>.020</td>
<td>.71</td>
<td>360</td>
</tr>
</tbody>
</table>

Table 1. Location and intensity of the maximum toroidal magnetic field for different choices of the profile of the α-effect. The location where the profile is peaked \( r_0 \) is given, along with the intensity \( \alpha_0 \). \( \gamma \) and \( T \) are the growth rate and the period of the solutions. \( B_r \) is the maximum value of the radial photospheric field averaged over time. \( B_T \) is the maximum value of the time average of the toroidal magnetic field intensity. When two values are given, the first one represents a secondary maximum close to the shear layer, see Figure 2. \( r_{\text{max}} \) is the location where the maximum is located, in units of \( R_\odot \). Numbers are dimensionless. Lengths have been scaled to \( R_\odot \) and time to \( R_\odot^2 / \eta \), where \( \eta \) is the value of turbulent diffusivity in the convection zone.
The α-effect, diffusivity and rotation profiles.

Figure 1. Radial distribution of the α-effect for the cases I-V. Values are not to scale. See Table 1 for the actual values used in the present simulations. The solid lines, dashed and continuous, represent the radial profile of the turbulent magnetic diffusivity η and the radial gradient of the rotation rate ∂Ω/∂r at the equator, respectively. They define for our model the Convection Zone and the shear layer (the Tachocline).

Figure 2. Surface plot of toroidal and radial field intensity integrated over time for Profile I. Values are scaled to the maximum of the radial surface field.

3. DISCUSSION OF THE RESULTS

Except for the case of Profile III, where α is concentrated in the surface layer, all the case studied display a local maximum of the toroidal field in the shear layer. Except for case V, this is not an absolute maximum which is instead achieved in the convection zone, within 0.8 ÷ 0.9 R☉, see Figure 2 and Figure 3.

The ratio of the maximum toroidal field to the surface radial field can vary from a few tens as in the case of the surface α-effect, case III, to a thousand, as in the case of the interface profile II. Both the profile I and V have ratios of the order of a few hundreds. No symmetry across the equator has been imposed on the solution which display a North-South asymmetry. Except for case of the surface α, the symmetry of the solution is mainly dipolar. Considering a mean radial surface field of the order of a Gauss (Schlichenmaier and Stix 1995) the range of ratios that we found would lead to an estimate of the large-scale mean toroidal field of the order of 10³ Gauss in the most favorable cases I,II,V. Both the surface (III) and top-hat (IV) profiles have smaller ratios of the toroidal to the surface fields.

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If one assumes that solar activity is originated in the shear layer, then it is possible to draw a butterfly diagram with the temporal evolution of the toroidal field near the shear layer. All those models show butterfly diagrams with activity at higher latitude then observed. This is a known feature of many dynamo models.

We have shown that different assumptions about the $\alpha$-effect give rise to very different spatial distributions of the magnetic fields inside the Sun. This message could also be read in reverse: should we be able to probe the field deeply inside the Sun, we could have information about the nature of the regeneration mechanism expressed by the $\alpha$-effect.

ACKNOWLEDGMENTS

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A NEW INSTRUMENT FOR SOUNDING THE SOLAR ATMOSPHERE

A. Cacciari\textsuperscript{1}, S. M. Jeffries\textsuperscript{2,3}, W. Finsterle\textsuperscript{2}, P. Rapex\textsuperscript{1}, A. Knox\textsuperscript{2}, C. Giebink\textsuperscript{2}, and V. Di Martino\textsuperscript{4}

\textsuperscript{1}Physics Departement, University La Sapienza, Piazzale Aldo Moro 2, I-00185 Rome
\textsuperscript{2}Maui Scientific Research Center, University of New Mexico, 590 Lipoa Pkwy, Ste. 262, Kihei, HI 96753
\textsuperscript{3}Steward Observatory, University of Arizona, Tucson AZ 85719
\textsuperscript{4}CASPUR, University La Sapienza, Piazzale Aldo Moro 2, I-00185 Rome

ABSTRACT

A new instrument based on Magneto-Optical Filters (MOFs) (Cacciari et al., 1994) will be used to simultaneously map the line-of-sight velocity at two heights in the solar atmosphere. Simultaneous Doppler images of 5 arc-seconds resolution will be taken in the K I (7699 Å) and Na I D\textsubscript{2} (5890 Å) lines, which are separated by a few hundred kilometers in the solar atmosphere (Grossman-Doerth, 1994). By cross correlating the signals of the K and Na channels we will be able to determine the travel time and thus the propagation speed of sound waves in the solar atmosphere. The experiment will be run at the South Pole during austral summer of 2002/2003.

Key words: magneto-optical filters; high-frequency acoustic waves; helioseismology.

1. DESCRIPTION OF THE INSTRUMENT

The Magneto-Optical filters at Two Heights (MOTH) instrument consists of two parallel optical benches, one for sodium (5890 Å) and one for potassium (7699 Å). Apart from the filters, the components on both optical benches are identical (see Figures 1, 2). Four CCD cameras acquire simultaneous images of the full solar disk in the red and blue wings of the solar K and Na D\textsubscript{2} lines. The difference of the intensities in the wings of both lines is taken as a measure of the line-of-sight velocity of the line-forming layer in the solar atmosphere (Cacciari & Fofi, 1978; Cacciari et al., 1988). The cameras are operated with 2×2 pixel binning to increase the SNR. The full disk image will cover about 340 pixels, thus each pixel corresponds to ≈5.3 arc-seconds, whereas the diffraction limit of the primary telescope is 3 arc-seconds and 4 arc-seconds for Na and K, respectively (see Table 1). The temporal sampling interval is 10 seconds with a duty cycle of 100%. The instrument box will be mounted on an actively guided tracking platform (Jeffries et al., 1989). The data acquisition system will be located in a buried building 30 meters away from the observing platform to reduce the impact of thermal turbulences. Optical fibers are used to transmit the signal as well as to control camera operations. The instrument has successfully proven its ability to acquire high quality Doppler images during test runs in Maui.

2. PROVIDING A NEW VIEW OF THE SOLAR ATMOSPHERE

Our comprehension of the Sun has increased dramatically in the last twenty years through the study of solar seismic waves (helioseismology). We have gained significant insights into the Sun's internal rotation and have been able to measure some of the
basic parameters of the solar model such as the primordial helium content and the depth of the convection zone. However, because the Sun's atmosphere is easily visible, seismic probing of its properties has not been pursued as rigorously as it has been for the solar interior. Consequently our understanding of the "acoustic properties" of the solar atmosphere is limited.

2.1. Scientific Goals

Acoustic waves with frequencies above the acoustic cut-off frequency for the solar atmosphere (\approx 5\text{mHz}) are able to propagate in the atmosphere (Jeffries, 1998). This property will allow us to use a time-distance analysis to determine the sound speed in the solar atmosphere by measuring the acoustic wave travel time of high-frequency waves travelling between two heights in the atmosphere. This can be done for all positions on the visible solar disk. We will then invert these data to produce a map of the average sound speed between the two heights. Although the resulting sound speed map will not provide accurate absolute sound speed values, due to the limitations of current models for the atmosphere (which are used in the inversion process), it will provide a first look at the horizontal sound speed inhomogeneities in the lower atmosphere. This information should spur improvements in the models. We will generate sound speed maps for several 17 hour stretches over a period of a month. These data will allow us to study any "short-term" variations that might be occurring in the atmosphere's properties. They will also provide a reference point for future studies of solar-cycle related variations.

2.2. Time-Distance Seismology

Computation of the acoustic ray paths in the atmosphere shows that, for the spatial resolution of our experiment, the paths can be considered as vertical in the atmosphere for all waves of interest (Figure 4). This suggests the analysis strategy shown in Figure 5. By cross-correlating the signal at D1 with the signal in the annulus at D we isolate waves with a certain horizontal wavelength (corresponding to the
Figure 4. Enlargement of the ray paths shown in figure 3. Since the angle subtended by a single resolution element in the instrument’s CCD camera will be \(\approx 0.5\) deg (i.e., approximately eight times the length of the horizontal axis), the ray paths in the atmosphere will appear to be vertical in our measurement.

Figure 5. Time-distance geometry for observations at two heights in the atmosphere. The wave travel-time across the atmosphere, \(t_{\text{atm}}\), is determined by subtracting the time taken for a wave packet to travel along the path \(D\) to \(D_1\), \(t_{\text{ex}}\), from the time taken for the path \(D\) to \(D_2\), \(t_{\text{ex}} + t_{\text{atm}}\). Because all possible annuli centered on \(D_1\) also have waves that travel through \(D_1\) and \(D_2\), (see Figure 4), this approach will produce a cleaner measurement of \(t_{\text{atm}}\), with superior signal-to-noise characteristics, than would a measurement based on the correlation of the signals over the disks \(D_1\) and \(D_2\).

skip distance \(D\). Identical wave discrimination is provided for the correlation of the signals at \(D_2\) and \(D\). Differencing the measured times for the peaks of the two cross-correlation functions gives the wave travel time between the two observing heights. Because the measurement is differential, we are insensitive to the lack of absolute travel-time measurement capability from a single measured correlation signal that is inherent when the raw data are filtered to isolate waves of a given frequency (D’Silva, 1998a,b). Since all waves travel vertically in the atmosphere, we can average the wave travel-time measurements for all horizontal wavelengths to improve the signal-to-noise ratio.

3. ACKNOWLEDGMENTS

We thank Sydney D’Silva for the ray path calculations and Marc Fimeri at Adept Electronic Solutions for many discussions related to the data acquisition system and for outstanding support.

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HIGH-FREQUENCY INTERFERENCE PEAKS IN BISON DATA

W. J. Chaplin¹, Y. Elsworth¹, G. R. Isaak¹, K. I. Marchenko¹, B. A. Miller¹, and R. New²
¹School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, U.K.
²School of Science and Mathematics, Sheffield Hallam University, Sheffield, S1 1WB, U.K.

ABSTRACT

We have analyzed 9 yr of non-imaged Doppler velocity observations of the visible disc of the Sun in an effort to search for pseudo-mode-like structure in the data above the acoustic cut-off frequency of the solar atmosphere (≈ 5400 μHz). These data were collected by the ground-based Birmingham Solar-Oscillations Network (BiSON) over the period 1992 January through 2000 December. Our analysis uncovers the presence of a pseudo-mode-like structure above the acoustic cut-off frequency that persists up to ≈ 8500 μHz, with a spacing between adjacent peaks (or troughs) of ~ 68 μHz. The signature – which disappears at higher frequencies – has a slightly different repeat period (i.e., frequency separation between successive peaks or troughs) to that found by Garcia et al. (1998) in full-disc GOLF data.

Key words: Sun: activity – Sun: oscillations.

1. INTRODUCTION

Acoustic waves are generated in the sub-surface convection zone of the Sun by stochastic excitation through turbulent motion. The solar atmosphere is vertically stratified under gravity, and there exists a characteristic 'acoustic cut-off' frequency (νac) above which periodic waves can no longer be sustained in an outward, radial direction. Near the solar surface, the decrease in density and temperature gives rise to a maximum in νac of ~ 5400 μHz. As such, outwardly propagating waves with characteristic frequencies less than this undergo total internal reflection; this defines the upper boundary of the resonant cavity in which the p-mode oscillations of the Sun are formed. In reality the transition is not instantaneous, i.e., as one passes through and increasingly above νac, the effective reflectivity of the upper boundary decreases, leading to a reduction in signal to noise and a decrease in the Q-factor (or sharpness) of the modes. For ν ≫ νac acoustic disturbances propagate as traveling waves, and a resonant signature is no longer expected. However, observations made at intermediate (Libbrecht 1988; Jeffries et al. 1988; Duvall et al. 1991; Ronan 1992), high (Fernandes et al. 1992) and most recently low (Garcia et al. 1998) angular degrees show clear evidence of such structure. Current prejudice favours an explanation for this based upon the geometric interference between waves that reach the observer directly (from the subphotospheric source) and indirectly, e.g., by partial reflection from the back side of the Sun (Kumar et al. 1990; Kumar 1994). Here, we build on the work of Chaplin et al. (2001) who found marginally significant evidence for the presence of high-frequency interference structure in low-ℓ BiSON data. Improvements in the analysis are made, and a larger quantity of data is used, with the result that we uncover prominent pseudo modes at high frequencies.

2. DATA AND ANALYSIS

The BiSON spectrometers observe the Sun in integrated light (i.e., a Sun-as-a-star view) and employ the technique of resonance scattering (Isaak 1961) to give a measure of the intensity in two narrow passbands in the wings of the 770-nm solar Fraunhofer line of potassium (Brookes, Isaak & van der Raay 1978a). With the blue and red-wing intensities designated IB and IR respectively, we form the well-known proxy of the topocentric velocity according to:

\[ S_\text{R}(t) = \frac{I_B - I_R}{I_B + I_R} \]  

The use of well-proven techniques – which rely upon the precisely known diurnal changes arising from the spin and orbital motion of the Earth – allow one to calibrate accurately each of the observed proxies to absolute velocity units (Elsworth et al. 1995). The orbital and diurnal variations – in addition to a large relativistic offset arising from the gravitational redshift – can then be removed to yield a set of velocity residuals, here with a cadence of 40 s. In order to facilitate the study, we have used data collected by the network over the period 1992 to 2001. This covers most of the epoch in the activity cycle of the Sun.


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Figure 1. Low-resolution BiSON power spectrum (smoothed with a 5-pt running mean) constructed from the incoherent addition of spectra made from independent, 1.9-d time-series segments. The peak at \( \sim 5500 \mu\text{Hz} \) is a gear-frequency artefact. Above this, in the region delineated by the two vertical lines, ‘pseudo modes’ are present whose signature persists up to a frequency of \( \sim 8500 \mu\text{Hz} \).

The comparative high-frequency quality of data from each BiSON site will inevitably vary (owing to slight variations in the detail and vintage of the instrumentation), with some stations better suited to the study of high-frequency phenomena than others. Here, we took data from our three best sites (those at dome-based observatories with high-throughput systems and photocell detectors), with objective criteria used to reject data of poor quality (e.g., see Chaplin et al. 2002 for a description of the technique as applied to the low-frequency regime). Those daily residuals from these sites that passed the rejection criteria were combined coherently to form a well-filled time series.

In addition to making use of a more extensive set of data, the other main improvement over our previous analysis (Chaplin et al. 2001) has been the application of a procedure to fill gaps in the time series of length up to 2 hr. The technique (Chaplin et al., in preparation) has some similarities with that of Fossat et al. (1999). The majority of gaps in any typical BiSON time string are of such short length; their removal is found to have a favourable impact on the contrast of any structure at high frequencies (e.g., from the substantial reduction of power aliased from the main body of the 5-minute spectrum).

The ‘filled’ time series was then divided into independent 1.9-d pieces (each composed of 4096 40-s points), but in a manner that sought to optimize the fractional fill of the selected sets. To do so a ‘sliding’ window was applied in the time domain that allowed us to pick out – and then ignore – any empty or very poorly filled segments of the time series. Power spectra of the selected segments were then added (incoherently) in the frequency domain. Fig. 1 shows the resulting low-resolution spectrum. The peak at \( \sim 5500 \mu\text{Hz} \) is a gear-frequency artefact. Above this, in the region delineated by the two vertical lines, a resonant-like structure is present that persists up to a frequency of \( \sim 8500 \mu\text{Hz} \).

3. DISCUSSION

The high-frequency structure is shown more clearly in Fig. 2. The mean separation between successive peaks, over the range \( 5700 \leq \nu \leq 7800 \mu\text{Hz} \), is found...
to be: $67.4 \pm 0.9 \, \mu Hz$ from an unweighted analysis of the differences; and $67.8 \pm 0.8 \, \mu Hz$ from a weighted mean that uses uncertainties extracted from fitting the peaks as a series of Lorentzians to fix the weights. The measured frequency spacing between successive even (left-hand panel) and odd-\ell (right-hand panel) pairs is plotted in Fig. 3. The average separations above the acoustic cut-off frequency are found to be: $135 \pm 1.6 \, \mu Hz$ for even pairs, and $135.1 \pm 1.1 \, \mu Hz$ for odd.

In the region where the pseudo modes are observed clearly, we find no evidence for any significant change in the separation between successive peaks (or troughs); however, a ‘phase’ change is observed as one moves downward through $\nu_{ac} \sim 5400 \, \mu Hz$ into the regime of trapped modes. Evidence for this is shown in Fig. 4 where the high-frequency part of the spectrum is plotted after the removal of the average power level. The dotted line is a simple sine-wave, with frequency period $68 \, \mu Hz$, whose phase has been fitted to match that of the high-frequency pseudo modes: it no longer matches (in phase) the structure below $\nu_{ac}$.

The characteristics of the structure we have uncovered are roughly similar (e.g., approximate peak spacing, phase change through $\nu_{ac}$ amplitudes) to those reported by Garcia et al. (1998), who were the first to find interference peaks above $\nu_{ac}$ at low \ell. However, a more detailed comparison reveals significant differences. The most prominent of these is the difference in mean spacing – the GOLF value is nearer $70 \, \mu Hz$ – with the result that if one compares the frequencies of the same pairs in the two sets, the difference in these is observed to increase to $\sim 60 \, \mu Hz$ at $\sim 7500 \, \mu Hz$.

The structure is also observed to persist to somewhat higher frequencies in the BISON data. Garcia et al. speculated that were the photosphere to be the only source of partial wave reflection for inward traveling waves (which would then interfere with those which travel straight out) the mean peak spacing would be expected to double as the reflectivity dropped to zero. We find no evidence for this above the suggested threshold of $\sim 7500 \, \mu Hz$.

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Figure 3. The measured frequency spacing between successive even (left-hand panel) and odd-$\ell$ (right-hand panel) pairs. Differences extracted over the range where the gear-frequency artefact may influence any fits to the data have not been plotted.

Figure 4. High-frequency part of the BiSON spectrum after the mean power has been removed. The prominent peak at $\sim 5500 \mu$Hz is a gear-frequency artefact. The dotted line is a simple sine-wave, with frequency period $68 \mu$Hz, whose phase has been fitted to match that of the high-frequency pseudo modes. A 'phase' change is clearly seen as one moves downward through $\nu_{ac} \sim 5400 \mu$Hz into the regime of trapped modes.
Correcting GONG+ Magnetograms for Instrumental Non-uniformities

R. Clark, J. Harvey, F. Hill, C. Toner

National Solar Observatory – GONG, P.O. Box 26732, Tucson, Az. 85726
rclark@noao.edu 520 318 8523 (voice) 520 318 8400 (fax)

The instruments in the GONG+ network are now producing magnetograms once a minute. These magnetograms are subject to a zero point uncertainty, varying across the field of view, of a few gauss. For a large number of images low order 2–dimensional polynomial surfaces are fit to quiet regions on the sun and averaged to determine this zero point error. Applying the resulting correction to individual magnetograms results in some improvement, as shown by inter–station comparisons. However, it is found that the zero point error has a time–varying component. A possible cause for this variation has been identified.

The recently upgraded instruments of the Global Oscillation Network Group (GONG) now routinely obtain magnetogram observations of the sun every minute. The high temporal and moderate spatial resolution of these observations offer a useful resource for the solar physics community. It is important to calibrate these data as accurately as possible to maximize their value.

An unknown factor impacting the quality of the magnetogram observations arises from non–uniformities and small imperfections in the magnetogram modulator,. These can introduce uncertainties of several gauss in the zero point of the magnetograms and impacts the quality of merged data from multiple GONG sites. We have tried various approaches to characterize this non–uniformity using available GONG data products.

Currently, the preferred method attempts to extract instrument related artifacts from actual magnetogram images. A low order 2–dimensional polynomial surface is fit to quiet regions of the sun. A threshold limit is used to limit contributions from active regions. This process is then repeated, using the departure from the fitted surface rather than from zero as the basis for rejecting points in active regions. In later iterations the threshold for rejection is tightened.

The GONG tracking turret rotates the solar image during the course of the day. The GONG camera is physically rotated to compensate and keep the image oriented with solar north aligned along the x direction on the CCD. The images from a day are numerically rotated back to the camera’s parked position. The surface fitting is then performed and the fits to these ‘derotated’ images are stacked and, for each pixel, the median is found and stored, with some further smoothing, to produce a daily stationary feature map. The ~90° variation in image rotation over the course of a day allows partial isolation of solar contributions from those that are fixed with respect to the modulator. These daily maps still show some contribution from large scale solar magnetic structure that leaks through the averaging process, and from seeing–induced noise. Several daily maps are averaged to produce a modulator correction map. Figure 1 shows part of this process.

The resulting zero point correction maps provide some improvement when applied to magnetogram images. The top row of images in figure 2 shows uncorrected magnetograms from the Big Bear and Mauna Loa GONG sites for the same minute (2001 October 8, 1850 UT), and the difference between them. The bottom row shows the same two magnetograms with the zero point corrections applied. The resulting difference image shows substantial improvement with respect to the whole–image bias correction, but only modest improvement in removing the net gradient across the image. Comparison of several such image pairs revealed variation in the quality of improvement during the day. Figure 3 shows the comparison for images at 2330 of the same day.

When a set of reference points from the individual fitted surface images were monitored during the day, systematic variations were noted. This could be produced by features from the rotating image of the sun being superimposed on the fixed modulator pattern. However, a trend was found to be permanent over an extended period of time, suggesting that the cause might arise from within the GONG instrument. Figure 4 shows an example of this from the Learmonth GONG site, where the behavior was first noticed.

Recent measurements by Harvey and Sudol (to be described elsewhere) at the GONG test site in Tucson confirm this interpretation of a time–varying modulator bias and suggest a possible cause. Their observations have established that:

- The modulator requires a finite switching time to
change between its +¼ to −¼ wave states.
• During this switching time the modulator is sensitive to linear polarization at the 1% level.
• The changing angles of reflection from the mirrors in the GONG tracking turret are a possible source of time-varying linear polarization at this level.

The investigation of this effect, and ways to measure and model it is now underway.

SUMMARY
• There is a zero point error in GONG magnetograms introduced by nonuniformity of the magnetogram modulator. This error can vary across the solar image.
• Modest improvement to GONG magnetograms can be achieved through the application of a fixed and constant correction to each station’s magnetograms.
• The magnetogram modulator has been found to be sensitive small amounts of linear polarization which may be introduced by the GONG tracking turret.
• For significant further improvement of GONG magnetograms a time varying correction will be necessary.

ACKNOWLEDGEMENTS
This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Interamerican Observatory.

Fig 1 At left is a magnetogram from Big Bear on October 8 2001. The greyscale saturates at approximately ±45 gauss. The sky background graylevel is set to 0. The center image shows the set of pixels included in the final iteration of the surface fitting process. The sky background and pixels excluded from the fit are set to black. Other pixels are shown with a greyscale between ±12 gauss. The initial pixel threshold to reject active areas was ±25 gauss relative to 0. The threshold for the final iteration was ±12 gauss relative to the fit. At right is the final surface fit to a 2–d quadratic polynomial shown with the same greyscale as the center image. The area outside the average position of the sun is set to black. Here solar north is toward the right.

Fig 2. The top row shows uncorrected magnetograms for Mauna Loa (left) and Big Bear (center) on October 8, 2001, 1850 UT. At right is the difference image (ML − BB). The bottom row shows the same images after modulator corrections (based on approximately 20 days of averaged surface fits) for each station is applied. The greyscale is linear between ±12 gauss. The sky background is set to 0. The black and white dots outside the solar disk are fixed in the instrument parked orientation. Images were scaled and registered with solar north toward the top for the inter-station comparisons.

Fig 3. Comparison of ML − BB uncorrected and corrected images for October 8, 2001 at 2330 UT.

Fig 4. The value (in gauss) at fixed points within the image on the modulator correction fitted surface changes through the day in a systematic manner. Nine reference points— the center, and eight points evenly spaced around the average location of the solar limb (R=80)% of the solar radius from the center) are plotted against time of day. Data from 10 days spanning the period October 2 – 31, 2001 are shown. The sensitivity of the modulator to linear polarization from the tracking turret varies across the field of view. This data is from Learmonth, where the time–dependent behavior was first recognized.
Learmonth

10 days near 011017

Fig 4
RING-DIAGRAM ANALYSIS WITH GONG++

T. Corbard\textsuperscript{1}, C. Toner\textsuperscript{1}, F. Hill\textsuperscript{1}, K. D. Hanna\textsuperscript{1}, D. A. Haber\textsuperscript{2}, B. W. Hindman\textsuperscript{2}, and R. S. Bogart\textsuperscript{3}

\textsuperscript{1}National Solar Observatory, 950 N. Cherry Ave., Tucson, AZ 85719, USA
\textsuperscript{2}JILA, Univ. of Colorado, Boulder CO 80309-0440, USA
\textsuperscript{3}Stanford University, CSSA-HEPL, Stanford, CA 94305-4085, USA

ABSTRACT

Images from the updated GONG network (GONG++) have been produced since July 2001. In order to treat individual site images and the merged images (Toner et al., 2003) for local helioseimology studies, we have developed an enhanced tracking/remapping code that is now part of the new GONG pipeline (GONG++) (Hill et al., 2003). We present here the data-cube, 3D power spectra and sub-surface flow maps which will become part of the new GONG++ products and compare the preliminary results with the ring diagram analysis of MDI images for the same days.

Key words: Sun; Ring-Diagram; Data Analysis.

1. INTRODUCTION

Ring-diagram analysis of MDI images carried out over the last few years has led to a breakthrough in helioseismology studies which now allow us to observe the solar sub-surface flows in both azimuthal and meridional directions and in both solar hemispheres. Using this analysis technique one can study plasma dynamics around and below surface magnetic features and their evolution during the solar cycle or at smaller time scales. It remains unclear however what the uncertainties and potential bias on these measurements might be. The new high resolution GONG+ observations will allow us to test these important results with independent datasets. We present in section 2 and 3 the principal features of the code developed to perform ring-diagram analysis of GONG+ images and give in section 4 the first qualitative comparison with MDI data analysis.

2. THE ‘DATA CUBE’

In order to estimate the mean 3D velocities of an area over the Sun we need to follow this area over time. Such a spatio-temporal area is defined by an array (or data-cube) of $N_x \times N_y \times N_t$ points $M_{ik}(t_k)$ identified by their heliographic latitudes $\psi_{ij}(t_k)$ and Carrington longitude $\varphi_{ij}(t_k)$ at different times $t_k$. In order to define such an area completely we need:

1. A reference time $t_0$ and a central position $M_{00}(t_0)$ given by $\psi_0 = \psi_{00}(t_0)$ and $\varphi_0 = \varphi_{00}(t_0)$

2. A sampling time $\Delta t$ so that $t_k = t_0 + k\Delta t$ for $k = -N_t/2 + 1..N_t/2$

3. The definition of a collection of points around the central position (remapping) and a tracking rate for following these points in time.

\[\begin{align*}
\psi_0 \quad \text{remapping} \quad \psi_{ij}(t_k) \\
\varphi_0 \quad \text{tracking} \quad \varphi_{ij}(t_k)
\end{align*}\]

4. An interpolator in order to find the values of the Doppler velocities at each point from the image pixels.

2.1. Remapping: The transverse cylindrical equidistant projection

For our local analysis, the high degree acoustic waves are assumed to be plane waves traveling across the surface of the Sun following geodesics i.e. great circles. Moreover we are going to use a 3D Fourier transform for which equidistance is assumed. Therefore we would like to have a grid on which each horizontal or vertical line is a great circle and where the distances are preserved in both directions (the equidistance in time being provided by the constant image rate $\Delta t$). In a gnomic projection each straight line would represent a great circle but the distance scale is greatly distorted. Unfortunately, no projection can be made that preserves distance along the entire extent of the line joining any two points. However the equidistance can be insured, at least in one direction, without too much distortion near the center of the projection.
Such a projection can be obtained by placing a point of the solar surface at the coordinates $X = \psi_i$, $Y = \pi/2 - \phi_i$, of the remapped area where $\psi_i$ and $\phi_i$ are respectively the latitude and longitude of that point in a system where the solar meridian passing through $M_0$ is the new equator. Since a line of constant longitude is a great circle, all horizontal lines of the remapped area are great circles and the distances are preserved along these lines. By noting $\Delta x$ and $\Delta y$ the angular spacings on the remapped area and reversing the projection, we obtain:

$$\begin{align*}
\psi_{ij}(t_0) &= \sin^{-1}\left(\cos X_i \sin(Y_j + \psi_0)\right) \\
\phi_{ij}(t_0) &= \sin^{-1}\left(\frac{\sin X_i}{\cos \psi_i(t_0)}\right) + \phi_0
\end{align*}$$  

(1)

where:

$$\begin{align*}
X_i &\equiv (i - (N_x + 1)/2) \Delta x & i &= 1...N_x \\
Y_j &\equiv (j - (N_y + 1)/2) \Delta y & j &= 1...N_y
\end{align*}$$  

(2)

We note that the azimuthal equidistant (or Postel) projection has also been used for ring analysis (e.g. Bogart et al., 1995). This projection is constructed by projecting on a plane tangent at $M_0$ whereas the projection defined above and used in Haber et al. (1995) is a transverse cylindrical equidistant projection that can be obtained by projecting on a cylinder. The Postel projection is equidistant on all lines passing through $M_0$ but not in $X$ nor in $Y$ (except for $X_i = 0$ and $Y_j = 0$). More discussion about the two projections is given in the Appendix.

2.2. Tracking

The main goal of tracking is to remove as much as possible the effect of the solar rotation which would otherwise hide the small horizontal and vertical flows of interest. We consider therefore that the latitude of a point is fixed in time but its Carrington longitude is changing at a rate that is not necessarily the Carrington rate and may depend on the latitude. We use:

$$\begin{align*}
\psi_{ij}(t_k) &= \psi_{ij}(t_0) \\
\phi_{ij}(t_k) &= \phi_{ij}(t_0) + (t_k - t_0) \times \\
&\quad \left[\Omega_0 - \Omega_1 \sin^2 \psi_0 - \Omega_2 \sin^4 \psi_0 - \Omega_3\right]
\end{align*}$$  

(3)

where $\Omega_0, \Omega_1$ and $\Omega_2$ are adjusted to match the sidereal differential rotation of the surface and $\Omega_3 \equiv 14.1844$ deg/day is the sidereal Carrington rate. We note that the tracking is done for all points at the rate corresponding to the heliographic latitude of the center of the area. All points lying on a great circle are therefore still on a great circle after tracking. For large areas the differential rotation may vary substantially across the area and therefore one may consider tracking each point using its heliographic latitude (i.e. $\psi_{ij}(t_0)$ instead of $\psi_{ij}$ in Eq. (3)). However, in the resulting sheared coordinate system, the $Y = \text{constant}$ lines would no longer describe great circles and it is probably better to still track the full area at a mean rate.

2.3. Interpolation

Once we know the heliographic coordinates of the points of interest we need to look for them on our solar images $I(t_k)$. The solar disc is shown on GONG+ merged images as a circle of radius $r_0$ pixels. They are registered so that the solar north is on top of the y axis and the east on the left. The coordinates $(\eta, \xi)$ from the lower left corner of the image taken at $t_k$ of the points $M_{ij}(t_k)$ are then obtained by doing a projection at Earth distance. If we note $\psi = \psi_{ij}(t_k)$ and $\phi = \phi_{ij}(t_k)$, we obtain:

$$M_{ij}(t_k) = I_{n\xi}(t_k)$$  

(4)

where:

$$\begin{align*}
\frac{n - N_x}{r_0} &= \sin \psi \cos(\xi - L_0)/(1 - \alpha) \\
\frac{\xi - L_0}{r_0} &= \sin \phi \cos B_0 - \cos \psi \sin B_0 \cos(\phi - L_0) \\
\alpha &= \sin \psi \sin B_0 + \cos \psi \cos B_0 \cos(\phi - L_0)
\end{align*}$$  

(5)

The position of the disc center $(\eta_0, \xi_0)$, its heliographic coordinates $(L_0, B_0)$, the Sun apparent semi-diameter in radians $\alpha$ and $r_0$ can be found in the FITS header of each image.

The values of $(\eta, \xi)$ computed from Eq. (5) are in general not integers and therefore we need to interpolate between the images’ pixels. We have implemented 3 different interpolators. The ‘ideal’ sinc interpolator is defined in 1D by:

$$I_x = \sum_{n = -N_s}^{N_s} \sin\left(\pi x - \pi n \frac{2N_s}{2N_x}\right) I_n$$  

(6)

where $N_s$ defines the length of the interpolator. The 2D interpolation is obtained by applying Eq. (6) successively in each direction. This can be seen as a tapered version of the sinc interpolator where the taper is ‘ideal’ for the Discrete Fourier Transform (DFT) in the sense that if we were to shift the image by an arbitrary fraction of a pixel using this interpolator, with $N_s$ equal to the number of pixels in both directions, the DFT would remain the same except for a phase difference. A similar interpolator (with a slightly different tapering function) is used during the merging of individual site images and it is probably a good idea to keep the same interpolator in both steps. However, in our current implementation, the computations needed for this interpolation are time consuming. This difficulty can be overcome by using a lookup table for the trigonometric quantities resulting in the so-called lookup sinc interpolator which has also been implemented. Finally, we have also implemented a spline interpolator based on a tensorial product of B-splines (De Boor, 1978) of arbitrary order $n_s$. It presents a good frequency response even for low order splines and its computation is based on finite differences which are very easy and fast to implement.

1 Respectively by the keywords: FNDLMBXC, FNDLMBYC, L0, B0, SEMIDIAM and C.MA.
By plotting the difference of the power spectra obtained from spline and sinc interpolator we realized that differences exist which are principally concentrated on the rings. These differences are however very small and the relative difference is essentially zero for absolute values of the horizontal wave number below 1 rad/Mm\(^{-1}\). Therefore it seems that only the very outer rings could potentially be affected by the choice of the interpolation.

2.4. The ‘dense pack’ of data cubes

In order to study an extended area of the solar surface we process a collection of 189 data cubes centered at different heliographic latitudes and longitudes \((\psi_0, \varphi_0)\) on the reference image \(I(t_0)\).

\[
\begin{align*}
\psi_0(u, v) &= u \times 7.5^\circ, \quad u = -7.7 \\
\varphi_0(u, v) &= L_0(t_0) + v \times 7.5^\circ, \quad v = -h_{[u]}, h_{[u]} \\
\end{align*}
\]

where \(h = [7, 7, 7, 7, 6, 6, 4, 3]\).

For the present work, we have done three dense-pack analyses for \(t_0\) corresponding to January 2002 11-13 with corresponding \(L_0(t_0) = 300^\circ, 285^\circ, 270^\circ\) respectively. We used the same parameters as in previous analyses of MDI data (e.g. Haber et al., 2002): the grid is defined by \(N_x = N_y = 128\), \(\Delta x = \Delta y = 0.125^\circ\), \(N_t = 1664\), \(\Delta t = 1\) mm; the tracking rate is given by \(\Omega_0 = 451.43\) nHz, \(\Omega_2 = 54.77\) nHz, \(\Omega_4 = 80.17\) nHz and the interpolator chosen is based on cubic splines \((m_s = 4)\).

3. MULTITAPER ANALYSIS, RING FITTING AND INVERSION

Each data cube is then Fourier transformed via a 3D FFT. This requires first a spatial and a temporal apodization in order to avoid truncation effects. We have chosen a 2D cosine bell apodization in the spatial direction reducing a \(18^\circ \times 16^\circ\) area to a circular patch of radius \(15^\circ\).

The temporal apodization is carried out using a multitaper technique which first multiplies a sequence of orthogonal sine tapers to the window function, reorthogonalizes them, applies a temporal FFT to each tapered time series and then averages the resultant power spectra. Doing so, we avoid the lost of data that would result from the use of a single taper and obtain a better distribution of the power along the rings. Applying too many tapers would result in an over-smoothed power spectrum and therefore a trade-off has to be found. In this work, we used 3 sine multitapers (Fodor & Stark, 1998).

The last steps of the analysis consist in fitting the rings in the power spectra (see Fig. 1) and inverting the inferred frequency shifts to obtain the depth dependence of the flow. This has been done using the code developed for MDI and described in Bogart et al. (1995).

4. FIRST RESULTS: COMPARISON WITH MDI

A study of the errors as a function of depth have shown that the lowest errors are reached at a depth of about 1-2 Mm. We have therefore chosen this depth for the first comparison between GONG and MDI ring diagram analysis. The differences between the flows obtained from the two datasets at a depth of 1.18 Mm are shown in Fig. 2. The amplitudes of the differences are clearly a function of the distance from the center of the disk. They remain small at the very center but can reach very high relative amplitudes at the edges. For the second day, a clear systematic difference in the orientation of the flow is found. This reflects the fact that the GONG flow for that day has been found with a significant curl. A problem with the P-angle could produce such apparent rotating flow but we couldn't identify clearly the source of the problem and why this happened only for that particular day which had a very good duty cycle of 91.2%. Clearly this is very preliminary work using the new GONG+ data product and more investigations and quantitative analysis are needed to better understand the systematic errors that may be present in both GONG and MDI data analysis.

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This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation.
APPENDIX: AZIMUTHAL VERSUS TRANSVERSE CYLINDRICAL

As stated above, the projection used is a transverse cylindrical equidistant projection (or transverse plate carrée). Cylindrical means that it is constructed by wrapping a cylinder around the sphere. All great circles orthogonal to the one tangent to the cylinder will be remapped as parallel straight lines. It is called transverse because the tangent great circle chosen is not the equator but rather the solar meridian passing through $M_0$. Equidistant means that distance is preserved along the great circles shown as horizontal lines on the remapped area. Finally we note that the points are not geometrically projected, in the true sense of the word, i.e. it is not a perspective projection.

The azimuthal equidistant or Postel projection is obtained by projecting on a plane tangent to the sphere at $M_0$ while preserving the distance along the great circles passing through $M_0$. It is also not a perspective projection. It is defined by: $X_p = \theta_p \cos \varphi_p$ and $Y_p = \theta_p \sin \varphi_p$, where $\theta_p$ and $\varphi_p$ are the colatitude and longitude in a system in which $M_0$ is at the new pole. In this system our cylindrical projection is defined by: $X = \sin^{-1}(\sin \theta_p \cos \varphi_p)$ and $Y = \tan^{-1}(\tan \theta_p \sin \varphi_p)$.

One can easily verify that $Y=$constant is the equation of a great circle. This also shows that for small $\theta_p$ (i.e. small areas), the vertical lines also become close to great circles and our projection becomes similar to a Postel projection. For larger areas, however, the cylindrical projection does a better job in having the lines parallel to one axis describe great circles, which may be a desired feature for local plane-wave analysis, at least along one direction of propagation.

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Figure 2. Difference between the flows at a depth of 1.18Mm inferred from GONG and MDI data for 3 consecutive days (January 11-13 2002 from top to bottom).
NONAXISYMMETRIC VARIATIONS DEEP IN THE CONVECTION ZONE

Thomas L. Duvall Jr.
Laboratory for Astronomy and Solar Physics, NASA Goddard Space Flight Center, Greenbelt, MD 20771 USA

ABSTRACT

Using a deep-focusing time-distance technique and the MDI medium-resolution data, a preliminary study of non-axisymmetric variability deep in the convection zone has been performed. The purpose of the present study is to see what signals might be present in raw travel times indicating variation, and what are the noise levels. Correlations with point separations in the range 40-50 deg have been measured for the entire 6+ year dataset over a significant fraction of the solar disk. Both flows and mean-time variations have been examined. Travel time maps are correlated from one day to the next, indicating real solar signals.

Key words: helioseismology.

One of the interesting properties of the Sun to study with time-distance helioseismology is nonaxisymmetric variations. These are not studied well with global modes, which average over the complete range of longitude. An interesting region to study is very deep in the Sun, near the tachocline. Axisymmetric rotation variations have been studied in this region with global modes (Howe et al., 2000) with very exciting results. Axisymmetric phenomena have also been studied in this region with time-distance helioseismology, including the meridional circulation (Giles, 1999) and the solar cycle variation of the mean travel time (Chou & Serebryanski, 2002).

A diagram showing the range of rays used in the study is shown in Figure 1. The rays are not focused at the bottom but instead all have their turning points below a single surface point, which will be the ultimate map point. Cross correlations are made for pairs of points at opposite ends of the rays. At the surface, the map point is the center of a circle in which cross correlations are done for pairs of points on opposite sides of the circle. In addition, the circle is broken up into north-south and east-west quadrants so that travel time differences can be derived to study flows as well as the mean travel times. In the study, two sets of travel times were computed from the cross correlations, one averaging over the entire distance range, 40.2-50.4 deg, and the other with the distance interval broken up into six equal intervals. Only the travel times averaged over the whole interval will be discussed here.

6+ years of MDI medium-resolution Dopplergrams were analyzed (May96-Aug02). Each 24-hour interval was analyzed separately to derive travel times. Images were tracked and interpolated onto a Postels azimuthal equidistant projection over the entire disk visible with the medium-resolution data (out to roughly r/R=0.90). The resultant day-long data cubes were Fourier-filtered with a high-pass cut at 1.5 mHz and a phase-speed filter that attempts to isolate the deep signals for separations in the range 40.2-50.4 degrees, the distance range used in the time-distance analysis. Cross correlations and travel times were computed as described previously (Kosovichev & Duvall, 1997). The results are three maps per day, one for the mean travel time, and two for the east-west and north-south difference signals. These maps are also in the Postels projection, have 32x32 pixels with angular sampling of 2.4 deg and are centered on 0 deg latitude and the longitude of disk center at the middle of the day.

One aspect of a preliminary study is to examine systematic problems with instrumental or analysis sources. To this end, the mean travel time over each day's map is plotted in Figure 2. In the dopplergrams, the radius is normally corrected for focus changes and other instrumental issues. The first time the data was analyzed, data from June-Aug02 did not have the radius corrected which led to the 6-sec jump on the far right of the plot. The radius error is 3 arc-sec out of 960. The purpose of this plot is to emphasize the need to do the geometry carefully. The data were reanalyzed with the correct radius with the results shown in Figure 3.

It is found in general that the travel times show systematic variations as a function of disk position, or field effects. Some of this could be due to real solar effects, such as the horizontal component of the node velocity, but I suspect that much of it is due to analysis or instrumental effects. The mean travel time shows a strong variation that is mostly a function of the center-to-limb distance. An average over the six years of the travel time versus center-to-limb distance is shown in Figure 4 (curve labeled 6-year average). This curve is easily reproduced with just
Figure 1. The range of rays used in this study, selected to bracket the bottom of the convection zone. The solid circle is at the level of the photosphere. The dashed circle is near the bottom of the convection zone.

Figure 2. The average travel time over each day's map (with a constant 99.5 minutes subtracted) over the 6+ years of the study (May 96 - Aug 02).
a single day’s results, as shown in the curve immediately below the 6-year average. This single day was late in the run in June 2002 when the mean value was below the 6-year average and hence the overall shift from the average. We tested whether the phase-speed filtering was contributing to this center-to-limb variation by trying some different filters for this one day. The first test was to remove the phase speed filter, leave only a high-pass temporal frequency filter with cut at 1.5 mHz. The resultant center-to-limb variation is shown by the top curve in Figure 4 showing a significant difference with the original filtering and now with the variation in the opposite sense. A second test was to double the width of the phase speed filter. The results for this test (curve labeled ‘less filtering’) show very little center-to-limb variation. The phase speed filter is done by multiplying the 3-d Fourier transform of the tracked data cube by a filter function and then inverse transforming. The Sun is spherical and does not flatten. This is probably the source of this systematic problem. It is useful to get rid of these kinds of variations as they lead to signals with a yearly period and other problems. Note the increasing size of the error bars for decreasing amount of phase speed filtering. The phase speed filter is used to isolate the desired signals before the cross correlation is computed. It improves the resultant signal-to-noise ratio, although also apparently causes some systematic problems.

To search for nonaxisymmetric noninstrumental variations in the solar interior, the map for a given day and latitude is cross correlated with the following day’s map as a function of longitude shift. A feature rotating with the Sun (and hence a real signal) will show the longitude shift of solar rotation. The average of 2299 cross correlations was computed to insure a statistically stable result. Peaks in the cross correlation indicating rotating solar features were found in all three variables. The results are shown in Figure 5.

Are the correlation peaks caused by perturbations deep in the Sun near the turning point of the rays or by a more shallow phenomenon? It is not obvious that a delta-function perturbation (in longitude) at the tachocline would cause such a narrow correlation peak. It is likely that shorter distances will need to be also used and that inversion techniques will need to be applied to do a really useful study.

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Figure 4. Center-to-limb variation of the mean travel time signal for the filter used in the analysis (bottom two curves) and some test cases (upper two curves). Apparently the phase speed filter causes a center-to-limb variation of the signal.

Figure 5. Cross correlation of one day's mean travel times as a function of longitude near the equator with the similar results from the next day, as a function of the longitude shift. The mean travel time is shown as the solid line, east-west difference as the long dashes and north-south difference as the short dashes. The mean signal is sensitive to temperature and magnetic field inhomogeneities. The east-west and north-south differences are sensitive to flows. The result shown is the six-year average of this one-day shift cross correlation. The peak near 13 deg shift is due to some real signal rotating with the Sun. The peak correlation near 0.1 means that 10% of the variance is correlated from one day to the next. The rms of the mean signal is 4.4 sec, while the rms is 12 sec for the difference signals. This correlation completely decays after three days. Similar correlation peaks are present at all latitudes studied (-39 to +39).
A SEARCH FOR THE RELATIONSHIP BETWEEN FLARING ACTIVITY AND SUBPHOTOSPHERIC FLOWS

E. Dzifčáková¹, A. Kulinová¹ and A. G. Kosovichev²

¹Astronomical Institute, FMPh Comenius University, Mlynská dolina, 842 48 Bratislava, Slovak Republic, E-mail: dzifcakova@fmph.uniba.sk, kulinova@fmph.uniba.sk
²Stanford University, 455 Via Palou, Stanford, CA 94305-4055, USA, E-mail: sasha@quake.stanford.edu

ABSTRACT

It is believed that subphotospheric shearing flows play important role in creating unstable magnetic topology that leads to initiation of flares and CME. However, the relationship between the flows and flaring activity is not well understood. Using the flow maps obtained by local helioseismology and magnetic and coronal data we attempt to search for this relationship. In particular, we study the evolution of active region NOAA 9393 in the context of changes in magnetic topology and flaring activity. SOHO/MDI and helioseismology data are used for determining the changes in morphology and are compared with changes of the topology as observed by TRACE.

Key words: flares; magnetic reconnection; subphotospheric velocity field.

1. INTRODUCTION

The mechanism of flaring activity is not well understood. It is widely believed that it is associated with deformations of coronal magnetic structures caused by various kind of foot-point motions (shearing, braiding or random walk), which lead to formation of current sheets. The current sheets tend to diffuse or become unstable, and via a reconnection process produce an energy release, we usually observe as flares. Theories and numerical simulations of 3-D magnetic reconnection are developing rather fast nowadays (e.g. Priest & Forbes 2000, Milano et al 1999, Galsgaard and Nordlund, 1996), and there is a demand to confront models with observations. The purpose for our study is to look for the relationship between subphotospheric velocity fields and motions of foot-points of magnetic structures producing flares. For the initial study, we have chosen one of the flares observed in active region (AR) NOAA 9393. We believe that this flare may be explained by the theory of magnetic flapping suggested by Priest & Démoulin (1995). This kind of 3-D reconnection requires the foot-points to move across quasi-separatrices (QSs), producing electric field parallel to the magnetic field lines. Then, the field lines may become disconnected from the plasma and rapidly flip through in the quasi-separatrix layers. We hope that local helioseismology data help to reveal this kind of foot-point motions. For this study we use the subphotospheric velocity maps inferred by time-distance (T-D) helioseismology (Duvall et al, 1993; Kosovichev & Duvall, 2002). Each of the flow maps was obtained by the T-D analysis of 8-hour series of MDI full-disk Dopplergrams, at 14 depths below the photosphere in the range 0-80 Mm. The horizontal resolution of these maps was approximately 2.9 Mm. For our analysis we used only the horizontal components of the flow field in the upper 10 Mm layers, which are determined most reliably (Kosovichev & Duvall, 1997).

2. ANALYSIS OF OBSERVATIONAL DATA

On March 28, 2001, NOAA AR 9393 was located near the center of the solar disk and about 8 flares occurred there. We have chosen a M4.3/SF flare for analysis, which started at 11:21 UT, reached the maximum of X-ray intensity at 12:40 UT and ended at 13:06 UT. The flare was observed in detail by TRACE in 1600 Å UV continuum and EUV line 171 Å (Fig. 1,2). The spatial resolution of TRACE images is 0.5 arcsec (0.36 Mm) per pixel. Also, MDI magnetograms were used to analyze the magnetic structure of the AR and for constructing the potential model (Fig.3). Basically, we used 3 magnetograms taken before the flare, at 4:00 UT, during the flare, at 12:00 UT, and after the flare, at 20:00 UT. Their spatial resolution is about 2 arcsec (1.5 Mm) per pixel. The subphotospheric velocity fields were obtained for 8-hour intervals centered at the same moments as the magnetograms. The observations show that during this period the magnetic field (MF) rapidly evolved in the lower part of AR (rectangle in Fig. 4a). In this part, the averages horizontal velocity vector was directed to the south-west (Fig. 4). The MF structures moved simultaneously with the plasma, so that the field lines became stretched. This kind of motion was observed just in few depths close to the surface, 3-12 Mm. In general, the velocity pattern seems to vary significantly with depth and not all flow patterns are connected with the MF movements.
2.1. Flare Structures in the Corona

The first flare brightenings appeared in UV continuum: one in a big positive polarity (BPP) spot and another one close to a smaller positive polarity feature (SPP, Fig. 1a). Similar behavior was observed in the EUV 171 Å line (Fig. 2a). At about 12:00 UT, the third brightening in the UV continuum appeared in the big negative polarity (BNP) spot (Fig. 1b). Simultaneously, a coronal loop system connecting the BPP and BNP spots became visible in the 171 Å line emission having a diffusive character (Fig. 2b). In addition, below this loop system two smaller and more clearly visible loop structures appeared in the EUV emission. However, they were short-living and disappeared about 30 minutes later. Near the maximum of the flare (12:40 UT), an additional loop system connecting BNP with SPP started to be visible. In the gradual phase of the flare, the evolution continued with the fragmentation of the diffusive loop systems to individual loops (Fig. 2c).

2.2. Magnetic Field Model and Flare Scenario

In order to compare the flare evolution with models we need to know field line connectivity in the observed AR. That is why we constructed a potential filed model from MDI magnetograms (Fig. 3) using magnetic charge method (Démoulin et al, 1993, Longcope, 1996). This model provides the coronal field structure in the absence of significant electric currents. It seems that during the flare evolution the global field structure remained largely unchanged. However, the detailed analysis of the sequence of model field revealed changes in connectivity of some field lines in the vicinity of quasi-separatrices (Qs) shown by the thick curves in Fig. 3a-c. We propose the following scenario for the M4.3/SF flare of March, 28, 2001. The first brightening appeared close to the Qs numbered 1 and 3 (Fig. 3). At this time, several field lines connecting the big and smaller positive magnetic polarities to the weak negative polarity at the right top corner of the image (see e.g. Fig. 3, 4) then changed their connection to the BNP spot in the center of AR NOAA 9393, where the Qs number 2 is situated. In our opinion, this flare evolution resembles the 3-D reconnection without null points via magnetic flipping (Priest & Démoulin, 1995). This theory requires plasma motions across the Qs. To search for the suitable plasma motions we used the local helioseismology results for the horizontal velocity fields obtained by the T-D technique. We analyzed the flow patterns in the center of the active region for the different depths below the surface (photosphere). According to the theory and TRACE observations we looked for flows crossing the Qs 1 and 3 depicted on Fig. 3 for modeled field configurations.

2.3. Analysis of Subphotospheric Flows

Generally, the most shallow depths have very similar flow pattern. This may be explained by the small depth difference among the depth levels so they represent flows in the same subphotospheric shell. As the depth increases, the flow pattern changes noticeably. Totally opposite plasma flows can be find in the same part of AR for deeper levels. The plasma flows suitable for our flare scenario have to be directed nearly to the center of AR. We have found this kind of flows at four depths: 3.0 Mm, 4.5 Mm, 6.4 Mm and 11.8 Mm below the surface. For these four depths the same flow pattern in the BPP spot prevails but the sought flow is not so evident for the region near the SPP. For instance, in 6.4 Mm there is just very weak sign of plasma motion across the QS number 3.

3. CONCLUSION

The NOAA AR 9393 was very complicated and rapidly evolving during the disk passage. It produced several flares. The initial analysis of M4.3/SF flare of March,
Figure 2. The flare evolution in 171 Å EUV line: a) pre-flare EUV image, b) EUV image with corresponding contours showing sunspot location, c) gradual phase of the flare. Solar north is up and west is to the right. Field of view is 300" x 300".

Figure 3. The potential magnetic field model of NOAA AR 9393: a) pre-flare configuration 4:00 UT, b) close to the maximum (12:40 UT) configuration 12:00 UT and c) post-flare configuration. The active quasi-separatrices are presented by heavy lines and are numbered 1, 2, 3, see text. Solar north is up and west is to the right. Field of view is 300" x 300".
28, 2001, have shown significant motions of the photospheric footpoints of the magnetic field structures associated with this flare. It seems that in this case the plasma flows that led to the global evolution of the lower part of the AR just deformed (stretched) the field lines, and the trigger of this flare should be plasma flows across the quasi-separatrices. A possible scenario of this flare, magnetic reconnection via magnetic flipping without null points, requires subphotospheric flows directed toward the center of the AR across the quasi-separatrices. We have searched for such flows using the flow maps obtained by the time-distance helioseismology and found these flows might be located at the depth of 0-3 Mm. However, the flows in the upper layers were in the opposite direction. Therefore, the precise relation between the subphotospheric flows and the reconnection scenario is not yet clear. This kind of study needs to be expanded to other suitable flares. Further analysis should be able to provide answers for the following questions: Should all plasma flows across the quasi-separatrices leading to flaring activity be observed in the same depth (below the photosphere)? What is the connection between the subphotospheric plasma flows and large-scale magnetic evolution of active regions?

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A. Eff-Darwich\textsuperscript{1} and S.G. Korzennik\textsuperscript{2}

\textsuperscript{1}Dept of Soil Sciences and Geology, University of La Laguna, Tenerife, Spain
\textsuperscript{1}Instituto de Astrofísica de Canarias, c/ Via Láctea s/n, 38205, Tenerife, Spain
\textsuperscript{2}Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, U.S.A.

ABSTRACT

We have inverted most of the available rotational frequency splittings at intermediate degrees. Namely, LOWL data from 1994 to 2000, GONG data from 1995 to 2001, and, MDI data from 1996 to 2002. Our purpose was to look for any temporal variation of the tachocline and its relation to the solar activity cycle. However, we did not find any significant change compatible with the three data sets. From our analysis, we estimated an upper limit on the temporal variation of $\Omega/2\pi$ of 3 nHz. This value is commensurable with the sensitivity of the present observational data to changes in the rotation rate of the tachocline.

Key words: oscillations, activity.

1. INTRODUCTION

Helioseismic data collected by the Global Oscillation Network Group (GONG), the Michelson Doppler Imager (MDI) and the LOWL instrument over the last eight years have allow us to infer the rotation rate in the solar interior and analyze it for any temporal evolution. The region where the rotation undergoes a sharp transition – from differential rotation in the convection zone to nearly rigid rotation in the radiative interior – is of particular interest. This region, known as the tachocline, is believed to corresponds to the location of the magnetic fields responsible for the solar dynamo. Hence any temporal variation associated with the solar activity cycle is likely to be located in this region.

Recently, Basu & Antia (2001, 2000), using GONG and MDI data, did not detect significant changes of the tachocline associated with the solar cycle. However, Howe et al. (2000) found a 1.3-year periodicity in the variation of the solar rotation rate of the tachocline estimated by inverting GONG and MDI data.

In the work presented here, we used the same mode sets for the MDI, GONG and LOWL frequency splittings to infer an upper limit to any temporal variations of the rotation rate at the tachocline, based on the Optimal Mesh Distribution (OMD) inversion technique (Eff-Darwich & Pérez-Hernández, 1997). The OMD technique, a variant of the regularized least-squares methodology, applies a different regularization factor at each depth and latitude, on an optimally chosen grid. This method produces inverted profiles with no region of the solar interior that is over-smoothed.

2. OBSERVATIONAL DATA

The observational data consist of frequency splittings computed from time series spanning different epochs and observed with different instruments. Namely, 57 sets based on 108-day-long time series derived from the GONG instruments (Leibacher et al., 1996) and spanning May 1995 to February 2001; 27 sets based on 72-day-long time series derived from the MDI instrument (Schou, 1999) and spanning May 1996 to November 2001; and 6 sets based on 1-year-long time series derived from the LOWL instrument (Tomczyk et al., 1995) and spanning 1994 to 1999.

In order to use temporally consistent data sets, only the modes common to all the sets for a given instrument were included in our inversions. As a consequence, the low degree modes ($l < 13$) present in some GONG data sets had to be rejected. Also this selection reduces the number of MDI and LOWL modes by 30% and 4% respectively.

The MDI and LOWL sets were further reduced to only include the modes common to both instruments. This could not be extended to the GONG data set due to the very small amount of modes common to all three instruments. Finally, and again for consistency, we deliberately restricted the range of degrees we included to correspond to the highest degree available in the LOWL data set ($i.e., l \leq 100$).

For each instrument and for each mode we computed...
the temporal averages of the frequency splittings. We subsequently subtracted the respective averaged splittings from each set, leaving us with frequency splitting changes with respect to this average as a function of epoch. For the GONG and MDI sets, we also computed averages corresponding to 1-year-long epochs. Such averaging reduced the scatter of the data while producing data sets comparable to the LOWL sets.

3. RESULTS

We have only analyzed the temporal variations of the rotation rate at the equator. Figure 1 shows the relative change of the rotation rate, as a function of radius, inferred from 1-year-long MDI, GONG and LOWL sets, all corresponding to time series collected in 1999. Figure 2 shows the relative change in the rotation profile, as a function of radius, for four 1-year-long MDI sets. All these profiles show no significant differences at the level of approximately 2 nHz at all radii.

We have checked that the precision and resolution of the inversion is good enough to detect small temporal variations of the tachocline. This is demonstrated in Figs. 3 and 4, where we show the rotation profiles inferred from the 1998 averaged MDI, GONG and LOWL data sets after injecting frequency splittings that would result from a small change in the rotation rate between 0.69 and 0.72 \( R_\odot \), of amplitude \( \varepsilon = 3 \) nHz. These figures clearly indicate that inversions of MDI and LOWL data are sensitive enough to detect relative variations as low as 3 nHz at the tachocline. The sensitivity of the GONG data to small variations at the depth of the tachocline is much lower than the one for the LOWL and MDI data. We attribute this to the small amount of GONG modes with \( \ell < 30 \) that are present in the data sets.

In order to look for temporal variations of the rotation rate of the tachocline, we examined the value of the equatorial rotation rate as a function of time at selected radii, as illustrated in Fig. 5. These curves correspond to inversions based on 1-year-long sets for all three instruments (GONG, MDI and LOWL), as well as on 108-day-long GONG sets and 72-day-long MDI sets. As expected, the inversions of one-year-long averaged MDI and GONG splittings dis-
play temporal variations similar to the inversions of the corresponding individual data sets.

We do not find any systematic temporal variations of the equatorial rotation rate at and below the tachocline that are compatible with all the data sets. A hint of a one year periodicity is present in both the GONG and MDI data but is only marginal. Fig. 6 show the Fourier transform of the temporal relative change of the rotation rate at selected depths. These curves indicate that this near one year periodicity is seen at nearly all depths where the inversions are reliable.

4. CONCLUSIONS

By including the same mode sets and using rotational splittings derived from three different instruments, we are led to conclude that there is not significant temporal variation of the equatorial rotation rate at depth of the tachocline. We also derived an upper limit of any temporal variation at the level of 3 nHz, a value that is constrained by the scatter present in the data.

ACKNOWLEDGMENTS

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The LOWL instrument has been operated by the High Altitude Observatory of the National Center for Atmospheric Research which is supported by the National Science Foundation.

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Figure 5. Temporal variations of the equatorial rotation rate at selected depths. Open and filled circles represent the solutions obtained from 108-day-long GONG and 72-day-long MDI data sets, whereas solid, dotted and dashed lines represent the yearly-averaged GONG, MDI and LOWL sets, respectively. At $r/R_\odot = 0.53$ the inversions of GONG data are not reliable and therefore they are not plotted.

Figure 6. Fourier spectra of the temporal variations of the equatorial rotation rate inferred from 108-day-long GONG (solid line) and 72-day-long MDI (dotted lines) data sets; shown at the same selected depths as in Fig. 5.
ANALYSIS OF ROTATIONAL FREQUENCY SPLITTINGS SENSITIVE TO THE
ROTATION RATE OF THE SOLAR CORE

R.A. García¹, A. Eff-Darwich²,³, S.G. Korzennik⁴, S. Couvidat⁵, C.J. Henney³, and S. Turck-Chièze¹
¹CEA/DSM/DAPNIA/SAp, CE Saclay, 91191 Gif-sur-Yvette CEDEX, France
²Dept of Soil Sciences and Geology, University of La Laguna, Tenerife, Spain
³Instituto de Astrofísica de Canarias, La Laguna, Tenerife, Spain
⁴Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, U.S.A.
⁵National Solar Observatory, Tucson, Arizona, 85726-6732, U.S.A.

ABSTRACT

Updated solar frequency splitting measurements suggest a slight decrease of the rotation rate below 0.25R⊙ and, albeit preliminary, rule out a core rotating faster than the upper radiative zone. The estimates of the rotation rate of the deep solar layers are based on new rotational frequency splittings computed using data from the GOLF and MDI instruments on board SoHO. Such results, provided they are confirmed after further analysis, give additional insight into the dynamics of the solar core.

Key words: solar oscillations; solar internal rotation.

1. INTRODUCTION

The rotation rate of the deepest layers of the sun is still a matter of great interest and debate. While a core rotating faster than the upper radiative zone has long been assumed, the very idea of a core rotating more slowly emerged only in the mid-1990’s. Rotation is key to constrain many dynamical phenomena of the deep solar interior, like the Eddington-Sweet circulation or the transport of angular momentum. Moreover, a precise knowledge of this rotation rate would help to derive the amplitude and profile of the inner magnetic field. After many years of helioseismic observations by space-borne instruments (i.e., GOLF and MDI) and ground-based networks (i.e., BISON, GONG, IRIS and LOWL) very high quality time series are available that should give us a detailed view into the deeply embedded solar core.

In a recent paper, Eff-Darwich & Korzennik (2002) carried out inversions of MDI, GONG and LOWL data for the time span 1996 to 1999. Their results are compatible with a radiative zone rotating like a rigid body at a rate of approximately 435 nHz, and a slight slow down in the core below 0.35R⊙. Results from Couvidat et al. (2002), based on the most recent GOLF data complemented by LOWL data for ℓ > 3, also suggest a decrease of the rotation rate in the core, as deep as 0.18R⊙.

Figure 1. Observed n-averaged sidereal rotational frequency splittings estimated from different data sets. The filled circles correspond to MDI data, while the triangles, open circles, squares and crosses correspond to splittings obtained with, respectively, BISON (Chaplin et al., 1999), GOLF Bertello et al. (2000), GONG (Ribelson-Soares & Appourchaux, 1999), and MDI reduced as an integrated-disk instrument (Bertello et al., 2000). Lines show theoretical splittings resulting from different rotation profiles below 0.25R⊙, namely Ω/2π = 230 (dotted line), 430 (solid line), or 630 nHz (dashed line).

We have concentrated our attention here on pressure modes, since no gravity mode has yet been positively detected. The rotational frequency splittings (hereafter splittings) of the low-degree p-modes hold all

the informations on the dynamics of the deep interior. Indeed, the lowest is the degree the deepest is the inner turning point. However, it is difficult to accurately measure these splittings: the uncertainties on estimates of low-degree modes splittings are significantly larger than the ones for intermediate- and high-degree modes. Moreover, the splittings for low-$m$ modes are as large as the linewidth of the modes itself. Finally, let us point out that there is also a large dispersion in the estimates of the $\ell = 1$ & 2 splittings. This scatter prevents us from drawing robust inferences on the value of the rotation below $0.2R_\odot$. This is illustrated in Fig. 1, where we show various set of observed splittings and theoretical predictions from different rates in the core. Unfortunately the picture of what really occurs inside the core is far from being clear.

To improve the signal-to-noise ratio at low frequency, linear combinations of GOLF and MDI velocity time series have been computed. This allows us to fit modes up to $\ell = 4$. These MDI time series are more sensitive to higher degree modes while the effect of adding GOLF cleans up the leakage of modes $\ell > 4$. However, for some of the $\ell = 4$ modes a combination of the latter two MDI masks provide a confirmation of the modes and their splittings (see Henney, 1999; Henney et al., 1999).

We used two different power spectral density estimators to compute power spectra: one being a fast Fourier transform with an oversampling factor of 4, the other being a 4th order sine multi-taper estimator.

In the paper, we have restricted ourselves to modes below 2 mHz. Indeed, above 2 mHz, the modes undergo substantial frequency shifts due to the change of activity level associated with the solar cycle. Moreover, the asymmetry of the line profile become substantial and must be taken into account.

Below 2 mHz, each multiplet was fitted to a set of Lorentzian profiles using a maximum likelihood method (Appourchaux et al., 1998). The parameters that were fitted are the amplitude of the mode, its frequency, the rotational frequency splitting, the linewidth, and the background noise.

As we are using solar-full-disk integrated data, all the even $\ell + m$ components of the multiplet are visible in the power spectrum. Thus, for the modes $\ell = 3$ and 4, two splittings have been fitted to take into account the difference between the sectoral and non sectoral components of the multiplet which have a different sensitivity to the equatorial rotation. Examples of some fits for $\ell=1$, 2, 3 and 4 modes are shown on Fig. 2 and the resulting splittings are given in Table 1.

All the splittings used are sectoral, (i.e. $\ell = m$); this implies that we are mainly sensitive to the rotation.
Table 1. Sectoral synodic splittings obtained from 2243 day-long velocity time series of GOLF and MDI.

<table>
<thead>
<tr>
<th>n</th>
<th>$\ell=1$</th>
<th>$\ell=2$</th>
<th>$\ell=3$</th>
<th>$\ell=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>398.488 ± 1.523</td>
<td>399.838 ± 2.228</td>
<td>403.693 ± 3.636</td>
<td>401.603 ± 3.724</td>
</tr>
<tr>
<td>8</td>
<td>398.694 ± 1.491</td>
<td>399.104 ± 2.834</td>
<td>405.226 ± 6.395</td>
<td>407.408 ± 4.927</td>
</tr>
<tr>
<td>10</td>
<td>412.368 ± 4.222</td>
<td>400.975 ± 5.063</td>
<td>404.787 ± 14.28</td>
<td>410.869 ± 13.89</td>
</tr>
<tr>
<td>11</td>
<td>401.959 ± 4.595</td>
<td>401.331 ± 6.453</td>
<td>400.579 ± 15.30</td>
<td>413.355 ± 20.76</td>
</tr>
</tbody>
</table>

near the equator. A rough estimate based on the rotational kernels show that only 25% of the value of the sectoral splitting of a low-degree mode is due to the rotation rate at latitudes higher than 35°.

3. RESULTS FROM THE MDI+GOLF DATA: A FORWARD APPROACH

We have analyzed our results using a forward approach, rather than an inverse one. We have computed the theoretical values of the sectoral splittings for different rotation profiles and compare them to the ones we computed. Figure 3 & 4 shows such comparisons and our new estimates of low degree splittings. These figures show that these new values are in good agreement with the model that includes a decrease of the rotation rate below 0.25$R_\odot$.

Actually, our estimates for the low-frequency $\ell = 1$ to 3 splittings are significantly lower than what is expected from a core rotating at a constant rate of 430 nHz (see Figs. 3 & 4). Our results rule out a core rotating faster than the upper layers of the radiative interior, and favor a decrease in the core rotation.

For the $\ell = 4$ modes we find results similar to those derived from time series based on MDI only and reduced as an integrated-disk instrument. Finally, the comparison between Fig. 1 and Fig. 4 shows the substantial decrease in the dispersion and the error bars of the data in relation to previous estimates: this strengthens the idea that we can rely on the quality of our data.

However, it is important to notice that the averaged splittings of our $\ell \leq 4$ modes correspond only to those below 2 mHz while for the high-degree modes, we are averaging up to $\sim 3.5$ mHz. Thus, the external turning point of our low-degree modes are deeper in the solar atmosphere and they are not sensitive to the external layers where the equatorial rotation increases (at least up to 0.9 $R_\odot$). Taking into account higher order modes for the $\ell = 3$ and 4 from LOWL, the averaged splittings are increased, partially filling the gap between the $\ell = 4$ and 5.

Before we can draw any firm conclusion, further tasks need to be performed:

- the analysis of GOLF+MDI data up to $\ell = 15$ (at least) must be carried out;
- the non-sectoral splittings must be fitted to study the dependence of the core rotation on the latitude;
- a full two-dimensional inversion must be performed to complement the forward approach.

ACKNOWLEDGMENTS

GOLF & MDI are cooperative efforts of many individuals to whom we are deeply indebted. The Solar Oscillations Investigation - Michelson Doppler Imager project on SoHO is supported by NASA grant
Figure 4. $n$-averaged sidereal rotational frequency splittings derived from the GOLF+MDI data for degrees $l \leq 4$ (open circles) and from MDI data for the modes $l \geq 5$ (filled circles, Schou et al. 1998). Continuous lines show the theoretical $n$-averaged splittings resulting from different rotation profiles below $0.25 R_\odot$, assuming either $\Omega/2\pi = 430$ nHz (solid line), or $\Omega/2\pi = 230$ nHz (dotted line).

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DIFFERENT PERIODICITIES IN THE ROTATION OF THE NORTHERN AND SOUTHERN SOLAR HEMISPHERES

K. Georgieva, B. Kirov

Solar-Terrestrial Influences Laboratory at the Bulgarian Academy of Sciences,
Bl.3 Acad.G.Bonchev str., 1113 Sofia, Bulgaria
tel.: +359-2-9793432/ fax: +359-2-700178/
e-mail: k.georgieva@bas.bg/ b.kirov@space.bas.bg/

ABSTRACT

Many terrestrial phenomena are governed by the Sun. Geomagnetic activity is related to the 22-year solar magnetic cycle, also obvious in solar wind parameters and the variations of the Earth’s rotation. A 22-year periodicity is found in solar rotation. The 47-year variation in the Earth rotation is highly correlated to the same variation in North-South asymmetry of solar rotation. A dominant 17-year cycle is observed in the exposure of the Earth to IMF sectors. The same cycle is found in hemispheric differences of coronal holes distributions. Here we determine the periodicities in the variations in the rotation of the two solar hemispheres. We find that not only the two hemispheres rotate differently, but have different periodicities in the variations of the rotation parameters. Our results confirm the weak magnetic coupling between them and show that the interplanetary magnetic field which is an extension of the solar coronal field, is related to the differential rotation in the more active solar hemisphere.

INTRODUCTION

The way in which Sun affects the Earth can tell us something about Sun itself. One of the much-debated problems in solar-terrestrial physics is the instability of the relations found between the solar activity and climate. Different authors have found positive, negative or missing correlations between solar activity and the surface air temperature in the 11-year solar cycle. A compilation of all published results, and a study of all available climate records from meteorological stations worldwide has shown that the correlation between the two depends on the period studied and not on the location, and changes sign in consecutive secular (Gleissberg) solar cycles. Moreover, it has been noted that the sign of the correlation depends on solar activity asymmetry. being positive with predominantly more active Northern solar hemisphere and negative when more active is the Southern hemisphere. This has lead to the idea of a secular variation of solar activity asymmetry (Georgieva and Kirov, 2000). The same conclusion has been reached based on the phase change of the semiannual variation of geomagnetic activity (Mursula and Zieger, 2001). Climate is closely related to atmospheric circulation, and long-term changes in atmospheric circulation have been supposed to be connected, either as a cause or as a consequence, to long-term changes in Earth’s rotation rate. However, this correlation has been demonstrated for the 20th century only, while the study of proxy records of atmospheric circulation has revealed that its correlation to the Earth’s rotation fluctuations changed sign in the same period in which both solar asymmetry and the correlation between solar activity and surface air temperature changed sign (Georgieva, 2002). The decadal variations in the Earth rotation rate have been found to be closely correlated to the North-South asymmetry of solar equatorial rotation, both showing dominant periodicities at about 47 years. The same periodicity has been found earlier in the Sun’s motion with respect to the mass center of the solar system (Bucha et al., 1985). Juckett (2000) has shown that, taking into account the Sun’s inertial movement and the tilt of the solar spin axis, solar differential rotation and North-South solar asymmetries can be driven by or modulated by planetary-induced spin-orbit coupling. A strong dependence was also demonstrated of the long-term changes of the correlation between solar and geomagnetic activity on solar activity asymmetry (Georgieva, 2002). The additional periodicity at 16–17 years found in solar activity asymmetry was also observed in the polarity of the long-term hemispheric differences in coronal hole distributions and in the IMF directions (Juckett, 1998).

The dependence of solar-terrestrial connections on solar asymmetry implies that the two solar hemispheres affect the Earth in a different way. Antonucci et al. (1990), based upon data for solar cycles 20 and 21, pointed out the different behavior of the interplanetary magnetic field originating from the Northern and Southern solar hemispheres, suggesting only a weak magnetic coupling between the two. Temmer et al. (2002) confirmed this conclusion for cycles 21, 22 and the current part of cycle 23. An important element of the solar dynamo which is responsible for the solar magnetic field is the solar differential rotation. The relation between the rotation and the magnetic field has been shown by many authors (i.e. Howard and LaBonte, 1983, Javariah and Gokhale, 1997 and references therein). It has long been known that the two solar hemispheres rotate at different rates. These differences...
have been studied by Balthazar and Wohl (1980), Arevalo et al. (1982), Balthazar et al. (1986), Hathaway and Wilson (1990), Pulkkinen and Tuominen (1998) and others. After all these studies, the interpretation of the time variation of the rotation differences between the northern and the southern hemispheres as well as its phase relative to the activity cycle still remains unclear.

DATA

In the present study we use A and B coefficients in the standard formula of solar differential rotation

$$\omega(\lambda) = A + B \sin^2 \lambda$$

calculated from GCR - Greenwich Photoheliographic Results (Javaraiah and Gokhale, 1995) for the two solar hemispheres, An, As, Bn and Bs, respectively, and kindly provided by Dr. Javaraiah from the Indian Institute of Astrophysics. GCR data cover the years 1874-1976, however because of the large uncertainties in 1878, the data for the years 1874-1878 have been omitted. A and B coefficient have been calculated for moving time intervals of 5 years successively shifted by 1 year (as pointed out by Javaraiah and Gokhale, 1997, their yearly values are inadequate to determine periodicities), so our data-set of middle years covers the interval 1881-1974. From them, we also calculate the asymmetries in the A and B coefficients,

$$A_a = (An-As)/(An+As), \quad B_a = (Bn Bs)/(Bn+Bs)$$

PERIODICITIES IN THE N-S ASYMMETRY
AND IN HEMISPHERIC ROTATION

The periodicities in the coefficient B (or B/A) for the whole solar sphere were determined by Javaraiah and Gokhale (1995), of the asymmetry of the coefficients of solar differential rotation, Aa and Ba – by Javaraiah and Gokhale (1997), and of the long-term variations of A and B – by Javaraiah (2000, 2002). In these papers, while computing the FFT of a sequence of values, the mean value has been subtracted from each value, and then the size of the sequence has been extended to the next power of 2 by adding zero values. To comply with our earlier results, we use the exact length of the data sets, so the derived periodicities differ slightly from the ones in the above-cited papers. As the variables are of different orders of magnitude, before computing the FFT we first standardize them.

In the upper and lower panel of figure 1, the FFT spectra of An and As are presented, respectively. The 47-year periodicity observed in Aa and in Earth rotation fluctuations, as well as the 18.8-year periodicity matching the $\approx 17$ year one in the IMF directions and coronal holes asymmetries and present also in Aa, are only seen in An and not in As. On the other hand, the $\approx 11$-year solar cycle period and the $\approx 22$-year Hale cycle are only found in An and not in As.

![Figure 1: FFT spectra of An (upper panel) and As (lower panel).](image1)

![Figure 2: Relation between the equatorial rotation rates in the Northern and Southern hemispheres, An and As, respectively.](image2)

As a whole, there is no good correspondence between the equatorial rotation rates in the two solar hemispheres – figure 2. While there is some weak though statistically significant correlation between the two ($r=0.36$ with $p=0.05$), there is absolutely no correlation between the latitudinal gradients in the two hemispheres, Bn and Bs. The correlation coefficient is only 0.06 – figure 3.

![Figure 3: Relation between the latitudinal rotation rate gradients in the Northern and Southern hemispheres, Bn and Bs, respectively.](image3)
In Figure 4 the FFT spectra of Bn and Bs are presented. The 47-year and the 10.4-year periodicities are seen only in the Northern hemisphere, while the dominant 18.8-year periodicity in the Southern hemisphere is much weaker in the Northern one.

![FFT spectra of Bn (upper panel) and Bs (lower panel).](image)

Figure 4: FFT spectra of Bn (upper panel) and Bs (lower panel).

The behavior of the B coefficient of the solar differential rotation is particularly important with relation to the dynamo theory of the solar magnetic field (i.e. Babcock, 1961). The interplanetary magnetic field (IMF), being an extension of the coronal field, results from this large-scale solar field. IMF has been measured directly since the beginning of the satellite era. For the period 1967-1994, we have data for both Bn and Bs coefficients of solar differential rotation derived by Mt Wilson Doppler shift measurements of photospheric line (Howard et al., 1984), and IMF parameters compiled in OMNI database of the National Space Science Data Center provided through their web-site [http://nssdc.gsfc.nasa.gov/omniweb/](http://nssdc.gsfc.nasa.gov/omniweb/).

The dominant periodicity in the average IMF magnitude in this period is 9.3 years coinciding with the dominant periodicity in Bs, while the dominant periodicity in Bn is 14 years. In this period predominantly more active is the Southern solar hemisphere. I.e., the dominant periodicity in the IMF seems to be the dominant periodicity of the differential rotation of the more active solar hemisphere.

To check this result, we compare the periodicities in Bn and Bs to the periodicity of aa index of geomagnetic activity in the periods 1881-1912 when more active is the Southern solar hemisphere, and 1913-1966 with more active Northern hemisphere. (We use aa index rather than the sunspot numbers as geomagnetic activity is related to the interplanetary magnetic field which is the extension of the coronal open magnetic field while sunspots are regions of closed magnetic field). In figure 5a, the FFT spectra of aa (respectively of IMF), Bn, and Bs are compared for the first period, 1881-1912, when more active is the Southern solar hemisphere.

![FFT spectra of aa-index of geomagnetic activity (solid line), Bn (broken line) and Bs (dotted line) for the period 1881-1912.](image)

Figure 5a: FFT spectra of aa-index of geomagnetic activity (solid line), Bn (broken line) and Bs (dotted line) for the period 1881-1912.

The dominant periodicity in aa is 16 years matching the dominant periodicity in Bs, while Bn varies with a period of 10.8 years.

![The same as figure 6a, for the period 1913-1966.](image)

Figure 5b: The same as figure 6a, for the period 1913-1966.

In the second period, 1913-1966, more active is the Northern solar hemisphere. The dominant periodicity in aa is 10.8 years. A strong peak at this periodicity is seen in Bn, and no peak in Bs – figure 5b.

![The same as figure 6a, for the period 1967-1994.](image)

Figure 5c: The same as figure 6a, for the period 1967-1994.

In the third period, 1967-1994, with more active Southern hemisphere, the dominant periodicity in aa, as in IMF, is 9.3 years, which is the Bs periodicity, while the Bn dominant periodicity is 14 years – figure 5c.
SUMMARY

Solar influences on some terrestrial phenomena depend on North-South solar asymmetry. This implies that the two solar hemispheres affect the Earth in a different way. The different behavior of the interplanetary magnetic field originating from the two solar hemispheres has been shown earlier. As the differential rotation is supposed to be an important element of the solar dynamo, we study the variations in the solar equatorial rotation rate and in the latitudinal gradient of the differential rotation in the two solar hemispheres matching similar periodicities in terrestrial phenomena. We find that the two hemispheres rotate differently, and have different periodicities in the variations of the rotation parameters which confirms the weak magnetic coupling between them. We show that the interplanetary magnetic field which is an extension of the solar coronal field, is related to the differential rotation in the more active solar hemisphere, which confirms the role of the differential rotation in the generation of the solar magnetic field.

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ASYMMETRY REVERSAL IN SOLAR ACOUSTIC MODES

Dali Georgobiani\textsuperscript{1}, Robert F. Stein\textsuperscript{1}, and Åke Nordlund\textsuperscript{2}

\textsuperscript{1}Michigan State University, Physics & Astronomy Department, Biomedical & Physical Sciences Bldg., East Lansing, MI 48824-2320, USA; phone (517) 355 9200 ext 2444, fax (517) 432 6191; emails dali@pa.msu.edu, steinr@pa.msu.edu
\textsuperscript{2}Teoretisk Astrofysik Center, Denmark Grundforskningsfond, Juliane Maries Vej 30, DK-2100 København Ø, Denmark; phone +45 353 25 968, fax +45 353 25 989; email ake@astro.ku.dk

ABSTRACT

The power spectra of solar acoustic modes are asymmetric, with velocity having more power on the low frequency side of the peak and intensity having more power on the high frequency side. This effect exists in both observations and simulations, and it is believed to be caused by the correlated background noise. We study the temperature near the solar surface by means of a 3D hydrodynamic simulation of convection with a detailed treatment of radiation. The temperature spectrum at optical depth $\tau_{\text{cont}} = 1$ has opposite asymmetry to the velocity spectrum, whereas the temperature measured at a fixed geometrical depth, corresponding to $< \tau_{\text{cont}} >/= 1$, has the same asymmetry as velocity. We believe that the asymmetry reversal in temperature at $\tau_{\text{cont}} = 1$ (and therefore in intensity) occurs partly because of the radiative transfer effects. High temperature sensitivity of the opacity suppresses temperature fluctuations on opposite sides of the mode peaks differently, thus causing the asymmetry reversal.

Key words: Sun: interior --- Sun: oscillations --- convection --- methods: numerical.

1. INTRODUCTION

For almost a decade, it has been known that the power spectra of solar acoustic modes are asymmetric, velocity has more power on the low frequency side and intensity has more power on the high frequency side of the power maxima (e.g. Duvall et al. 1993). The asymmetry reversal between velocity and intensity is thought to be due to the correlated background noise contribution to the intensity power spectra (Nigam et al. 1998). It is debated whether the asymmetry reversal occurs in velocity (Roxburgh & Vorontsov 1997) or intensity (Nigam et al. 1998) or both (Kumar & Basu 1999, Skartlien & Rast 2000). Theoretical models predict asymmetries that depend on the source depth and type (Kumar & Basu 1999, Georgobiani et al. 2000b, Skartlien & Rast 2000). Roxburgh & Vorontsov (1997) considered the superposition of dipole and quadrupole sources; Nigam et al. (1998) used a combination of monopole and dipole terms; Kumar and Basu (1999) show that the asymmetry reversal could be triggered even by dipole or quadrupole sources alone.

Simulations of the shallow upper layer of the solar convective zone have resonant acoustic modes like the Sun. The emergent intensity and the velocity in the photosphere have asymmetric spectra with the opposite asymmetry (Georgobiani et al. 2000a). In this letter we calculate the temperature and velocity power spectra at the continuum optical depth $\tau = 1$ and at the geometrical depth corresponding to $< \tau >/= 1$. At unit continuum optical depth the velocity and temperature have opposite asymmetry, with the velocity having more low-frequency power and the temperature more high frequency power. At fixed geometrical depth, however, the velocity and temperature have the same asymmetry, with more low frequency power for modes at low frequencies. These results indicate that the asymmetry reversal might be caused by radiative transfer effects.

2. MODEL OF THE SOLAR CONVECTION

We use a three-dimensional hydrodynamic code of Stein and Nordlund (1998 and references therein) to simulate the upper layers of the solar convection zone. The computational domain covers 6 Mm by 6 Mm horizontally and 3 Mm in vertical direction, from 0.5 Mm above the $< \tau >/= 1$ surface to 2.5 Mm beneath it. The model includes non-gray, LTE radiative transfer. Horizontal boundaries are periodic, while vertical ones are transmitting. The spatial resolution is 100 km horizontally and $\sim 50$ km vertically, with a finer grid interpolated for solving the radiative transfer equation. The radiation field is calculated by solving the Feautrier equation along a vertical and 4 straight inclined rays, after averag-
ing the Planck function into four bins by wavelength sorted according to opacity (cf. Nordlund 1982, Stein & Nordlund 2003). Snapshots are saved at 30 s intervals. We have simulated 43 hours of solar time.

3. CALCULATION OF POWER SPECTRA

We compare power spectra of intensity, temperature and velocity in order to shed light on a cause of the p-mode line profile asymmetry and its reversal between velocity and intensity. From our simulations, we have coordinate- and time-dependent quantities: vertical velocity \(V(x, y, z, t)\), temperature \(T(x, y, z, t)\), emergent intensity \(I(x, y, t)\), etc. To obtain a power spectrum at a particular geometrical depth \(z_0\), we use \(V\) and \(T\) at \(z = z_0\). To obtain \(V\) and \(T\) at a particular optical depth, say, \(\tau = 1\), we calculate \(\tau(x, y, z, t)\) and interpolate \(V\) and \(T\) to the height at which \(\tau(x, y, z, t) = 1\) for each position \((x, y)\) and time \(t\). We choose \(z_0\) to be the depth for which \(<\tau> = 1\). Clearly, \(V(x, y)\) or \(T(x, y)\) at a geometrical depth \(z_0\) will not be the same as \(V(x, y)\) or \(T(x, y)\) at \(\tau = 1\): we see deeper in the cooler intergranular lanes compared to the hotter granules. We investigate if this makes a difference in the power spectra and somehow affects line asymmetries.

We separate the oscillation modes into radial modes, for which we average our data horizontally, and non-radial modes, for which we multiply the data by the corresponding spatial sines or cosines and then average horizontally (for more details, see Georgobiani et al., 2000a). We then Fourier - transform these time strings to get power spectra and phase relations. In our simulations, the first nonradial mode (horizontal wavelength \(\lambda_h = 6\) Mm) corresponds to a harmonic degree \(\ell = 740\), because the simulation box represents a small fraction of the solar surface. We investigate the behavior of the emergent intensity, plasma temperature and plasma velocity for this first nonradial mode of the simulation (see also Georgobiani et al. 2000a).

4. RESULTS

The emergent continuum intensity is a good measure of the plasma temperature at local instantaneous continuum optical depth unity, \(\tau(x, y, t) = 1\), as expected from the Eddington-Barbier relations. The temperature at \(\tau = 1\) has the same spectrum (Fig. 1) and phase as the emergent intensity. The spectra of the emergent intensity and plasma temperature at \(\tau = 1\) have opposite asymmetry to the plasma velocity (Fig. 2) as is observed (e.g. Duvall et al. 1993) and as was discussed by Georgobiani et al. (2000a).

The spectrum of the temperature at fixed geometrical depth \(z_0\), corresponding to the average continuum optical depth unity, \(<\tau_{x, y, t}> = 1\), (Fig. 3) is, however, rather different from its spectrum at local instantaneous unit optical depth (Fig. 1), with diff-

\[\begin{align*}
\text{Figure 1.} & \quad \text{Spectrum of temperature, } T_\nu, \text{ at } \tau(x, y, t) = 1. \quad \text{The solid curve is boxcar smoothed over } \Delta \nu = 65\mu\text{Hz.}
\end{align*}\]

\[\begin{align*}
\text{Figure 2.} & \quad \text{Spectrum of velocity, } V_\nu, \text{ at } \tau(x, y, t) = 1. \quad \text{Its spectrum is almost the same at fixed geometrical depth corresponding to } <\tau_{x, y, t}> = 1.
\end{align*}\]

ferent asymmetries especially noticeable for the fundamental mode. The temperature at \(<\tau> = 1\) has the same asymmetry as the velocity.

5. DISCUSSION

What changes the asymmetry of the temperature spectrum between measuring it at local \(\tau = 1\) and average \(<\tau> = 1\)? We analyze the fundamental mode that has the most prominent asymmetry and agrees closely with the corresponding solar \(\ell = 740\) mode. Figure 4 shows the velocity and temperature profile of this mode at average continuum optical depth one. It is clear that the velocity and temperature have the same profiles. Figure 5 shows the velocity and temperature profiles measured at local optical depth one, which is where one would observe them. The velocity spectrum is hardly changed, but the amplitude of the temperature fluctuations is reduced by more than an order of magnitude. The high temperature sensitivity of the \(H^-\) opacity obscures high
Figure 3. Spectrum of temperature, $T_\nu$, at $<\tau> = 1$. The temperature at $<\tau> = 1$ has the same asymmetry as the velocity.

Figure 4. The temperature (dotted) and velocity (solid) spectrum for the f-mode measured at $<\tau> = 1$. Here and after, diamonds represent unsmoothed temperature profiles, whereas stars correspond to the parameter on the left y-axis. The temperature and velocity profiles look very similar to each other. Curves are smoothed over 32 $\mu$Hz.

温度 gas and alters the height at which the gas is observed. This reduces the magnitude of the observed temperature fluctuations, but this reduction is not as great on the high frequency side as on the low frequency side. Hence, the mode asymmetry is changed.

Why is the reduction of the temperature fluctuations different at high and low frequencies? At the fixed geometrical height $<\tau> = 1$, the temperature fluctuations were larger on the low frequency side of the mode. This produces a larger opacity variation on the low frequency side of the mode (Fig. 6), which in turn leads to a larger variation in the height where local $\tau(x, y, t) = 1$ (Fig. 7). The radiation temperature we see is equal to the gas temperature at optical depth unity, according to the Eddington-Barbier relations. Since the gas temperature is decreasing outward, the larger variation in the location where the radiation originates on the low frequency side of the mode leads to a smaller temperature variation there, while the smaller variation in the location where radiation originates on the high frequency side of the mode leads to a larger temperature variation there. We therefore observe a larger intensity variation on the high frequency side of the mode than on the low frequency side of the mode and thus the asymmetry is reversed from that observed in the velocity (and temperature when measured at a fixed geometrical depth).

Figure 5. The temperature (dotted) and velocity (solid) spectrum for the f-mode measured at $\tau = 1$. The amplitude of the temperature fluctuations is nonuniformly reduced across the mode peak.

Figure 6. The temperature (dotted) and opacity (solid) spectrum for the f-mode measured at $<\tau> = 1$. The larger temperature fluctuations on the low frequency side of the mode produce larger opacity variations.
6. CONCLUSION

We have found that the emergent intensity and the temperature spectra at local instantaneous optical depth unity have the opposite asymmetry to the velocity as is observed, while the temperature at fixed geometrical depth corresponding to average optical depth unity has the same asymmetry as the velocity. This indicates that radiation transfer plays a role in the asymmetry reversal observed between the intensity and Doppler velocity. Correlated noise can produce the observed mode asymmetries for the case of a simple localized δ-function source (Kumar & Basu 1999, Georgobiani et al. 2000b, Skartlien & Rast 2000). However, the p-mode excitation is actually spread out over several hundred kilometers (Stein & Nordlund 2001). Oscillation induced opacity changes vary the location of radiation emission (τ = 1) in a way that reduces the magnitude of the temperature fluctuations and reverses their asymmetry.

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MERIDIONAL MOTION AND THE SLOPE OF ISORotation CONTOURS

P. A. Gilman¹ and R. Howe²

¹High Altitude Observatory, National Center for Atmospheric Research, 3450 Mitchell Lane, Boulder, CO 80301, USA; email gilman@hao.ucar.edu
²National Solar Observatory P.O. Box 26732, Tucson, Arizona, 85726-6732, USA; email rhowe@noao.edu

ABSTRACT

As helioseismic inversions have continued to improve in precision, it has become apparent that in the bulk of the solar convection zone the contours of rotation, while certainly not parallel to the rotation axis as predicted in earlier global convection simulations, are also not radial, as has often been stated based on earlier inversions. Instead, between 15 and about 55 degrees latitude (measured at \( r = 0.8R \)) the rotation contours make an angle with the rotation axis of about 25 degrees, which does not appear to vary systematically with latitude in this latitude range. What then determines this angle? By use of an extremely simple dynamical equilibrium model, we show that coriolis forces from the observed meridional circulation could be responsible for this angle if, without this effect, the contours of rotation would be radial.

1. INTRODUCTION

Global compressible convection models for the solar convection zone have yet to reproduce well the radial gradient of rotation within the convection zone, though they appear to be getting somewhat better (Brun & Toomre, 2002). Since helioseismically determined profiles of solar rotation have become plentiful, it has been common to characterize the bulk of the convection zone as having very little radial rotation gradient, with sharp radial gradients confined to near the photosphere on the top, and in the tachocline on the bottom. In reality this appears to be a slight oversimplification, in that the latest helioseismic profiles are refined enough to see that in low and mid-latitudes, the rotation contours instead seem to be oriented at an approximately constant angle to the solar rotation axis, of about 25 degrees (e.g., Schou et al., 1998, 2002).

At the same time, helioseismic measures of meridional flow show it is usually toward the poles through the bulk of the convection zone, as far down as can be measured (Haber, 2002).

Leaving aside the detailed complexity of full-blown 3D spherical shell convection models, we show here that taking into account only the influence of coriolis forces from the meridional circulation, and a simple drag on the differential rotation, we can reproduce rather well the angle the rotation contours make with the rotation axis, if we assume that in the absence of meridional circulation, the rotation contours would indeed have been radial, as has been previously inferred from helioseismic measurements.

2. THE DATA

Figure 1 shows the mean rotation profile derived from 66 overlapping 108-day periods of GONG data starting in May 1995 and continuing through December 2001. Approximately 30,000 splitting measurements were used for each inversion, covering the range of degree \( l \) from 3 to 150. The profiles were derived using a 2-dimensional, Regularized Least Squares inversion (as used, for example, by Howe et al., 2000) of the individual-\( m \) rotational splittings for each time period. The contours in the bulk of the convection zone, below the subsurface shear layer and above the tachocline, maintain almost the same angle to the vertical over a wide range of latitudes.
3. THE MODEL

We can find steady solutions for the differential rotation from a simplified form of the longitudinal equation of motion, in which the only physical processes acting are the coriolis forces from the meridional circulation, and a drag-type process, representing the tendency of the convection to maintain a differential rotation in the absence of meridional circulation. We assume this latter process can be represented by the product of the difference between the calculated differential rotation, and the one prescribed by the convection (which convection is not modelled explicitly), and a ‘drag’ or ‘rate’ coefficient \( d \), which just defines the time it takes for the convection to reestablish the assumed differential rotation profile. For this study we assume the convectively maintained rotation profile \( \Omega_0(\theta) \) is independent of radius in the bulk of the convection zone. Then the steady differential rotation \( \Omega(r, \theta) \) that results from the balance between coriolis forces and the drag process is given by

\[
\Omega(r, \theta) = \Omega_0(\theta) - \frac{2\Omega_c}{rd\sin \theta}(v_\theta \cos \theta + v_r \sin \theta)
\]

in which \( \Omega_c \) is the constant rotation rate of the coordinate system, conveniently taken as the rotation of the interior below the tachocline, \( r \) is the radial co-ordinate, \( \theta \) is colatitude, and \( v_\theta, v_r \) are the radial and latitudinal components of the meridional flow field.

In the absence of well-established measurements of the meridional circulation in the bulk of the convection zone, we use a simple model due to van Ballegooijen & Choudhuri (1988). In this model, the density profile is approximated by \( \rho(r) \propto (R/r - 1)^n \), with \( n = 1.5 \), where \( R \) is the solar radius, and a flow field satisfying the continuity equation is given by

\[
v_r(r, \theta) = u_0 \left( \frac{R}{r} \right)^2 \times \left( -1 + \frac{c_1}{2n+1} \xi^n + \frac{c_2}{2n+k+1} \xi^{n+k} \right) \times \xi \sin^m \theta \left( m + 2 \right) \cos^2 \theta - \sin^2 \theta,
\]

\[
v_\theta(r, \theta) = u_0 \left( \frac{R}{r} \right)^3 \times \left( -1 + c_1 \xi^n - c_2 \xi^{n+k} \right) \sin^{m+1} \theta \cos \theta,
\]

where \( \xi = (R/r - 1) \), \( u_0 \) gives the scale of the flow field and \( k > 0 \) and \( m > 0 \) are adjustable parameters governing the radial and latitudinal dependence of the flow field. The constants \( c_1 \) and \( c_2 \) are defined as

\[
c_1 = \frac{(2n+1)(n+k)}{(n+1)k} \xi_n^{(n+k)}, \quad c_2 = \frac{(2n+k+1)n}{(n+1)k} \xi_k^{-(n+k)}
\]

where \( \xi_n \) is the value of \( \xi \) at the base of the convection zone.

4. RESULTS

The best values of the parameters \( k, m, \) and \( u_0 \) were found by a minimization procedure. The \( \chi^2 \) parameter was defined as the sum of squares of the difference between the observations and the model, weighted by the inverse squares of the formal errors on the inversion results over radial and latitudinal mesh points in the range \( 0.8R < r < 0.95R \) and \( \theta > 15^\circ \). The first step in matching the model to the data is to find the ‘neutral line’ where the Coriolis term in equation 1 evaluates to zero. This line lies mostly at approximately 0.82R, moving towards the surface at lower latitudes. The underlying radial rotation pattern is then taken to be such that the rotation rate at each angular grid point is equal to that observed where the radial line for that angle crosses the neutral line.

Under this criterion the best fit was found with \( k = -0.49 \) and \( m = 5.59 \), and a \( u_0 \) value that, given the observed peak meridional flow near the surface of about 15 m/s, corresponds to \( d \approx 0.3 \) nHz. \( d \) can be related to the ‘eddy viscosity’ \( \eta \) as \( (2\pi)dL^2 \), where \( L \) is the eddy scale. The value of \( L \) is not well determined, but could be related to the scale height. \( L = 0.5 \times 10^8 \) cm would give \( \eta \approx 5 \times 10^{10} \) cm²/sec, while \( L = 10^9 \) cm would correspond to \( \eta \approx 2 \times 10^{11} \) cm²/sec.

Figure 2 shows the angles of the contour lines in the unperturbed and perturbed radial patterns corresponding to these parameter choices, with the observed values shown for comparison. Figure 3 shows the corresponding meridional flow pattern and the perturbed radial flow pattern. The latitude corresponding to each rotation contour was determined about the middle of the convection zone, and the gradients were measured over the range 0.8 ≤ r/R ≤ 0.95.

5. DISCUSSION

The coupling of convection, meridional circulation and differential rotation in the solar convection zone is inherently complex, so understanding their interplay invites a
Figure 3. The meridional flow field (left), and the perturbed radial-pattern flow field, with the contours of the unperturbed radial field as dashed lines, (right) for the model with the $u_0, k, m$ parameters chosen to give the best match to the observations.

A variety of approaches, including some that do not rely exclusively on full 3D numerical simulations. We offer the above modest diagnostic analysis as an example of what might be done, now that the details of differential rotation and meridional circulation below the solar surface are becoming better known from helioseismic inferences. At best, this simple approach applies only to the bulk of the convection zone, not to the shear layer near the top, nor to the tachocline.

The meridional circulation profile used here is essentially theoretical; even its amplitude is not constrained until we specify the drag parameter, which is not well known. We need to develop a more complete description of the meridional circulation from observations, inferring the radial flow from the observed latitudinal flow, to see how the the angle of the rotation contours changes. The observations suggest that the poleward flow extends beyond that shown here in Figure 3. This will certainly affect the contour angle at high latitudes.

Our best fit to the differential rotation contour angle also yields a meridional circulation of considerably smaller latitudinal extent than works best in flux-transport type dynamo models (Dikpati & Charbonneau, 1999). We have found that reducing the $m$ value in Equations 2 and 3 down to 1 from the optimal 5.59 degrades the results somewhat in low latitudes, but the midlatitude fit is still rather good. Further study of both dynamo models and observed meridional circulation is appropriate to understand whether there is any real conflict here.

The possible relationship between meridional circulation and differential rotation we have defined suggests that it would be worth while to determine, with a 3D global convection simulation, under what conditions our starting assumption (no radial rotation gradient when meridional flow absent) would be true. It would also be useful to see whether imposing on such a model a meridional circulation like that observed would result in differential rotation contours closer to the observed profile than are now generated without this constraint. If this latter test were successful, it could mean that the weak spot in global convection models for the solar convection zone is to be found in the calculation of meridional circulation. One point is quite clear even without such a numerical calculation: adding a fixed meridional circulation of the type represented by Equations 2 and 3 to a 3D global convection simulation in which the rotation contours are nearly constant on cylinders, rather than radial, will not result in reproducing the correct rotation contour tilt angle. Such a meridional circulation would instead tilt the rotation contours past being parallel to the rotation axis, exactly the opposite of what is needed.

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MONTE CARLO FILTERING FOR THE DETECTION OF PHASE-COHERENT SOLAR SIGNAL IN THE LOW-FREQUENCY DOMAIN.

G. Grec and C. Renaud
Département Cassini, UMR 6529 du CNRS, Observatoire de la Côte d’Azur, 06304 Nice, France.

ABSTRACT

The detection of the global solar modes is mostly made thanks to the associated local maximum in the power spectrum of the signal. This analysis has not been very successful below 1.3 mHz, due probably to the random signals generated by the surface perturbations emerging from the convective zone. We propose an heuristic description of an alternate analysis using a mean phase evaluation over a set of cross-correlation of 2 randomly generated complementary functions, themselves extracted from the helioseismic signal. This analysis is used to explore the frequency range below 1.5 mHz, where we expect the gravity modes together with the low-order acoustic modes. The first preliminary results obtained between 100 μHz and 300 μHz for GOLF (Gabriel et al. 1995) are briefly shown.

1. INTRODUCTION

One of the major objectives of the helioseismic instruments aboard the SoHO space solar observatory (Domingo et al. 1995) is to determine the frequency of the low radial order acoustic modes and of the gravity modes. These modes are all related to the deep layers of the sun, down to the core. Then the measurement of their frequency will constrain more tightly models of the internal structure. During the study of the instrumental specifications, the detection of low-frequency solar oscillations (below 1 mHz) was supposed to be a simple problem of the detection of periodic signals and to be achievable as the result of a long set of observations. With the assumption of a solar noise composed of random components coming from all possible other sources, a simple periodic signal should grows up above the background in the observed power spectrum as the length of the time series grows.

Data are now available almost continuously since Feb. 1996, but little major progress concerning solar mode detection at low frequencies has resulted.

The initial statement is in fact related to the simple hypothesis of solar modes being detectable as perfectly coherent periodic oscillations. After 5 years of SoHO operations, the apparent failure of this simple approach is questionable. We can then take it for granted that the spectral analysis has reached a limit: no stable frequency can be identified apart from the acoustic modes, for which the level of excitation is high enough to produce a broad energy peak above the noise. At low frequency, the energy of a mode decreases, the life time increases and finally the p-mode sequence is lost below the background noise. In this part of the spectrum, below 1.2 mHz, an improved method of detection is needed.

2. THE HELIOSEISMIC SIGNAL

We use here 5 years of data coming from GOLF (Gabriel et al. 1995), covering the period from Apr. 1996 to Apr. 2001, with 2 significative gaps due to the temporary losts of SoHO in 1998. The photometric signal is normalized for the energy in the frequency domain of the “5 min” acoustical modes. This calculation gives a function $S(t)$, corrected for the solar distance, the radial velocity and the instrumental drifts. No assumption is made about the physical origin of the signal (Grec et al. 1999). A relation of this signal to the disk-averaged velocity can only be made for the $p$ modes. The other components are related to any source of variation of the monochromatic intensity, to the instrumental noise, and to the photon noise.

3. METHOD OF ANALYSIS

Let first consider the benefit of an hypothetical stereoscopic observation made from 2 planets on opposite sides of the Sun: the signal of the global oscillations like the acoustical modes would be correlated, while the signal from the convection or other local stochastic sources should be independent. The correct way to detect the components common to the 2 observations is to compute the cross-correlation of


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the 2 signals, noted \( F(t) \) and \( G(t) \). In Fourier space, the related function is the cross-spectrum (CS), i.e., the complex product of the 2 Fourier transforms (FT). When 2 signals at a given frequency are related in time, the time lag corresponds to a stable relative phase for the corresponding spectral components, the phase being measurable in the CS. We extend this notion to the case of the single signal from the observable solar disk.

In order to make an analysis on several independent observations, the function \( S(t) \) is split into a series \( S_i(t) \) on successive 101 d long time intervals, then \( 1 \leq i \leq 17 \). The 101 d duration of a single observation results in an acceptable frequency resolution.

Each function \( S_i(t) \) is then used independently to make a series of \( N \) experiments using the functions \( F_{i,n}(t) \) and \( G_{i,n}(t) \), complementary in time, as defined in Eq. 1 and Eq. 2, where \( 0 \leq n \leq N \).

\[
F_{i,n}(t) + G_{i,n}(t) = S_i(t) \\
F_{i,n}(t) \cdot G_{i,n}(t) = 0
\]  

(1)  
(2)

To be free of systematic effects, as the obvious components related to a periodic window, we use random numbers to produce a set of functions \( \Pi_{i,n}(t) \) randomly distributed in time to compute \( F_{i,n}(t) \) and \( \Pi_{i,n}(t) \) (Eq. 3 and Eq. 4).

\[
F_{i,n}(t) = S_i(t) \cdot (\Pi_P(t) \ast \Pi_{i,n}(t)) \\
G_{i,n}(t) = S_i(t) \cdot (1 - \Pi_P(t) \ast \Pi_{i,n}(t))
\]  

(3)  
(4)

where \( \Pi_P(t) = 1 \) for \( 0 \leq t \leq P \) and \( \Pi_P(t) = 0 \) elsewhere, \( \Pi_{i,n}(t) = \sum_{k=-\infty}^{\infty} \epsilon_{k,n} \delta(t-kP) \) where \( \delta(t) \) is the Dirac distribution and the random series \( \epsilon_{k,n} = 1 \) or \( \epsilon_{k,n} = 0 \) with the probability distribution \( \psi(0) = \psi(1) = 0.5 \).

Are the \( F_{i,n}(t) \) and \( G_{i,n}(t) \) statistically independent? The heuristic answer is "likely yes" for the stochastic components (in the simple example of a system of fringes due to a pair of random pulses, the signal still exists in 1 case out of 4 when the 2 pulses are in a given window, but in this case they disappear from the complementary signal and then the fringes disappear in the CS. On the other hand, for any periodic component, the signals in \( F_{i,n}(t) \) and \( G_{i,n}(t) \) remain identical.

As \( F_{i,n}(t) \) and \( G_{i,n}(t) \) are almost simultaneous, the CS \( \mathcal{P}_{i,n}(\nu) \) (Eq. 5) is a real number for any frequency related to a periodic component.

\[
\mathcal{P}_{i,n}(\nu) = \mathcal{F}_{i,n}(\nu) \cdot \mathcal{G}_{i,n}^{*}(\nu)
\]  

(5)

where \( \mathcal{F}_{i,n}(\nu) \) is the TF of \( F_{i,n}(t) \) and \( \mathcal{G}_{i,n}^{*}(\nu) \) is the conjugate of the TF of \( G_{i,n}(t) \).

Practically a small imaginary component in \( \mathcal{P}_{i,n}(\nu) \) should be expected, due to the noise in the signal at the same frequency and to the small random difference of the time averages over the 2 windows. We then search for any component following Eq. 6 and

\[
\mathcal{R}(\mathcal{P}_{i,n}(\nu)) > 0 \quad (6) \\
| \Im(\mathcal{P}_{i,n}(\nu)) / \Re(\mathcal{P}_{i,n}(\nu)) | < d \quad (7)
\]

In our present work, the duration of the elementary window is \( P = 6 \) h, the number of experiments for each 101 d sample is \( N = 4500 \), the maximum selected for \( 3/\mathcal{R} \) is \( d = 0.1 \). The spectral resolution of the calculation is \( \rho = 3.6 \) nHz.

4. COMMENTS ON ARTIFACTS

An FFT algorithm gives a sampled FT, the sampling in frequency being related to the length of the input file. For a periodic component, the width of the line depends on the real duration of the signal. There is obviously no relation between any frequency we are looking for and the duration of the observation defining the frequency resolution. We have then to oversample the calculation of the FT, as we do in the simple case of the calculation of a spectral density.

The calculation of \( F_{i,n}(t) \) and \( G_{i,n}(t) \) from \( S_i(t) \) (Eq. 1 and Eq. 2) obviously produces sidelobs in \( \mathcal{F}_{i,n}(\nu) \) and \( \mathcal{G}_{i,n}(\nu) \). As \( \mathcal{F}_{i,n}(\nu) + \mathcal{G}_{i,n}(\nu) = \mathcal{S}_i(\nu) \) those sidelobs are opposite, resulting in the CS in a negative number rejected in the Eq. 6. Then the side lobes cannot produce a false detection. Nevertheless, blind intervals associated to the strong spectral components in the signal could be present in the CS, accordingly to the \( P \) duration of the elementary window.

5. SIGNAL DETECTION

For each function \( \mathcal{P}_{i,n}(\nu) \) we select the bins for which Eq. 7 is satisfied and count \( R_{i,N}(\nu) \) the total number of selected bin for the \( N \) experiments. For the same selected bins, we calculate \( A_{i,N}(\nu) = \sum_{n=1}^{N} \mathcal{R}(\mathcal{P}_{i,n}(\nu)) \). \( R_{i,N}(\nu) \) shows several lines above a random distribution. \( A_{i,N}(\nu) \) increases at low frequency, similarly to the plain PS of \( S_i(t) \).

In order to identify the frequency intervals where the signal goes above the noise, we define for each section a cumulative function \( S_j(f_j) = \sum_{n=1}^{N} \sum_{\nu=f_j+w}^{f_j+w} R_{i,N}(\nu) \) on successive frequency intervals of width \( w = 0.2 \) nHz (corresponding to a 50 d lifetime for the signal). After the selection of the higher values of \( S_j(f_j) \), we obtain a short set of discrete values \( \Phi_j \) for the frequency \( f_j \).

Finally, a solar signal related to a resonant mode is suspected if the same frequency \( \Phi_j \) exists in the result of the calculation for several temporal sections \( S_i(t) \) of the original data. We can then calculate a weighted frequency of the detected lines, as

\[
\Phi_j = \frac{\sum_{n=1}^{N} \sum_{\nu=f_j}^{f_j+w} \nu R_{i,N}(\nu)}{\sum_{n=1}^{N} \sum_{\nu=f_j}^{f_j+w} R_{i,N}(\nu)}
\]

(8)
6. USE FOR POWER SPECTRUM AVERAGING

At this point, we can simply go back to the standard PS. The additional information we have now extracted from the phase of the CS indicates the segments $S_{i}(t)$ (defined in Sec. 3) in which a given periodic signal is detected. We can then compare 2 averaged PS: Firstly we select the sections of signal on which a given solar period is detected and compute the mean PS. Secondly we compute the mean PS for the remaining observations.

Fig. 1 is an example of a peak in the mean PS of the selected sections compared to a flat power for the other time section. The statistical significance of this result is the next point to study: as we use now an averaged PS, the distribution of amplitude of a detected line is a gaussian (central limit theorem). Then this simple analysis allows to detect lines significantly above the noise after the selection of the PS. Fig. 2 gives the mean PS for the frequency domain of a p modes. The detection of the p-mode doublet shows that our fully automated analysis cross-check the result resulting from a careful selection of time sections made by observers following the frequency recurrence of the solar p modes (Lazrek et al. 1997).

7. FREQUENCY OF THE DETECTED OSCILLATIONS

The value $\phi$ is not a true measurement for the solar mode frequency. Several choices exist to define a mean frequency for the detected signal, involving $R_{i,N}(\nu)$ and $A_{i,N}(\nu)$ defined Sec. 5 or not.

8. COMPARISON WITH ANALYTIC MODELS

The identification of a solar signal with a solar mode must be supported by an approximate knowledge of a set of frequencies, as was the case the first time a regular spectrum corresponding to p-modes was detected (Grec et al. 1980). That is at least true for GOLF, where the sun is observed as a star and no information on the geometrical pattern is available. We use the results from (Provost et al. 2000) to relate our present results with the calculated frequencies (result translated in periods as shown in Fig. 3). Several close coincidences exist\(^2\).

\(^2\)The margin is not discussed here. The authors (Provost et al. 2000) estimate the uncertainty related to the solar model used for the calculation at 3 $\mu$Hz, the rotational splitting extends this range dependently on the degree of the mode.
9. DISCUSSION AND CONCLUSION

We there detect a periodic signal using his phase coherence. In comparison the power spectrum used for the classical analysis gives only an information about the energy in a given bandpass: for a periodic signal that is a sharp line, exactly like any fringe due to the random background.

It is clear from the results from 300 $\mu$Hz to 1000 $\mu$Hz that the convective background contributes to our filtered signal. Then, the most we can expect for the resonant modes is to detect them with an higher probability than the other pseudo-periodic events, as the resonance may extend the life time and the level of excitation of the related signal. Our first results favour this hypothesis.

The lines in $R_N(\nu)$ have a different amplitude distribution than in a PS. The noise in $R_N(\nu)$ is not frequency dependent, allowing a direct statistical study, free of observer's bias and model fitting.

At the present level of analysis, we conclude that we have detected sporadic quasi-periodic signals. Several oscillations at same frequency are detected in different sections of the solar signal. Several but not all of those frequencies correspond to those calculated for the g modes and the low-order p modes.

There is no evidence for a permanent periodic solar signal. The search for oscillations with a very long coherence time, an up to now expected property of the solar modes, seems then not to be an adequate approach. The detection of solar g modes has been somewhat suspected in the PS of the signal (Gabriel et al. 1999). We can now explain why the corresponding peak does not become higher when the observation time increases: our present result indicates that a given mode is hardly visible continuously for as long as half a year.

The physical nature of the GOLF signal is unknown, apart from the p-mode oscillations for which the velocity component has been identified (Pallé et al. 1999 and Renaud et al. 1999). A forthcoming analysis of the measurements from SOI (Scherrer et al. 1995) and VIRGO (Fröhlich et al. 1995) should give complementary informations, as they give data on the surface velocity and the brightness variations averaged over the solar disk.

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TORSIONAL OSCILLATIONS IN SUNSPOTS DERIVED FROM SOHO/MDI DATA

V.I. Haneychuk and L.V. Didkovsky

1Crimean Astrophysical Observatory, Nauchny Crimea 98409 Ukraine
2Big Bear Solar Observatory, North Shore Lane, Big Bear City, CA 40386 USA

ABSTRACT

High-resolution (0.6 arc sec/pixel) MDI data maps with two single sunspots seen in brightness, line-of-sight velocity and magnetic field are analyzed. Using the assumption of the cylindrical symmetry, the radial, tangential and vertical components of the velocities are calculated for a number of distances from the center of the sunspot to its penumbral outer limit. Time variations of these components show significant power in a period range near 2-5 days and in the well-known 5-min band. The power in the 5-min region would be associated with interaction between acoustic oscillations and the sunspot structure.

Analysis of the velocity components vs. distance from the center of spot shows no substantial temporal variations for both radial and vertical ones. Nevertheless, the tangential velocity component is changed significantly and reverses its sign to the opposite one. The most important velocity changes are located near the outer limit of the spot’s umbral. This phenomenon can be interpreted as a torsional oscillations of the sunspot. An explanation of these oscillations feature is presented and discussed.

Key words: solar spots; torsional oscillations.

1. INTRODUCTION

The torsional oscillations of the solar spots were detected as a change of sign of the tangential (azimuth) component of velocity, \( V_t \), in the single spot or in the simple spot group. Using the data of solar velocity maps, obtained for each day of observations, Gopasyuk (1981) has observed the simultaneous variations of tangential velocity in the preceding and following spots. Period of this oscillations was estimated as approximately 6 days. The difficulties of these approaches are that the detailed maps of velocities were obtained one time during each day and the set of observations was taken during 4 days only.

There was an attempt of Gopasyuk (1985) to show faster torsional oscillations in a sunspot, with periods near 40 min. These variations were found on a non-significant level due to their non-regular behaviour.

Due to several reasons (see discussion) the more reliable periods of torsional oscillations looked like a few days.

This fact was confirmed by Gopasyuk & Lyamova (1987), Gopasyuk et al. (1988) through a study of long-lived sunspots using the photosphericograms. The values of periods of torsional oscillations were detected in a range of 2-26 days.

Study of that kind of oscillations is important for the physical processes in the sunspots as well as determining of deepness of magnetic flux tubes under the photosphere (see Gopasyuk et al., 1988).

Uninterruptible data set of solar surface seen in line-of-sight velocity, magnetogram and brightness, obtained from SOHO/MDI with high resolution (0.6 arcsec) gives us practically complete information to study the solar sunspots and the possible oscillations in detail.

2. DATA OF OBSERVATIONS

We have used the high resolution SOI/MDI data obtained from December 5, 16^{41}m, to December 7, 21^{55}m, 1998 UT with the total length of set 53^{15}m or 2.22 days. As an example, the full maps of 1024x500 pixels or 620x303 arcsec near the central meridian seen in the line-of-sight velocity, magnetic field and brightness are presented in Figure 1. A set of these maps was obtained each minute with some gaps (see for example Figure 2).

For our analysis we have selected two single spots with their form close to cylindrical symmetry. They belong to the sunspot groups NOAA 8402, 8403 and are marked in Figure 1 as SP1 and SP2 correspondingly. The first spot, SP1, seems to be the preceding spot of another than NOAA 8402 group because its polarity is the same as polarity of the real big leader in this group. The second cylindrical spot, SP2, is
For this purpose in each image we must determine the direction to the solar center, i.e. the angle $\alpha$ between the $X$-axis of the original image and the direction to the solar disk center from the spot. The simple formula (Gopasyuk, 1985)
\[
\cos \alpha = \cos \varphi \frac{\sin L}{\sin \theta}
\]
was obtained in assumption that solar $B_0$ angle is always equals to 0 ($\varphi$, $L$ are the latitude and longitude of the sunspot center). More accurate procedure of calculation $\alpha$ taking into account the spherical geometry is following.

1) Calculation of the heliographic distance
\[
\cos \theta = \sin \varphi \sin B_0 + \cos \varphi \cos B_0 \cos L \tag{1}
\]
and the “incorrect” latitude $\varphi'$ (when angle $B_0$ is always 0)
\[
\sin \varphi' = \sin \varphi \cos B_0 - \cos \varphi \sin B_0 \cos L. \tag{2}
\]

2) Calculation of the additional values
\[
\cos \varphi' = \sqrt{1 - \sin^2 \varphi'}, \quad \sin \theta = \sqrt{1 - \cos^2 \varphi},
\]
\[
\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \tan \varphi' = \frac{\sin \varphi'}{\cos \varphi'}. \tag{3}
\]

3) Determination of the angle $\beta$ between the meridian of the spot and direction to the solar disk
\[
\sin \beta = \frac{\sin L \cos \varphi}{\sin \theta \cos \varphi'}, \quad \cos \beta = \frac{\tan \varphi'}{\tan \theta}. \tag{4}
\]
The sign of angle $\beta$ is completely determined by these equations.

4) Determination of the angle $\alpha$
\[
\alpha = 270^\circ - \beta. \tag{5}
\]

In the new coordinate system we can determine the components of $V_r$, $V_\varphi$ and $V_\psi$ on each distance $\rho$ from the center of sunspot using the final formulae

\[
V_r(\rho) = \int_0^{2\pi} V_{r||}(\rho, \psi) \cos \psi d\psi \frac{\pi \sin \theta}{2 \pi \cos \theta}, \tag{6}
\]

\[
V_\varphi(\rho) = \int_0^{2\pi} V_{\varphi||}(\rho, \psi) \sin \psi d\psi \frac{\pi \sin \theta}{2 \pi \cos \theta}, \tag{7}
\]

\[
V_\psi(\rho) = \int_0^{2\pi} V_{\psi||}(\rho, \psi) d\psi \frac{\pi \sin \theta}{2 \pi \cos \theta}. \tag{8}
\]

where integration is over the circle of radius $\rho$ from the sunspot center, $\psi$ is polar angle. Due to the effect of the projection this circle is transformed into the ellipse at the real Sun image. By transforming the solar image into the heliographic coordinates, ($\varphi$, $L$) this effect is automatically taken into account and coordinates $x$, $y$ of the new image can be determined as
\[
x = \rho \cos(\psi - \alpha) + x_c, \quad y = \rho \sin(\psi - \alpha) + y_c, \tag{9}
\]
where $x_c$ and $y_c$ are the coordinates of the sunspot center in the image.
Figure 2. Temporal variations of averaged over the whole spot SP1 the tangential components of a) velocity $V_t$ and b) magnetic field $H_t$, the radial components of c) $V_r$ and d) $H_r$, the vertical components of e) $V_z$ and f) $H_z$. The sine-wave is the best fit for a period near 3 days. The vertical lines indicate the selected time moments for Figure 4.

Figure 3. The same as in the Figure 2 variations over the whole spot SP2. The sine-wave is the best fit for period near 5.2 days.

Table 1. The value of phases, expressed in fraction of period $\varphi/2\pi$ of fitted sin-wave $y = A \sin\left(\frac{2\pi t}{P} - \varphi\right) + C$.

<table>
<thead>
<tr>
<th>Component</th>
<th>SP1</th>
<th>SP2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>0.51</td>
<td>0.36</td>
</tr>
<tr>
<td>$H_t$</td>
<td>0.22</td>
<td>0.82</td>
</tr>
<tr>
<td>$V_r$</td>
<td>0.82</td>
<td>0.71</td>
</tr>
<tr>
<td>$H_r$</td>
<td>0.27</td>
<td>0.46</td>
</tr>
</tbody>
</table>

In the Figures 2,3 averaged over the whole spot the corresponded components of velocity and magnetic field are shown in time. The first spot, SP1, was observed during almost 53.2 hours and the main period of oscillation was detected near 73 hours or 3 days. The second spot, SP2, was observed during 28.7 hours, but the period of the oscillations was found to be near 125 hours or 5.2 days. Spot 1, therefore, was observed during 0.73 of main period while the spot 2 was seen only during 0.23 of period of oscillation and the errors in the determination of parameters for for the spot SP2 are higher.

As we can see from the Figure 2, the main variations of velocity and magnetic fields are observed on the tangential and radial components while the vertical one shows preferably the linear trend only. Short-time variations of velocities show the oscillations in the 5-min region which can be interpreted as a penetration of global oscillations into a sunspot (see components $V_t$, $V_r$ and $V_z$). Influence of that penetration and investigation of the obtained spectra are the subject of special considerations shown in a number of papers.

Special attention ought to be paid to the phase differences between the components of velocities and magnetic field. The phases $\varphi$ for each fitted sin-wave $y = A \sin\left(\frac{2\pi t}{P} - \varphi\right) + C$ (where $A$ is amplitude, $P$ is period and $C$ is constant) is presented in Table 1. We can see that for spot SP1 the torsional $H_t$ and radial $H_r$ components of magnetic field have practically the same phase (see Figure 2). The phase difference between velocity components $V_t$ and $V_r$ is approximately 0.30 or close to the 1/4 of period. Radial velocities $V_r$ and magnetic field $H_r$ are practically in anti-phase. The difference between torsional components of velocity $V_t$ and magnetic field $H_t$ is approximately 1/4 of period.

5. DISCUSSION

Phenomenon of torsional oscillations is qualitatively described by Priest (1982). Usually it connected with the propagation of the Alfvén wave inside the spot (Gopasyuk et al., 1988). But this phenomenon seems to be more complex than in simple case of the pure Alfvén wave, where the component of velocity $V_t$ and magnetic field $H_t$ must vary in anti-phase (Priest,
1982). Such a variation we observe in the spot SP2, but the accuracy of the period and phase determination is not sufficient here to make that conclusion on a significant level. In the spot SP1 we see that velocities and magnetic field variations are not in antiphase, but with phase difference near 1/4 of period.

Moreover, we can see the variation of radial velocities $V_r$ and magnetic field $H_r$ components which can be explained by a possible compression-expansion of the magnetic flux tube. This variation is associated with the whole spot as we can see from the Figure 4b, where the profile of corresponded components of velocity are plotted along with radius for selected time moments.

From the Figure 4a we also can see that torsional component of velocity $V_t$ varies very significantly over the spot. The sign and profile of this single component is changed to the opposite one with the change of velocity semi-wave (see Figure 2). So we can conclude that the torsional component is really predominant in the phenomenon of the sunspot torsional oscillations. Also we can see (from the Figure 4a) that the main changes of this velocity is located near the umbra-penumbra limit as it was observed before by Gopasyuk (1977). Other essential changes of velocity is near the penumbra limit, where the maximal velocity of radial component $V_r$ is observed.

Such a behavior of velocity and magnetic field in the solar spot, especially the fact that $V_t$ and $H_t$ show anti-phase leads to an assumption that we observe real Alfvén wave in sunspot at the photosphere level. Meanwhile, the phase difference of $\approx \pi/2$ between components $V_t$ and $H_t$ may indicate the important role of non-homogeneous atmosphere at that level. Really, if the propagation velocity of the Alfvén wave in the photosphere is $v_A \approx 10$ km/s (Priest, 1982), then its wavelength will be $\lambda_A = v_A P \approx 2.6 \cdot 10^6$ km ($P \approx 3$ days $\approx 2.6 \cdot 10^5$ s is a period of observed oscillations). Meantime the characteristic height scale in photosphere $h \approx 300$ km, and we can see that $\lambda_A \gg h$ which means that the simple approach to the Alfvén wave propagation is not valid here. Quantitative model of this phenomenon can be built taking into account the non-homogeneous (with height) atmosphere and real solution of MHD-equations for this case.

So we can see that long-term high-accuracy data of observations, obtained with SOHO/MDI, gives the possibility to investigate in details the physics of the phenomenon of the torsional oscillations of the magnetic flux tubes as a propagation of Alfvén wave at the photosphere level.

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THE GONG++ DATA PROCESSING PIPELINE

Frank Hill, John Bolding, Clifford Toner, Thierry Corbard, Steve Wampler, Bret Goodrich, Jean Goodrich, Patricia Eliason, and Kerri Donaldson Hanna

National Solar Observatory, 950 N. Cherry Ave., Tucson, Arizona, USA, 85719
Tel: +1-520-318-8318 Fax: +1-520-318-8278
E-mail: fhill@noao.edu

ABSTRACT

We describe the hardware and software for the new GONG++ data processing system, and discuss our current experience in developing a pipeline for local helioseismology.

INTRODUCTION

The recent upgrade of the GONG detectors from 256 × 256 to 1024 × 1024 pixel cameras (known as GONG+, Harvey, Tucker & Britanik 1998) can only reach its full scientific potential with a corresponding major upgrade to the data processing system. This system, known as GONG++, is currently under development. It will produce new data products, including subsurface synoptic flow maps over continuous rotations, subsurface active region flow maps, farside images for space weather applications, tracked and remapped data cubes for general local helioseismology, ring diagrams, time-distance diagrams, holographic images, high-degree mode parameter estimates, merged velocity images, merged magnetic field images, and magnetic synoptic charts. In order to deal with the greatly increased volume and complexity of the data processing, as compared to the earlier GONG Classic processing task, we have adopted an automated pipeline approach. This approach was employed by the Stanford MDI/SOI Project and has been shown to be extremely effective in dealing with the varied demands and volume of helioseismic data processing and scientific analysis (Kosovichev et al. 1997). Here we describe the GONG++ hardware and present our current experience with the integration of a “mini-pipeline”, which produces subsurface synoptic flow maps using ring diagram analysis.

HARDWARE

The GONG++ hardware system had to meet a minimum set of requirements: it had to be reliable, provide high-performance computing capability and substantial data storage resources, and allow the use of existing code without major revision. The selected configuration is comprised of a symmetric multi-processor SunFire 4800 with twelve 900-MHz CPUs and 24 GB RAM. The data storage requirement has been met with six T3 disk arrays holding a total capacity of 4.7 TB, and a StorEdge L180 Tape Library, with a capacity of 18 TB on 180 LTO (Linear Tape Open) cartridges and four LTO drives. Figure 1 shows the system, which is now installed at Tucson.

Figure 1: The GONG++ processing hardware. The rack on the left contains the SunFire 4800 with twelve 900-MHz CPUs and 24 GB of memory; the middle rack holds the six T3 disk arrays and the UPS systems, and the box on the right is the L180 StorEdge tape library.

The hardware can adequately fulfill the performance requirement of keeping cadence with the data collection. The 22.7-TB storage capacity is sufficient for about 650 days of anticipated GONG++ data products, which are expected to accumulate at a rate of about 35 GB per day. GONG’s considerable experience with Sun equipment and the Solaris operating system, should expedite the task of porting usable code to the new system. The processing pipeline system and the tape library archive control system needed to be created. In order to accelerate development, commercial products will be used for these components.


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Figure 2: A flow chart/block diagram of the GONG++ data processing pipeline. The square blocks represent analysis modules. The arrows represent the flow of the data, which will be controlled by the OPUS pipeline package and the VERITAS™ hierarchical storage management system. The circles represent the accumulator stages that monitor the temporal coverage of the data products and initiate processing steps as needed. The trapezoids represent the input to the pipeline, and the cylinder represents the on-line archive.

SOFTWARE

The GONG++ system is essentially comprised of analysis modules, accumulators, a pipeline structure, and an archive. The processing data flow, as shown in Figure 2, begins with the ingestion of the data from the upstream calibration process. The data proceeds through an optional restoration stage and is then remapped to a common registered sky image. The registered images either enter a global analysis pipeline via a spherical harmonic transform (SHT) or are merged for use by local methods. As the data flows through the pipeline, the accumulator modules monitor the progress and control the processing schedule by periodically checking for sufficient time coverage before initiating the next stage. The accumulator modules are an integral part of the local helioseismology pipeline — it would be a formidable task for an operator to successfully keep track of the large number of areas in a dense-pack ring diagram synoptic flow map.

Each local helioseismology module in Figure 2 actually contains several sub-modules. For example, the ring diagram box produces an image cube (i.e. a dense-pack set, Haber et al. 2002), computes the 3-dimensional power spectrum, fits the rings, and performs the inversion. The time-distance (Zhao, Kosovichev & Duvall 2002) and acoustic holography (Braun & Lindsey, 2001) modules will also contain several steps, as will the production of magnetograms. All valuable and computationally expensive products will be archived, among them, merged images, data cubes, fitted parameters, and inversion output.

The pipeline also provides an entry point to reprocess data starting from intermediate data products that can be extracted from the archive. Finally, users will have access to the archive and the ability to run their own processing, albeit at a lower priority than the production process.

Two essential pieces of this data handling software are the pipeline control system and the tape library and
archive management system. Both of these components are being constructed using commercially available software, whereas the analysis modules and accumulators are being written in house. The use of commercial products for the core systems greatly accelerates the pipeline development timeline and reduces the overall cost. We have selected OPUS for the pipeline control system, and VERITAS™ as the tape library and file management system. The web sites for these packages are listed in the references section of this paper.

Figure 3: The OPUS Process Manager screen. This shows an operational ring-fitting pipeline composed of seven steps while it is processing the 189 areas of a dense pack. In this snapshot, two of the processes are idle, while the remaining five are working on five different files in the pack.

Figure 4: The OPUS Observation Manager screen. This shows a portion of the list of the 189 files/regions in the dense pack. On the right are seven columns, one for each of the seven steps in the pipeline, showing the status of the file as it moves through the steps. In these columns, the letter c indicates completion, p indicates processing, and w shows a file waiting to begin the step. Here, 3 of the 189 files have completed the entire processing chain.

OPUS was originally developed at the Space Telescope Science Institute to process HST telemetry data, but is actually a powerful, general-purpose tool that provides a fully distributed pipeline processing system for any series of applications. The OPUS environment allows multiple instances of multiple processes to run in multiple pipelines on multiple nodes. It is particularly well suited for GONG++ local helioseismology reduction, which typically utilizes hundreds of small areas on the disk. Figures 3 and 4 show a snippet of the OPUS interfaces – the Process Manager and the Observation Manager.

VERITAS™ is a commercial software package that is used primarily to operate tape library systems and provide automated backup services. It can also be used as a hierarchical storage management (HSM) system wherein the tape library behaves as a very large near-line disk partition. VERITAS™ can be called from a command line, making its integration with OPUS easily accomplished, and the distribution of files across the tape library is completely controllable, which is essential for helioseismic processing.

MINI-PIPELINE EXPERIENCE

We found that the construction and implementation of a mini-pipeline to produce flow maps from tracked and remapped data cubes was extremely easy. An example of one of the flow maps produced by the pipeline is shown in Figure 5. Because OPUS only requires that the code be executed via a Unix shell script, it was simple to link together analysis modules written in IRAF, Fortran 90, Fortran 77, and C. The pipeline makes full use of all available processors that are assigned to the task without the need to translate the code into distributed parallel processes. We have recently installed the process to merge images, which completes the core steps in the ring-diagram pipeline. The next major task will be the integration of OPUS and the VERITAS™ HSM system.

ACKNOWLEDGEMENTS

This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Interamerican Observatory.

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Figure 5: An example dense-pack flow map from a single day of merged GONG+ images, produced through the GONG++ ring-fitting pipeline. The background image is the simultaneous solar magnetic field, obtained from a GONG+ magnetogram.
COMPARISON OF NEAR-SURFACE FLOWS ASSESSED BY RING-DIAGRAM AND F-MODE TIME-DISTANCE ANALYSES

Bradley Hindman1, Laurent Gizon2, Deborah Haber1, Thomas Duvall, Jr.3, and Juri Toomre1

1JILA and Dept. Astrophysical and Planetary Sciences, Univ. of Colorado, Boulder CO 80309-0440, USA
2W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA
3Lab. for Astronomy and Solar Physics, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA

ABSTRACT

The near-surface shear layer exhibits a rich medley of flows that are now being measured by time-distance and ring analysis techniques. We present comparisons of the flows obtained with the two techniques using SOI–MDI Dynamics Program data from the years 1998 and 1999. The time-distance analyses utilize f-mode data without depth inversion. The flows deduced with the two methods are remarkably similar, with common inflow and outflow sites as well as agreement in the general flow directions. The direct correspondence of features in the flows is realized in both quiet and active regions.

1. INTRODUCTION

The local helioseismic methods of ring and time-distance analysis have reached a level of maturity in which measurements of subsurface flows are regularly generated with a spatial coverage that spans the majority of the solar disk. Both techniques are presently being applied with great zeal to data from the MDI instrument aboard the SOHO spacecraft. Several of these studies have mapped the wind patterns associated with solar subsurface weather (SSW) over a few Carrington rotations within each of the last seven years. The resulting flow fields evince complex and dynamic behavior. At the smallest scales, time-distance has successfully measured supergranular flows (Duvall & Gizon 2000) and the outflow around sunspots (Gizon et al. 2001). At larger scales, likely connected to giant cell convection, both the time-distance and ring analysis techniques have detected strong interactions with magnetic features with converging flows around active regions. Finally, on global scales, the local techniques have confirmed the presence of the migrating ‘torsional oscillations’ (e.g. Hathaway et al. 1996, Schou 1999), and have shown that meridional circulations evolve with the solar cycle and may change direction with increasing depth (Giles 1999; Haber et al. 2000, 2002; Beck et al. 2002).

Most of this work has been performed independently with little validation of the resulting measurements through comparisons with other techniques. Before, we can fully trust the exciting discoveries of local helioseismology, we must verify that the different local helioseismic techniques are reliable and robust. In this paper we make the first direct comparison of flows obtained through ring and time-distance analyses.

2. RING AND TIME-DISTANCE ANALYSES

Ring analysis assesses the speed and direction of horizontal winds below the solar surface by measuring the Doppler shift of ambient acoustic waves that are advected by the flow. Frequency splittings between waves propagating in opposite directions are a direct measure of the flow velocity averaged over the depths where the measured acoustic mode has significant amplitude.

Using the procedures detailed in Haber et al. (2002), we measure the Doppler shifts by careful fitting of the peaks within a 3–D power spectra. Through 1-D RLS inversion of the integral equation which relates the frequency splittings to the flow properties, the horizontal velocity is computed as a function of depth below the photosphere (Hill 1988, Thompson et al. 1996, Haber et al. 2002). Using Dynamics Program data from SOI–MDI, we apply this mode fitting and inversion procedure to different patches on the solar surface on a daily basis to build a map of the local flow field as a function of time and position on the solar disk. Typically, each region consists of a 15° diameter disk. A mosaic of such tiles fills the solar disk out to 60° from the center, with tile centers separated by 7.5° in longitude and latitude. Before analysis, each tile is tracked at the surface rotation rate (Snodgrass 1984) to remove the effects of differential rotation. The end result is a measurement of the flow field as a function of time, depth, and position on the solar disk. The horizontal resolution is roughly 15° and the vertical resolution is a few Mm near the surface.

Time-distance helioseismology (Duvall et al. 1993) applied to f modes provides information about the two components of the horizontal flows in a 1 Mm-deep layer beneath the surface (Duvall & Gizon 2000). The flows are estimated by measuring the difference in travel-time for f modes propagating in opposite directions. Travel-times are procured by cross-correlating a central pixel on the Sun with nearby quadrants centered on the four cardinal directions. Instead of a full inversion (Gizon et al. 2000), we use a simple calibration to convert from travel-time perturbations to velocities.

A short report on the data analysis can be found in Gizon et al. (2001). For a given year, Doppler images from the MDI Dynamics Program are tracked at the Carrington rotation rate. For every 8 hr-long section of the data, we obtain a 90° × 90° map of the horizontal flows. From each map we subtract the mean flow map for the Dynamics period to remove systematic errors across the field of view. The residual flows are then averaged together on a Carrington longitude-latitude grid with spatial sampling of 0.24° in both coordinates. For comparison with the ring analyses which have far coarser resolution, the resolution of the time-distance measurements has been degraded by spatial averaging. The averaging was performed over each of the 15° diameter tiles in the ring-analysis mosaic, and the flows were weighted by the spatial apodization function used to generate the 3-D spectra employed in the ring analysis. The end result is a set of measurements with the same coverage and horizontal resolution as the ring-analysis. Figures 1 and 3 are synoptic maps of the residual flow field that remains after the longitudinal mean has been removed from both the time-distance and ring-analysis flows. Figures 2 and 4 show the longitudinal means of the zonal and meridional flows for both techniques.

Both techniques generate zonal flows that have consistent shape as a function of latitude. However, there appears to be a substantial constant offset of 15 m/s between the rotation rates. This offset represents about 0.8% of the sun’s equatorial rotation rate. We do not fully understand the source of this offset, although we suspect that it is due to the different tracking and remapping methods used to remove rotation and sphericity. The time-distance analyses for a given day are performed on a single region, 90° square, that is tracked at the Carrington rate and remapped using an equal area projection. The ring analysis is executed on many different regions each 15° in diameter, each tracked at the local surface differential rotation rate and remapped using Postel’s projection. The mean meridional flows obtained by both techniques and shown in Fig. 4 are roughly consistent when the time-distance results are compared to the 0.9 Mm depth of the ring analysis measurements. However, major deviations between the meridional flows occurs at latitudes greater than 30° in the year 1998 (shown in Fig. 2).

The similarities between the residual flows obtained with f-mode time-distance and ring analysis are heartening. Both techniques appear to provide reliable measurements of large scale flows. It is now appropriate that we seek the source of the remaining systematic differences, thereby hoping to make improvements to both helioseismic techniques.

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Figure 1. Synoptic maps of near-surface flows for Carrington rotation 1932 spanning 21 Jan – 17 Feb 1998. The flow field shown is the residual that remains after the mean meridional and zonal flows have been subtracted. The upper map was obtained using ring analyses with RLS inversion. The lower map was generated from time-distance analyses of f-mode data without depth inversion. The time-distance measurements have been averaged spatially such that the two analysis schemes have the same horizontal resolution of 15°.

Figure 2. The mean zonal flows (left panel) and meridional flows (right panel) as a function of latitude obtained using ring analysis and time-distance f-mode analysis of Dynamics Program data from Carrington rotation 1932 (in year 1988). The zonal flow is measured relative to the surface differential rotation rate of Snodgrass (1984). The thick solid curve is the flow obtained with time-distance analysis. The remaining curves were obtained using ring analysis, and correspond to the flow at different depths: solid 0.9 Mm, dotted 2.0 Mm, dashed 4.4 Mm, and dot-dashed 7.1 Mm.
Figure 3. Synoptic maps of near-surface flows for Carrington rotation 1948 spanning 3 Apr – 29 Apr 1999. As in Fig. 1, the flow field shown is the residual flow field and the spatial resolution of the time-distance measurements have been degraded to match that of the ring analyses. The tiles used in the ring analyses overlap by 7.5°, and the resulting flow field has been interpolated twofold to generate arrows with a spacing of 3.75°.

Figure 4. As in Fig. 2, showing the mean zonal flows (left) and meridional flows (right) from Carrington rotation 1948 (in year 1999). These flows were removed from the full flow field to obtain the synoptic map shown in Fig. 3. Such a subtraction procedure was also followed separately in Fig. 1 using the mean flows shown in Fig. 2.
COMPARING RESULTS FROM THE GONG $L = 0$ AND BISON TIME SERIES

R. Howe$^2$, W. J. Chaplin$^1$, Y. Elsworth$^1$, G. R. Isaak$^1$, R. W. Komm$^2$, and R. New$^3$

$^1$School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
$^2$National Solar Observatory P.O. Box 26732, Tucson, Arizona, 85726-6732, USA
$^3$School of Science and Mathematics, Sheffield Hallam University, Sheffield S1 1WB, UK

ABSTRACT

Approximately 5 years of the $l = 0$ time series from the GONG project have been analysed using the algorithm developed for the BiSON 0-dimensional data. The data cover the period 1995-2000. The results are compared with those from a parallel analysis of contemporaneous BiSON data, and also with the results of the traditional GONG analysis of the low-degree time series. The spectra analysed were prepared using the multitaper spectral analysis technique used in the recent re-analysis of the GONG data. We consider both solar-cycle trends and temporally averaged values for mode frequencies, linewidths, amplitudes and asymmetry parameters.

1. INTRODUCTION

The BiSON project has collected unresolved helioseismic data for well over two decades, and has operated a full six-station network of automated and semi-automated stations since 1991. The GONG project, on the other hand, has collected resolved helioseismic images since 1995, using a six-station network of 256 × 256 pixel cameras. The $l = 0$ time series of the GONG data is analogous to the unresolved observations, though the visibility of the higher $l$ modes falls off faster with $l$ in the GONG data. The GONG PEAKFIND algorithm is not highly optimized for the lowest-degree modes, so it is interesting to see whether we can improve our results by using the BiSON algorithms.

2. THE DATA

For this exercise, we have chosen 1728 days of data, starting on 1995 September 28 and ending on 2000 June 20. The data were subdivided into various multiples of 36 days (1 GONG month) for the analysis, and the spectra were prepared using multitaper spectral analysis with five sine tapers (Komm et al., 1999) as used in the latest re-analysis of the GONG data. Figure 1 shows the BiSON and GONG spectra for the first 864 days. In addition to the relative attenuation of the GONG $l = 2$ and $l = 3$ peaks, there is marked attenuation of the GONG power and inflation of the BiSON power at low frequencies. These effects are due to the filtering methods used on the time series – a two-point difference filter for GONG and a 25-point moving mean for BiSON – and were corrected for in the mode height estimates.

3. ANALYSIS

In the BiSON fitting (Chaplin et al., 2000) the peaks are fitted in pairs, $l = 0/2$ and $l = 1/3$, using an asymmetric peak profile for each peak. The relative height of the temporal sidebands for each fitting run was fixed at a value determined by the fill. The power $P$ for a peak in this...
model is represented as

\[ P(\xi) = \left( \frac{h}{1 + \xi^2} \right) \times [(1 + \alpha \xi)^2 + \alpha^2], \]  

(1)

where,

\[ \xi = 2(\nu - \nu_0)/\Gamma \]  

(2)

\( \nu_0 \) is the frequency of the Lorentzian component, \( \Gamma \) its width, \( h \) its height, and \( \alpha \) a fractional parameter characterizing the asymmetry (Nigam & Kosovichev, 1998). This expression simplifies to the normal Lorentzian for \( \alpha = 0 \). In the fitting algorithm, the natural logarithms of the width \( \Gamma \) and height \( h \) were varied rather than the parameters themselves, and these parameters are plotted throughout this paper in logarithmic form.

4. RESULTS

4.1. Absolute Values

We plot in Figure 2 the values of \( h, \Gamma \) and \( \alpha \), for all modes that were successfully fit in all the six nonoverlapping 288-day periods. In the height parameter the relative attenuation of the GONG \( l = 2 \) modes is clearly visible. The \( l = 3 \) modes are not shown because they are so attenuated in the GONG spectrum that none of them were successfully fitted in all the spectra.

4.2. Temporal Variations

Peak parameter shifts were found by identifying modes that were successfully fitted in all the 144-day time periods, then comparing the values at each epoch with the temporal mean and averaging over the available modes. An absolute shift in the logarithm of a parameter corresponds to the logarithm of the fractional shift in the absolute value of the parameter, and for small values approximates to the fractional shift in the parameter itself. Figure 3 shows the shifts in \( \nu_0, \log(h), \log(\Gamma) \) and \( \alpha \). The frequency shifts show a clear upward trend with increasing solar activity, while the mode height shows a decreasing trend and the linewidth a weak increasing one. Both the height and width measurements are affected by the varying duty cycle of the data. The asymmetry parameter shows fluctuations but no clear solar-cycle trend. The frequency trends are similar to those seen by Chaplin et al. (2001) for BiSON and by Howe et al. (1999) and Howe et al. (2002) for medium-f GONG data, while the linewidth and amplitude changes are consistent with the results of Chaplin et al. (2000) for BiSON and Komm et al. (2000) for GONG.

4.3. Comparison with GONG pipeline results

In order to compare with the results from the standard GONG pipeline, we consider overlapping 108-day time periods and use only those modes which are common to all the time periods in all three analyses. As the GONG

Figure 2. Mean values for mode height (top), mode width (middle) and asymmetry parameter (bottom) for GONG (filled symbols) and BiSON (open symbols) in nonoverlapping 288 day periods. Circles indicate \( l = 0 \) modes, diamonds \( l = 1 \) and triangles \( l = 2 \).
analysis uses symmetrical Lorentzian peaks, there is no asymmetry parameter to consider.

Figure 4 shows the frequency variation, peak height and linewidth variation for the three sets of data. Similar trends are seen in all three sets.

4.4. Consistent anomaly in frequency shifts

Although the frequency shifts seen are highly correlated with the global magnetic flux index, a linear fit to this parameter leaves residuals (Fig. 5) showing a strong quasi-periodic variation, with the residuals themselves being well correlated between the two projects. Similar effects have not been found in high-degree modes (Howe et al., 2002).

5. DISCUSSION

The GONG and BiSON $l = 0$ spectra are too different to allow either to be trivially used to improve the fill of the other.

The GONG and BiSON data show broadly similar temporal trends in frequency, mode width and mode height.

The difference in the $l = 0$ asymmetry, if genuine, might reflect a difference in the way the observing spectral lines of the two projects are formed, with different limb-darkening effects preferentially showing up in the radial modes which is more heavily weighted away from disk center.

The fluctuations in the mode frequency that do not correlate with the activity index may be related to high-latitude magnetic flux changes that are sensed by the low-degree modes but not well detected in the magnetogram measurements.

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This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísico de Canarias, and Cerro Tololo Interamerican Observatory.

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Figure 4. Temporal shifts in $l = 0$ $\nu_0$, $h$ and $\Gamma$ for 108-day periods, for GONG (solid circles) and BiSON (open circles) data processed through the BiSON pipeline, and for GONG data processed with the GONG algorithm (filled squares).

Figure 5. Residuals after subtraction of linear fit to global magnetic flux, for GONG (filled) and BiSON (open).
TIME-DISTANCE HELIOSEISMOLOGY OF SUBSURFACE FLOWS

Stephen J. Hughes and Michael J. Thompson
Space & Atmospheric Physics Group, The Blackett Laboratory, Imperial College, London, SW7 2BW, UK

ABSTRACT

We revisit the work of Giles (1999) in an attempt to extend the work on large scale flows using the technique of time-distance helioseismology. The basic process and techniques are discussed and some initial results are shown. The behaviour of the meridional and zonal flows is found to be similar to that found by Giles and separately by ring diagram methods (Haber et al. 2002)

Key words: Time-distance helioseismology, meridional flows, zonal flows.

1. OBSERVATIONAL DATA TO TIME ANOMALIES

The key to time-distance helioseismology is obtaining travel times from observations of the oscillations of the solar surface. These travel times must then be related to the subsurface conditions governing the propagation of waves in the subsurface region. Subsurface flows manifest themselves in a manner that depends on the direction of propagation of the waves, and so can be inferred observationally from the difference in travel times between forward- and backward-propagating energy.

Dopplergrams produced from the MDI instrument aboard the SOHO satellite are filtered and corrected to provide data for particular areas of the solar disc. The following description is after Giles (1999). The initial data cubes are two dimensional in space with a third temporal axis. These data are first background-subtracted to remove the Doppler signal of the solar rotation, and then tracking is performed by remapping the image using cylindrical equal areas, i.e. the remapped image has points spaced equally in longitude and in sine of latitude. The data is then filtered to remove unwanted input sources of supergranulation and surface gravity waves. Supergranulation represents a coherent velocity structure and as such points in the same region would have highly correlated motions, the properties of supergranular cells are known, so a high-pass filter may be used with full transmission above 1.7 mHz and an FWHM of 0.40 mHz. To remove the f mode, the ridge of the surface gravity wave can be approximated in k-ω space, and a low-pass filter used between the p1 and the f modes to attenuate the signal of the latter sufficiently, but to allow all acoustic frequencies through. Further selections are made to limit the 3D power spectrum to the cones of interest; to pick out waves that travel the appropriate distance, and that travel in the required direction.

Correction of several effects is also necessary. The Modulation Transfer Function (MTF) of MDI attenuates the power at high wavenumber, which limits the resolution as waves that travel short distances as more difficult to observe. Correction is achieved with an MTF derived from measurements of the behaviour of the instrument. Finally the power astigmatism of MDI – the instrument sees more apparent power along the East-West axis than along the North-South axis – can be measured and corrected.

With the data filtered and corrected, cross-correlation of these data is performed to obtain local seismograms of the Sun. This is achieved in the Fourier domain to increase the speed of the process, and a point is correlated with an arc of points in the direction of interest to help drive down the noise. We used the cross-correlations of Giles (1999).

The correlated data is fitted using a Gabor wavelet (Giles 1999):

\[ G(\tau) = A \exp \left[ -\frac{\delta \omega^2}{4} (\tau - \tau_p)^2 \right] \cos (\omega_0 (\tau - \tau_p)) , \]

which is obtained from analysing the normal modes under a cross-correlation. Here, \( \tau \) is the delay time, \( \omega_0 \) and \( \delta \omega \) are the central frequency and FWHM of the power spectrum of the solar oscillations, \( A \) is an amplitude, and \( \tau_p \) and \( \tau_g \) are the phase and group travel times. We choose a peak in the envelope of the first bounce and use estimated values for the free parameters \( (A, \omega_0, \delta \omega, \tau_p, \tau_g) \). This fitting recovers \( \tau_p \) more accurately than \( \tau_g \) so we use this as our measurement of ‘travel time’. An example of a fit is given in Fig. 1.
2. FORWARD MODELLING

Forward modelling requires assumptions to be made about the subsurface conditions. We assume an equilibrium model and then compute the sensitivity along a raypath that join two points separated by a distance Δ. The basic problem is

$$t_i = \int K_i u \cdot ds ,$$

(2)

where the flow (u) along the raypath produces a time lag (ti) due to the sensitivity of the Sun (Ki). In the ray approximation we compute this kernel (Ki) by assuming that the sensitivity lies purely along the ray path. This equation includes the contribution of horizontal and vertical flows to the sensitivity, in practice it is difficult to resolve the vertical flow and we compute a kernel purely for the horizontal component; see for example Fig. 2, which shows two one-dimensional kernels - for two different Δs - as a function of depth.

Ray theory holds in the high-frequency limit, which is not necessarily a valid approximation in the case of the Sun. A step to improving on this approximation is to consider the path of acoustic energy that is scattered at some third point before reaching our ‘receiver’: this can be achieved using the Rytov Approximation (Jensen & Pijpers 2002). The sensitivity to subsurface conditions of wave-energy travelling between two points is now a three dimensional structure, yet the sensitivity is actually zero along the analogous ray path. This behaviour is known as the Banana-Doughnut paradox (Marquiering 1998).

3. MEASUREMENT RESULTS

Data from MDI were processed at Stanford by Giles (1999) and these data were used to explore the large-scale flows of the Sun. To compare with the results of ring analysis we used the analysed data from the Dynamics programme. The North-South and East-West components were analysed separately to explore the character of meridional and zonal flows respectively. The data plotted in Fig. 3 (meridional) and Fig. 4 (zonal) are the difference in time between rays travelling in opposite directions: this reveals the perturbation due to flows, since their effect will be anisotropic. The time-differences imply flows of a certain magnitude, but are not resolved into horizontal and vertical components.

The data were tracked at the equatorial Snodgrass rate (Snodgrass 1984). The effect of differential rotation with latitude remains in the data, however. Thus the inferred zonal flow measurement (Fig. 4) reflects the lower zonal flow speeds at higher latitudes relative to the equatorial rate, as predicted by the Snodgrass rotation rate

$$\Omega = A_0 + A_2 \sin^2(\vartheta) + A_4 \sin^4(\vartheta) ,$$

(3)
where $\theta$ is latitude, with $A_0$ set to zero. The zonal flow measurement is consistently faster (in the prograde sense) than this surface rate, however, because the solar rotation increases with depth in the near-surface shear layer (at least at low- and mid-latitudes).

4. INVERSION

We invert the travel-time anomalies measured from the fitted cross-correlations using kernels derived using a model of the solar radial structure to infer the subsurface flows. The inversion is achieved via singular value decomposition (SVD) of the matrix that represents the discretised kernels. SVD produces $K = AB^T$, where $\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_N)$, and $A$ and $B$ are orthogonal matrices. A simple regularisation scheme acts to limit the higher order terms in the inversion that would amplify noise. Our solution is then

$$u_i = \sum_{j=1}^{N} \frac{\sigma_j}{\sigma_j^2 + \lambda^2} (B A^T)_{ij} t_j.$$  \hspace{1cm} (4)

Here $\lambda$ represents the trade-off parameter between smoothness of fit, and accuracy. This is equivalent to regularized least-squares inversions with zero-derivative smoothing. We carried out 1D depth inversions with 14 points in acoustic radius and 15 travel times, independently of other latitudes and longitudes. Some of the results of the inversion are shown in Fig. 5 whilst the effect of different trade-off parameters is seen in Fig. 6.

5. CONCLUSIONS

The measurement results of the meridional flow seem to agree reasonably well with those of Haber et al. (2002) from ring analysis. They agree less well with those of Giles (1999), where a sharper downturn in the time anomalies is seen at about 40° latitude.

The inversion results appear to track down with some success to 17 Mm, but it would be useful to have more data for the inversion. Thus an important next step will be to investigate the feasibility of analysing the cross-correlations before averaging.

Another key step will be to implement the banana-doughnut kernels in the inversion procedure.

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Figure 5. Inversion of the 1997 (top) and 1998 (bottom) meridional flows with depth for averaged longitude and time with the $\lambda^2 = 10^{-3}$. The solid line is at 3.5 Mm depth; the dotted line at 5.8 Mm and the dashed line at 8.9 Mm. The flow seems to track down with perhaps a slight increase in magnitude at greater depths, which is consistent with the findings of Giles using MDI Structure-Program data (Giles 1999).

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Figure 6. Comparison of different values of $\lambda^2$ from a value of $10^{-5}$ (solid line) through $10^{-4}$ (dotted line), $10^{-3}$ (dashed line) and $10^{-2}$ (dash-dot line) for a depth inversion of synthetic data. We used a synthetic flow of 20 Mm$^{-1}$ at all depths. Note the poorly recovered flows at depths smaller than 5 Mm: our inversion is based on rays travelling 1$^\circ$ to 6$^\circ$ so the resolution is poor where the rays do not sample; a 1$^\circ$ has a lower turning point at approximately 4 Mm. It seems a good choice to carry out inversion with a $\lambda^2$ of $10^{-3}$ or $10^{-4}$ based on the level of noise in the data.
COMMENTS ON INFERRING THE PROPERTIES OF THE SOLAR ACOUSTIC SOURCES BY MODELING THE VELOCITY AND/OR INTENSITY FLUCTUATIONS

Stuart M. Jefferies\textsuperscript{1,3}, Pier-Francesco Moretti\textsuperscript{2}, Maurizio Oliviero\textsuperscript{2}, and Cynthia Giebink\textsuperscript{1}

\textsuperscript{1} Maui Scientific Research Center, University of New Mexico, 590 Lipoa Parkway, Ste 264, Kihei HI 96753
\textsuperscript{2}INAF-Osservatorio Astronomico di Capodimonte, Via Moiariello 16, I-80131 Napoli, Italy
\textsuperscript{3}Steward Observatory, University of Arizona, Tucson AZ 85719

ABSTRACT

We model the observed velocity and intensity power spectra and the intensity-velocity cross-spectrum using an updated version of the Severino et al. (2001) model that includes the effects of the acoustic source. We find that in order to accurately describe the data it is necessary to include a correlated background component in both the V and I signals at low frequencies, and in the I signal at high frequencies. Preliminary results show that even using the new model we can not uniquely determine the phase that is related to the acoustic source at high frequencies, or the amplitudes and phases of the individual correlated background signals. It appears that further physical or observational constraints are needed before we can obtain this information.

Key words: Sun: oscillations, convection, line asymmetry, acoustic sources.

1. INTRODUCTION

The resonant line profiles in the velocity (V) and intensity (I) power spectra of the solar acoustic modes are well-known to be asymmetric at low frequencies (lf). The amount of asymmetry is believed to be related to the type and location of the acoustic sources (Duvall et al. 1993; Gabriel 1995; Abrams & Kumar 1996) and to the level of the interaction of the waves emitted by these sources with convection (Roxburgh & Vorontsov 1997; Nigam et al. 1998; Kumar & Basu 1999a). The latter phenomenon, which manifests itself as a coherent signal that is correlated with the oscillation signal (hereafter referred to as a “background” signal as it is a non-resonant phenomenon), is responsible for the differing sense of asymmetry observed in the V and I power spectra for the same spectral region. Although the details of the correlated background signal are not yet fully understood, the consensus seems to be that this signal most likely only affects I observations. As a consequence, there have been several determinations of the properties of the lf acoustic sources that are based on measurements of the line asymmetry in the V power spectrum (Chaplin & Appourchaux 1999; Nigam & Kosovichev 1999b; Kumar & Basu 2000). Line asymmetry is also present in the high-frequency (hf) part of the oscillation spectrum (above 5 mHz) where it is, again, related to the properties of the acoustic sources. However, in this region of the spectrum the frequencies of the spectral peaks (in particular, the inter-peak spacing) also depend on the acoustic source properties (as well as the Sun’s internal structure) and provide a complimentary diagnostic of the source. Indeed, Kumar and coworkers (Kumar 1994; Kumar & Basu 2000) have used this fact to infer the depth of the hf sources. By modeling the lf line asymmetry behavior and the hf peak spacing in the V power spectrum - with the assumption that there is no contamination from correlated background signals - Kumar & Basu (2000) find that hf oscillations are excited closer to the solar surface than the lf oscillations. This finding is in accord with theoretical expectations based on the mixing-length theory for convection.

However, recent studies of the complex I-V cross-spectrum, in conjunction with the V and I power spectra, suggest that the V signal does contain a significant correlated background component (Skartlien & Rast 2000; Severino et al. 2001; Barban & Hill 2002). Obviously, this casts some doubt on the validity of any inference about the depth and type of the lf acoustic sources that is based on measurements of the line asymmetry in the V power spectrum and which does not account for the effects of a correlated background signal. In addition, we note the presence of a correlated background signal at hf may also affect the determination of the source properties in this part of the spectrum.

Here we further investigate the power of the observational constraints, provided by the combined use of the V and I power and the I-V coherence and phase-difference spectra, to yield information about the presence (or lack) of correlated background sig-

nals at both low and high frequencies. We also comment on the information that can be inferred on the properties of the acoustic sources and the correlated background signal using our fitting model.

We will start from the model of Severino et al. (2001) (hereafter called paper I).

2. FITTING MODEL

Paper I has identified the different components in the solar signal which are necessary to describe the total V and I power spectra and the complex I-V cross-spectrum. Following the terminology in paper I, we use "correlated" to mean that there is a certain time lag between the appearance of the convective signal and the oscillation signal, and "coherent" to mean that the fluctuations of the V and I signals are linearly related with a well-defined phase difference. We can then decompose the observed signal into four different components: (1) a coherent resonant p-mode signal (p); a coherent background which comprises (2) a correlated (cc) and (3) an uncorrelated component (cu); and finally, (4) the uncoherent noise. Using this model we can then describe the velocity \( P_V \) and intensity \( P_I \) power spectra at a given spherical harmonic degree \( \ell \) using

\[
P_V(\nu) = |V_{cc}|^2 + |V_{cu}|^2 + |V_{n}|^2
\]

and

\[
P_I(\nu) = |I_{cc}|^2 + |I_{cu}|^2 + |I_{n}|^2
\]

where \( V_{cc} \) and \( I_{cc} \) are the total correlated signals, i.e. the sum of the p-mode signal and the correlated background,

\[
V_{cu}(\nu) = |V_{cu}| e^{i\phi_{vcu}}
\]

and

\[
I_{cu}(\nu) = |I_{cu}| e^{i\phi_{vcu}},
\]

are the coherent, uncorrelated components, \( |V_{n}| \) and \( |I_{n}| \) are the incoherent noise components. The I-V coherence (\( \rho \)) and phase difference (\( \Phi \)) spectra are modeled as

\[
\rho(\nu) = \frac{|X|}{\sqrt{P_V P_I}}
\]

and

\[
\Phi(\nu) = \text{tan}^{-1} \left( \frac{\text{Im}(X)}{\text{Re}(X)} \right)
\]

where

\[
X(\nu) = I_{cc} V_{cc}^* + I_{cu} V_{cu}^*
\]

is the complex cross-spectrum and \( * \) denotes complex conjugation. Surprisingly enough, the model of paper I, which is based on a symmetric (Lorentzian) line profile for the p-mode signal, is able to produce good fits to all four of the observed spectra without considering any "intrinsic line asymmetry" (i.e. the component of the line asymmetry caused by the presence of the source). Obviously, this model is inadequate for determining the source depths at \( hf \) and \( hf \). It also cannot model the behavior of the \( hf \) interference peaks. We must therefore introduce both a source and the ability to accurately describe the \( hf \) interference peaks into our model. We achieve this by describing the resonant p-mode signal in terms of a two-wave interference model in which the source lies outside the resonant cavity (Duvall et al. 1993; Meunier & Jeffries 2000). In this case, the total coherent, correlated signals \( V_{cc} \) and \( I_{cc} \) become:

\[
V_{cc}(\nu) = A_V \left[ 1 + \frac{D e^{-i(\theta - \delta \theta)}}{1 - R e^{-i2\delta \theta}} \right] + B_V e^{i\phi_{vcu}}
\]

and

\[
I_{cc}(\nu) = A_I \left[ 1 + \frac{D e^{-i(\theta - \delta \theta)}}{1 - R e^{-i2\delta \theta}} \right] e^{i\phi_p} + B_I e^{i\phi_{tec}}
\]

Here \( A \) is the amplitude of the upward emitted wave from the source (the subscripts \( V \) and \( I \) denote the quantity as measured in velocity and intensity, respectively), \( R \) is the wave reflection coefficient at the solar surface, \( D \) is the amplitude of the downward emitted wave (with respect to the upward emitted wave), \( \theta \) is the phase delay of a wave incurred in traveling from the top of the acoustic cavity to the bottom, \( \delta \theta \) is the phase delay between the acoustic source and the top of the acoustic cavity, \( \phi_p \) is the phase difference between an evanescent wave measured in intensity and velocity, \( B_V \) and \( B_I \) are the amplitudes of the correlated components of the background signals and \( \phi_{vcu} \) and \( \phi_{tec} \) are the phases of these signals with respect to the oscillation mode. The parameter \( \theta \) is related to the mode frequency, \( \nu_0 \), through the expression

\[
\theta(\nu) = \pi(\nu_0 - \nu) \left( \frac{dn}{dr} \right)^{-1}_\ell,
\]

where \( (dn/dr)_\ell \) is the separation between radial orders (\( n \)) at constant degree \( \ell \). We note that the difference between this source-outside-the-cavity [SOC] model and a source-inside-the-cavity [SIC] model is that in the latter, both terms in the square brackets in equations (8) and (9) are divided by \( (1 - R e^{-i2\delta \theta}) \). At \( hf \), \( R \approx 0 \) and both the SOC and SIC models reduce to the same "geometric" two-wave interference model (Kumar & Lu 1991). We have chosen to use the SOC model in this work as it represents the expected scenario for the \( hf \) waves (Kumar & Basu 1999a).

3. RESULTS

To ascertain the effects of ignoring any correlated background signal in V when determining the acoustic source position, we exercised our model on nine months of V and I observations taken by the Global Oscillation Network Group's instruments (see Oliviero et al. 2001) during the period 1996 October 28 to 1997 September 16. The rotation corrected,
Figure 1. Fit to the \( n = 9, l = 15 \) mode. The V and I power spectra are normalized such that the first point in the fitting interval has unit amplitude.

\( m \)-averaged, V and I power spectra and the I-V coherence and phase difference spectra were generated from the raw V and I coefficients as described in Severino et al. (2001), and then averaged in frequency to a final resolution of 0.4 \( \mu \)Hz.

Fits were made with and without a correlated background signal in V using a conjugate gradients routine to minimize a \( \chi^2 \) error metric. Figures (1) and (2) show typical fits to the \( I_f \) and \( H_f \) oscillation signals as measured in the four observables \( P_V, P_I, \rho \) and \( \Phi \). Figure 3 shows that at \( I_f \) the fits with \( B_V = 0 \) are systematically worse than those obtained with \( B_V \) as a free parameter, while at \( H_f \) there is no systematic difference either way. To interpret these results we need to look at equations (8) and (9) in more detail with respect to the model of paper I. The first terms in the brackets of equations (8) and (9), which represent the upward propagating wave from the source, can obviously be considered as an additional correlated background component. This becomes more evident by rewriting equations (8) and (9) to highlight the dependence on source depth, i.e.,

\[
V_{cc}(\nu) = e^{i2\delta\theta} \left[ C_V e^{i\psi_V} + \frac{A_V'}{1 - R e^{-i2\delta\theta}} \right]
\]

and

\[
I_{cc}(\nu) = e^{i2\delta\theta} \left[ C_I e^{i\psi_I} + \frac{A_I'}{1 - R e^{-i2\delta\theta}} \right] e^{i\phi_{\nu}}
\]

where

\[
C_V e^{i\psi_V} = A_V e^{-i2\delta\theta} + B_V e^{-i(2\delta\theta - \phi_{V,cc})}
\]

\[
C_I e^{i\psi_I} = A_I e^{-i2\delta\theta} + B_I e^{-i(2\delta\theta - \phi_{I,cc} + \phi_V)}
\]

\( A_V' = A_V D \) and \( A_I' = A_I D \). The upward emitted wave and the “convective” correlated background term show up in the equations in the same manner, i.e. as a (complex) additive constant \( (C) \) to a now symmetric line profile. Thus one might doubt if the decomposition into two separate terms is supported by the observed data. Indeed, the \( H_f \) results in Figure 3 do not support such a decomposition. As the upward wave is the fundamental part of the explanation of the \( H_f \) interference peaks, this probably means that there is no correlated background in V at \( H_f \). However, the \( I_f \) results do support such a distinction and show that a correlated background is needed to model the observed V signal. Going back to equations (11) - (14), this now raises the question: “Can we uniquely determine the amplitudes and phases of the individual components of the complex constants?” To address this question we performed fits to a \( I_f \) mode with different fixed values of \( \delta \theta \). Figure 4 shows that the resulting goodness-of-fit values are essentially identical. This suggests that the answer to the above question is “no.” (We note that we performed the same test with a SIC model and obtained identical results.) The result can be understood by studying equations (13) and (14). Here we can see that if the individual amplitudes and phases in the complex constants are free parameters, then even if there is the constraint that the source phase be common to both constants, the amplitudes and phases of the correlated background components are free to adjust to give the appropriate complex constant for the fit. That is, although the complex I-V cross-spectrum places a strong constraint on the I-V phase difference between the correlated background signals, it does not constrain the amplitudes and phases of the individual components. Having said this, we cannot exclude the possibility that the source depth and the properties of the correlated backgrounds can be uniquely determined using other information that is in the data (e.g., the absolute phases of the V and I signals which are lost in the power and cross spectra), or by using additional physical/mathematical constraints (e.g., by using a larger portion of the spectrum in the modeling (Skartlien & Rast 2000)) to transform an ill-posed problem into a well-posed problem.
2003ESASP.517D...9G

Figure 3. The difference $\Delta \chi^2/\chi^2 = (\chi^2_{B_V=0} - \chi^2)/\chi^2$. Here $\chi^2_{B_V=0}$ and $\chi^2$ are measures of the goodness-of-fit obtained with $B_V = 0$ and with $B_V$ as a free parameter. The $F_\chi$ ratio, which can be used to determine the validity of adding extra parameters to the fitting model (Bevington & Robinson 1992), shows that the fits made with $B_V = 0$ are significantly worse than the fits where $B_V$ is allowed to vary for all but two of the $\ell$ modes.

4. SUMMARY

1) We confirm the presence of a correlated background signal in the V data at low frequencies. Thus source depths that have been inferred from modeling the $\ell$ modes without including the effects of a correlated background in the model, should be treated with caution.

2) Preliminary results show that we can not uniquely determine the phase associated with the source location and the amplitudes and phases of the correlated background components at $\ell$ using our two-wave interference model and the four observed spectra ($P_V$, $P_R$, $\rho$, $\Phi$). However, this certainly does not mean that these quantities cannot be determined from the observed data: we may just need additional constraints.

3) We better understand [i.e., equations (11) - (14)] why the model of Severino et al. (2001) is able to describe the observed spectra even though it does not include any “source” contribution to the line asymmetry.

5. ACKNOWLEDGMENTS

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NOISE PROPAGATION IN INVERSION OF HELIOSEISMIC TIME-DISTANCE DATA

Jesper Munk Jensen\textsuperscript{1}, Thomas L. Duvall Jr.\textsuperscript{2}, and Bo Holm Jacobsen\textsuperscript{3}

\textsuperscript{1}Theoretical Astrophysics Center, Danish National Research Foundation, DK-8000 Aarhus C, Denmark and Department of Earth Sciences, University of Aarhus, Denmark
\textsuperscript{2}NASA/Goddard Space Flight Center, Greenbelt, MD 20771 USA.
\textsuperscript{3}Department of Earth Sciences, University of Aarhus, Denmark

ABSTRACT

We present an analysis of noise propagation in time-distance inversion. The data covariance is estimated from a quiet region. We obtain estimates for the correlation and uncertainties of the inversion result both from theoretical propagation of the covariance matrix and from inversion of realizations of the noise model. Inversion of data containing a sun spot is shown along with the estimates of the uncertainties.

1. INTRODUCTION

When interpreting inversion results it is important to have an estimate of the uncertainty on the result. So far this has been missing for inversions of time-distance data looking for structural changes in the solar convection zone (Kosovichev 1996, Kosovichev & Duvall 1997, Jensen et al. 2001). Giles (1999) obtained error estimates for time-distance difference data used to investigate the meridional flow and from these estimates of the uncertainty on the inversion results. Baudin & Korzennik (1998) showed results of trying to estimate errors for the travel times and found then to be strongly correlated. In this paper we obtain estimates of noise levels and correlations from a quite solar region and use these to investigate the uncertainty on the inferred models. The uncertainty estimates are obtained using both a theoretical estimation of the covariance matrix of the inversion result and by realization and inversion of a medium with characteristics similar to the observed noise. The effect of correlated errors on inversion of global helioseismic data has been discussed by Gough & Sekii (2001).

The data used in this paper was obtained from the MDI instrument onboard the SOHO satellite. It consists of 23 eight hours block obtained between Jan. 9, 1998 and Jan. 17,1998. Time-distance data was obtained from each data block individually for 12 different annuli with radii between 0.3 and 4.38 heliospheric degrees. Part of this data set has previ-

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Estimates of the data variance as a function of mean annulus radius.}
\end{figure}

ously been inverted by Kosovichev et al. (2000) and Jensen et al. (2001). During the time of observation a large active region emerged in the field of observation. This emerging region was studied by both Kosovichev et al. (2000) and Jensen et al. (2001).

2. ESTIMATION OF THE DATA ERRORS

Estimations of the error statistics was obtained directly from the data. To find the error statistics of the data a quite area without magnetic activity was identified. Since there was no signal coming from magnetic activity this area was taken to be representative of the measurement error in the data. The data from this region was then analyzed for all 23 data blocks to obtain the error statistics. Figure 1 shows the standard deviation of the noise. We see that the noise first decreases with increasing radius of the annulus before it starts to increase with a minimum for the noise around 16 Mn. This is in agreement with the finding of Giles (1999) that the observational errors are largest for short and long distances. To fully specify the co-variance matrix of

\cite{Proc. of SOHO 12/GONG+ 2002 'Local and Global Helioseismology: The Present and Future', Big Bear Lake, California (U.S.A.), 27 October – 1 November 2002 (ESA SP-517, February 2003)}
the data the power spectrum of the noise was analyzed. We found that noise power spectrum for the first annuli could be fitted nicely by a power law. For the larger annuli there was a deficit of power at large wave-numbers and therefore a second power law was fitted in this area. Figure 2 shows the measured power spectra and corresponding fits. Random media obeying the obtained power laws was then calculated. An example of such a realization for different annuli is shown in Figure 3 along with data from the same annulus. Here we see that the random media is in good agreement with the noise part of the data.

3. ESTIMATION OF MODEL ERRORS

We now wish to invert the data to obtain estimations of the subsurface wave speed. This is done using conventional linear inversion where the model estimate, $m_{est}$, is given as

$$ m_{est} = H d_{obs} , $$

where $d_{obs}$ is the observed data and $H$ an inverse operator. The co-variance matrix of the estimated model is given as

$$ C_m = HC_eH^T , $$

where $^T$ denotes the transpose. We use a Regularized Least-Squares inversion operator chosen using the Tichonov approach to regularization (e.g. Hansen 1996). Here the idea is to minimize

$$ \min \left\{ \| G m - d \|^2 + \epsilon^2 \| L m \|^2 \right\} , $$

where $\| \cdot \|$ is the $L_2$ norm and $L$ is an operator that determines what aspect of the model is regularized.

$$ H = (G^T G + \epsilon^2 L^T L)^{-1} G^T $$

$$ L = \text{diag}(s_{0,k}') $$

where $s_{0,k}$ is the smoothness in the $k$'th layer of the reference model. This choice of $L$ corresponds to minimizing the norm of the relative smoothness perturbation. The trade-off parameter, $\epsilon$, is chosen using a L-curve plot of the data misfit versus the regularized norm of the model. The trade-off parameter should be chosen near the bend of the L-curve (Hansen 1996). The spatial resolution of the inversion is characterized by the resolution matrix given as

$$ R = HG . $$

The rows of $R$ are the averaging kernels of the inversion which show how well localized the model estimates are.

Eq. 2 gives one way of estimating the co-variance of the inversion results. Another way is to do a Monte Carlo simulation where a number of realization of a random medium with the same properties as the noise are inverted. The co-variance of the model estimations can then be found from this statistical properties of these inversions.
It can be difficult to use Eq. 2 to estimate the model co-variance matrix since the co-variance matrix for the data can be huge. We have used the MCD technique described by Jacobsen et al. (1999) to perform the inversions. This technique assumes that the sensitivity kernels and data co-variance are translationally invariant in the horizontal directions. The inversions can then be done in the Fourier domain which enables one to solve the full inverse problem including the co-variance matrices at a very modest computational expense.

4. RESULTS

We have inverted one of the data blocks using the methods described above and sensitivity kernels given by Jensen & Pijpers 2002. The data was obtained January 13 from 11 A.M. UT to 7 P.M. UT. Thus the active region is studied 24 hrs. later than Kosovichev et al. 2001 and Jensen et al. 2001. Figure 4 shows the model estimate at different depths along with an example of the inverted noise realization. Here one can see how the spatial correlations change with depth and that inverted noise realization corresponds nicely to the background in the inversion of the data. In the upper two layers shown in Figure 4 there is clearly a significant wave-speed perturbation below the sunspot. A vague signal is seen in the deepest layer below the active region but when compared to the noise level in the simulated data the anomaly does not seem significant. The standard deviation on the inversion can be calculated in two ways as mentioned earlier. Using Eq. 2 we obtained an estimated of the co-variance matrix. We also calculated 10 realizations of the noise model, inverted these and calculated the standard deviation for these models. The results are shown in Figure 5. Here we can see that the simulated errors and the theoretical ones from the co-variance matrix agree very well. Also shown is the error estimate obtained when assuming that the noise is uncorrelated. This leads to an underestimation of the standard deviation of the model. The high spatial frequencies are effectively filtered away by the inversion whereas the lower frequencies in the correlated noise remain. In
horizontal errorbars here does not indicated a good depth resolution. Near the surface we see a negative wave-speed perturbation of -0.3 km/s. If this is taken to be a temperature effect due to the cold sun spot it corresponds to a temperature decrease of around 700K. The wave-speed perturbation then increases up to around 0.8 km/s at a depth of 6 Mm. To explain this wave-speed increase a magnetic field of around 25kG or a temperature increase of 2500K should be present at this depth. The wave-speed perturbation then decreases down to 25 Mm below which we see no significant anomaly.

5. CONCLUSIONS

In this paper we have presented an analysis of the uncertainties in the inversion of helioseismic time-distance data. We have taken the spatial correlation of the data into account and estimated the full covariance matrix of the inversion results. We have also presented inversions of synthetic noise models with the same statistical properties as the observational noise. Uncertainty estimates obtained from these two approaches give the same results. The results show that assuming uncorrelated measurement errors leads to an underestimation of the model estimation error. The full covariance matrix should therefore be used when calculating errors for time-distance inversion results.

REFERENCES


TEST OF HELIOSEISMIC TIME-DISTANCE INVERSION USING 3-D
FINITE-DIFFERENCE WAVEFIELD MODELING

Jesper Munk Jensen\textsuperscript{1}, Kim Bak Olsen\textsuperscript{2}, Thomas L. Duvall Jr.\textsuperscript{3}, and Bo Holm Jacobsen\textsuperscript{4}

\textsuperscript{1}Theoretical Astrophysics Center, Danish National Research Foundation, DK-8000 Aarhus C, Denmark
\textsuperscript{2}Institute for Crustal Studies, University of California at Santa Barbara
\textsuperscript{3}NASA/Goddard Space Flight Center, Greenbelt, MD 20771 USA.
\textsuperscript{4}Department of Earth Sciences, University of Aarhus, Denmark

ABSTRACT

Here we present the first validation test of helioseismic time-distance inversion which includes a stochastic waveform computation in a 3-D solar model. For a given velocity structure a stochastic shallow source is propagated by finite-difference acoustic computation to generate random 3-D acoustic vibrations for which synthetic Dopplergrams are computed. The preliminary results indicate that state-of-the-art processing and inversion may recover only a noisy picture of interior sound-speed perturbations even for essentially noise-free data.

1. INTRODUCTION

Time-distance helioseismology has within the last decade emerged as an important tool for investigating the solar convection zone (see e.g., Jensen 2002 and references therein). An important application of time-distance inversion has been tomographic inversion for wave-speed structures in the solar interior.

To help validate the results obtained from inversions of observed data, tests using synthetic data are required. Most often the synthetic data inverted has been generated by forward calculation using the same sensitivity kernels as employed in the inverse procedure (e.g., Kosovichev & Duvall 1997, Korzennik 2001 and Zhao et al. 2001). Jensen et al. (2003) derived synthetic traveltimes from 2D finite-difference modeling of the acoustic wavefield of an impulse source.

However, a more independent and complete validation is achieved by modeling more closely the type of data observed, in this case the Dopplergrams. The surface velocities are generated by a stochastic wavefield believed to be excited by a near surface random source. As a first attempt at a such more truthful validation we derived synthetic Dopplergrams by performing a 3-D random wavefield computation in the acoustic approximation using finite differences in space and time for a known interior wave velocity distribution.

2. MODELING THE DATA

The wavefield calculations were done using 3-D finite-difference acoustic modeling (Olsen 1993). The model used in the calculations was 160Mm by 160Mm by 75Mm with a discretization of 250km, with boundary conditions being free at the surface, absorbing at the bottom and periodic on the vertical faces. A time step of 1 second was used to satisfy the stability criteria. The sound-speed and density profiles from the solar model of Christensen-Dalsgaard et al. (1996) were used in the modeling. In a layer 500km below the surface acoustic energy was generated each time step on a 500km by 500km grid. A source of random strength was introduced and a Ricker wavelet with a central frequency of 3.15mHz was used as source function. The top panel of Figure 1 shows a cross section of the perturbation to the solar model. The full perturbation is rotational symmetric around \(x = 0\)Mm and has the shape of a stack of pennies. The perturbation was chosen to emulate the observed wave-speed structure below a sunspot (e.g., Kosovichev et al. 2000). The movement of the surface was sampled with a spatial and temporal resolution corresponding to the high resolution of the MDI instrument. From 8 hours of these surface oscillations we calculated time-distance data using the cross-correlation technique (Duvall et al. 1997).

3. INVERSION AND RESULTS

The inversion was carried out using the MCD method (Jensen et al. 1998, Jacobsen et al. 1999) and wave-theoretical sensitivity kernels (Jensen and
Pijpers 2002). The regularization parameter was chosen using a L-curve criterion. Figure 1 bottom panel shows a cross section through the result of the inversion. We see that it is possible to recover the general structure of the anomaly. Both the sound speed decrease near the surface and the sound-speed maximum below are recovered, but it is not possible to resolve the tail of increased sound-speed further down. There was no noise added to data so much of the noise in the inferred model must be due to the limited time used to calculate the data. We expect that by increasing the length of time window used in the cross-correlation the estimation noise would improve at the expense of temporal resolution.

This study may be important by its result but perhaps even more important by its principle. When a specific processing and inversion procedure is to be applied to observed data it is important that synthetic data simulate these data as closely as possible. Then the degree of recovery of a known interior structure can be taken as a good and independent validation measure of the performance to be expected on the real data. We believe that future studies using synthetic data calculated by finite-difference modeling, possibly including flows and magnetic fields, will improve our understanding of both time-distance data and the obtained inversion results.

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LOW DEGREE P-MODES AND THE SOLAR CYCLE

S.J. Jiménez-Reyes\textsuperscript{1}, A. Jiménez\textsuperscript{2}, and R.A. García\textsuperscript{3}

\textsuperscript{1}Themos, Instituto de Astrofísica de Canarias, La Laguna, Tenerife, Spain
\textsuperscript{2}Instituto de Astrofísica de Canarias, La Laguna, Tenerife, Spain
\textsuperscript{3}CEA/DSM/DAPNIA/SAp, CE Saclay, 91191 Gif-sur-Yvette CEDEX, France

ABSTRACT

GOLF and VIRGO/SPM instruments aboard SOHO satellite have collected more than 6 years of high quality data which make possible a detailed study of the p-mode parameters during the solar activity cycle. With the data of both instruments from 1996 to 2002, the variations of frequencies, linewidth, power, energy and energy rate of the low \( \ell \) p-modes are studied along the solar cycle using both velocity and intensity observations. The results of these variations are presented and discussed.

Key words: Sun: oscillations - Sun: activity.

1. INTRODUCTION

GOLF and VIRGO/SPM are helioseismic instruments aboard SOHO satellite which measure low degree Sun oscillations. GOLF and its performances are described in Gabriel et al., 1997 and VIRGO in Fröhlich et al., 1997. The variations of p-mode parameters have been studied by different authors and at different periods (Anguera Gubau et al., 1992; Régulo et al., 1994; Jiménez-Reyes et al., 1998, 2001; Chaplin et al., 2000; Kom et al., 2000; Appourchaux, 2000; Toutain and Kosovichev, 2000; Jiménez et al., 2002) and using different instrumentation from ground and space. The special analysis made in this work for the GOLF data make now possible to add the GOLF information to the previous one. This have been also done with the simultaneous data of VIRGO/SPM.

2. DATA ANALYSIS

The data used in this work goes from 11 April 1996 up to 9 January 2002, the whole time series have been divided in 100 day time series and computed the power spectra for each one. A total number of 21 power spectra have been obtained from solar minimum to solar maximum. The first analysis use these 100 days power spectra to study the variations of p-mode parameters with time. The second analysis use the same spectra but averaged each three, resulting in a total of 7 power spectra covering 300 days each one to study the variation of p-mode parameters per Radio Flux unit versus frequency.

For VIRGO the three SPM channels have been used (402nm, 500nm and 862nm) and the results of the three wavelengths have been averaged yielding to a mean value for VIRGO/SPM. The VIRGO/SPM data reduction consist of fitting first-order polynomial to correct the aging and applying a moving averaged (the effect of SPM “Attractors” have been minimized as described in Jiménez et al., 2002).

On the other hand the GOLF data need a special analysis when looking for p-mode variations during the SOHO mission. Before the SOHO lost (summer 1998) the GOLF time series were obtained from the blue wing of the Na D lines while after, the data came from the red wing. This variation in the configuration of the instrument changes the observed depth in the solar atmosphere and modifies the solar power spectrum—both, the solar background and the power of the individual modes—as a function of frequency. To correct this effect, we compute the average ratio between the 800 days power spectra of the two periods (see Fig.1a), we fit it to a 10 order polynomial function and we normalize the red wing data by this fitted function.

There is a second problem on the GOLF data due to the dependence on the orbital radial velocity of the velocity derived from one wing of the solar profile. Once more, the observed depth in the solar atmosphere is modified with a one year modulation. As we use series of 100 day long, there is a small residual of this modulation in the power of the modes which can hide the searched solar cycle effect. To minimize it, we normalize each 100 day power spectrum to the first one. In Fig.1b we show the integrated power between 1.5 and 5 mHz of each 100 day subseries before (continuous line) and after (dashed line) the normalization procedure.
Once the power spectra have been obtained, the p-mode parameters are computed fitting Lorentzian profiles using a Maximum Likelihood Estimation.

3. RESULTS AND CONCLUSIONS

The results of the first analysis are shown in Fig.2 versus time. In the left the results corresponding to GOLF and in the right the ones for VIRGO/SPM. No distinction between different $\ell$ have been used. From top to bottom are the variations (left $y$ axis) in nHz for frequency and in % for linewidth, power, energy and energy rate respectively related to the mean value. To see how correlated is the Radio Flux to the variation of the p-mode parameters, each parameter is fitted to a straight line versus this solar index (21 points) and then scaled using the resulted parameters for the fit, this is represented by the continuous line in the plots (units in the right $y$-axis). The numbers inside are the global change of the parameters between extreme phase of the solar activity and the correlation factor with the level of significance of it.

For the second analysis we proceed as follows. For each p-mode we fit the seven obtained values of the parameter as a linear function of the Radio Flux (RF) corresponding to each of the seven time series. The slope of the fit give us the variation of the parameter per Radio Flux unit, nHz/RF for frequency and %/RF for the rest. Plotting these slopes versus frequency we can obtain the variation of the p-mode parameters for a certain value of RF. This is what is shown in Fig.3, in the left for GOLF and in the right for VIRGO/SPM. From top to bottom as in Fig.2. Different symbols are used for different $\ell$.

The variation of low degree p-mode parameters along the solar cycle (11 April 1996 to 9 January 2002) obtained with the velocity data of GOLF (after the correction of the observed depth) and the intensity data of VIRGO/SPM perfectly agree and yields to the same conclusions. The temporal variation of the p-modes parameters are highly correlated with solar activity except the energy fed in the mode. The frequency shifts to higher frequencies at a rate of 1nHz/RF below 2500 $\mu$Hz, 3nHz/RF at 3000 $\mu$Hz, 5nHz/RF at 3500 $\mu$Hz and 8 nHz/RF at 3700 $\mu$Hz.

This frequency shifts are explained by the influence of magnetic activity in the uppermost layers of the Sun and it depends of the frequency of the modes. The p-modes linewidths increase at a rate of 0.15%/RF and these changes are associated with changes in the parameters of the convection zone which affects mainly the plateau in the linewidths shape. Power and energy mode decrease at a rate of -0.3 %/RF and -0.1 %/RF respectively, while the energy fed in the modes remain constant. The energy deficit can be associated with energy transfer to flux tubes in high activity periods to excite oscillations in magnetic elements.

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Figure 2. Variation of p-mode parameters versus time. (a): GOLF results, (b): VIRGO/SPM results (average of the three channels). From top to bottom, variation (left y-axis) of frequency, linewidth, power, energy and energy rate, related to the mean value. Full lines are the Radio Flux scaled to the best fit between the p-mode parameter and Radio Flux (right y-axis). The numbers inside are the variation from minimum to maximum, the correlation factor and the level of significance. No distinction between p-mode degrees have been used.
Figure 3. Variation of p-mode parameters per unit of Radio Flux (RF) versus frequency. (a): GOLF results, (b): VIRGO/SPM results (average of the three channels). From top to bottom, variation of frequency, linewidth, power, energy and energy rate. Different symbols for different p-mode degrees.
ACTIVITY RELATED VARIATION OF WIDTH AND ENERGY OF GLOBAL P-MODES

R. Komm, R. Howe, and F. Hill
National Solar Observatory, 950 N. Cherry Ave., Tucson, AZ 85719

ABSTRACT

We derived mode width, energy, and energy supply rate from 66 108-day GONG time series currently processed with multitapers. We show the temporal variation of these mode parameters from the previous minimum to the maximum of the current solar cycle localized in latitude. Mode width and energy of global modes clearly sense the local distribution of surface magnetic activity. The relation between magnetic activity and localized mode energy and width is linear within the measurement uncertainties. The energy supply rate however does not show such a relation with the latitudinal distribution of surface magnetic activity. The results presented here are consistent with previously published results, where we analyzed periodograms instead of multitapered spectra.

1. INTRODUCTION

We present the temporal variation of mode energy, energy supply rate, and lifetime of global acoustic modes localized in latitude. This is the first time that we use multitapered spectra instead of periodograms as in previous work to measure mode width, \( \Gamma \), and amplitude, \( A \), of individual modes. The mode width is inversely proportional to the lifetime and contains information about the damping of modes, while the mode area \( (A \Gamma) \) is a measure of the acoustic energy of the mode and thus contains information about the excitation of modes. The \( p \) modes are thought to be stochastically excited by the release of acoustic energy from sources near the top of the turbulent convection zone (see, for example, Rast, 1999; Rimmele, Goode, Harold, & Stebbins, 1995, and references therein). The study of mode width and energy can thus contribute to the understanding of the upper part of the convection zone near the solar surface. For example, Houdek et al. (2001) investigate the effect of changing the horizontal length scale of convective eddies on the damping of acoustic modes and discuss whether the observed solar-cycle variation of the mode width might be caused by a change in the horizontal length scale of the solar granulation.

In the present study, we focus on the latitudinal response of these global mode parameters by using their \( m \) dependence. (Each mode is characterized by radial order \( n \), spherical harmonic degree \( l \), and azimuthal order \( m \).) The different \( m \) values of an \((l, n)\) multiplet sample different ranges in latitudes, as seen for example in the “hooks” found in frequency multiplets of high magnetic activity (Howe, Landy, Komm, & Hill, 2001; Howe, Komm, & Hill, 2002). The \( m \) dependence of frequency shifts is of course the basic information used to derive the latitudinal variation of the internal solar rotation. Here, we analyze mode area and width derived from multitapered spectra and map their response in latitude and time. This method is in some sense the global analog of the ring diagram analysis that has shown that active regions produce large localized shifts in frequency (Hindman et al., 2000). To improve the resolution in latitude and to reduce the influence of noise, we average over several latitude-time maps derived from different multiplets, after normalizing the mode parameters by mode inertia. We create maps of these mode parameters using the same latitudinal grid as magnetograms and study the relation between magnetic activity and the excitation of global modes.

2. DATA AND METHOD

We use 66 GONG data sets processed through the GONG pipeline (Hill et al., 1996) covering the rising phase of the current solar cycle from mid-1995 to late-2001. (This time period includes the operation of GONG in ‘classic’ mode before the cameras were upgraded in March 2001 and the operation in ‘blended’ mode during the transition.) Each time series has a length of 108 days to achieve good frequency resolution and consecutive data sets are shifted by 36 days to improve the temporal resolution. The mode parameters for each mode, characterized by radial order \( n \), spherical harmonic degree \( l \), and azimuthal order \( m \), were estimated from 108-day multitaper power spectra using the standard GONG analysis (Anderson, Duvall, & Jeffreys, 1990), which

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fits modes up to \( l = 150 \). We use only fits that passed the quality tests of the peak-fitting algorithm described in detail by Hill et al. (1998). In this study, we use the time series processed with multitapers (Kras, Howe, Komm, & Hill, 2002; Komm et al., 1998) which makes peak-fitting easier and leads to more good fits.

We analyze mode width, \( \Gamma_{nlm} \), and amplitude, \( A_{nlm} \), as provided by the peak-fitting algorithm, and combine them to calculate the area under the mode and then multiply it by the corresponding mode mass, \( M_{nl} \), to calculate the mode energy, \( E_{nlm} \), the sum of kinetic and potential energy (Goldreich, Murray, & Kumar, 1994):

\[
E_{nlm} = \frac{\pi}{2} C_{vis} M_{nl} A_{nlm} \Gamma_{nlm}.
\]  

The factor \( (\pi/2) \) is a scaling factor for the area under a Lorentz profile and \( C_{vis} = 3.33 \) corrects for the reduced visibility due to leakage (Hill & Howe, 1998). The mode mass is defined as the product of mode inertia, \( I_{nl} \), and solar mass, \( M_\odot \):

\[
M_{nl} = 4\pi M_\odot I_{nl}.
\]

We use the mode inertia values, normalized at the solar surface, from the model S of Christensen-Dalsgaard et al. (1996) defined as

\[
I_{nl} = \frac{1}{M_\odot |\nabla \rho_{phot}|^2} \int \rho |\nabla \rho|^2 dV,
\]

where \( \rho \) is the density, \( |\nabla \rho| \) is the displacement associated with the oscillation, \( |\nabla \rho_{phot}| \) is the displacement at the photospheric radius, and integration is over the volume \( V \) of the Sun (Christensen-Dalsgaard & Berthomieu, 1991). We also calculate the quantity \( A_{nlm} \Gamma_{nlm}^2 \), which is proportional to the energy supply rate, \( dE_{nlm}/dt \) (Goldreich, Murray, & Kumar, 1994):

\[
\frac{dE_{nlm}}{dt} = 2\pi E_{nlm} \Gamma_{nlm}.
\]

To derive the latitudinal dependence of these quantities, we calculate one-dimensional SOLA (Subtractive Optimally Localized Averages) inversions applying the algorithm introduced by Howe, Komm, & Hill (2002) using the formulation of Phipps & Thompson (1994). The inversions are calculated on a grid of 51 positions equidistant in sine latitude from equator to pole. We define Gaussian kernels centered at different latitudes and find the appropriate weights to reproduce as closely as possible these kernels with the distributions of the squared spherical harmonics of the available modes.

The measured mode parameters have to be corrected for a variety of effects, as explained in detail in Komm, Howe, & Hill (2002). For each multiplet, we correct the mode area for a bowl-shaped \( (m/l) \) dependence which is caused by the fact that only part of the solar surface is observed. We also correct the mode parameters for gaps in the temporal window function which lead to modes with increased width and reduced amplitude.

3. RESULTS

Figure 1 shows the average mode width, energy, and energy supply rate as a function of frequency derived from all fitted multiplets. Mode width and energy supply rate are scaled with \( Q_{nl} \) which is defined as the ratio between \( I_{nl} \) and \( I_0(r_{nl}) \) where \( I_0(\nu) \) is the inertia of radial modes interpolated to the frequency \( \nu \) (see, for example, Christensen-Dalsgaard et al., 1996). The average mode parameters shown in Figure 1 are solely a function of frequency.

Figure 2 shows the mode parameters as a function of time and latitude derived from 1-dim SOLA inversion. For this step, we use only modes with \( l = 40 - 79 \) and \( n = 9 - 11 \) which are well-fit and are present at all time samples. The mode width
increases with time at all latitudes and the mode energy decreases, which simply reflects that the magnetic activity increases at all latitudes shown. The larger-than-average increase or decrease at latitudes with large magnetic activity indicates that width and energy of global modes sense the latitudinal distribution of magnetic activity. The energy supply rate decreases with time but its variation in latitude appears to be unaffected by the distribution of magnetic activity.

Figure 3 shows the residual mode parameters as a function of magnetic activity for all time samples and latitudes equatorward of 60°. The residuals are calculated by normalizing the mode parameters with their quiet-sun values (measured before mid-1997 and below a median activity of 4 G) after subtracting the global temporal variation. The reasons for the relatively large scatter for B < 7 G were discussed in Komm, Howe, & Hill (2002). The linear regression curves provide a good fit to mode width and energy within the scatter of the measurements. This indicates that the relation between magnetic activity and these mode parameters is linear. The zero slope of the energy supply rate indicates that this mode parameter is insensitive to the local distribution of magnetic activity.

4. SUMMARY AND DISCUSSION

The results presented here are consistent with previously published results (Komm, Howe, & Hill, 2002), where we analyzed periodograms instead of multitapered spectra. But, we have not checked whether...
there might be any small systematic differences. We intend to do a more in-depth comparison between the results of periodograms and multitapered spectra in the near future.

Width and energy of global modes vary on average by about +6% and −16% respectively from minimum to maximum of the current cycle for modes near 3 mHz. These mode parameters respond to the local distribution of surface magnetic activity. The relation between them and magnetic activity is linear within the measurement uncertainties. These results thus confirm 'globally' the well-known fact that magnetic features such as sunspots and plages absorb $p$ modes (Braun, Lindsey, Fan, & Fagan, 1998; Braun, 1995; Bogdan, Brown, Lites, & Thomas, 1993; Braun, Duval, & LaBonte, 1987, and references therein).

For the same multiplets, the energy supply rate decreases on average by about −7% from minimum to maximum of the current cycle (which is small compared to the average decrease of mode energy). However, the supply rate does not show a localized decrease related to the latitudinal distribution of surface magnetic activity.

In this study, we have focused on mode parameters derived from multiplets near 3 mHz since the average mode energy and width show the largest activity-related variation in this frequency range. Our next step will be to analyze more multiplets in order to cover a broader range in frequency and to investigate whether the localization of these mode parameters is frequency dependent.

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HELOISEMIC TESTS OF SOLAR VARIABILITY MODELS

L.H. Li\textsuperscript{1}, S. Basu\textsuperscript{1}, S. Sofia\textsuperscript{1}, F.J. Robinson\textsuperscript{1}, P. Demarque\textsuperscript{1} and D. B. Guenther\textsuperscript{2}

\textsuperscript{1}Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.
\textsuperscript{2}email: li@astro.yale.edu

\textsuperscript{2}Department of Astronomy and Physics, Saint Mary’s University, Halifax, Nova Scotia B3H 3C3, Canada

ABSTRACT

We construct models of the structure and evolution of the Sun which includes variable magnetic fields and turbulence. The magnetic effects included are: (1) magnetic pressure, (2) magnetic energy, and (3) magnetic modulation of turbulence. The effects of turbulence are: (1) turbulent pressure, (2) turbulent kinetic energy, (3) inhibition of radiative losses from a convective eddy, and (4) generation of magnetic fields. Using these ingredients we construct various types of solar models and test them against helioseismic data. We find that only a solar variability model which includes a magnetically-modulated mechanism to generate turbulence can agree with all the current available observational data.

Key words: Sun: oscillations; Sun: magnetic fields; Sun: interior.

1. INTRODUCTION

Following the suggestion by Sofia et al. (1979) that any change in the solar luminosity, \( L \), must be accompanied by a change in the radius, \( R \), a number of theoretical investigations have attempted to establish the relationship between these changes, denoted as \( W = \Delta \ln R / \Delta \ln L \). In each case, the response of a solar model to a particular type of perturbation was calculated, with the perturbation applied at a specific location in the model. From these calculations, it is clear that the derived value of \( W \) depends on the form of the perturbation applied to the stellar structure equations and on the location (depth) in the model where the perturbation appears (see Endal et al. 1985 and references cited therein, and Spruit 1991). Therefore, it is crucial to avoid perturbation treatments for magnetic fields and turbulence in construction of solar variability models.

In order to avoid perturbation treatments, we have to introduce magnetic (and turbulent) stellar structure variables and then reformulate the stellar structure equations. This was initiated by Lydon and Sofia in 1995, updated by Li and Sofia in 2001, applied to turbulence by Li et al. in 2002. All the conservation requirements for a star (mass, momentum, local and global energy conservation) are guaranteed in the reformulated stellar structure equations. In this paper, we

(i) compute solar models at different phases of a solar activity cycle by modifying the stellar structure equations to include (1) a variable magnetic field, (2) turbulence, and (3) feedback between turbulence and magnetic fields.

(ii) test the solar variability models by reproducing the cycle variations of the total solar irradiance (e.g., Fröhlich 2000), photospheric temperature (Gray and Livingston 1997), solar radius and the observed solar low- and medium-degree \( p \)-mode frequencies at different phases of a solar activity cycle (data from the MDI on board SOHO).

The parameters of the model are chosen to satisfy the irradiance changes and temperature changes as a function of solar activity. The \( p \)-mode frequencies are a test of the model.

2. STELLAR STRUCTURE EQUATIONS WITH MAGNETIC FIELDS AND TURBULENCE

The detailed derivation is given in Lydon and Sofia (1995), Li and Sofia (2001) and Li et al. (2002). Here we summarize the main points.

2.1. Magnetic (and turbulent) structure variables

Since magnetic fields are vectors, we have to introduce two magnetic structure variables in the one-dimensional stellar modeling because there are two independent components for the magnetic field: the horizontal \( (B_h) \) and vertical \( (B_z) \) components. Instead of using these two components as the magnetic variables, we use the magnetic energy per unit mass \( \chi_{\text{mag}} = (B_h^2 + B_z^2)/(8\pi\rho) \) and the ratio of the specific heats due to the magnetic field \( \gamma_{\text{mag}} = 1 + B_h^2/(B_h^2 + B_z^2) \) as the independent variables to describe the magnetic structure of a star.
Similarly, the variables that govern turbulence can be defined by the turbulent kinetic energy per unit mass $\chi_{\text{turb}} = \frac{1}{2}(v_h^2 + v_z^2)$ and the ratio of specific heats due to turbulence $\gamma_{\text{turb}} = 1 + 2v_z^2/(v_h^2 + v_z^2)$, where $v_h$ and $v_z$ are the horizontal and vertical turbulent velocity components given by three-dimensional numerical simulations, respectively.

Figure 1. The observed relative p-mode frequency variations by MDI as functions of frequency and angular degree.

2.2. Modified stellar structure equations

The stellar structure equations modified by magnetic fields, or turbulence, or both, have the same appearance as the standard equations. However, pressure and entropy include contributions from the magnetic field and Turbulence. Details may be found in Lydon & Sofia (1995), Li & Sofia (2001), Li et al. (2002), and Sofia & Li (2001).

3. FREQUENCY DATA FROM MDI

In this study, we use 26 MDI data sets containing centroid frequencies determined from measurements made between 1996 May 1 and 2001 April 21 with a break between June 16 and October 22 in 1998, when there was no contact with SOHO. Our purpose is to find out how the cycle variations of the p-mode frequencies constrain solar structural cycle variations. Therefore, we first sort out the common modes for all data sets, which contain 1381 p and f modes with $4 \leq l \leq 286$. We then take yearly averages. In this way we obtain 6 data sets, each for years 1996-2001. Taking the data set for 1996 (the activity minimum) as the reference set, we obtain frequency changes for each year by subtracting the reference frequencies from that year's data set, as shown in Fig. 1.

From Fig. 1 we can see that the frequency differences increase monotonically with solar activity and frequency $\nu$. The degree-dependence is known to be an effect of the mode mass only. The frequency differences do not seem to imply any changes below the sub-surface layers of the Sun (see Basu & Antia, this volume).

Figure 2. Magnetic field distributions at four depths. The profiles are Gaussians. A and B are close to the base of the convection zone, while E and F are close to the surface. $B_{96}$ and $B_{00}$ stand for the magnetic induction in 1996 and 2000, respectively.

Figure 3. Calculated relative frequency variations from the solar models with the magnetic fields shown in Fig. 2.

4. MODELS WITH A PURE MAGNETIC FIELD

We characterize localized fields as having Gaussian profiles with peaks at different depths, as shown in Figure 2. The amplitude of the field is determined by matching the observed solar-cycle variation of the total solar irradiance (e.g., Fröhlich 2000). The temporal dependence of the amplitude is a nonlinear function of the sunspot number (see Li & Sofia 2001)

$$B_m = B_0 \left(190 + [1 + \log(1 + R(Z))]^5\right).$$

In order to compare with MDI observations, we compute the relative p-mode frequency differences for
different levels of magnetic activity for the models with the applied fields shown in Fig. 2. The results are shown in Fig. 3. Obviously, none of these models agree with the MDI observations shown in Fig. 1.

Figure 4. The magnetic field obtained when the magnetic energy density is proportional to the turbulent kinetic energy density.

If small-scale magnetic fields are generated by turbulence, the magnetic energy density should be proportional to the turbulent kinetic energy density. We use the turbulent kinetic energy density from three-dimensional simulations of realistic turbulent convection of the Sun (Li et al. 2002; Robinson et al. 2002). Figure 4 shows the magnetic induction defined by the turbulent kinetic energy density $E_{\text{turb}}$:

$$B = \sqrt{8\pi E_{\text{turb}}}.$$

This model is not self-consistent since turbulence itself is not included. The p-mode frequency changes of the model do not match the observations, as shown in Fig. 5. The frequency and degree dependence of the frequency differences is, however, similar to the MDI observations (Fig. 1) if we reflect the figure about the frequency axis. This suggests that the required magnetic field for the MDI observations must have something to do with turbulence.

5. MODELS WITH BOTH MAGNETIC FIELD AND TURBULENCE

We construct a model with (1) a magnetic field, which increases with solar activity (See Fig. 6):

$$\chi_{\text{mag}} = 0.07(R_z/157)^{2/5}\chi_0,$$

where $\chi_0$ is the turbulent kinetic energy per unit mass obtained by 3D simulations (Robinson et al. 2002), $R_z$ is the sunspot number; (2) turbulence that decreases with solar activity:

$$\chi_{\text{turb}} = [1 - 0.155(R_z/157)^{2/5}]\chi_0$$

when log $P < 6.4$; and (3) we assume that the sum of $\chi_{\text{mag}}$ and $\chi_{\text{turb}}$ decreases with solar activity to satisfy the constraints on radius changes (See Fig. 6); (4) we define $\gamma_{\text{mag}} = \gamma_{\text{turb}} = \gamma_0$, where $\gamma_0$ is obtained from 3D simulations (Robinson et al. 2002). The total $\gamma$ is defined as follows

$$\gamma = (\gamma_{\text{mag}}\chi_{\text{mag}} + \gamma_{\text{turb}}\chi_{\text{turb}})/\chi.$$

We find that this model (a) agrees with the cycle variations of global solar parameters such as TSI (e.g., Fröhlich 2000), phosphoric temperature (Gray & Livingston 1997), and radius (Dziembowska et al. 2001; Antia et al. 2001); (b) implies no variation for the depth of the convection zone (Basu and Antia 2000); (c) reproduces the observed MDI data at least qualitatively, see Fig. 7. This model however, does have one parameter that needs to be adjusted to get the correct form of p-mode variation.
Figure 6. $\chi_{\text{mag}}$ and $\chi_{\text{mag}} + \chi_{\text{turb}}$ as a function of depth. Magnetic energy increases from the solar activity minimum to maximum, while the combined magnetic and turbulent energy decreases from the activity minimum to maximum. The temporal dependence is nonlinear.

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8. LIMITATIONS

Our models have several limitations that need further work in the following directions: (a) Both mag-
MODIFICATIONS OF THE EQUATION OF STATE TO ACHIEVE DESIRED CHANGES IN THERMODYNAMIC QUANTITIES

Aihua Liang and Werner Däppen
University of Southern California, Los Angeles, California, 90089, U.S.A.
email: aliang@usc.edu / dappen@usc.edu

ABSTRACT

Various helioseismological applications are subject to the uncertainty in the equation of state. A typical example is the helioseismic determination of the helium abundance in the convection zone. Only confidence in the quality of the equation of state will lead to a reliable result. There are several methods to improve the quality, depending on the type of the equation of state. Some equations of state are rather fundamental, but become very complicated for realistic astrophysical matter. An example is the OPAL equation of state developed at Livermore. Other equations of state are more intuitive and less rigorous, but easier in their practical realization. An example is the Mihalas-Hummer-Däppen (MHD) equation of state. It makes sense therefore to try to modify the physical parameters in MHD, such that it can emulate OPAL. The result will be a flexible tool for stellar modelers. We report on preliminary work in this direction, based on a systematic study of the response of the thermodynamic quantities to several classes of modifications of the occupation probabilities, as well as different truncations of the internal partition functions. We have also re-visited the issue of the so-called Planck-Larkin partition function (PLPF).

Key words: Equation of state, partition function; helioseismology.

1. MOTIVATION

Helioseismic data are being obtained with ever increasing accuracy, from SOHO and GONG, and other helioseismic observatories. The increased accuracy translates into tougher observational constraints on solar models. The equation of state is one of the crucial ingredients from input physics. Especially in the convection zone, due to its largely adiabatic stratification, the influence of the equation of state can be largely disentangled from the effect of opacity. Below the convection zone, of course, opacity is the key physical quantity that determines the temperature gradient. Among the two most popular equations of state that are currently used in solar models are the MHD and OPAL equations of state. They are at the base of two respective opacity computation efforts, the international Opacity Project (OP) (see, e.g., Seaton 1995), and the OPAL project pursued at Livermore (see, e.g., Iglesias & Rogers 1991, 1996, and references therein). OP uses the Mihalas-Hummer-Däppen (MHD) equation of state (Däppen et al. 1988; Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988), realized with the free-energy-minimization method in the so-called “chemical picture” (e.g. Däppen 1980). OPAL uses an activity-expansion equation of state, ACTEX, which is also known as the OPAL equation of state (Rogers & Iglesias 1992; Rogers 1986; Rogers, Swenson, and Iglesias 1996), realized in the so-called “physical picture”. Both equation-of-state approaches have their strengths and weaknesses. The advantage for MHD mainly is its flexibility and practicability, for OPAL it is its conceptual conciseness and its more rigorous theoretical foundation.

In the last few years, it had turned out that overall, in solar models, the OPAL equation of state appears to be the closer match to reality than MHD. However, even for OPAL, there remain significant discrepancies between models and observations. In addition, there are indications that in the top 2 radius per cent of the Sun, corresponding to the helium and hydrogen ionization zones, MHD might be a better match than OPAL. Therefore, improvements and higher accuracy is required for from both formalisms. Here, we are focusing on modifications of the MHD equation of state, with a first goal to bring it closer in line with OPAL for the physical conditions, where the latter is demonstrably better, such as in the deeper layers of the Sun. A second and more ambitious goal will be to modify MHD such that it will reflect the real equation of state of the Sun better than OPAL.

2. APPROACHES

Although we are working essentially within the framework of the chemical picture, more specifically the MHD equation of state, for our purposes, we have also developed a simplified physical-picture model in order to obtain guidelines for the direction in which modifications of MHD should be attempted. Such comparisons will respond to our first task, which is seeking modifications of MHD to simulate OPAL.
However, for our second task, which is seeking modifications of MHD to simulate empirical data, from the Sun, that is, we have to proceed differently. We introduce a systematic examination of a set of different formalism of the MHD equation of state, in which we incorporate a wide range of possible modifications, and different parameters. Since the crucial feature of the MHD equation of state is its temperature-and-density-dependent internal partition function for atoms, ions and molecules, we concentrate on the specific choice of the partition function.

By considering all sorts of dependencies of the partition function on temperature, density and chemical composition, with our set of different formalisms, we study the direction and sensitivity of changes in the partition functions in the resulting thermodynamic quantities; knowing how an MHD model deviates from the real solar structure, we will target the modifications of the partition function to construct an equation of state that matches the solar values.

3. WORK DONE SO FAR

3.1. Simulating OPAL

As mentioned in part 2, one of our thrusts is dedicated to mimic OPAL with a modified MHD equation of state. For the purposes of this study, we have written our own version of a very simple, hydrogen-only, activity expansion program [so far, the original ACTEX results are available only in form of the tables computed and distributed by the Livermore group (Rogers & DeWitt 1973; Rogers 1974)]. Despite the fact that our realization of the OPAL equation of state only describes a pure hydrogen plasma, here we believe that for the present goal of finding the appropriate MHD modifications, our version includes the essential physics.

We have first tried the obvious, which is the so-called Planck-Larkin partition function PLPF,

\[ Z_{\text{int}} = \sum_{nl} g_{nl} \left( \exp\left( -\frac{E_{nl}}{kT} \right) - 1 + \frac{E_{nl}}{kT} \right) \]  

(1)

Where, \( Z_{\text{int}} \) is the internal partition function, \( g_{nl} \) is bound state weight, \( E_{nl} \) is bound state energy and \( T \) is the temperature.

The PLPF is the result of cancellations of contributions in the expansions of the physical picture. By its definition, and its nature, it is not obvious that it is a partition function in the sense of the chemical picture. In the chemical picture a partition function has a distinct interpretation, closely related to the occupation probability of excited levels of bound species.

Several authors pursuing the physical-picture approach have been suggesting a stronger version of the aforementioned statement. Those authors (Rogers 1986; Rogers 1994; Ebeling et al. 1985) have insisted that if one does realize a free-energy-minimization-formalism one must use the PLPF to be consistent with a physical-picture equation of state, and thus ultimately with a more consistent description of the thermodynamics of a dense plasma. Until now, this suggestion had not really been followed, because some advocates of the chemical-picture approach saw no reason to adopt a part (considered merely ancillary) which results from a different approach (see, e.g., Däppen, Anderson & Mihalas 1987).

We have found that replacing the MHD partition function with the PLPF works remarkably well. Figure 1 shows how close the MHD equation of state with a Planck-Larkin partition function comes to the OPAL equation of state in the hydrogen ionization zone.

3.2. Simulating the Solar Equation of State

As also mentioned in part 2, our other thrust is dedicated to a more systematic consideration of a collection of rather arbitrary, not necessary physical, partition functions, which we choose to serve as open parameters. The goal is to adjust them to obtain a given thermodynamic quantity. From the result above, we conclude that the PLPF is the solution if one wants to mimic the OPAL equation of state (at least our own, simplified version of it).

This project is only at its beginning. So far we have tried out the following: for 2 kinds of partition functions, the original MHD partition function, and the Planck-Larkin partition function, we have been adding excited states one by one. This gives as a simple parameterization of two families of different partition functions. In addition, we can learn how many excited states in a plasma one has to include to obtain results which are de-facto indistinguishable from those of complete partition functions. A particular case is the ground-state only partition function, here denoted SAHA, since most plasma thermodynamics based on simple Saha equations of the ionization equilibrium include only the ground-state contribution.

Although the work is still much in progress, we have already found some interesting features. We first examined the respective speed of convergence of a thermodynamic quantity (here mainly the adiabatic exponent \( \gamma_1 \) because of its solar relevance) as a function of the number of excited states in the partition function. We found that in the hydrogen ionization zone, along the solar track, the MHD ionization rate converges blazingly fast, as can be seen from Figure 2; Figure 3 shows \( \gamma_1 \). It is obvious that the 1st excited state produces the largest outer envelope, the 2nd excited state produces the first fine detail, a twin peak, while the 3rd excited state produces something even more refined: a little wiggle around valley which pushes the figure very close to 9th excited state (which for our analysis correspond to including an infinite number of excited states). In other words, all excited-states features are largely, and highly accurately, built up by the 1st, 2nd, and 3rd excited states, while from the 4th excited state onward only minor nuances are produced.

In this context we have noticed another interesting feature of the MHD equation of state: it turns out that the quick convergence in the summation of excited states in MHD is not general, it is true only for one class
of temperature-density dependence; for the other class MHD converges the same way as OPAL equation of state, which means that higher and higher excited states are not becoming irrelevant like in Figure 2 and Figure 3, but rather contribute significantly to the end result. Incidentally, the solar conditions in the hydrogen ionization zone are placed right at the separation between those two classes. Figure 4 and Figure 5 illustrate these ideas.

4. CONCLUSION

We have begun to examine the result of a rather arbitrary set of equation-of-state modifications with a “forward problem”, that is, a systematic study of several classes of modifications of the occupation probabilities, as well as different truncations of the internal partition functions. We anticipate that the resulting changes in the thermodynamic quantities, especially in the seismologically important adiabatic exponent, will help us guide towards the next step, which will be an “inversion problem” to find the appropriate modifications of the MHD occupation probabilities such that the resulting thermodynamic quantities will match the value of OPAL, and eventually the real solar data. As a first practical result of this study, we have re-visited the crucial, and promising, role that the Plank-Larkin partition function could play in chemical-picture equations of state such as MHD.

Acknowledgments: This work was supported by the grant AST-9987391 of the National Science Foundation.

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Figure 2. Ionization degree $x$ calculated with the MHD equation of state relative to the Saha equation of state, for an increasing number of excited states.

Figure 3. Adiabatic exponent $\gamma_1$ calculated with the MHD equation of state relative to Saha equation of state, for an increasing number of excited states.

Figure 4. Ionization degree $x$ calculated with the MHD equation of state along the density-temperature track $\log \rho = \log T - 10.56132$, relative to the ground-state-only MHD, for an increasing number of excited states.

Figure 5. Same as Fig. 4, but along the track $\log \rho = \log T - 21.84654$.
HELOSEISMIC CONSTRAINTS ON THE CHEMICAL COMPOSITION
AND THE EQUATION OF STATE

Chia-Hsien Lin and Werner Däppen

Department of Physics and Astronomy, University of Southern California, Los Angeles, CA90089-1342, U.S.A.
email: chlin@physics.usc.edu, dappen@usc.edu

ABSTRACT

The intrinsic $\gamma_1$ difference is defined as the discrepancy in the adiabatic exponent $\gamma_1$, resulting from the discrepancies in the equation of state and the chemical composition. The other part of the $\gamma_1$ difference, "induced" by the discrepancy in the internal structure (i.e., pressure, density or temperature) is excluded. Our examination of solar models revealed that the discrepancy in the heavy-element composition and the discrepancy in the equation of state result in different and independent features in the functional form of the intrinsic $\gamma_1$ difference. Therefore, the two discrepancies can be distinguished by looking for the features that do not change when only either the equation of state or the chemical composition is being varied. The possibility of such a strategy was already discussed in an earlier study (Gong, Däppen and Nayfonov 2001), although only theoretically. Our inversion tests demonstrate the potential of its realization. In this paper, we present our initial inversion results between the Sun and various solar models and discuss the strengths and limits of our inversion code.

Key words: Helioseismology; Sun - chemical composition and equation of state; Inversion - method and uncertainty.

1. THE INVERSION METHOD AND THE PROBLEM

The inversion method adapted in this study is the Subtractive Optimally Localized Averaging method (Pijpers & Thompson 1994). The Sun is represented by the observed data obtained from the MDI instrument (Rhodes et al. 1998). The denotation of the models and the mode sets are described in another paper in these proceedings (Lin & Däppen).

Figure 5 is an example in which the inverted results from using different mode sets are inconsistent. Therefore, prior any further investigation, a scheme must be developed to determine whether or not an inverted result is credible. The mode set which yields the most credible result will be used in our further investigation.

2. THE EXAMINATION SCHEME

- The scheme consists of the following steps:
  1. From the results of the model study,
     1a. select two reference models (e.g., model A and model B) and the mode set that gives the best inverted result;
     1b. determine the region in which the inverted curve is reasonably consistent with the expected curve.
  2. Examine the inverted values of (Sun - A), (Sun - B) and (A - B) to identify a region in which the superposition relation (eqn. (2)) is satisfied.
  3. The inverted results of the three sets are considered credible only in the overlapping region of step 1 and step 2.

- The superposition relation among the inverted values is derived below:

$$\gamma_{\alpha \beta} \equiv \frac{\gamma_{\alpha} - \gamma_{\beta}}{\gamma_{\beta}}$$

$\gamma_{\alpha}$ : a physical quantity $\gamma$ of model $\alpha$;

$\gamma_{\alpha \beta}$ : the relative difference of $\gamma$ between the two models, of which model $\beta$ is the reference model.

$$\gamma_{SB} - \gamma_{AB} = \frac{\gamma_S - \gamma_B}{\gamma_B} - \frac{\gamma_A - \gamma_B}{\gamma_B}$$

$$= \frac{\gamma_S - \gamma_A}{\gamma_B}$$

$$= \frac{\gamma_S - \gamma_A}{\gamma_A} \times \frac{\gamma_A}{\gamma_B}$$

$$= \frac{\gamma_S - \gamma_A}{\gamma_A} \times \left(1 + \frac{\gamma_A - \gamma_B}{\gamma_B}\right)$$

$$= \frac{\gamma_S - \gamma_A}{\gamma_A} + \frac{\gamma_S - \gamma_A}{\gamma_A} \times \frac{\gamma_A - \gamma_B}{\gamma_B}$$

(1)

$$\approx \gamma_{SA}$$

(2)

The second term in (1), of order $O(\gamma_{\alpha \beta}^2)$, is generally negligible, provided $\gamma_{\alpha \beta} \ll 1$.
3. RESULTS AND DISCUSSION

The vertical error in all results is defined as the measure of $\chi^2$ and the contribution from the cross term. The vertical error in the region $0.6R - 0.92R$ ($R$ being the radius of the Sun) in all the figures shown in this paper is of the order of $10^{-4}$.

The results from our model study (i.e., inverting for the difference between two known solar models) indicate that the model set 1200d12 is the best mode set for the inversion through most part of the Sun, except in the region $r > 0.99R$, where the result of the mode set 1500d12 becomes the best. Therefore, we have selected the mode set 1200d12 for the inversion shown in the Figure 1-4. An alternative selection would be to use the mode set 1200d12 for the inversion in the region $0.6R < r < 0.99R$ and the mode set 1500d12 for the inversion in the region $r > 0.99R$.

1. Figure 1 is an example in which the three sets of inversion have passed the examination scheme described above. Hence, the inversion with respect to the models NONe and CNONE are reliable down to the depth of $r = 0.6R$.

2. In Figure 2, even though the inverted values of the three sets do satisfy the linear superposition relation, the result of the model study (the middle panel) indicates that the inversion with respect to the model X7419X15 is invalid below 0.8R. The middle panel also shows the resolution limit of our inversion code: the narrow feature in the region $0.97R < r < 1.00R$ was not resolved.

3. Figure 3 is an example in which the three sets of the inverted values do not satisfy the mathematical relation Eqn. (2). Since the validity of the inversion (mdi98 - CNONE) was justified in Figure 1, we can conclude that our inversion (mdi98 - model S) was incorrect.

4. Figure 4 shows the inverted results with respect to various models. The inversions using C, CNONE and NONe as reference models have been justified by the examination scheme. The clear possibility to distinguish the three curves suggests that the effect of a variation of the chemical composition can be resolved by our inversion.

5. Figure 4 also reveals an intriguing feature: the model excluding carbon appears to be more similar to the Sun than are the models which including C, N, O, Ne is. This points to the possibility that the currently used value for the carbon abundance might be overestimated.

4. CONCLUSION

- A scheme to identify credible inversion results has been developed.

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Figure 2. The stars represent the inverted values. The solid lines in the two middle panels are the exact difference computed from subtracting one model from the other. The reference model in this panel is X7419Z15 (i.e., X=0.7419, Z=0.015).

Figure 3. The inverted results are represented by solid lines. The fact that the results of the top two panels do not add up to that of the bottom panel signals an inconsistency (see text, section 3.3).
Figure 4. The inverted results between the Sun and various models.

Figure 5. The inversion between the Sun and the model CNONE.
INVESTIGATING INVERSION UNCERTAINTIES RESULTING FROM MODE SELECTIONS

Chia-Hsien Lin and Werner Däppen

Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-1342, U.S.A.
email: chlin@physics.usc.edu, dappen@usc.edu

ABSTRACT

The quality of an inversion depends on many factors. In addition to the well known ones, such as the target function, the surface term and the suppression parameters, different inversion codes utilizing different matrix inversion algorithms could also bring in an additional uncertainty. Therefore, it is important to assess the uncertainty and the stability of an inversion code prior using it to obtain any reliable result.

The examination of our inversion code suggested the following:

- low-degree low-order modes are essential for an accurate inversion;
- when the quantity being inverted contains narrow features near the surface, the inversions using different mode sets would be inconsistent.

In this paper, we show some representative results, in order to invite stimulating discussions and insightful suggestions.

Key words: Sun; Helioseismology; Inversion; Uncertainties; Singular Value Decomposition.

1. THE INVERSION METHOD

The inversion technique adapted in our inversion code is Subtractive Optimally Localized Averages (SOLA) (Pijpers & Thompsoon 1994). To compute the inverse of a matrix, we implemented the Singular Value Decomposition (SVD) algorithm (Numerical Recipes in C) in order to avoid the degeneracy among modes.

The physical quantity being inverted is the intrinsic $\gamma_1$ difference. The intrinsic difference is the part of the difference of the adiabatic exponent that results from the difference in the equation of state itself. The other part of the $\gamma_1$ difference, "induced" by the difference in the equation of state through a change of the internal structure (denses and temperatures) is excluded.

2. THE MODELS AND THE MODE SETS

The aim of this paper is to examine the effect of different mode selections on the quality of an inversion. The mode sets used in our analysis are described below.

The investigation consists of two parts:

1. Model Study:
   This part is to assess the accuracy of the inversion code. The frequency difference between two models have been used as the input data for the inversion program, which delivers the difference between the two models. The inverted values are then compared with the exact difference obtained by subtracting one model from the other.

   - Models:
     All models, except Model S, use OPAL opacities (1995) and the CEFF equation of state without the $\tau$ correction of the Coulomb pressure (treated in Debye-Hückel theory). The chemical composition in each model incorporates a slight variation, indicated in the notation, with respect to the composition used in OPAL equation of state.
     - CNOnE:
       The chemical composition is the same as in OPAL equation of state;
     - C:
       The only heavy element is carbon;
     - NONe:
       The heavy elements do not contain carbon. The ratio among N, O, Ne remains the same as that in CNOnE;
     - X7419Z15:
       Hydrogen abundance X=0.7419 and the heavy element abundance Z=0.015 (implying a helium abundance Y=0.2431.
     - Y2462Z15:
       Helium abundance Y=0.2462 and the heavy element abundance Z=0.015 (implying a hydrogen abundance X=0.7388.
     - Model S:
       From Christensen-Dalsgaard et al. (1996)
• Mode sets:
The inversion of each pair of models was repeated with the following mode sets.
- 1200d12: \( l = 0 - 200, \text{steps} \ dl = 2 \);
- 1300d12: \( l = 0 - 300, \text{dl} = 2 \);
- 1400d12: \( l = 0 - 400, \text{dl} = 2 \);
- 1500d12: \( l = 0 - 500, \text{dl} = 2 \);

In each mode set, two different selections of mode were used:

(A) all the modes computed from the solar models;
(B) only the modes that appear in the observed data.

2. Real-Sun Study:

• Real Sun (mdl98): the observed data obtained from the MDI instrument (Rhodes et al. 1998).
• Reference Models: The solar models described in Model Study were used as reference models to invert for the difference between the real Sun and each model.
• Mode Sets: the inversion with respect to each reference model was repeated with the mode sets described in Model Study.

3. RESULTS AND DISCUSSION

1. Model S - CNONe
(the lower two panels in Figure 1 and Figure 2):

- The inverted results from using different mode sets are inconsistent. The discrepancy among the results, despite being small, is of the same order of magnitude as the intrinsic \( \gamma_1 \) difference in this region. Therefore, the discrepancy should not be ignored.
- In the region \( 0.6R < r < 0.99R \) (\( R \) being the solar radius), the mode set 1200d12 produces the best inversion results. Adding higher degree modes enlarges the discrepancy between the inverted curve and the expected curve.
- However, in the region \( r > 0.99R \), the inverted curve of mode set 1500d12 becomes the best one.
- The discrepancy among different mode sets, despite being small, is of similar order of magnitude as the intrinsic \( \gamma_1 \) difference in this region. Therefore, the discrepancy should not be ignored.

2. Comparing Figure 1 and 2, it is clear that the results from using mode selection (A) (all the modes) is better than from using mode selection (B) (only the modes having a counterpart in the observed data). The major difference between the two mode selections is that the low-degree low-order modes are missing in mode selection (B).

3. Figure 3 is to examine the importance of low-degree, high-order modes. In the testing mode set (the result is indicated as circles), the modes \( n > 10 \) for \( l < 80 \) were removed. The figure implies that the absence of these modes only slightly degrades the inversion in the region \( 0.7R - 0.8R \).

4. The two observations above suggest that the low-order modes are more important than the high-order modes for an accurate inversion.

5. Figure 4 is another example in which the inverted results from different mode sets are inconsistent. A careful examination of the three figures, 1, 2 and 4, reveals the following:

The intrinsic \( \gamma_1 \) differences of the three pairs of model, (CNONe - NONe), (Model S - CNONe) and (CNONe - X7419Z15), are all smaller than 0.001. However, the differences of the last two pairs, which show the inconsistency, contain narrow features near the surface.
This implies that our inversion method would be inaccurate if the difference to be inverted is concentrated in a narrow region near the surface.

6. Figure 5 shows the inverted result between real solar data and the model CNONe. The inverted results from using different mode sets are inconsistent. Following the discussion 5, we suspect that the intrinsic \( \gamma_1 \) difference between the Sun and the model CNONe is concentrated in a narrow region near the surface.

4. CONCLUSIONS

- Absence of low-degree, low-order modes can lead to a significant discrepancy between inverted and true values. Therefore, it is important to improve the observational instruments to include these modes in the observed data.
- The inconsistency of the inversion results among different mode sets is most prominent when the difference to be inverted is narrowly concentrated near the surface. A scheme to determine which inverted curve is most credible has been developed (Lin & Däppen, these proceedings).

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Lin C.-H. and Däppen W., in these proceedings.

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Figure 1. Inversion results using all the modes computed from a model. The top panel shows the result of (CNO\textsc{Ne} - NONE), and the lower two panels are the results of (Model S - CNO\textsc{Ne}). NONE and CNO\textsc{Ne} were respectively the reference models in the two sets of inversion. The solid line is the expected intrinsic $\gamma_1$ difference.

Figure 2. Inversion results using the modes contained in our set of observational data, that is, excluding low-degree low-order modes. The solid line is the expected intrinsic $\gamma_1$ difference.
**Figure 3.** The solid line is the expected intrinsic $\gamma_1$ difference. The squares are the inverted values using a mode set that does not contain low-degree high-order modes.

**Figure 4.** The inversion result of (CNONE - X7419Z15) with X7419Z15 (i.e., $X=0.7419$, $Z=0.015$) as the reference. The solid line is the expected intrinsic $\gamma_1$ difference.

**Figure 5.** The inversion result of (md198 - CNONE).
COMPOSITION OF THE SOLAR INTERIOR: INFORMATION FROM ISOPOE RATIOS


[1]University of Missouri, Nuclear Chemistry, Rolla, MO 65409 U.S.A.
tel: +1 537-341-4420 / fax: +1 537-341-6033 / e-mail: om@umr.edu
[2]Clarkson University, Physical Chemistry, 641 Spring Hill Estate, Eminence, KY 13699-5814 U.S.A.

ABSTRACT

Measurements are reviewed showing that the interior of the Sun, the inner planets, and ordinary meteorites consist mostly of the same elements: Iron, oxygen, nickel, silicon, magnesium, sulfur and calcium [1]. These results do not support the standard solar model.

INTRODUCTION

Reynolds found the decay product of extinct $^{129}$I and an unusual abundance pattern of the other xenon isotopes in meteorites in 1960 [2,3]. Since $^{129}$I exceeded that expected if the solar system formed from an interstellar cloud, Fowler et al. [4] suggested that D, Li, Be, B and extinct $^{129}$I and $^{107}$Pd might have been produced locally in the early solar system.

In 1965 the decay product of extinct $^{244}$Pu [5], a nuclide that could only be made by rapid neutron-capture in a supernova (SN), was found in meteorites.

In 1972 two major types of xenon were discovered in meteorites [6]: "Normal" xenon (Xe-1) at the lower left of Fig. 1 and "strange" xenon (Xe-2) from a SN at the upper right. Xe-1 includes fractionated forms along a dashed line in the lower left corner. In 1975 primordial helium was shown to have accompanied Xe-2, not Xe-1, at the birth of the solar system [7,8].

This link of primordial helium with Xe-2, shown in Fig. 2 for the Allende meteorite [7,8], was later confirmed in diverse types of meteorites [10,11]. The amount of Xe-2 in the early solar system was sufficient to shift the composition of bulk xenon in meteorites (Point AVCC in Fig. 1) away from Xe-1.

![Figure 1: Normal and "strange" xenon in meteorites.](image1)

![Figure 2: The link of primordial helium with xenon isotopes in the Allende meteorite.](image2)

These results from 1960 to 1975 suggested that local element synthesis produced more elements than Fowler et al. [4] had imagined: Debris from a single supernova (SN) may have formed the entire solar system [7,8]. The Sun formed on the collapsed SN core. Cores of inner planets formed out of iron-rich material in the central region. The outer planets formed out of light weight elements from the outer SN layers. Fig. 3 outlines this scenario [7,8].

Measurements after 1975 confirmed that Xe-1 is linked with iron and that Xe-1 is dominant in the inner solar system. They showed that Xe-2 is linked with light elements and with gaseous planets in the outer solar system. These measurements also yielded affirmative answers to the following questions:

A. Is a supernova the only viable source for Xe-2?
B. Does radioactive dating indicate a supernova at the birth of the solar system?
C. Are elements in the outer planets different from those in the Sun and the inner planets?
D. Is the interior of the Sun iron-rich, like ordinary meteorites and the inner planets?
E. Can luminosity and solar neutrinos be explained if the Sun formed on a collapsed supernova core?
Figure 3: Birth of the solar system from a supernova

POST-1975 RESULTS

A. Some proponents of the standard model continued to attribute Xe-2 to super-heavy element fission until the early 1980s [12], but there is now agreement that Xe-2 contains r-products from a supernova rather than fission products. Relic interstellar grains that formed near a supernova did not carry Xe-2 and other elements with excess r-, p- and/or s-products into the early solar system [13]. This would not explain the link of Xe-2 with primordial helium. Further, there is no convincing evidence of interstellar grains that are older than the host meteorites or were irradiated with cosmic rays before being embedded in them. A supernova is now accepted as the only viable source for Xe-2.

B. Kuroda and Myers [14,15] combined $^{244}\text{Pu}$-$^{136}\text{Xe}$ and U, Th-Pb dating methods to show the presence of a supernova explosion about 5 Gy ago, at the birth of the solar system. This is shown in Fig. 4. Other laboratories found evidence of a supernova in isotopic anomalies and short-lived nuclides trapped in meteorites. Isotopes of molybdenum remained unmixed, even in massive iron meteorites [16]. Chemical separations occurred before many short-lived r-products had decayed away, within 10,000 sec of the supernova explosion [17,18].

Figure 4: Five Gy ago a supernova produced $^{244}\text{Pu}$.

C. Measurements revealed Xe-1 trapped in troilite (FeS) inclusions of diverse meteorites, including the Allende meteorite [19-22]. Xe-1 is also in Earth and Mars, planets rich in Fe and S. Xe-2 was found to accompany primordial helium in different types of meteorites, including carbonaceous chondrites [10,11]. The Galileo mission also found Xe-2 in the helium-rich atmosphere of Jupiter [23,24]. In the solar wind, in Xe-1, in Xe-2, and in Jupiter, $(^{136}\text{Xe}/^{134}\text{Xe}) = 0.80, 0.84, 1.04,$ and $1.04 \pm 0.06$, respectively. Experimental data from the Galileo probe into Jupiter can be viewed on the web at www.uml.edu/om/abstracts2001/windleranlalysis.pdf.

D. Measurements showed the presence of Xe-1 in the Sun, but light (L) xenon isotopes in the solar wind are enriched relative to the heavy (H) ones by about 3.5% per amu [25]. As shown in Fig. 5, light isotopes of He, Ne, Ar, Kr and Xe in the solar wind all follow a common mass-dependent fractionation power law, where the fractionation factor, $f$, is

$$\log (f) = 4.56 \log (H/L)$$

Application of Eq. (1) to elemental abundance in the photosphere [25] shows that the seven most abundant elements in the interior of the Sun are the same ones that comprise 99% of meteorites: Iron, nickel, oxygen, silicon, sulfur, magnesium and calcium [1].
Table 1 shows that light isotopes are less abundant in solar flares, as if flares by-pass 3.4 stages of mass fractionation [26]. Heavy elements are enriched systematically, by several orders of magnitude, in material ejected from the interior of the Sun by an impulsive solar flare [27].

Table 1. He, Ne, Mg, and Ar in solar wind and flares

<table>
<thead>
<tr>
<th>Isotopic Ratio</th>
<th>Solar Wind</th>
<th>Solar Flares</th>
<th>SW/SF</th>
<th>Expected**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$He/$^4$He</td>
<td>0.00041</td>
<td>0.00026</td>
<td>1.58</td>
<td>1.63</td>
</tr>
<tr>
<td>$^{20}$Ne/$^{22}$Ne</td>
<td>13.6</td>
<td>11.6</td>
<td>1.17</td>
<td>1.18</td>
</tr>
<tr>
<td>$^{24}$Mg/$^{26}$Mg</td>
<td>7.0</td>
<td>6.0</td>
<td>1.17</td>
<td>1.15</td>
</tr>
<tr>
<td>$^{36}$Ar/$^{38}$Ar</td>
<td>5.3</td>
<td>4.8</td>
<td>1.10</td>
<td>1.10</td>
</tr>
</tbody>
</table>

**If solar flares by-pass 3.4 stages of fractionation

The probability is almost zero (P < 2 x 10^{-33}) that Eq. (1) by chance selects from the solar atmosphere seven trace elements that a) all have even atomic numbers, b) are made deep in supernovae, and c) are the same elements that comprise 99% of ordinary meteorites [1].

E. Systematic properties [28] of 2,850 known nuclides (Fig. 6) reveal an inherent instability in assemblages of neutrons relative to neutron emission. This may explain luminosity [29-31] of an iron-rich Sun. Neutrons emitted from the collapsed SN core may initiate a chain of reactions that generate luminosity, solar neutrinos, and an outpouring in the solar wind of 3 x 10^{15} \ ^{1}H^+ per year.

- Escape of neutrons from the collapsed solar core $\langle q n \rangle \rightarrow q n + 10^{-22} \text{MeV}$
- Neutron decay or capture by other nuclides $^{0}_{\text{n}} \rightarrow ^{1}_{\text{H}^+} + e^- + \text{anti-v} + 1 \text{MeV}$

\[ 4 \ ^{1}H^+ + 2 e \rightarrow ^{2}He^{*} + 2 \nu + 27 \text{MeV} \]

- Escape of excess $H^+$ in the solar wind
  $3 \times 10^{15} \ ^{1}H^+$year depart in the solar wind

The hydrogen-filled universe may be the result of this outflow of protons from the Sun and other stars. A summary is on the web <http://www.baloliron.com>

CONCLUSIONS AND PROPOSED TESTS

The link of Xe-1 with iron extends to the Sun: Iron is its most abundant element. Fusion in the parent star (Fig. 3) likely depleted light elements from the material that formed the Sun and the inner planets.

The following measurements are proposed to test our conclusion of an iron-rich Sun:
1. Measure anti-neutrinos (3 x 10^{38} s^{-1}, E < 0.782 MeV) from neutron decay at the solar core. Low E targets for inverse \(\beta\)-decay are the Homestake Mine $^{35}$Cl $\rightarrow ^{35}$S reaction [32], the $^{14}$N $\rightarrow ^{14}$C or $^{3}$He $\rightarrow ^{3}$H reactions.
2. Measure neutrinos from reactions that increased the $^{15}$N/$^{14}$N ratio [33] and produced excess $^{6}$Li and $^{10}$Be in the outer layers of the Sun [34,35].
4. Measure gravity anomalies, magnetic fields, the quadrupole moment, or circular polarized light [37] from a compact object (~10 km) in the Sun.
5. Measure other properties that constrain mass segregation in the Sun and other stars [38-40].
6. Look for excess heavy elements in the fast-moving solar wind, e.g., from the Sun's poles.
7. Use a narrowly focused laser beam to penetrate the Sun's hydrogen-rich veneer.

ACKNOWLEDGEMENT

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See also <http://www.ballofiron.com>
USING THE BISON SIGNAL TO PROBE THE PHOTOSPHERE.

B. McCarty, W. J. Chaplin, and Y. Elsworth

School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, U.K.

ABSTRACT

Red and blue-wing velocity residuals have been generated from each side of the 770-nm potassium line profile as observed by BISON. Analysis of these single-sided residuals allows an estimate of the velocity scale height, $H$, to be made. An initial investigation using data from one BISON station gives a value of $H \sim 530 \pm 45$ km.

Key words: Sun: activity – Sun: oscillations.

1. BACKGROUND

Light from the Sun is received by resonant-scattering spectrometers at each of the Birmingham Solar Oscillations Network (BISON) observatories (Chaplin et al. 1997). These instruments are sensitive to the potassium absorption line at 770 nm in the solar atmosphere. During an oscillation the measured wavelength of the solar potassium line will move relative to a fixed laboratory reference. Each spectrometer contains a cell of potassium vapour which is subjected to a large longitudinal magnetic field. This applied field splits the potassium line into two components via the anomalous Zeeman effect. The varying solar absorption line alters the measured intensity at each passband and this is used to determine the Doppler velocity of the oscillation. In the standard BISON analysis the following ratio is formed:

$$S_R(t) = \frac{I_B - I_R}{I_B + I_R}, \quad (1)$$

where $I_B$ and $I_R$ are the intensities measured at the blue and red wings of the spectrometer respectively. A calibration technique (Elsworth et al. 1995) is used to translate this measured ratio into a velocity.

2. NEW ANALYSIS

In order to investigate height dependent characteristics of the solar oscillations it is desirable to have independent sets of velocity residuals from each wing of the solar potassium line. The resulting blue and red-wing time series, $O_B$ and $O_R$, (similar to those obtained by GOLF in its single-wing mode of operation, e.g., Pallé et al. 1999) are obtained via the relations:

$$O_B = \frac{I_B}{I_T}, \quad (2)$$

$$O_R = \frac{I_R}{I_T}, \quad (3)$$

where, again, $I_B$ and $I_R$ are the observed intensities from the blue and red passbands respectively and $I_T$ is a normalisation factor that comes from a transmission monitor device in each of the spectrometers. The transmission monitor supplies a throughput range of wavelengths much larger than the passband being used. Both $O_B$ and $O_R$ are subsequently translated into velocity residuals using the same basic technique applied to the normal ratio data. Figure 1 shows the velocity oscillations as measured at the red and blue passbands. Note that the signals from each wing are clearly out of phase.

3. VELOCITY TO HEIGHT MAPPING

The BISON velocity signal can be mapped to heights in the photosphere as follows. Photons are increasingly absorbed by material in the photosphere as they travel up through the solar atmosphere. The characteristic absorption-line profile reflects the amount of photon absorption with increasing height. The deeper part of the profile represents regions higher in the atmosphere whereas the outer wings represent those lower down (Figure 2). As the observed line profile moves back and forth, different positions on each wing of the line profile, and therefore different depths in the atmosphere, are sampled by the instrument. Consequently a range of heights in the solar atmosphere can be ‘probed’ using this technique. Figure 3 shows how a point in the solar atmosphere is mapped to the velocity signal recorded by one of the BISON spectrometers (Underhill 1998). In this case the mapping given is as seen by each of the two passbands in the instruments. Notice that the blue-wing signal is predominantly mapped to lower regions of the atmosphere, whereas the red wing maps to those higher up.
OBTAINING THE SCALE HEIGHT OF THE SOLAR ATMOSPHERE

Using the above technique it is possible to infer experimentally the velocity scale height of the solar atmosphere. If no radiative losses are assumed, the amplitude of solar oscillations should have a height dependence of the form $V = V_0 \cdot \exp(z/H)$, where $H$ is the scale height. If one takes the natural logarithm of both sides we have:

$$\log_e(V) = \log_e(V_0) + \frac{1}{H} \cdot z.$$  \hspace{1cm} (4)

By plotting the natural logarithm of the RMS amplitude of the oscillations as a function of height (Figure 4), one can obtain a value for $H$ from the reciprocal of the fitted gradient of a linear regression of the data. In this investigation over 8000 one-hour segments of red-wing data, recorded at the Las Campanas station in 1994, were used. Over the course of the year the measured amplitudes vary as a function of height in the atmosphere as the BiSON instrument samples different points on the observed solar absorption line. A linear fit to the resulting RMS amplitudes give a value for $H \sim 530 \pm 45$ km. The analysis given here does not take into account the effects of Doppler imaging (Brookes, Isaak & van der Raay 1978); neglecting this in the analysis leads to an overestimation of $H$ by a factor of roughly $\sim 15$ per cent. This suggests the ‘unbiased’ value may be nearer 460 km.

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Figure 4. Amplitude of solar oscillations as a function of height (Las Campanas 1994).
THE PROBLEM OF EXTERNAL BOUNDARY CONDITION IN ACOUSTIC MODE CALCULATION

P. A. P. Nghiem
CEA/DSM/DAPNIA/SAp, CE de Saclay, 91191 Gif sur Yvette Cedex, France
tel: +33 (0)1 69 08 92 64 / fax: +33 (0)1 69 08 65 77
e-mail: papnghiem@cea.fr

ABSTRACT

As for any other problem in physics, the calculation of solar acoustic modes implies the resolution of differential equations under certain boundary conditions. In the frame of linear adiabatic oscillations, the set of equations is very well known and the Isothermal Atmosphere Approximation (IsAtAp) is generally used for external boundary conditions. The resulting eigenfrequencies obtained by numerical codes are in very good agreement with observational ones in the range 800-2500 μHz. But at larger frequencies, the discrepancies become more and more important to reach tens or hundreds of microHz, respectively for low or high-degree modes. Those disagreements are commonly supposed to come from problems in the near surface layers, but numerous attempts to introduce alternative physics at the surface do not succeed to really explain them. In this work, I first point out the drawbacks of the IsAtAp or any similar type of approximation using a specific atmosphere. Then, I propose to use a new boundary condition based on the relative importance of wavelength vs. pressure-height. This method allows to better characterize the near surface layers while it cancels the disagreement at large frequencies, even for high-degree modes.

INTRODUCTION

Often in Physics, one has to derive differential equations and then to solve them. The problem is that in addition the physicist has to decide which boundary conditions to use, conditions that are necessarily not contained in the initial equations. Of course, that choice should be the most relevant possible, but it remains inevitably a certain part of arbitratition. A rigorous justification can be available only with an other study, which may implies the solving of other differential equations and the choice of other boundary conditions...

In this paper, we discuss the choice of boundary conditions in the calculation of stellar acoustic modes. We recall the status of this problem, in relation with the discrepancy between theoretical and observational frequencies. A critical analysis is then attempted. Finally a new choice of boundary condition is suggested, which may better take into account the input stellar structure, and consequently may help to deduce the real surface conditions from mode observations.

The differential equations governing the propagation of acoustic waves are well known, in the case of linear-adiabatic oscillations under spherical symmetry (see e.g. Christensen-Dalsgaard 1998). But to calculate the stellar eigenmodes, it is necessary to determine the cavity limit where the propagation is trapped, that is the boundary conditions. Due to the spherical geometry, the wave naturally turns back to the surface at a certain depth, where there is in principle no discontinuity and no real problem of boundary condition. The problem must rather be examined at the surface, where reflections with brutal phase change occur.

Several choices of external boundary conditions was tried until now. The ones using an outer atmosphere finishing abruptly are found to be not realistic, and was discarded. Until now, the most commonly used is the so-called Isothermal Atmosphere Approximation (IsAtAp) (Unno et al., 1989) which gives the best results. Let us first examine its foundation. At the surface, the terms in 1/r together with the gravity can be legitimately neglected, and the above noticed differential equations can be reduced to give:

\[
\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \xi_r - \frac{1}{\Gamma_1 p_0} \frac{\partial (\Gamma_1 p_0)}{\partial r} \frac{\partial \xi_r}{\partial r} + \left(\frac{1}{\Gamma_1 H_p H_P} - \frac{1}{\Gamma_1 p_0} \frac{\partial^2}{\partial r^2}\right) \xi_r = 0
\]  

(1)

where \( \xi_r \) stands for the radial displacement; \( r \) the radius, \( t \) the time; \( H_P \) and \( H_P \) the pressure and density scale height; \( c_0 \), \( p_0 \), and \( \Gamma_1 \) the equilibrium sound speed, pressure and adiabatic exponent. By searching a local solution of the form

\[
\xi_r = A \exp(i\omega t - \alpha r)
\]

with \( k \) the wave number, \( \omega \) the angular frequency, one obtains

\[
k = \frac{1}{2} \left[ \frac{1}{\Gamma_1 p_0} \frac{\partial (\Gamma_1 p_0)}{\partial r} \right]^{\frac{1}{2}} - \frac{4}{\Gamma_1} \frac{\omega^2}{c_0^2} + \frac{4}{\Gamma_1 H_p H_P}
\]

(3)

The IsAtAp assumes that an isothermal atmosphere can be fitted at a location \( \gamma \) to the initial atmosphere so that:

\[
\Gamma_1 = \text{const}, \quad H_p = H_p = H = \text{const}, \quad \frac{\partial^2}{\partial r^2} \frac{p_0}{H_p^2} = \frac{p_0}{H_p^2}
\]

(4)

Thus (3) becomes:

\[
\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \xi_r - \frac{1}{\Gamma_1 p_0} \frac{\partial (\Gamma_1 p_0)}{\partial r} \frac{\partial \xi_r}{\partial r} + \frac{4}{\Gamma_1 p_0} \frac{\partial^2}{\partial r^2} \xi_r = 0
\]
\[
k_{\text{atoh}} = \frac{i}{2H} \left[ -1 + \sqrt{1 - 4 \frac{\omega^2 H^2}{c_0^2}} \right]
\]

which is the expression used in Gough and Thompson (1990), or Christensen-Dalsgaard (1998). It is generally admitted that the term under the square root must be positive, if not there would exist a propagative component into the atmosphere and that means that the wave is not trapped. The boundary condition can be deduced, along with a cut-off frequency:

\[
\omega \leq \omega_c = c / 2H.
\]

In spite of that reasoning, that adoption of boundary condition contains also naturally a dose of arbitation: the first part of the above \( k_r \) is anyway non-propagative, whatever its second part is or is not propagative, the formation of a standing wave is not possible.

The problem is more disturbing when observing that this boundary condition depends on the product \( \Gamma_I H_P \) (or \( \Gamma_P \)) everywhere, except at \( r_f \) where it depends on the individual values of \( \Gamma_I \) and \( H_P \) (or \( p \)) separately. Let us take for example the case of the Sun where \( r_f \) is far outside the external limit of all the eigenmodes. A change of \( \Gamma_I \) and \( H_P \) while keeping their product the same, at one unique point well outside all the resonant cavities, will change all the eigenfrequencies. But the same change everywhere else, including inside the cavities, will let the eigenfrequencies unchanged. This behavior seems to be not physically satisfying.

This notion of cavity limit is furthermore blurred, or even erased, when the ISAtAp is applied in numerical codes with the so-called shooting technique. The final solution is obtained by fitting at \( r_f \) the wave coming from the interior to the one coming from infinity. So a model change very far outside, even infinitely far, can determine the eigenfrequencies. In fact, stellar eigenmodes are equivalent to eigenmodes of musical wind instruments, which have an outer free end not very well defined like a stellar atmosphere. Now, we know that the sound amplitude of a trumpet can be affected by other musicians, or the acoustic of the concert hall, but never its musical notes themselves (i.e. the sound frequency). The question worth be debated in the seismology community about the existence or not of a resonant cavity. If there is really no cavity, how can be understood the existence of eigenmodes? If resonant cavities exist, where are their limits, or at least their "equivalent" limits? Once it is determined, how can be explained that outside, the evanescent part of the wave that does not come back to the cavity, can report the environment parameters there to the cavity for changing its eigenmodes?

About the concrete results obtained until now with calculation codes for the Sun, the numerical eigenfrequencies agree within the \( \mu \)Hz with observational ones in the range 800-2500 \( \mu \)Hz. But the disagreement increases beyond, to reach 20 \( \mu \)Hz at 5000 \( \mu \)Hz for low degrees \( \ell = 0, 10 \), and hundreds of \( \mu \)Hz for high degrees \( \ell > 200 \). And the cut-off frequency given by (6) is 5038 \( \mu \)Hz, while the observational one is larger than 5500 \( \mu \)Hz. These discrepancies are suspected to come from a bad description of the very near surface physics. Many attempts have been made to reduce or explain them, invoking the convection, the turbulent pressure, the non-adiabaticity, etc... (see e.g. Rosenthal et al. 1995, 1999; Kosovichev 1995). In spite of that effort, no satisfying solution was found, at most the discrepancy at large frequencies can be reduced at the cost of deterioring the agreement at low frequencies. In this last case, it can also be remarked that the change of the Solar model is situated in the very external layers corresponding to the cavities of the modes from 2500 to 5000 \( \mu \)Hz; the question is how it can perturb the lower frequencies for which the external limits are more inside.

The solar cycle is also known to be due to magnetic change (among others) at very near surface. Many studies have been devoted to extract the structure change related to the cycle (see references in Ngheim et al. 2002). The results on the properties of the surface magnetic field are still in discussion. Two disturbing points often arise in those kind of studies. The first one is that numerical simulations are made by invoking also magnetic changes in the evanescent part of the waves, even in the chromosphere. There is still to find the physical mechanism through which changes in the environment 1000 km away from a cavity can perturb its eigenfrequencies. The second point on the contrary deals with the analytical expression integrating until \( R \), the photometric radius, the effects of the structure on the frequency. The wave being acoustical ignores totally what is a photometric radius. Although the external limits of the cavities lay between \( -0.9 R \) and \( -1.0005R \), a change of these limits of only \( 10^{-4} \) leads easily to a change of tens of \( \mu \)Hz. So the integration until \( R \) for all the modes remains to be justified.

Clearly, the existence of resonant cavities and their limits are somewhat questionable with the ISAtAp. More generally, it can be said that such a boundary condition using an extra atmosphere, either less or more sophisticated than the isothermal atmosphere, will not help to understand any difference with observations. Indeed, a special atmosphere must be fitted to the parameters of the initial atmosphere coming from e.g. a star evolution code. When, as presently, there are disagreements between theoretical eigenfrequencies and observational ones, too many reasons can be invoked:

- the special atmosphere is not a good approximation for the initial atmosphere
- the initial atmosphere does not have the good parameters at the fitting point
- another fitting point is to be chosen
the initial atmosphere has not good parameters in a larger range. It is difficult to decide to change either the special or the initial atmosphere, or the two, and at what location. The use of only one structure seems to be preferable.

ANOTHER BOUNDARY CONDITION

Coming back now to a more physical understanding of acoustic waves, another boundary condition can be derived. Let us use the concept of local wave developed in Nghiem (2002), where notions of trapped waves and cavities are clearly claimed. Starting from the beginning, a wave is a propagation of an oscillation around an equilibrium state. If there is no more equilibrium environment, no oscillation is possible, thus no propagation, and the wave is reflected. In the case of local acoustic wave, the equilibrium state is the local pressure homogeneity, i.e. when the wavelength is small compared to the pressure spatial change:

\[ \frac{2\pi}{k c_0} < a_E H_p \quad (k) \]

(7)

where \( \lambda \) is the wavelength, \( k \) the wavenumber, \( H_p \) the pressure scale height projected on the propagation direction, and \( a_E \) a constant to be determined. When that condition is no more fulfilled, there is reflection. Such successive reflections determine the resonant cavity where trapped waves take place. Taking into account that at the surface \( k = \alpha c_0 \), and neglecting the surface curvature, (7) becomes:

\[ \frac{1}{k_r} \frac{\omega^2}{c_0^2} > \frac{2\pi}{a_E H_p} \quad . \]

(8)

As there is only one coefficient to determine, we have only to use one condition: the observational cut-off frequency of 5600 \( \mu \text{Hz} \) for the Sun (García et al. 1998). In this frequency range, the order can be neglected, and (8) can be simplified to

\[ \frac{\omega}{c_0} > \frac{2\pi}{a_E H_p} \quad . \]

(9)

In order that the last trapped mode is at 5600 \( \mu \text{Hz} \), Fig. 1 indicates that one must have

\[ a_E = 11.3 \quad . \]

(10)

This coefficient can be considered as the first of several coefficients involved in a Taylor expansion in \( \lambda \) and/or \( H_p \). Those coefficients could be determined by experiments on Earth, or spatial observations, or theoretical methods. But they are not specific to a star. The external turning points are obtained by equalizing the two members of (8). They are different for each frequency, and depend on the surface profiles of \( c_0 \) and \( H_p \). (For large degrees, \( l > 15 \), they also depend on the degrees). On the contrary, the phase at those limits is always the same. Indeed, the wave is reflected because as regard to its wavelength, there is no more matter, and in the absence of any constraint, the phase must be so that there is an antinode, like the free end of an oscillation rope.

Generally, existing semi-analytical methods lead to an opposite result: the external limit is always the same, the photometric radius \( R \), while the phases there depend on the frequencies. It is possible that this way of doing implies a variable change in wave integration, which makes the formalism much more complicated. The study of structure perturbation on the modes could also be importantly biased when believing that every mode is stopped at \( R \).

The relations (9) and (6) look very similar, but they are conceptually very different. The first one comes from a fit with another atmosphere and determines only the cut-off frequency, while the second one uses directly the initial structure and is employed to determine the external limit for every frequency \( \omega \).

RESULTS

The proposed external boundary condition along with the local wave formalism allow to derive a very simple set of equations (Nghiem 2002). Its resolution for the solar model called Btz (Brun, Turck-Chièze & Zahn 1999) and the modes with degrees \( l = 0 \) to 10, gives the external turning points \( r_2 \) of Fig 2, and the eigenfrequencies that are compared in Fig. 3 to the numerical results (code ADIPLS, Christensen-Dalsgaard 1997). The differences \( \Delta \nu \) between semi-analytical and numerical frequencies depend strongly on the frequency, but very few on \( l \). As this behaviour is specific to \( r_2 \), excluding all other parameters (internal turning points, phase changes), it is straightforward to conclude that the discrepancy comes from the one on \( r_2 \). Let us precise that the numerical code employs the IsAtAp and thus uses artificial \( H_p \) and \( \Gamma_\nu \) profiles different from the initial ones.

From \( \Delta \nu \), it is in fact very easy to inverse the \( H_p \) profile of the IsAtAp (the \( \Gamma_\nu \) profile is then deduced by maintaining unchanged the product \( \Gamma_\nu H_p \)), because there is a one-to-one correspondence between \( \nu \) and \( r_2 \) (Fig. 2), and between \( r_2 \) and \( H_p \) (Eq. (8)). That inverse \( H_p \) profile is given in Fig. 4 where it can be seen that the IsAtAp mainly consists in smoothing the initial \( H_p \) profile. These two profiles will provide rigorously the same eigenfrequencies from the numerical code. With the present formalism, the second one will give frequencies whose difference with numerical ones are contained within \( \pm 2.5 \mu \text{Hz} \) in the range 900-5000 \( \mu \text{Hz} \) (Fig 5).

To validate the above \( H_p \) inversion, we need to calculate by an other way the \( H_p \) used by the IsAtAp. This last method should search to fit at \( r_2 \), the evanescent wave coming from \( r_2 \), and the wave solution in an isothermal
atmosphere coming from a “surface” point \( r_s \). It is equivalent to search for \( r_s \) so that:

\[
\int kdr = i k_{\text{iso}}(r_f - r_s) + \text{Const}
\]

where \( k \) and \( k_{\text{iso}} \) are given by (3) and (5). The equivalent \( H_{\text{iso}} \) seen by this approximation can be induced with

\[
H_{\text{iso}}(r) = \frac{\Gamma_1 H_P(r)}{\Gamma_1(r_f)}, \text{ or with } \rho_{\text{iso}}(r) = \frac{\Gamma_1 \rho(r)}{\Gamma_1(r_f)}
\]

where \( r \) is the mean position between \( r_s \) and \( r_f \). In Fig. 6 with \( \text{Const} = 0.2 \), it can be seen that the equivalent \( H_{\text{iso}} \) derived from (12) (crosses and circles) agrees in two different regions of radius with the previously inversed \( H_P \) (continuous line).

The boundary condition together with the inversion method proposed here being validated via the numerical calculation, we are now ready to apply them to solar observations. It is done in Ngheim (2002) to deduce the surface profiles of \( H_P \) and \( \Gamma_1 \), and in Ngheim et al. (2002) to estimate the magnetic field profile related to the solar cycle.

Finally, the resulting \( \Delta \nu \) between semi-analytical and observational results agrees within ±2.5 \( \mu \)Hz in the range 1000-5000 \( \mu \)Hz. If the numerical calculation adopts the same boundary condition and the same \( H_P \) than here, the agreement would be better than the \( \mu \)Hz in the whole frequency range.

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MAGNETIC FIELD STRENGTH IMPLIED IN THE EIGENFREQUENCY VARIATION RELATED TO THE SOLAR CYCLE

P. A. P. Nghiem¹, R. A. García¹, S. Turck-Chièze¹, S. J. Jiménez-Reyes²

¹CEA/DSM/DAPNIA/SAp, CE de Saclay, France
²Themis, Instituto de Astrofísica de Canarias, La Laguna, Tenerife, Spain

E-mail: papnghiem@cea.fr, rgarcia@cea.fr, cturck@cea.fr, sjimenez@themis.iac.es

ABSTRACT

During the solar cycle, the acoustic eigenfrequency shift is roughly independent of the degree, for modes of degrees less than 10. Following the approach of Nghiem (SOHO/GONG 2000 and this conference), that very typical behavior can be interpreted as a change in the mode cavity size. We consider here the case where it is mainly due to a pressure scale height variation, while the sound speed change can be neglected. Using the formalism developed and the LOWL data obtained from 1996 to 2000, the change in surface pressure with the solar cycle is deduced. Assuming that all this variation has a magnetic origin, we infer the related profile of magnetic field change with the cycle. From SOHO data, we also give an upper limit of the absolute magnetic field.

INTRODUCTION

Since the publication of Woodard and Noyes (1985), many helioseismic measurements from Earth and space convincingly showed the increase of acoustic eigenfrequencies with the magnetic activity. Many attempts have been performed to interpret that change during a solar cycle. See e.g. Goldreich et al. 1991, Wright and Thompson 1992, Kush 1998, Moreno-Insertis and Solanki 2000, ... and references therein. It seems that only one general consensus can be drawn: the mechanism at the origin of the frequency change is situated very close to the surface. Regarding the physical cause itself, mainly the magnetic field is invoked, but entropy, transport pressure, luminosity, asphericity, radius, etc. were also studied. No certitude is still available.

The problem is to relate a structure change to an eigenfrequency change. That implies the calculation of eigenmodes, which until now, has a crude approximation right at the surface itself, namely the isothermal atmosphere approximation: a specific atmosphere lying to infinity is automatically substituted to the atmosphere coming from a Sun model. That means that a change introduced in the initial model will not be seen really like it is when calculating the frequencies. Another consequence is the lack of accuracy when defining the acoustic cavity limits. Two opposite behaviors can be noted:

- In semi-analytical calculations, the effect of a structure change is integrated up to \( R \), the photometric radius, which has no special sense for acoustic waves.
- In numerical calculations, phenomena very far from \( R \), e.g. in the chromosphere, affect the eigenmodes.

In the present paper, we will try first to clarify the problem at the cavity limit, which helps to determine with a high precision the location where the waves should have been perturbed. Then a first attempt is made to infer a profile of magnetic field change due to the cycle.

THE ACOUSTIC CAVITY

The concepts used in the following are based on the approach of local waves developed in Nghiem (2002a). The external boundary conditions are discussed in more details in Nghiem (2002b). Basically, it is admitted that eigenfrequencies are the signatures of trapped waves in a cavity. As acoustic oscillations occur around an equilibrium state which is the local pressure homogeneity, they can no more exist where such an equilibrium ends. That location marks the cavity limit because the acoustic wave is reflected there. So this latter can only propagate in an homogeneous environment, or at least locally homogeneous regarding its wavelength, that is when:

\[
2nk_c \frac{c_0^2}{\omega^2} < 11.3H_p
\]

(1)

where \( \omega \) is the pulsation, \( c_0 \) the sound speed, \( H_p \) the pressure scale height, and \( k_c \) the radial wavenumber. At the surface where the gravitation effect is negligible, \( k_c \) is given by

\[
k_c = \sqrt{\frac{\omega^2}{c_0^2} \cdot \frac{l(l+1)}{r^2}}
\]

(2)

with \( l \) the degree, \( r \) the distance from the solar centre.

The cavity limits are determined by the points \( r_2 \) where (1) becomes an equality. On Fig. 1 that external cavity profile for the modes \( l = 0 \) to 10 is given, using the Sun structure called Btz (Brun, Turck-Chièze & Zahn 1999).

Several remarks are worth to be noted:
- For those modes, the second part of (2) becomes negligible at the surface. The external cavity limit is thus only given by the profiles of \( H_p \) and \( c_0 \). When those latter are known, \( r_2 \) mainly varies with the
frequency, and is almost independent of the degree (for
$l > 15$, $r_2$ depends more and more on $l$).
- There is practically a one-to-one correspondence
between the frequency and $r_2$. To the range $[-500,$
$-5000]$ μHz, corresponds the range $[-0.9,$ $-1.0005]$ $R$.
A change of $10^4$ in $r_2$ will lead to a change of tens
or hundreds μHz in the frequency. In those conditions,
an integration right up to $R$ for all the modes remains to be
justified.
- Following the trapped wave concept, a perturbation
anywhere inside a resonant cavity, its limits included,
will be seen by the entire cavity, due to the travelling
forth and back of the wave, and will induce an
eigenfrequency change. On the contrary, a perturbation
strictly outside the cavity limits cannot impact on
the cavity, thus cannot change its eigenmodes, because
the evanescent wave does not go back to the cavity.

The external cavity limits being known precisely for
each frequency, let us look now at the effect of the solar
cycle on the eigenfrequencies.

THE SOLAR CYCLE EFFECT

We use the LOWL frequency measurements (S. J.
Jiménez-Reyes 2001) from the year 1996-1997
(hereafter indice 1,) to the year 1999-2000 (hereafter
indice 2). Fig. 2 gives the frequency change for the
modes with $l = 1$ to 10. It can be noticed that those
curves vary with the frequency, but are roughly
independent of $l$. For larger $l$, the variation progressively
increases with $l$. That is exactly the behavior of the
external turning points $r_2$, to the exclusion of all other
wave parameters like the internal turning point or the
phase advance. The conclusion is straightforward: the
effect of the solar cycle is to shift the external cavity
limits of the modes.

That variation seems to affect the modes of frequencies
larger than roughly 2000 μHz. When looking at the
correspondence frequency- $r_2$ in that range:

\[
\begin{array}{cc}
\nu (\mu Hz) & r_2 / R \\
1905 & 0.99594 \\
1944 & 0.99616 \\
1983 & 0.99636 \\
2003 & 0.99646 \\
2044 & 0.99665 \\
2084 & 0.99682
\end{array}
\]

it can be estimated that the effect of the solar cycle is
situated in the region outside of $r_2 / R = 0.9965$, which
extends from 2450 km below the photometric radius.

The one-to-one correspondence between the frequency
and $r_2$, can be employed to deduce the $r_2$ displacement
by the cycle. For that, we must start from a Sun model

\[\text{Fig. 1. External turning points } r_2 \text{ for the modes } l = 0 \text{ to } 10.\]

\[\text{Fig. 2. Frequency change, year 99-00 minus year 96-97. Points of same } l \text{ are joined by continuous then progressively discontinuous lines, for } l = 1 \text{ to } 10.\]

\[\text{Fig. 3. Frequency discrepancy model-observation (year 96-97).}\]

\[\text{Fig. 4. Mean position of the envelopes of the frequency curves, year 99-00 minus 96-97, before(→→) and after(→0) cancellation.}\]
96-97. The semi-analytical approach in Nghiem (2002a) allows to do it. From the initial solar model Bt, we determine the surface $H_p$ profile so that the discrepancy between theoretical and observed frequencies is limited in $\pm 1.9$ $\mu$Hz, without any general shift (Fig.3). Then, by cancelling the global shift (independent of $l$) in frequencies observed for the year 99-00, the variation in $r_2$ is deduced. On Fig.4, the mean position of the envelopes of the frequency curves before and after cancellation are displayed.

The small oscillations in these curves come from a lack of precision. It is also noticeable that when only working with the degrees $l = 1$ to 10, the frequency shifts of the degrees until 99 are also cancelled. This demonstrates that the effect of the cycle is clearly a displacement of the external cavity limit $r_2$.

It remains to find out the mechanism at the origin of that $r_2$ displacement. Eq. (1) indicates that $r_2$ depends on the two profiles of $c_0$ and $H_p$ at the surface. A change in $c_0$ at one point will change $r_2$ for the given mode, but also for all the modes that pass through that point, because their propagation speed has changed. On the contrary, a change in $H_p$ will only change the stopping condition at a given location. In other words, there is a one-to-one correspondence between $r_2$ and $H_p$. In Nghiem (2002b), the inversion method changing $H_p$ while keeping the product $\Gamma_2 H_p$ constant (to keep $c_0$ unchanged), is validated for the case of the isothermal atmosphere. If the same assumption can be applied here for the solar cycle, the inferred $H_p$ profile change can be obtained (Fig.5), then the pressure profile change can be deduced (Fig.6).

To a first approximation, a magnetic field tends to change the solar environment in this sense. An horizontal field $B_h$ introduces a magnetic pressure which decreases the gas pressure by the same amount, given by (see e.g. Goldreich et al. 1991):

$$p_2 - p_1 = -p_{B_h} = \frac{B_h^2}{8\pi}.$$  \hspace{1cm} (3)

That will decrease the sound speed, but the Alfvén speed due to the magnetic field partly compensates it (Kuhn 1998):

$$c_A^2 - c_s^2 = \frac{B_h^2}{8\pi} \left(\frac{2 - \Gamma_2}{\rho_0}\right).$$  \hspace{1cm} (4)

If the variation of the sound speed can be neglected, and if all the pressure change of Fig.6 can be attributed to the magnetic field as described by Eq. (3), the change in the magnetic field profile with the cycle can be deduced (Fig.7). That variation reaches the maximum of $+800$ G at $r/R = 0.9965$, while very near the photometric radius the maximum change is $-80$ G.

**PRELIMINARY CONCLUSIONS**

The local wave formalism describes the eigenfrequencies in terms of trapped waves in a cavity with clearly defined external limits. That allows to study near surface phenomena: the effect of the solar cycle.

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**Fig.5.** Variation of the pressure scale height due to the cycle.

**Fig.6.** Variation of the pressure due to the cycle.

**Fig.7.** Variation of the magnetic field due to the cycle.

**Fig.8.** Profile of the mean magnetic field at the surface.
can be precisely located, and can be identified as a shift of the external limits of the mode cavities.
Several questions are still to be studied concerning the physical mechanism at the origin of that displacement. Its effect on the surface profiles of \( H_p \) and \( c_0 \) are to be determined. If \( H_p \) is changed while the sound speed is kept constant, it will be straightforward to infer the variations with the cycle of the \( H_p \) and \( T_1 \) profiles, and then of the searched mechanism. If the sound speed is also changed, the problem becomes more complicated but its resolution may not be impossible.
The magnetic field profile given here should be considered only as a first attempt toward the solution. On the one hand, the change of \( c_0 \) is weak compared to the change of \( H_p \), but is not really zero, so it should be taken into account. On the other hand, the calculation here of the variation of \( B_0 \) supposes that the mean value of \( B_0 \) is negligible. This remains to be confirmed.
In fact, the same procedure than above can be applied to determine the mean magnetic field for the Sun. The model-Sun discrepancy in frequency is also independent of \( l \), for \( l = 0 \) to 10. With the same assumptions than above, the magnetic field (not taken into account by the Sun model) can be deduced. Its profile is given on Fig.8 when comparing theoretical frequencies (Ngheim 2002a) to observational ones (SOHO spacecraft; MDI data from Rhodes et al. 1997; GOLF data from Bertello et al. 2000, and from García et al. 2001). The mean magnetic field would increase from zero to a maximum of \( 3.4 \times 10^5 \) G, when going from \( r/R = 1.0008 \) down to 0.97. The rather high values of this field should be e.g. interpreted as maximum values, and other physical mechanisms should perhaps be invoked.

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PSF RECONSTRUCTION FROM THE SOLAR DISK EDGE OBSERVATION

Konstantin V. Parchevsky
Crimean Astrophysical Observatory, 98409 Crimea, Ukraine

ABSTRACT

Instrumental smoothing leads to image quality degradation and decreasing of the resolution. A Point Spread Function (PSF) is an important characteristic of optical device which can be considered as measure of image blurring. There are algorithms which permit to reconstruct an original image from observations if we know the PSF. A new algorithm of the PSF reconstruction from observations of quiet solar limb is proposed. Algorithm is based on solution of the convolutional Fredholm integral equation of the 1-st kind with the observed blurred image of solar edge in the right hand side. Is is assumed that 2D PSF can be presented as $K(x, y) = X(x)Y(y)$. Original limb image in the wing of $H_d$ has a sharp cut-off. In case of big magnification we can neglect limb curvature and consider it as a half-plane. It is shown, that the solution of integral equation can be found as $X(x) = \frac{d}{dz}f(x)$, where $f(x)$ is an observed blurred solar disk image averaged along $y$ direction. Algorithm does not require any additional apriory assumptions on the PSF functional dependence. PSF obtained in such a way can be used for reconstruction of solar images to increase resolution. In the Crimean Astrophysical Observatory this method is used for improving images of solar prominences. Reconstructed images show fine fiber-structure of prominences which is not seen in original observations.

Key words: Methods: Data analyses, Image reconstruction, Sun: Observations.

1. INTRODUCTION

The image, obtained from telescope (or other optical device), is not a real (original) image, because telescope distorts the image. Let us imagine that we observe just a point object. Ideal telescope gives us a point image, but if we use a real one, we shall obtain not a point but shaded region Fig. 1. Ideal point is blurred. The function which describes brightness distribution obtained from the observation of the single point is called the Point Spread Function (PSF). This is an important characteristic of the optical device.

In case when the image contains more than one point such blurring must be applied to each point of the original image. It is similar to the smoothing by the running average but in two dimensions. Such instrumental smoothing leads to image quality degradation and decreasing of the resolution. It is easy to write the equation which relate the original $g(x', y')$ and observed (smoothed) $f(x, y)$ images

$$ Af \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(x - x', y - y')g(x', y') \, dx'\, dy' = f(x, y), $$

where the right hand side of the integral equation $f(x, y)$ is the observed image, kernel $K(x, y)$ is the PSF, $g(x', y')$ is the original image. The right hand side and the kernel of the integral operator $A$ are known functions. The kernel can be either calculated or obtained by observing of the point object. To obtain an original image $g(x', y')$ we must solve Fredholm integral equation of the 1-st kind (Tikhonov & Arsenin). The problem of the image reconstruction represents ill-posed inverse problem.
2. 2D IMAGE RECONSTRUCTION

To solve the ill-posed problem of image reconstruction we have to use regularization. In this paper we used Tikhonov regularization method to stabilize the solution (Tikhonov et al., 1990). In case of real images functions $K(u, v)$, $g(\xi, \eta)$ and $f(x, y)$ have bounded supports and, therefore, integrals in Eq. (1) have finite limits. Assume that the exact kernel $K$, the right hand side $f$ and solution $g$ belong to the following functional spaces

\[
\begin{align*}
&f(x, y), \quad K(u, v) \in L_2\left(\mathbb{R} \times (\mathbb{R} \times \mathbb{R})\right), \\
&g(\xi, \eta) \in W_2^2((\mathbb{R} \times \mathbb{R}) \times \mathbb{R} \times \mathbb{R})
\end{align*}
\]  

(2)

The uniform convergence of $g(\xi, \eta)$ in arbitrary rectangle $[a, b] \times [c, d]$ follows from the convergence in $W_2^2$ space. Assume that instead of exact $K$ and $f$ we know their approximate values $K_h$ and $f_h$ such that

\[
\|f_h - f\|_{L_2} \leq \delta, \quad \|A - A_h\|_{W_2^2} \leq \varepsilon
\]  

(3)

In this case Tikhonov algorithm is reduced to the minimization of the following smoothing functional

\[
M^\alpha[g] = \|A_h g - f_h\|_{L_2} + \alpha \|g\|_{W_2^2}
\]  

(4)

Due to special form of the kernel of convolution equation (1) (it depends only on argument difference) we can use highly effective algorithm based on Fourier transformation (Verlan & Sizikov, 1986). Introduce Fourier images of the right hand side and the kernel as follows

\[
\begin{align*}
\hat{f}_h(\omega, \theta) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_h(x, y) e^{-i(\omega x + \omega y)} dxdy, \\
\hat{K}_h(\omega, \theta) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_h(u, v) e^{-i(\omega u + \omega v)} du dv.
\end{align*}
\]  

(5)

The extremal which minimizes (4) can be easily written as follows

\[
g^\alpha(\xi, \eta) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\hat{K}_h^*(\omega, \theta) \hat{f}_h(\omega, \theta) e^{i(\omega \xi + \omega \eta)}}{L(\omega, \theta) + \alpha \left[1 + (\omega^2 + \theta^2)^2\right]} d\omega d\theta,
\]

where \(\hat{K}_h^*(\omega, \theta) = \hat{K}_h(-\omega, -\theta), \quad L(\omega, \theta) = |\hat{K}_h(\omega, \theta)|^2\).

Introducing uniform grid on $(x, y)$ and $(\xi, \eta)$, using discrete Fourier transformation and approximating integrals in Eq. (1) by rectangle formula, smoothing functional (2) can be rewritten in a discrete form as

\[
M^\alpha[g] = \frac{\Delta x \Delta y}{N_1 N_2} \sum_{m=0}^{N_1-1} \sum_{n=0}^{N_2-1} \left\{ |\hat{K}_{mn} \hat{g}_{mn} \Delta x \Delta y - \hat{f}_{mn}|^2 + \alpha \left[1 + (\omega_m^2 + \theta_n^2)^2\right] \right\}.
\]

(6)

Minimum of this functional is reached on the vector with the following Fourier coefficients

\[
\hat{g}_{mn} = \frac{\hat{K}_{mn} \hat{f}_{mn} \Delta x \Delta y}{|\hat{K}_{mn}|^2 \Delta x \Delta y + \alpha \left[1 + (\omega_m^2 + \theta_n^2)^2\right]},
\]

\[
m = 0, 1, \ldots, N_1 - 1, \\
n = 0, 1, \ldots, N_2 - 1.
\]

Optimal value of regularizing parameter can be found from the principle of generalized residual. Discrete Fourier transformation was calculated by FFT.

As a result we shall obtain the image with better resolution. It is worth to emphasize that this is not a simple brightness/contrast changing. It is real enhancement of the image. We use an additional information (PSF) and it permitted us to improve the image. Results of image reconstruction are shown on Fig. 2. Original image (a) was blurred (b) by convolution with the kernel. 2D Gaussian curve was chosen as the kernel. It blurs one pixel to a round region approximately 30 pixels in diameter. Noise was added to simulate an instrumental errors. Image (c) represents a result of image reconstruction. It has much better resolution then the blurred image (b).
Figure 3. A quiet part of the solar disk edge is used for the PSF reconstruction.

3. THE PSF RECONSTRUCTION

This algorithm of the image reconstruction essentially uses information about the PSF. There is no problem to obtain the PSF for stellar telescopes. It is sufficiently to observe a bright star. Observed image is the PSF, because we can consider a star as a point object very accurately. Usually, it is difficult to extract the point sources from solar observations. We propose to use the image of solar disk edge for obtaining the PSF. In fact, if we know the original image \( g(x', y') \) equation (1) can be considered as integral equation for obtaining the kernel \( K(x, y) \) from the observed image. In case of big magnification and narrow field of view we can consider the solar edge as a half-plane. In this case the original image does not depend on vertical coordinate

\[
g(x', y') = \theta(x'), \quad \text{where} \quad \theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}
\]

Assume, that the kernel \( K(x, y) \) can be represented as a product of two functions each of them being dependent on only one coordinate

\[
K(x, y) = X(x)Y(y)
\]

and normalized as follows:

\[
\int_{-\infty}^{\infty} X(x) \, dx = \int_{-\infty}^{\infty} Y(y) \, dy = 1. \quad (10)
\]

In this case Eq. (1) can be rewritten as

\[
\int_{-\infty}^{\infty} Y(\eta) \, d\eta \int_{-\infty}^{\infty} X(\xi)\theta(x - \xi) \, d\xi = f(x). \quad (11)
\]

The first integral equals unity in accordance with the normality condition (10). The second integrand in Eq. (11) is non-zero if \( x \geq \xi \). Hence, the problem of the PSF reconstruction is reduced to calculation of the first derivative from the observed image of solar disk edge and equation (11) can be rewritten as

\[
\int_{-\infty}^{\infty} X(\xi) \, d\xi = f(x), \quad X(x) = f'(x). \quad (12)
\]

To obtain 2D kernel \( K(x, y) \) we must know \( Y(y) \) as well. Usually, the horizontal \( X(x) \) and the vertical \( Y(y) \) cuts of the kernel \( K(x, y) \) are not differ drastically. We have made an additional assumption about symmetry of the kernel

\[
K(x, y) = X(x)Y(y). \quad (13)
\]

It permits us to reconstruct PSF and image using the same picture. Kernel asymmetry can be easily taken into account. One can obtain \( Y(y) \) by repeating the PSF reconstruction procedure for the horizontal image of solar edge. Further improvement of the PSF reconstruction procedure is connected with the assumption that the edge of the solar disk has no uniform brightness. In this case \( g(x, y) \) cannot be reduced to the theta-function and we must solve 2D integral equation (1) relative to \( K \) numerically.
4. RESULTS AND DISCUSSION

The algorithm of image reconstruction described above was realized as a package of programs on MatLab 6.0 and Visual C++ 5.0. We used subroutines from (Press et al., 1992). It is successfully used for enhancement of solar images and spectra obtained on Tower Solar Telescope (TST-2) and coronograph (KG-1) of the Crimean Astrophysical Observatory. For illustration of the method, proposed technique is applied to the reconstruction of $H_\alpha$ image of prominence. The solar image was obtained on the coronograph (KG-1) of the Crimean Astrophysical Observatory by A. Shakhovskaya in $H_\alpha$ spectral line. For the PSF reconstruction a quiet part of solar edge near the top of photograph was chosen (Fig. 3). One dimensional function $f(x)$ of the edge blurring is shown on Fig. 4. It is obtained by averaging Fig. 3 over the columns with further normalization. This function is used as the right-hand side in the 1D integral equation (12). Result of the PSF reconstruction is shown on

To obtain the horizontal cut $X(x)$ (Fig. 5a) of the PSF we must calculate the derivative from the $f(x)$ (see Fig. 4). For numerical calculation of the derivative we used method based on solution of integral equation (12) by Tikhonov regularization method (details of numerical realization can be seen in (Parchevsky, 2000)). 2D kernel $K(x,y)$ (Fig. 5b) is assumed to be symmetrical (formula (13) is used).

Now we can start the image reconstruction procedure described in Section 2. Observed $H_\alpha$ image of the prominence is shown on Fig. 6. Using the kernel $K(x,y)$ obtained from observation of solar disk edge and observed image as the right-hand side we can solve integral equation (1) and obtain reconstructed image $g(x,y)$. Result of this reconstruction is presented on Fig. 7. Optimal value of regularization parameter is equal to $\alpha = 0.002$. On Fig. 7 one can see fine fiber structure of the prominence. This structure cannot be seen in the original image due to instrumental smoothing.

There is a natural resolution limit due to the diffraction. Optical device cannot give us the image with the resolution better than the diffraction limit, but after such reconstruction we shall obtain the image better than the Nature permits. No physical laws are violated, because it is a computed image (not observed).

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THE IMPACT OF SMALL-SCALE STRUCTURES ON RAY-BASED TIME-DISTANCE INVERSIONS

Gary H. Price
SRI International, Menlo Park, CA 94025, USA
tel: 650-859-4820 / email: gprice@solar.stanford.edu

ABSTRACT

The insensitivity of wave travel times to small-scale perturbations of the medium encountered along a ray path has been appreciated for some time, and has been demonstrated quantitatively both in geophysical and helioseismic contexts using a variety of 'toy' problems for which the actual wave behavior can be assessed with high fidelity. I explore here, in the context of one such problem, the hypothesis that ray-based inversions—which nominally remain sensitive to such perturbations—noneetheless can, even in their presence, usefully characterize perturbations to the medium at scales larger than that bounding the ray domain of validity. The results from the ray-based inversions undertaken here approximate a smoothed (at the bounding scale) representation of the actual perturbations, which range in scale across this limit. Thus, although constituting only a small and highly idealized sample, they tend to support this hypothesis, suggesting that ray-based inversions may provide a viable option in situations where computational efficiency, or the adequacy of first-order results, is at issue. Issues raised by the incorporation of higher-order effects such as refraction into ray inversions are also addressed.

Key words: helioseismology; time-distance; ray inversions.

1. INTRODUCTION

Time-distance helioseismology attempts to infer localized departures from a nominal state of the solar interior through examination of the departure of observed wave-packet travel times from those expected for the nominal state. Initially, time-distance interpretations of observational data generally were undertaken in the context of the ray approximation (e.g. Duvall et al. 1996; Kosovichev 1996), which neglects finite-wavelength effects. One of these effects is that wave travel times are insensitive to small-scale perturbations of the medium that lie too near the ray path—the so-called banana-doughnut effect (Marquering et al. 1999), as has been demonstrated quantitatively in both geophysical and helioseismic contexts (e.g. Hung et al. 2000; Birch et al. 2001). As ray inversion kernels sample only along the ray path, appreciation of the banana-doughnut effect has reinforced previously-expressed concerns (Wielandt 1987; Bogdan 1997) as to the validity of ray-based analyses in the presence of relatively small-scale structures such as, in the case of the Sun, active regions and sunspots. In order to elucidate further the impact of such structure on ray inversions, I examine here ray-based inversion results for a very simple problem, namely propagation of adiabatic acoustic waves in the presence of a spherically symmetric sound-speed perturbation to an otherwise uniform medium.

2. COMPUTATIONAL MODELS

2.1. Field Calculation

In the field calculation, the total field, as well as incident and scattered fields, are determined for a point-source acoustic wave incident upon a spherically symmetric sound-speed perturbation $\Delta c$ to a uniform medium with sound speed $c_0 = 10 \text{ km/s}$, as given by

$$\left( \frac{\partial^2}{\partial t^2} - \frac{\Delta c(|r - r_0|)^2 \nabla^2}{c_0^2} \right) p(r, t) = w(t) \delta(r - r_s) \tag{1}$$

with $p = p_{\text{inc}} + p_{\text{heat}}$, $r_s$ the source location, and $r_0$ the center of the scatterer. The sound-speed perturbation is approximated as a sequence of uniform shells, as illustrated in Figure 1, in a multilayer elaboration, much as envisioned by Wielandt (1987), of a spherical-harmonic expansion of the fields devised previously for a homogeneous scatterer (Birch et al. 2001). The consequent multilayer representation is broadly analogous to those developed for planar model atmospheres by Pfeffer (1962) and by (Press & Harkrider 1962). The number of layers required to accurately approximate the smooth perturbation was determined by repeatedly halving the layer thicknesses in successive trial calculations of the travel-time perturbation at a point symmetrically across the perturbed region from the source until the result stabilized; 32 layers usually sufficed to obtain an accuracy of a tenth of a percent or better in this quantity for the perturbations discussed here, with twice this number occasionally being needed.

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The spherical-harmonic expansion is, as previously noted by Wielandt (1987), not constrained as to source and observer locations by its well-known convergence difficulties, because convergence of the scattered-field expansion is dictated by the size of the scatterer and the incident-field expansion need be used only at the outer boundary of the scatterer. The program organization, in which the expansion coefficients are first evaluated before any field computations are undertaken, lends itself to use of the program as a coroutine in the iterative minimization whereby the ray inversion is performed.

The incident wave field is taken to be a band-limited wave packet of the form

\[ w(t) = \exp\left(-\frac{(\pi \Delta f t)^2}{2 \log 2}\right) \cos(2\pi f_0 t), \]

with a center frequency \(f_0\) of 3 mHz and a full width at half maximum \(\Delta f\) of 0.75 mHz.

The travel-time perturbation is obtained by cross correlation of the incident and total fields, as outlined in Birch et al. (2001). Specifically, the cross correlation is calculated using complex field representations, and the offset from zero of its envelope peak is taken to be the perturbation to the group travel time of the wave packet.\(^1\) In order to minimize sampling effects, the correlation envelope is also Fourier interpolated before its largest sample and the two samples bracketing it are fit with a parabola, the peak of which is the quantity finally used.

### 2.2. Ray Inversion

The ray inversion is achieved by minimization of the sum of the squared differences between the travel-time perturbations calculated by tracing a fan of rays through a model of the perturbation, as is illustrated in Figure 2 for the unperturbed medium, and those obtained from a full field calculation at the far end of each ray. The

\(^1\)Cross correlation of the real fields yields an oscillatory function, the peak of which gives the perturbation in phase travel time; this quantity shifts more widely than does the group-time perturbation, to the point of exhibiting cycle jumps in extreme cases.

\(^2\)Actually, both the outermost rays shown in Figure 2 are superfluous, as was realized only after the calculations had been completed.

For the layered perturbation model adopted here, the first term on the right side of Equation 3 reduces to a sum over the incremental delays \(\Delta s_i/c_i\) accumulated on the path segments \(\Delta s_i\) within each uniform layer of sound speed \(c_i\), with the path of the ray determined by application of Snell's law at the layer interfaces, while the second term becomes simply \(\Delta c_0 = [r_2 - r_1]/c_0\) with \(r_1\) and \(r_2\) the locations of the path end points.

The sound-speed perturbations \(\Delta c_i\) in the ensemble of layers through which the rays are traced constitute the minimization parameters. The layer thicknesses are set equal to the radii of the central Fresnel zones of rays that in the unperturbed medium just graze the inner boundary of each layer; these thicknesses substantially exceed those ultimately used in the multilayer field calculations.

In the inversions reported here, knowledge of the perturbation symmetry has been assumed. Consequently the fan of rays that samples the volume occupied by the perturbation can be restricted to a single plane that also contains the center of the perturbation, and the layers need not be subdivided laterally in the \(\theta\) and \(\phi\) directions. Additionally, only one ray of each pair of symmetrically disposed rays shown in Figure 2 needs to be traced.\(^2\) The angular spacing between rays in the fan was chosen, as shown in the figure, to be half that between rays that graze successive layer interfaces in the unperturbed medium; thus the perturbed region was somewhat oversampled.
Rather than undertake the uncertain task of homing the rays to fixed end points in the presence of refraction, the locations of the ray end points were allowed to vary, while the ray directions at the source were held fixed, during the course of the inversion. Consequently a set of field calculations is required at each iteration of the downhill simplex minimization algorithm (Press et al. 1999, Sec. 10.4) used to perform the inversion. The far end points of the rays are, however, constrained to lie at the same distance from the perturbation center as the source point, as shown in Figure 2; this simplifies the treatment of any backscattered rays.3

3They then can be treated the same as any other ray.

3. RESULTS

Figure 3 presents results from a set of inversions that involved a range of perturbation half widths at half maximum (HWHM) that bracket the radius of the first Fresnel zone ($r_f = 8.7$ Mm) for the ray that passes through the center of the perturbation. Each vertical pair of plots show results for positive (upper plot) and negative (lower plot) perturbations, indicated by the dashed lines, having the same strength (±10% at the center of the perturbation) and width. The ratio of half width to Fresnel-zone radius, indicated at the top of each column, increases to the right. The solid lines show the inferred perturbation strength in each layer.

For the narrowest perturbations, shown in the leftmost column of Figure 3, the perturbation is largely washed out in the inversion result. As the width of the perturbation increases, however, the inversion yields a perturbation that approximates increasingly well—within the limits afforded by a model consisting of an ensemble of uniform layers—that actually present.

The inversion results for positive and negative pairs of inversions do not quite mirror each other, as they would be expected to do were the travel-time perturbations strictly linear in perturbation strength. The possibility that this is an artifact of the simplex minimization used to accomplish the inversion has not entirely been ruled out, and the non-linearity in perturbation strength of second-order contributions to the travel-time perturbation could exacerbate such behavior. In any event, the asymmetry in the inversion results for a given perturbation size together with their variation from one perturbation size to another provides some indication of the uncertainty in the result.
4. DISCUSSION

Variations of a medium transverse to a ray path on scales smaller than the size of the central Fresnel zone surrounding the path give rise to wave diffraction, a finite-wavelength effect that is not addressed by ray theory. This circumstance motivated my choice of $r_F = (c_0 s_0^2 / f_0)^{1/2}$, the radius of this zone at midpath ($s_0^2 / f_0 = 45$ Mm) for the ray through the center of the perturbation, to characterize the perturbation sizes in Figure 3. It also underlies the concerns expressed by Bogdan (1997) as to the substantial extent of the region about the ray path that is sampled by wave packets representative of those present in the Sun.

In the work presented here, I have addressed these concerns by insisting that the perturbation model used in the inversion not contain scales smaller than this limit. Thus, the model that is fit to the data to effect the inversion is consistent with the use of ray theory, notwithstanding that the actual medium may not be. As regards the impact of the banana-doughnut effect on the inversion should smaller-scale perturbations actually be present, the validity of the ray kernels hinges only on whether the medium to which they are applied varies slowly enough in the vicinity of their ray paths that the sampling provided by these paths suffices to characterize this variation at the frequencies of interest, which here has been forced to be the case.

An obvious consequence of this approach is that the inversion result is inherently incapable of representing variations of the medium at scales smaller than the imposed limit. In the results presented here, isolated perturbations appear consequently to become increasingly averaged out as the perturbation size decreases below this limit. This behavior may also be encouraged by the modest oversampling of the perturbed region that was incorporated into the inversions. Such behavior seems appropriate if one is seeking only to recover variations at the larger scales from the inversion.

Linearized inversions generally are understood to be poorly conditioned because the inversion kernel constitutes a smoothing operator, the undoing of which represents at small scales an attempt to recover information that has been filtered out of the data (Giles 1999). Although there is no explicit regularization incorporated into the non-linear inversions undertaken here, constraint of the perturbation model to appropriately large scales would appear to address this circumstance in a physically appropriate fashion.

The decision to work with varying ray end points was, as noted above, motivated by a desire to avoid the vagaries of ray homing, which at its most bellicose can involve situations such as on occasion during the inversion finding the homing target to have fallen within a shadow zone. Such complications are not beyond the range of possibility when working real problems, so should a need to go beyond first-order effects become evident in time-distance helioseismology, it would seem worthwhile to explore whether an approach similar to that adopted here remains feasible.

5. CONCLUSION

The results from the ray-based inversions undertaken here approximate a smoothed (at the bounding scale) representation of the imposed sound-speed perturbations, which range in scale across this limit. Thus, they support the view that ray-based inversions are a viable option in situations where computational efficiency, or the adequacy of first-order results, is at issue.

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ACCURATE MEASUREMENTS OF SOI/MDI HIGH-DEGREE FREQUENCIES AND FREQUENCY SPLITTINGS

J. Reiter\textsuperscript{1}, A. G. Kosovichev\textsuperscript{2}, E. J. Rhodes, Jr\textsuperscript{3}, and J. Schou\textsuperscript{2}

\textsuperscript{1}Zentrum Mathematik, Technische Universität München, D-85747 Garching, Germany
\textsuperscript{2}W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085, USA
\textsuperscript{3}Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-1342, USA

ABSTRACT

We present accurate measurements of high-degree $p$-mode frequencies and frequency splittings obtained from the Full-Disk Program of the Michelson Doppler Imager (MDI) experiment onboard the Solar and Heliospheric Observatory (SOHO). The frequencies and frequency splittings are computed from unaveraged zonal, tesseral, and sectoral power spectra using a new fitting method of Reiter et al. (2002) based upon a maximum-likelihood fitting approach. In this method, both the spectral power distribution and contributions of the various observational and instrumental effects to the spatial leakage matrices are modelled accurately. We demonstrate that one of the most long-standing problems in high-degree helioseismology, viz. the jumps in the frequency splitting coefficients, can be solved by taking into account the distortion of the leakage matrix by the solar differential rotation. The results of inversion of the initial frequency set determined using this new method in the range of angular degree $\ell = 45 - 300$ show a substantially better resolution of the subsurface layers compared to the previous studies with $\ell$ below 220.

Key words: Methods: Data analysis, Sun: oscillations.

1. INTRODUCTION

Since the acoustic modes of high angular degree are trapped near the solar surface they represent an important diagnostic tool for probing the structure and dynamics of the upper convective boundary layer. This region is of great interest for many reasons. For example, in this region the ionization zones of hydrogen and helium are located, and they can provide rather sensitive tests of the solar equation of state. Also, solar cycle variations and complex near-surface rotational dynamics detected in low- and intermediate-degree modes can only be fully understood if high-degree modes are included in the analysis. In addition, the high-degree modes have the potential to significantly improve the resolution of the solar structure in the near-surface region, provide detailed tests of the equation of state and constrain the envelope helium abundance (e.g. Rabello-Soares et al., 2000; Di Mauro et al., 2002).

However, the high-degree mode frequencies are difficult to measure reliably. This is because high-degree mode parameters cannot be determined in a straightforward manner using simple peak-finding algorithms. A power spectrum computed for a specific target mode with degree $\ell$ and azimuthal order $m$ contains contributions of power from modes with neighbouring $\ell$ and $m$ as well. These contributions are measured by the so-called leakage matrix, which is essentially defined as the overlap integrals between the target mode and modes with neighbouring $(\ell, m)$. With increasing degree the separation of the spatial leaks decreases. Moreover, the widths of the lines increase with frequency and degree. As a consequence, individual modes blend into ridges of power. Since the amplitudes of the spatial leaks are asymmetric the central frequency of a ridge is significantly offset from the frequency of the targeted individual mode frequency, resulting in unphysical jumps in the fitted parameters (e.g. Korzennik, 1990). The $p$-modes typically begin to blend into ridges for degrees ranging anywhere from $\ell = 140$ to $\ell = 210$ or thereabouts depending on the overtone number $n$. We show that to resolve the offset problem the leakage matrix calculations at high degrees have to take into account the distortion of the eigenfunctions by the solar differential rotation first noted by Woodard (1989).

2. DATA AND FITTING TECHNIQUE

The results presented in this paper are based upon spectra that were created from observations obtained from the MDI Full-Disk Program during the 1996 MDI Dynamics Run (Scherrer et al., 1995). Specifically, a 60.75-day time series of full-disk MDI Dopplergrams taken at a cadence of one minute was converted into $2\ell + 1$ time series of spherical harmonic coefficients. The initial time series had a common duty cycle of 95.2$.\%$. After filling a few of the shortest gaps using standard gap-filling techniques, the duty cycle of the final time series was increased to 97.3$.\%$. These time series of spherical harmonic coefficients were used to calculate a set of $2\ell + 1$ zonal, tesseral, and sectoral power spectra for $0 \leq \ell \leq 1000$ by a stan-
standard FFT method. The initial results presented here are obtained for modes in the range $45 \leq \ell \leq 300$.

The zonal, tesseral, and sectoral power spectra generated in the above-described manner were fitted employing the so-called “Maximum-Likelihood, Multi-Peak, Tesseral-Spectrum” method (Reiter et al., 2002). Unless otherwise stated a theoretical model profile based upon the asymmetrical line profile of Nigam & Kosovichev (1998), an expansion of the rotational frequency splittings in orthogonal polynomials first suggested by Ritzwoller & Lavelle (1991), the theoretical value of $c_1$, the ratio of the horizontal and vertical components of the solar $p$-mode eigenfunctions, and the so-called reference set of leakage matrices (cf. Section 4) were used in the calculations presented in this paper.

3. EFFECT OF PERTURBATION OF THE LEAKAGE MATRIX BY SOLAR DIFFERENTIAL ROTATION

One of the long-standing problems in high-degree helioseismology is the fact that the rotational splitting coefficients and the mean multiplet frequencies appear to undergo jumps or offsets for degrees ranging anywhere from $\ell = 140$ to $\ell = 250$. The exact location of the range of $\ell$ values where the jumps occur depends on the radial order $n$. These discontinuities were noticed by Korzennik (1990) more than a decade ago, and more recently by Rhodes et al. (1999) in attempts to generate rotational splitting coefficients employing the cross-correlation method of Brown (1985) and Tomczyk (1988). Most often mode-blending effects were blamed for causing these jumps. Reiter et al. (2002) speculated that one effect that may cause these discontinuities is the coupling of modes due to the differential rotation of the Sun (see also Korzennik et al., 2002).

Woodard (1989) has shown that the distortion of high-degree $p$-mode eigenfunctions by a slow, axisymmetric differential rotation can be expressed as a superposition of the unperturbed eigenfunctions of the same radial order $n$ if Coriolis forces are neglected. Applying Woodard's theory, we have found that taking into account this effect in the leakage matrix has a very significant impact upon the resulting frequency-splitting coefficients. This is demonstrated in Figure 1. While for the lower degrees the splitting coefficients remain almost unchanged the notorious jumps can be seen to disappear if the mode coupling due to the differential rotation of the Sun is taken into account.

There is also an effect to be noted upon the fitted mean multiplet frequencies. This is shown in Figure 2 where we have compared the average multiplet frequencies with the theoretical frequencies of Model S calculated by Christensen-Dalsgaard et al. (1996). As can be seen the inclusion of the perturbation of the leakage matrix by the solar differential rotation smoothes the distribution of the fitted frequencies significantly.

![Figure 1](image_url)  
Figure 1. Degree dependence of two sets of $a_1$, $a_3$, and $a_5$ splitting coefficients for the $n = 1$ ridge. The splitting coefficients marked with the diamonds and the error bars were generated using a leakage matrix which took into account only the corrections for the distortion due to latitudinal differential rotation. The triangles were computed without the inclusion of these corrections. The error bars are $\pm 1 \sigma$.

4. EFFECT OF INSTRUMENTAL IMAGE DISTORTION

In addition to taking into account the distortion of the eigenfunctions by the solar differential rotation, several instrumental effects and processing problems, which have been disregarded in the initial generation of the spectra, have to be taken into account. However, rather than re-doing the spherical harmonic transform (SHT) which is computationally expensive we instead chose to correct the leakage matrix by passing distorted spherical harmonic images through the SHT code. Among the problems encountered are the plate scale error, the fact...
Figure 2. Degree dependence of differences between average multiplet frequencies and the theoretical frequencies calculated using Model S of Christensen-Dalsgaard et al. (1996). The top panel is for the $n = 1$ ridge, while the bottom panel is for the $n = 2$ ridge. Note the different vertical scales in the plots of the two panels. The frequency differences marked with the diamonds and the error bars were generated with the inclusion of the corrections to the leakage matrices due to differential rotation. The triangles were computed without the inclusion of these corrections. The error bars are $\pm 1 \sigma$.

Figure 3. Same as Figure 2 aside from the fact that the frequency differences marked with diamonds and error bars were generated by taking into account both the solar differential rotation and the instrumental effects in the leakage matrix employed. The leakage matrix used in computing the triangles included neither the corrections for the effects of differential rotation nor any corrections for the instrumental problems. The triangles shown here are identical to those shown in Figure 2.

Figure 4. Degree dependence of the difference of $a_1$ splitting coefficients obtained by employing two different sets of leakage matrices, for the $n = 1$ ridge. The splitting coefficients denoted by $a_1(1kref)$ were generated using the so-called reference set of leakage matrices, which did not include any corrections for the instrumental problems, while the coefficients denoted by $a_1(1kfull)$ were generated using leakage matrices which did include such instrumental corrections. The corrections for the solar latitudinal differential rotation were included in both cases. The error bars are $\pm (\sigma_1^2 + \sigma_2^2)^{1/2}$ where $\sigma_1$ and $\sigma_2$ are the standard deviations of the measurements of the two sets of $a_1$ splitting coefficients.

that the MDI optics introduces a cubic distortion, the fact that the CCD is tilted and the width and spatial nonuniformity of the instrumental point spread function (PSF). In the following we compare the results of using the so-called reference set of leaks, i.e. leaks which have been generated without taking into account any of these effects, with those using our best estimate of the cubic distortion and a previous estimate of the best plate scale (radius) correction for the MDI 1996 Dynamics Run. Given the problems of estimating the CCD tilt and the point spread function we have not yet corrected for them.

The inclusion of both the plate scale error and the image distortion as well as the solar differential rotation effect in the leakage matrix employed has a rather strong impact upon the fitted frequencies. This is shown in Figure 3. As can be seen by comparing Figures 2 and 3 the frequencies become much smoother if both the plate scale error and the image distortion are taken into account in the leakage matrix. The impact of these instrumental effects upon the $a_1$ splitting coefficient is shown for the $n = 1$ ridge in Figure 4. As can be seen the $a_1$ splitting coefficient is strongly affected for $\ell \geq 200$. Similar effects are to be noted on the higher-order splitting coefficients as well as on the splitting coefficients for other ridges.
While the inversion shown in Rhodes et al. (1997) stops at 0.96 \( R_\odot \), the new inversion goes out to 0.99 \( R_\odot \) and, hence, includes the turn-around in the difference of \( c^2 \) at 0.98 \( R_\odot \). Resolving the structure of this feature in detail is important for studying the equation of state and the chemical composition of the Sun.

Finally, this method provides also accurate estimates of rotational frequency splittings which are important for understanding the angular momentum transport and dynamics of the near-surface shear layer. The range of our measurements can be extended up to at least \( \ell = 1000 \) for the MDI Dynamics data.

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STUDIES OF THE SENSITIVITY OF P-MODE OSCILLATION FREQUENCIES TO CHANGING LEVELS OF SOLAR ACTIVITY

P. Rose\textsuperscript{1}, E.J. Rhodes, Jr.\textsuperscript{1,2}, J. Reiter\textsuperscript{3}, W. Rudnisky\textsuperscript{1}

\textsuperscript{1} Dept. of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089, USA
\textsuperscript{2} Space Physics and Astrophysics Research Element, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109-8099, USA
\textsuperscript{3} Zentrum Mathematik, Technische Universität München, D-80747 Garching, Germany

ABSTRACT

Most studies of the solar cycle dependence of the p-mode oscillation frequencies have employed long-duration observing runs. Recently, Rhodes \textit{et al.} (2002) have employed observing runs as short as three days in length using MDI data sets. The use of such short time series has resulted in a higher sensitivity (as measured in the slope of the linear regression of frequency differences upon the 10.7-cm flux differences) than did the use of long time series that averaged over regions varying widely in their levels of activity. Here we address the question of whether we can confirm this increased sensitivity by employing observing runs as short as three days in duration by using GONG+ data. We have employed GONG+ observations obtained during two different intervals in 2001 for these studies.

Key words: solar oscillations; frequency shifts.

1. INTRODUCTION

While it is now a well-established fact that the frequencies of the low- and intermediate-degree solar p-mode oscillations do indeed change with time in response to changing levels of solar activity, there is currently no consensus as to the solar origin of these changes. This situation was summarized clearly by Kuhn (2001), who described the various changes which have been seen in solar $f$- and $p$-mode frequencies, frequency splittings, horizontal flow velocities, and solar diameter measurements as diagnostics of what he called the "acoustic solar cycle".

2. GENERATION OF POWER SPECTRA FROM GONG+ VELOCITY IMAGES

The data sets for this analysis came from two separate runs in 2001: March 23 through March 26, and June 4 through June 13. During the March run, GONG+ was not fully established and full-disk velocity images were available from the Big Bear site alone. During the 12-day run in June, observational data was available from the sites at Big Bear, Learmonth, and Cerro Tololo. When the data was retrieved through the GONG Data Storage and Distribution System (DSDS) over a year ago, velocity images from the remaining GONG+ sites were not yet available. These sets of data represent some of the very first high-resolution data collected by the GONG+ program. Figure 1 shows a plot of all the available data for 2001. The 5-day March run from Big Bear covers days 82 to 86 and the 12-day June run covers days 155 to 166.

The GONG+ images were processed utilizing code developed for analyzing images taken at the 60-Foot Solar Tower at the Mt. Wilson Observatory. Since the local data reduction software does not employ FITS images, the first processing step was the con-


\textbf{Figure 1.} GONG+ duty cycle coverage for 2001. Big Bear was the first site to come on line on March 23, day 81, followed by Learmonth and Cerro Tololo on June 1. The fourth site (El Teide) starts to fill in the middle gap by July 30, day 211. The days processed for this paper are 82-86 and 155-166.


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version, for each minute, of the GONG+ FITS images into a 860x860 pixel, 16-bit integer array and a separate file containing the FITS keywords. These FITS headers were kept for inclusion of the keywords into subsequent processing steps.

The next step involved a multistage calibration procedure performed on the GONG+ full-disk velocity images. Each of the calibration steps was performed using specific keywords from the FITS headers. These calibration steps were performed separately and prior to the remaining reduction of the images. The first step was the conversion of the pixel values in the images into units of m/s. The second step accounted for the motion of each site with respect to the Sun, and the third step accounted for the velocity offset between sites. Once all of these calibration steps were performed, the component of the observer’s motion due to rotation was removed. The component of motion due to Earth’s revolution about the Sun was not taken into account.

Upon completion of the calibration procedure, the spherical harmonic decomposition step was performed using a modified version of the 60-Foot-Solar-Tower SHC-code. The SHC-code was modified to read the heliographic coordinates from the extracted FITS header and to apply these values to each individual velocity image. When we processed the March data we employed the same upper degree limit of 800 that we employ for our Mt. Wilson data. By the time we began processing the June data, we had modified our code to extend the maximum $\ell$-value to 1000. The time series of spherical harmonic coefficients from each observing day were transposed into one-dimensional time series for each $(\ell, m)$ pair. The time series was extended with zeroes to a length of 32,768 minutes for each of the three-day sets. The one-dimensional power spectra were then computed from these time series in standard fashion.

3. GENERATION OF P-MODE DATASETS FOR TEMPORAL STUDIES

The $p$-mode frequency datasets which we employed in this study were computed with our WMLTP, “Windowed, Multiple-Peak, Averaged-Spectrum”, fitting method. As its name implies, the WMLTP method is applied to $m$-averaged power spectra. This method employs a sum of as many as nine such peaks and it convolves the resulting theoretical profile with the power spectrum of the temporal window function of each observing run. The details of the WMLTP method are given by Rhodes et al. (2001). This fitting method was applied to a total of six sets (2 in March and 4 in June) of $m$-averaged GONG+ power spectra which were computed from observing runs that were 4320 minutes in length. The WMLTP fitting method used the theoretically-predicted $p$-mode velocity eigenfunction component ratios along with a leakage matrix which had been generated using a small radius error and a small amount of optical distortion to generate a complete set of $p$-mode fitting parameters for degrees up through 800 from the March runs and up through 1000 from the June runs. Figure 2 shows a $\ell$-$v$ diagram for one of these four three-day June runs.

Figure 2. $\ell$-$v$ plot for one of the four, 3-day June 2001 GONG+ runs computed for the range June 10 through June 12.

4. SOLAR CYCLE-DEPENDENT FREQUENCY SHIFTS

To extend previous studies of $p$-mode frequency shifts so that both the high-frequency and high-degree modes would be included in the comparisons, we inter-compared all of our various GONG+ datasets, yielding a total of fifteen tables of frequency shifts. In order to study the dependence of these frequency shifts upon changing levels of solar activity, we also computed the average 10.7-cm radio flux and the average magnetic plage strength index for each observing interval. In the top panel of Figure 3 we show the average 10.7-cm radio flux values as a function of time for all of our different frequency datasets computed to date. These include longer-duration observing runs available to us from MDI and MWO in addition to the GONG+ observing runs. The points on the plot span the same time as the frequency datasets in order to place the levels of solar activity in the context of the most recent 11 years. This figure indicates that we have been able to obtain frequency datasets covering more than one entire solar cycle. In the bottom panel of Figure 3 we show the daily 10.7-cm flux values for each of the 5 days in March 2001 and each of the 12 days in June 2001 which we have analyzed.

In Figure 4 we show the frequency dependence of the unbinned and binned frequency shifts between the first and third of our four June datasets. Since the 10.7-cm flux was higher during the third of our four 3-day runs in June than it was during the first of these four runs, most past studies of the temporal behavior of the $p$-mode frequencies would predict that those frequencies would have increased along with the rising level of activity between these two runs. Figure 4 shows that such frequency increases were seen, but only up to a frequency of about 5000 $\mu$Hz. On the other hand, for frequencies between 5000 and about 6200 $\mu$Hz the frequencies actually went down from the first interval to the second.

To better illustrate this anti-correlation in the behavior of the temporal frequency shifts in the lower and higher frequency regimes, we have plotted the
Figure 3. (top) Average 10.7-cm radio flux (corrected to 1 AU) for different frequency datasets versus time over the past 12 years. (bottom) Daily 10.7-cm flux values during each of the 17 GONG+ observing days.

Figure 4. Frequency dependence of unbinned and binned frequency differences between GONG+ (06/10-12/01) - GONG+ (06/04-06/01). a.(left), shows the frequency differences vs. frequency for all values up to 7000 \( \mu \text{Hz} \). b.(right), shows the frequency dependence of the average frequency differences in bins that were 250 \( \mu \text{Hz} \) wide. Note, that the vertical scales are different in these two panels.

The frequency dependence of the four sets of binned frequency differences which resulted when we compared the frequencies of the four June runs with those of the March 26-27 run, in Figure 5. These latter two days of the March runs corresponded to a period of high 10.7-cm flux, as indicated in Figure 3. In fact, the level of 10.7-cm flux during both of these two days was considerably higher than it was during any of the four 3-day runs in June. By subtracting the set of frequencies computed from the 3/26-27/01 run, from the sets of frequencies computed from each of the four June runs, we were able to compute four different sets of frequency differences, each set of which corresponded to a time interval of decreasing solar activity. For such intervals of decreasing solar activity, we would have expected the frequency differences to be negative up to \( \nu = 5000 \mu \text{Hz} \) and to become positive for higher frequencies. This was indeed the case, as can be seen here in Figure 5. Furthermore, since the 06/04/01 - 06/06/01 run had the lowest 10.7-cm flux of any of the June runs, while the March 26-27 run had the highest such flux, we would have expected that the frequency differences between these two runs would have produced the strongest dip at 5000 \( \mu \text{Hz} \), as illustrated by the dashed curve in Figure 5. On the other hand, the comparison of the two June datasets which we illustrated in Figure 4 corresponded to an increase in the level of solar activity between the two runs, and the entire curve in that figure can be seen to anti-correlate with all four of the curves in Figure 5.

In order to study the response of these GONG+ frequency shifts to changing levels of solar activity in more detail, we first selected four different points along the curves of Figure 5. Specifically, we selected the frequencies of 3625, 4875, 5625 and 5875 \( \mu \text{Hz} \) as being representative of the low- and high-frequency regimes of our curves. Next we subtracted the 10.7-cm flux values of the different observing runs from one another and we generated a table of differences in the radio flux. Next, we performed linear regression analyses of the four sets of frequency shifts upon the differences in 10.7-cm flux. The results of regressing the binned shifts for the four frequencies at 3625, 4875, 5625 and 5875 \( \mu \text{Hz} \) upon the differences in the 10.7-cm flux are shown in Figure 6. As with most past studies of the low- and intermediate-degree modes, the frequency shifts at 3625 and at 4875 \( \mu \text{Hz} \) show positive slopes, with values of 8.65 and 25.21 nHz/SFU, respectively (where SFU=1 solar flux unit). Interestingly, these slopes are relatively shallow with respect to the slopes which resulted from the comparison of MDI 3-day runs, reported elsewhere during this meeting by Rhodes et al. (2003). These slopes are also more similar to those using long-duration runs where modes have died out or have been averaged over a long period of time. For the frequency shifts at 5625 and 5875 \( \mu \text{Hz} \) the slopes are -17.92 and -44.58 nHz/SFU, respectively. These negative slopes indicate an anti-correlation with the 10.7-cm solar flux. In all four panels of Figure 6 the open-squares are representative of frequencies computed for \( \ell \) up to 800, while the filled-squares are representative of frequencies computed for \( \ell \) up to 1000.
Finally, to obtain even more sensitivity to the changes in solar activity, we subtracted the frequency shifts observed at $\nu = 5625$ $\mu$Hz from the frequency shifts observed at $\nu = 4875$ $\mu$Hz. These differences in the frequency shifts are plotted versus the differences in the 10.7-cm flux in Figure 7. Because both sets of frequency shifts are sensitive to changes in solar activity, but in an opposite sense, the difference in these two shifts is even more sensitive to changes in activity than is either frequency shift alone. The linear regression fit to all of the pairs of points has a slope of 43.13 nHz/SFU. This value is again, similar to results obtained from long-term runs. Rhodes et al. (2002) computed a slope of 58 nHz/SFU for the same $\Delta(\nu = 4875 - \nu = 5625)$ $\mu$Hz versus $\Delta$10.7-cm flux, for all of the MDI short-term and long-term runs combined. Since these latter two slopes are in rough agreement, we can say that the $p$-modes seen in the GONG+ spectra correlate with short-term changes in the level of solar activity with a slope similar to that found by comparing observing runs that were long in comparison with the 3-day runs employed in this study. The fact that our GONG+ slopes were all smaller than were the corresponding slopes reported by Rhodes et al. (2003) from their comparisons of MDI 3-day runs, indicates that our GONG+ comparisons did not confirm the expectations we had when we began our study of the GONG+ runs. We are now trying to understand these differences. Perhaps the differences are due to the fact that several of our GONG+ tables of frequencies were only computed for degrees ranging up to 800. Another alternative is that perhaps the mechanism which causes the $p$-mode frequencies to shift in response to changing levels of solar activity might saturate at the relatively high levels of activity encountered during 2001 in comparison with the much lower levels of activity which corresponded to our 1996 MDI frequency tables.

5. CONCLUSIONS

In spite of the fact that this study did not confirm the larger regression slopes found by Rhodes et al. (2003), we still believe that the keys to resolving the current controversy over the mechanism of the frequency shifts are the use of short time series, including both the high-frequency and high-degree modes. Inter-comparisons of different sets of $p$-mode frequencies obtained from the GONG+ 2001 data, show that frequencies above 5000 $\mu$Hz are anti-correlated with changes in solar activity and they are more sensitive to changes in activity than the frequencies below 4000 $\mu$Hz. Also, the different sets of frequencies of both the intermediate- and high-degree modes have high enough signal to noise ratios that we have been able to compare time series as short as three days in duration. It will be desirable to study additional 3-day runs, as well as one-day runs in the future.

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A FIRST STUDY OF THE EXCITATION AND DAMPING RATE VARIATIONS EXTRACTED FROM IRIS$$^{++}$$ OBSERVATIONS.

D. Salabert¹, S. J. Jiménez-Reyes², and S. Tomczyk³

¹Département d’Astrophysique, UMR 6525, Université de Nice-Sophia Antipolis, 06108 Nice Cedex 2, France
²Themis, Instituto de Astrofísica de Canarias, E-38701, La Laguna, Tenerife, Spain
³High Altitude Observatory, NCAR, PO Box 3000, Boulder, CO 80307 USA

ABSTRACT

The IRIS$$^{++}$$ network (International Research of the Interior of the Sun) have collected 11 years of low-$$\ell$$ helioseismic data. Annual time series with an overlapping of 6 months are used to study the variations over the solar activity cycle of the different p-mode parameters. We find a global change of about -26% in the amplitudes, 14% in the linewidth and -12% in the velocity power while the rate at which the energy is supplied remains constant.

Key words: helioseismology, low-$$\ell$$ p-modes, solar activity.

1. INTRODUCTION

The p-mode parameters are demonstrated to be very sensitive to the solar activity cycle. The first report given by Woodard & Noyes (1985) uncovered a frequency shift for low degree of around 0.4$$\mu$$Hz. Afterwards, these results were soon confirmed by other works for low and intermediate degree revealing moreover news aspects of the mode parameter variations. Thus Pallé et al. (1990a,b) first and Anguera Gubau et al. (1992) later on found an important decrease of the amplitude with increasing activity.

In the present work we study the signature of the solar activity for the p-mode parameters. By means of these results, we analyze the variation of the velocity power of the low degree modes and the energy supplied to them. To achieve this goals, we make use of IRIS$$^{++}$$ database which has been carried on solar observations for almost a complete solar cycle.

2. DATA ANALYSIS

The IRIS$$^{++}$$ network is composed of the original IRIS sodium data which has been merged with the Mark-I resonant scattering spectrophotometer sited at the Observatorio del Teide (Tenerife, Spain) and the Magneto-Optical Filter LOWL instrument located in Mauna Loa Observatory (Hawai, USA). The merging of these three different observations in a consistent manner (Salabert et al., 2002) results in an important improvement in network duty cycle thanks to the well distributed location of the instruments. The observations carried on 11 years since 1989 July 1st up to 1999 November 5, spanning the maximum and the falling phase of solar cycle 22 and the rising phase of the current solar cycle 23 are divided in 20 time series of 360 days with an overlapping of 180 days.

3. MODELING THE ACOUSTIC MODE SPECTRUM

A solar eigenmode can be considered as a damped one-dimensional oscillator. This model allows us to assume that the peaks in the Fourier spectrum are asymptotically described by Lorentz profiles:

$$M_{n,\ell}(\nu) = \frac{A_{n,\ell}(\Gamma_{n,\ell}/2)^2}{(\Gamma_{n,\ell}/2)^2 + (\nu - \nu_{n,\ell})^2}$$

where $$\nu_{n,\ell}$$ is the central resonance frequency, $$A_{n,\ell}$$ is the power at the resonance and $$\Gamma_{n,\ell}$$ is the linewidth.

The total velocity power $$V^2_{n,\ell}$$ for one mode is proportional to the amplitude $$A_{n,\ell}$$ times the linewidth $$\Gamma_{n,\ell}$$, which corresponds to the area under the mode such as,

$$V^2_{n,\ell} \propto A_{n,\ell}\Gamma_{n,\ell}.$$  

Regarding the total energy in the modes $$E_{n,\ell}$$ is the sum of kinetic and potential energy and it can be written as

$$E_{n,\ell} = M_{n,\ell}V^2_{n,\ell}$$

where $$M_{n,\ell}$$ is the corresponding mode mass. The energy of a damped oscillator can be derived by using the oscillator analogy. This combined with Eq. 2
and 3 give us an estimation of the rate at which the energy is supplied to the modes which can be written as

$$\frac{dE_{n,t}}{dt} \propto A_{n,t}^2 \nu_{n,t}.$$  (4)

These parameters provide information about different phenomena. Thus, the linewidth is a direct measure of the damping rate while the velocity power represents the equilibrium between the excitation and the damping of the modes. The energy supply rate contains information about the excitation of the modes.

The extraction of the mode parameters have been done as follows: pairs of even and odd degree modes were fitted together, because the presence of the 11.57 µHz side-lobes which complicate the structure of the power spectrum. The rotational splitting is taken to be constant and equal to 400 nHz. We constrained the linewidth to be the same for all the components of the two multiplets. The relative m-components height ratio $A_{n,t}/A_{n,1}$ are constrained to take the theoretical value for an instrument without spatial resolution for the case of observations using sodium line (Christensen-Dalsgaard, 1989). The p-mode parameters are then extracted by minimizing a standard maximum likelihood function with a $\chi^2$ distribution. The natural logarithm of the amplitudes, linewidths and background noise have been fitted and not the straightforward parameters themselves to give a normal fitting distribution. Doing this, the uncertainties on each parameter can then be determined from the inverted Hessian matrix.

Due to the effect of a non-ideal window function the power is redistributed from the main lobe into side-lobes and into the background. Therefore, the amplitude of a peak in the power spectrum depends on the characteristics of the window function, especially the duty cycle. To simulate the effect of the window we firstly computed a window function for the time series normalized it to unity. Then this was convolved with a Lorentzian profile with half-width of 0.5 µHz and unit amplitude. Finally, the simulated spectrum was fitted to determine the peak amplitude, the side-lobe amplitude, and the linewidth of the Lorentzian to determine the correction factors.

The results confirm that the fitting procedure overestimate the linewidths and underestimate the amplitudes, this effect being larger on the amplitudes than on the linewidths.

4. RESULTS

4.1. Time variation

Once the amplitudes and the linewidths are corrected from the window function effect, we have computed the mean values of each parameter for those modes in the frequency range between 2.6 and 3.6 µHz. The difference with a reference taken as the average of 6 consecutive power spectra during the solar minima (1994-1997), weighted by the uncertainties, is calculated. Thus we obtain the fractional changes in percent along the 11 years analyzed.

We have also fitted the time variation of each parameter as linear function of the corresponding mean value of the Radio Flux to each time series. The slope of this fit multiplied by the difference of Radio Flux at extreme phase of the solar activity gives us an estimation of the global change. The global change for each parameter as well as the correlation factor and the level of significance are shown in Table 1.

Figure 1 shows the well known frequency shifts changes over the solar activity. The solid line corresponds to the mean value of degrees $0 \leq \ell \leq 3$ for each order $n$, weighted by the uncertainties. A global change of $0.35 \pm 0.01$ µHz between the maximum and the minimum of solar activity is found. The upper panels of Fig. 2 shows the amplitude and the linewidth variations respectively. The evolution with the solar activity is very clear, the amplitudes being well anti-correlated and the linewidths being correlated with the solar activity cycle. We found a global changes of $-25.76 \pm 1.85\%$ for the amplitudes and $14.19 \pm 1.08\%$ for the linewidth. Regarding the velocity power (also shown in Fig. 2), it is well anti-correlated with the solar activity, showing a variation of $-11.61 \pm 1.40\%$. As for the energy supply rate, even a slight positive variation of $2.12 \pm 1.68\%$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Global change</th>
<th>$r_S$</th>
<th>$P_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{n,t}$</td>
<td>$0.35 \pm 0.01$ µHz</td>
<td>0.82</td>
<td>3.63e-4</td>
</tr>
<tr>
<td>$A_{n,t}$</td>
<td>$-25.76 \pm 1.85%$</td>
<td>0.82</td>
<td>2.36e-4</td>
</tr>
<tr>
<td>$\Gamma_{n,t}$</td>
<td>$14.19 \pm 1.08%$</td>
<td>0.82</td>
<td>3.54e-4</td>
</tr>
<tr>
<td>$V_{n,t}$</td>
<td>$-11.61 \pm 1.40%$</td>
<td>-0.83</td>
<td>3.12e-4</td>
</tr>
<tr>
<td>$E_{n,t}$</td>
<td>$2.12 \pm 1.68%$</td>
<td>-0.09</td>
<td>0.69</td>
</tr>
</tbody>
</table>

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Figure 2. Relative differences versus time for the amplitude, linewidth, velocity power and energy rate respectively.

Figure 3. Relative mode frequency differences per unit of Radio Flux versus frequency.

is found, we can consider that no variation is observed since the correlation factor is practically null. Therefore we conclude the energy rate remains constant with the solar activity.

4.2. Frequency dependence

We now study the frequency dependence. To do so, a weighted linear fit is computed at each (n, l) between each parameter and the corresponding values of the mean Radio Flux. The gradient of the fit represents the variation of the parameter per unit of the Radio Flux used here again as a proxy of the solar cycle. In Fig. 3 the frequency shifts versus frequency can be seen. It shows an important increase with frequency between 2.5 mHz and 3.6 mHz; below 2.5 mHz the frequency shifts are small but positive. A change of about 4 mHz/RF at 3.6 mHz is observed. The upper panels of Fig. 3 shows the changes obtained for the amplitudes and the linewidths respectively over the frequency range from 1.8 mHz to 3.8 mHz. The amplitudes are clearly negative, as for the linewidths, they are clearly positive implying an increase in the damping rate with solar activity. The lower panels of Fig. 4 illustrate the changes in the velocity power and in the energy supply rate respectively. The velocity power decreases with an increasing solar activity, whereas no variation is observed in the energy supply rate, meaning that the net forcing function remains constant with solar activity.

The frequency dependence in these changes is less clear, except for the frequency shifts. But the changes in amplitude, linewidth and velocity power seems to reach a maximum between 2.6 and 3.1 mHz, peaked near ~ 2.8 mHz. Only the energy supply rate seems to be not frequency dependent.

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5. CONCLUSION

The merging of IRIS, Mark-I and LOWL database has provided 11 years of full disk observations. The reasonably good quality of the data, the annual duty cycles of around 60% and the extent of it covering a complete solar cycle, make this database particularly suitable to exploit the variation of the p-mode parameters.

Using time series of one year period with an overlapping of six months we have analyzed the p-mode parameter changes along the solar activity cycle. Besides the well known frequency shift, we found also clear evidences of the p-mode amplitude and linewidth variations, uncovering a global change of about −26% and 14% respectively correlated with the solar activity cycle. The frequency dependence is less clear in both parameters, even a maximum near 2.8 mHz seems to be discerned.

By means of these two parameters, we have calculated the velocity power of the modes as well as the energy supplied to them. In the case of the velocity power, we found a change of about −12% while the energy rate does not show an important variation correlated with the solar cycle, concluding that this remains constant with solar activity.

The results reported in this analysis are in very good agreement with those found recently by Jiménez-Reyes et al. (2002) using data from two different space experiments, VIRGO and GOLF on-board the SoHO spacecraft.

REFERENCES

SUPERGRANULAR WAVES OBSERVED USING MDI SURFACE DOPPLER SHIFT DATA

Jesper Schou
Stanford University, HEPL Annex A201, Stanford, CA 94305-4085, USA, jschou@solar.stanford.edu

ABSTRACT

Recently Gizon, Duvall and Schou (2002) suggested that supergranulation has a wave-like component. Here I show that the phenomenon can also be observed using surface Doppler shift data and extend their results. I also show results for rotation and meridional flows beyond ±70° latitude inferred using the supergranular waves and look for temporal variations in the various properties.

Key words: Supergranulation.

1. INTRODUCTION

Since supergranulation was discovered more than 40 years ago by Leighton, Noyes and Simon (1962) the details of the physical origin of the phenomenon have remained elusive. Gizon, Duvall and Schou (2002, in the following GDS, see also Gizon and Duvall, 2003) found evidence that there is a wave like component to the supergranulation and recently Schou (2002) confirmed this using surface Doppler shift data. Before GDS the rotation rates and meridional flows inferred using the supergranulation were different from those inferred by other means, and in some cases difficult to reconcile with other measurements. With the realization that there is a wave like component it is now possible to use the supergranulation to infer flows which appear to be more physically reasonable. In addition to demonstrating that the wave like phenomenon is visible in surface Doppler shift, this paper presents a first attempt at using the waves to look for temporal variations in the inferred flows and for variations in other properties of the waves.

2. RESULTS

The results shown here were obtained using 60 days of MDI full disk Doppler shift data from each of the 1996, 1997 and 1998 dynamics runs. These are the same data used in Beck and Schou (2000, in the following BS), which provides further details. The 1996 data were also used by GDS. These data were first detorated and averaged over 1 hour intervals to remove the f- and p-mode signals. Strips in longitude with a length of ≈ 150° and a width of 10° in latitude were then extracted, multiplied by a weighting function, tracked, padded and passed through a 3 dimensional Fourier transform. The results from one choice of weighting function, averaged over frequency and degree, are shown as a function of azimuth in Figure 1. This figure shows that there is more prograde power than retrograde power, as was seen by GDS. As can also be seen there is significantly more power in the longitudinal direction than in the latitudinal direction. Compare this with GDS, who used a time-distance method sensitive to both components of the horizontal velocity, and who saw significant power in the latitudinal direction at the equator. This indicates that the displacement is predominantly in the direction of propagation (for further details see Schou, 2002).

That the displacement is predominantly in the direction of propagation means that the observed Doppler shift from waves traveling in longitude will change sign as they cross the the central meridian and that waves traveling in latitude will not. In the following only waves traveling in (or near) these two directions will be considered. For the waves traveling in longitude the weighting function contains a sign flip at the
Figure 2. Left: Cut in the 3 dimensional power spectrum at the equator and around $k_\parallel = 0$ (waves traveling near the longitude direction). The grayscale is logarithmic with black 300 times the power of white. Right: Cut at $40^\circ$ latitude around $k_\perp = 0$ (waves traveling near the latitude direction). These plots were made using the average of the 1996, 1997 and 1998 data.

The advection of a wave pattern will, at a given azimuth, lead to two peaks in the power spectra at frequencies $\omega_\pm = \pm \omega_0(k) + kv$, where $\omega_0 = 2\pi v_\theta$ is the oscillation frequency, $k$ is the wavenumber (given by the degree $l = kR_\odot$, where $R_\odot$ is the solar radius) and $v$ is the component of the velocity in the direction of propagation. A model consisting of two Lorentzians and a background was thus fitted to the spectra obtained by, at each $k$, averaging over a small range ($\pm 15^\circ$) of propagation angles. From these fits the oscillation frequency $\omega_0$ and flow velocity may be determined.

Figure 3 shows $\omega_0(k)$ for waves near the equator. Note that the frequencies appear to follow a power law well. Points below $l \approx 30$ are likely to be unreliable.

Figure 4 shows the rotation rate at the equator as inferred from the fits.

Finally Figures 5 and 6 show the rotation rate and meridional flow inferred from fitting the power spectra for the three different years.

3. DISCUSSION

As can be seen, and as was shown in Schou (2002), the waves observed in the time distance analysis are easily visible in the raw Doppler shift data and the results generally confirm those in GDS. This is significant since there was some concern based on the fact that the phenomenon had not been observed previously.
It is interesting that the dispersion relation is so close to a square root and that it appears to depend relatively little on latitude and direction. However, it is clear from Figure 3 that there is some dependence on the latitude, especially for 1998. That the latitude variation varies with time is somewhat suspicious and could indicate that it is an artifact of the data analysis. However, whether these variations are real or an artifact remains to be shown.

The rotation rate shown in Figure 4 is much less \( l \) dependent than that previously measured from the same data in BS (which had a variation of the order 10mHz between \( l=50 \) and \( l=200 \)). This is consistent with a weak variation of the rotation rate with depth and allows for the numbers to be averaged over \( l \) as done in the following figures.

The inferred rotation rate is remarkably close to the magnetic tracer rate of Komm, Howard and Harvey (1993). Given that the advection of the magnetic field at the surface is closely associated with the supergranulation, this is not very surprising. But it is in stark contrast to the conclusions from earlier measurements of the supergranular rotation rate.

The three years show very similar rotation rates, which is again not very surprising. The main difference is in the stability of the fits at high latitude. The cause of this has not been determined, but could be related to the different B0-angles.

To show the differences more clearly a (common) polynomial in \( \sin(\text{latitude})^2 \) was subtracted from each year. Results are shown in Figure 7. With some imagination the effect of the torsional oscillation can be seen in the movement towards the equator of the maximum around 20°. These residuals are, by the way, similar to those shown in Schou and Beck (2001), except for the poorer latitude resolution here.

The meridional flow results appear to be quite stable up to around 75° latitude with a clear turnover around 20° latitude. It is interesting to note that the meridional flow is not perfectly antisymmetric across the equator. As shown this is largely explained by a P-angle error caused by the misalignment of the MDI instrument on the SOHO spacecraft (C. Toner, private communication) and by the difference between

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Figure 7. Top: The rotation rates from Figure 5 with a fit of a second order polynomial in \(\sin(\text{latitude})^2\) subtracted. Bottom: Same plot symmetrized.

Figure 8. The results from Figure 6 corrected for the effect of the incorrect P-angle.

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ACKNOWLEDGMENTS

The author thanks Laurent Gizon and Tom Duvall for helpful discussions. The SOHO SOI/MDI project is supported by NASA grant NAG5-10483 to Stanford University. SOHO is a project of international cooperation between ESA and NASA.
AUTOCORRELATION ANALYSIS OF MDI HIGH-FREQUENCY DATA

T. Sekii\textsuperscript{1}, H. Shibahashi\textsuperscript{2}, and A.G. Kosovichev\textsuperscript{3}

\textsuperscript{1}Solar Physics Division, National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan
\textsuperscript{2}Department of Astronomy, University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{3}W.W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA

ABSTRACT

We have analyzed MDI data using time-distance autocorrelation function, in a high-frequency range above the acoustic cut-off frequency of the solar atmosphere. The MDI velocity, intensity and line-depth signals were looked at. The wave reflection rate at the photosphere has been found to be around 10 per cent for \( l = 125 \) and \( v = 6.75 \mu \text{Hz} \). The result is compared with a previous measurement.

1. INTRODUCTION

It has been more than a decade since the so-called high-frequency interference peaks (HIPs) were found in solar oscillation spectra above the atmospheric cut-off frequencies (Jeffries et al. 1988, Duvall et al. 1991; also see Jeffries 1998). The peaks cannot be due to eigenscissors since the sun does not provide cavity for such high-frequency acoustic waves. The presence of the peaks were then explained by interference of waves emitted from acoustic sources in a radially thin layer (Kumar et al. 1990).

Jeffries et al. (1997) used the same model to analyze and interpret their 18-day long South Pole data to find evidence of partial reflection at the photosphere (cf. Garcia et al. 1998). An extra reflecting layer in the chromosphere was also required to explain the double-ridge structure in their time-distance autocorrelation. They also measured the reflection rates, which were 13–22 per cent at the photosphere and 3–9 per cent at the chromospheric layer. The reflection rates, for waves with given property, are determined by the local thermal structure. For the photospheric case, convective energy transport in the upper convection zone plays an important role. Detailed measurement of the photospheric reflection rate can be used as a test for the mixing-length and other convection theories.

The height of the chromospheric reflecting layer was estimated from the separation of the main and the satellite feature in the double-ridge structure of the time-distance autocorrelation. The layer is \( \sim 1000 \text{km} \) above the photosphere, compared to 2000km, the standard figure for the thickness of the chromosphere. Why there is a reflecting layer in the middle of the chromosphere remains largely unexplained. Further study of this layer might be useful or even essential in understanding the solar chromosphere.

From what in the above we have decided that it is worthwhile to repeat a similar analysis on MDI velocity, intensity and line-depth data.

2. ANALYSIS OF MDI DATA

The datasets we analyzed are spherical-harmonic time-series for velocity, intensity and line depth signals. The velocity and intensity datasets are both
61-day long and coeval. The line-depth set is 63-day long. First, m-averaged power spectra, corrected for rotational splitting, were produced up to \( l = 200 \).

Figures 1 and 2 show m-averaged power spectra for \( l = 125 \) obtained from velocity signal and line-depth signal, as well as wide-range (200–8000 \( \mu \)Hz) temporal autocorrelation functions (ACFs) and high-frequency (5500–7000 \( \mu \)Hz) temporal autocorrelation functions. The ACFs were obtained simply by Fourier transform of the power spectra after applying a box-car frequency filter. The wide-range temporal ACFs are dominated by 5-minutes band signals and indeed, the spacing between the strong (amplitudes of) correlations is consistent with the asymptotic return trip time for \( l = 125 \), \( \nu = 3000 \mu \)Hz wave. The temporal ACFs for 5500–7000 \( \mu \)Hz range are the basis for our current study. For the line-depth case, there is an increase in correlation, although not very strong, for the time delay that corresponds to the wave return-trip time for \( l = 125 \) and \( \nu = 6250 \mu \)Hz (the centre of the frequency range).

By comparing the power spectra for velocity signal and line-depth signal, one notes that the underside of both spectra have broad peaks around 3000 \( \mu \)Hz but that line-depth spectrum also has an even broader second peak between 5000 \( \mu \)Hz and 5500 \( \mu \)Hz. This is probably a signature of chromospheric mode previously reported by Harvey et al. (1993). The signature is visible neither in the velocity signal nor in the intensity signal (now shown). This implies that for studying waves in the chromosphere, the line-depth data are the best among the three classes of the MDI oscillation data. The reason is not well understood.

Figure 3 shows another set of velocity power spectrum and two ACFs for \( l = 18 \). Obviously m-averaging is not working too well. Aside from that, the wide-range temporal ACF shows an interesting behaviour: the correlation decays very quickly and then comes back. In fact the behaviour was shown by \( l = 125 \) case too, to a lesser degree. There, the correlation decays too quickly compared to the p-mode linewidths in 3000 \( \mu \)Hz range. In fact the correlation comes back much later.

This behaviour of the temporal ACF is because of what we call geometrical factor. In terms of the simplest of the asymptotic pictures, a wave packet starts at a point on the surface travels towards the centre of the sun, but refracts back eventually to the surface to reach another point on the surface. The angular distance between the two surface points may be called skip angle. Observed ACFs are affected by how many pairs of points, separated by the corresponding skip angle, are seen on the visible side of the sun. If the angle is small, there are many pairs and the correlation signal can be strong. For the \( l = 18 \) case in the above, the skip angle for \( \nu = 3000 \mu \)Hz is close to 90° which is already large enough to damp the correlation significantly. For a pair of points separated by two skips, the skip angle is \( \sim 180° \), which means essentially no pair is visible. This explains the almost total lack of signal for the second skip location. For a triple skip, the skip angle is \( \sim 270° \), so the amplitude is as strong as that for a single skip, and for a quadruple skip the angle is \( \sim 360° \) to enable the coming back of the correlation amplitude. The \( l = 125 \) case can be similarly explained; the effect of dispersion seems not very important during the first few skips.

The rate of this artificial damping we call geometrical factor. Figure 4 shows asymptotic geometrical factors for intensity signal and (line-of-sight) velocity signal, the latter being affected by projection an-
Figure 4. Asymptotic geometrical factor for intensity (upper curve) and velocity (lower curve) signals. Apodization is not taken account.

Figure 5. The demodulated time-distance autocorrelation function for 63-day MDI line-depth data.

Figure 6. Slices at the travel time of 80 minutes and 160 minutes, taken from the autocorrelation function in Figure 5 (envelope), and the correlation before demodulation.

Table 1. The wave reflection rate for velocity (V), intensity (I) and line-depth (LD) signals. The amplitudes are compared at the travel time of 70 minutes and 140 minutes, 80 minutes and 160 minutes, and 90 minutes and 180 minutes.

<table>
<thead>
<tr>
<th>Pairing</th>
<th>V</th>
<th>I</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>70/140</td>
<td>9.7%</td>
<td>9.4%</td>
<td>10.3%</td>
</tr>
<tr>
<td>80/160</td>
<td>9.1%</td>
<td>9.0%</td>
<td>10.2%</td>
</tr>
<tr>
<td>90/180</td>
<td>9.4%</td>
<td>8.1%</td>
<td>9.8%</td>
</tr>
</tbody>
</table>
4. DISCUSSION

It is not clear if the geometrical factor was taken into account in the analysis of the South Pole data by Jefferies et al. (1997). If it was not, then the reflection rate must have been underestimated since for the second-skip ridge the skip angle is larger (essentially twice the skip angle for the first) and the geometrical factor is smaller. We found, however, reflection rates that were smaller than the South Pole figures. This was unexpected.

It is unlikely that the unexpected rates were due to solar cycle variation. The South Pole data were acquired during December 1994 – January 1995 period. The MDI velocity and intensity data were from April – June 1997, and the MDI line-depth data were from May – July 1996. We do not think the difference in the solar-cycle phase is large enough.

The most likely reason is that this is because of our failure to separate the main feature (due to the photospheric reflection) and the satellite feature (due to the chromospheric reflection). Our ridges are mixtures of these two, and by comparing the first-skip ridge and the second-skip ridge we obtain only a certain average of the photospheric and the chromospheric reflection rates.

Even if above is correct, we still have to understand why we failed to separate the main and the satellite features. It could be due to the fact the MDI uses a photospheric Ni line. The South Pole observation used K line, which is chromospheric and therefore better suited for studying high-frequency wave. We have started to study TON data (see, e.g., Chou et al, these proceedings), which are also from observation of K line.

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We thank Tom Duvall, Stewart Jefferies, Jesper Schou and Phil Scherrrer for useful discussions.

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ON AVERAGED TIME-DISTANCE AUTOCORRELATION DIAGRAMS

T. Sekii\textsuperscript{1} and H. Shibahashi\textsuperscript{2}

\textsuperscript{1}Solar Physics Division, National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan
\textsuperscript{2}Department of Astronomy, University of Tokyo, Tokyo 113-0033, Japan

ABSTRACT

To calculate time-distance autocorrelation for local-helioseismic analyses, the correlation function is often put in a form which permits application of the convolution theorem so that CPU-intensive direct integration is alleviated. It is normally justified by a statistical argument with a certain assumption on properties of power distribution of the wavefield; the results are then interpreted as ensemble averages of the autocorrelation. However, the same results can also be obtained by taking spatial averages without any assumption on the power distribution, thereby providing a different interpretation to the results obtained through convolution theorem. It is straightforward to demonstrate the above in the case of wavefield in infinite two-dimensional space. Here we present a direct demonstration of the same equivalence of the averages in the case of spherical geometry, which has turned out to be less straightforward.

1. INTRODUCTION

One of the most successful methods of local helioseismology is based on inversion of time-distance diagram (Kosovichev 2002), which is calculated through autocorrelation of observed wavefield.

Ultimately, local helioseismology might not be concerned with any kind of average. In practice, however, some kind of averages are almost always taken to improve signal-to-noise ratio. Certain kind of average can also be used as a reference, local deviation from which is then measured. Averages were used for demonstration purpose, too.

Here we concentrate on the cases where one calculates two-dimensional autocorrelation, i.e., autocorrelation of wavefields as a function of travel distance and travel time. Since the wavefield here is observed on two-dimensional solar surface, this requires integration in time and in direction of the travel. A direct integration is highly CPU intensive, though in time domain we can almost always apply convolution theorem so that we can use FFT to reduce the flop count. The autocorrelation function can be expressed as a Fourier-type transform of wave power in the spatial domain as well. In the next section describe how this is done, for the case for infinite two-dimensional space, first by taking ensemble average, which seems to be how it is done by most of the community, and then by taking spatial average. We then note that these two are in fact equivalent. In section 3 we demonstrate the same equivalence for the case of spherical geometry. In section 4 the results are summarized and implications are discussed.

2. INFINITE TWO-DIMENSIONAL SPACE

We denote observed wavefield by \( u(x, t) \), where \( x \) is position vector and \( t \) is time. Two-point correlation for displacement vector \( d \) and time delay \( \tau \), measured at position \( x \), is defined as

\[
C(x, d, \tau) \equiv \int dt u^*(x, t)u(x + d, t + \tau). 
\]

Through out the paper, integral is taken over the entire range unless otherwise stated, and in the above the range is \((-\infty, \infty)\). By using the following expression, based on Fourier transform \( a(\omega, k) \):

\[
u(x, t) = \int d\omega \int dk a(\omega, k)e^{i(k \cdot x - \omega t)},
\]

the two-point correlation is now

\[
C(x, d, \tau) = \int dt \int d\omega \int dk \int d\omega' \int dk' a^*(\omega, k)a(\omega', k')
\times e^{i[(k' - k) \cdot x - (\omega' - \omega)t + k' \cdot d - \omega' \tau]}.
\]

By noting

\[
\int e^{-i(\omega' - \omega)t} dt = 2\pi \delta(\omega' - \omega)
\]

this is further reduced to the form

\[
C(x, d, \tau) = 2\pi \int d\omega \int dk \int dk' a^*(\omega, k)a(\omega, k')
\times e^{i[(k' - k) \cdot x + k' \cdot d - \omega \tau]}.
\]

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The following assumption on ensemble averages:

\[ \langle a^*(\omega, k)a(\omega, k') \rangle \propto \delta(k - k')|a(\omega, k)|^2, \]

which has been known to be adequate for real data, is equivalent to the assumption of statistical translational invariance, i.e., to the assumption that \( \langle u^*(x, t)u(x + d, t') \rangle \) is independent of \( x \). In fact, from this requirement, through the inverse Fourier transform

\[ a(\omega, k) = \frac{1}{(2\pi)^3} \int dx \int dt u(x, t)e^{-i(kx - \omega t)}, \]

we obtain

\[ |a(\omega, k)|^2 = \frac{1}{(2\pi)^2} \int dx \int dk |a^*(\omega, k)a(\omega, k')| \]

We can therefore somewhat symbolically write

\[ \langle a^*(\omega, k)a(\omega, k') \rangle = \frac{(2\pi)^2}{S} |a(\omega, k)|^2 \delta(k - k'), \]

where \( S \) is the (large enough) area of spatial integration (we have been considering the limit of \( S \to \infty \) which is obviously inconvenient here; here we step back a little). By using this relation, ensemble average of the two-point correlation is

\[ \langle C(x, d, \tau) \rangle = \frac{(2\pi)^3}{S} \int d\omega \int dk |a(\omega, k)|^2 e^{i(kd - \omega \tau)}, \]

which does not depend on \( x \). After statistical averaging, the time-distance autocorrelation is Fourier transform of the observed power spectrum.

On the other hand, the spatial integral of the two-point correlation is calculated to be

\[ \tilde{C}(d, \tau) = \frac{1}{S} \int dx \langle C(x, d, \tau) \rangle \]

\[ = \frac{(2\pi)^3}{S} \int d\omega \int dk |a(\omega, k)|^2 e^{i(kd - \omega \tau)}. \]

The two formulae are identical, once the observed power is identified to its ensemble average.

The final two-dimensional autocorrelation function is obtained by the further averaging on the direction angle \( \Theta \) of \( d \):

\[ \tilde{C}(d, \tau) = \frac{1}{2\pi} \int d\Theta \tilde{C}(d, \tau) \]

\[ = 4\pi \int d\omega \int dk |a(\omega, k)|^2 k J_0(kd) e^{-i\omega \tau}. \]

Here \( J_0 \) is a Bessel function and

\[ |a(\omega, k)|^2 = \frac{1}{2\pi} \int |a(\omega, k)|^2 d\Theta, \]

where \( \Theta \) is the angle of \( k \).

3. SPHERICAL GEOMETRY

Let us move on the case of spherical geometry, which is more relevant to local helioseismology. We assume, however, that wavefield is observed on the entire surface of the sun, rather than on the visible side only.

We introduce Fourier/Spherical-Harmonic expansion of the wavefield:

\[ u(\theta, \phi, t) = \int d\omega \sum_{l,m} a_{lm}(\omega) Y_{lm}(\theta, \phi)e^{-i\omega t}, \]

where \( \theta \) and \( \phi \) are spherical coordinates and \( Y_{lm}(\theta, \phi) \) is the spherical harmonics of degree \( l \) and azimuthal order \( m \). We then take two-point correlation between \( (\theta, \phi) \) and \( (\theta', \phi') \), which are separated by angular distance \( \Theta \). The azimuthal angle of the latter, with respect to the former, is denoted by \( \Phi \), where the angle is measured from the meridian that pass through the former (Figure 1). Then the two-point correlation is

\[ C(\theta, \phi, \Theta, \Phi, \tau) \]

\[ \equiv \int du^*u(\theta, \phi, t)u(\theta', \phi', t + \tau) \]

\[ = 2\pi \sum_{l,m' \neq 0} \left[ \int d\omega a_{lm}^*(\omega)a_{lm'}(\omega) \right. \]

\[ \times Y_{lm}^*(\theta, \phi)Y_{lm'}(\theta', \phi')e^{-i\omega \tau}. \]

The following assumption on ensemble averages

\[ \langle a_{lm}^*(\omega)a_{lm'}(\omega) \rangle = \delta_{ll'}\delta_{mm'}|a_{lm}(\omega)|^2 \]

produces

\[ \langle C(\theta, \phi, \Theta, \Phi, \tau) \rangle = 2\pi \sum_{l,m} \int d\omega |a_{lm}(\omega)|^2 \]

\[ \times Y_{lm}^*(\theta, \phi)Y_{lm}(\theta', \phi')e^{-i\omega \tau}. \]

Let us note that

\[ \sum_{m} Y_{lm}^*(\theta, \phi)Y_{lm}(\theta', \phi') = \frac{2l + 1}{4\pi} P_l(\cos \Theta). \]
Then application of statistical isotropic condition (in practice, this is valid for $m$-averaged power spectra after they are corrected for rotation)

$$\langle |a_{lm}(\omega)|^2 \rangle = \langle |a_{l}(\omega)|^2 \rangle ,$$

leads to

$$\langle C(\theta, \phi, \Theta, \Phi, \tau) \rangle = \sum_{l} (l + 1/2) \int d\omega \langle |a_{l}(\omega)|^2 \rangle P_{l}(\cos \Theta) e^{-i\omega \tau} ,$$

which does not depend on $(\theta, \phi)$ (nor $\Phi$). Note that a similar formula for the case of theoretical time-distance correlation, taking account of eigenscillation components only, is derived from the assumption that there is no frequency degeneracy (Kosovichev & Duvall 1997).

Calculating spatial average requires rotating the coordinate system. The original pole of the spherical coordinate is at the position $P$. We first rotate the coordinate around its axis by angle $\phi$, so that the point $A$ is now on the prime meridian. Then we rotate the coordinate along PA by angle $\theta$ so that the pole moves to the point A. Now the coordinate of the point B changes from $(\theta', \phi')$ to $(\Theta, \Phi)$, and the relation between the spherical harmonics before and after the rotation is given by

$$Y_{l'}m'\phi'(\theta', \phi') = \sum_{m''} d'_{m'm''}(\theta, \phi) Y_{l''}m''(\Theta, \Phi) e^{im\phi'} ,$$

where $d'_{m'm''}(\theta)$ is related to a Wigner D-function (Edmonds 1960):

$$d'_{m'm''}(\theta) = D_{m'm''}(0, \theta, 0) .$$

We also introduce

$$\gamma_{lm} = \frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!} ,$$

so that

$$Y_{lm}(\theta, \phi) = \gamma_{lm} Y_{l}^{m}(\cos \theta) e^{im\phi} .$$

Then the spatial average is ($d\Omega = \sin \theta d\theta d\phi$)

$$\langle \tilde{C}(\tau) \rangle = \frac{1}{4\pi} \int d\Omega \langle C(\theta, \phi, \Theta, \Phi, \tau) \rangle$$

$$= \frac{1}{2} \sum_{lm} \int d\omega a_{lm}(\omega) a_{lm}^{*}(\omega) Y_{lm}(\Theta, \Phi) \times \gamma_{lm} \int d\phi Y_{lm}(\Theta, \Phi) e^{im\phi} \times e^{-i\omega \tau}$$

$$= \pi \sum_{lm} \int d\omega a_{lm}(\omega) a_{lm}(\omega) Y_{lm}(\Theta, \Phi) \times \gamma_{lm} \int d\phi Y_{lm}(\Theta, \Phi) e^{im\phi} \times e^{-i\omega \tau} ,$$

where the contribution from $m' \neq m$ dropped because of the integration over $\phi$. By taking angular average over $\Phi$, we can further drop any contribution from $m'' \neq 0$:

$$\tilde{C}(\tau) = \frac{1}{2\pi} \int d\Phi \tilde{C}(\Phi, \tau)$$

$$= \pi \sum_{lm} \gamma_{lm} \gamma_{l'm'} \int d\omega a_{lm}(\omega) a_{l'm'}(\omega) \times P_{l'}(\cos \Theta) \times \int Y_{l'm'}^{*}(\theta) Y_{l'm'}(\cos \theta) \sin \theta d\theta e^{-i\omega \tau} .$$

Since

$$d'_{l'm'}(\theta) = \sqrt{(l' - m')! (l' + m')!} P_{l'}(\cos \theta)$$

(Edmonds 1960), we have

$$\int d\phi Y_{l}^{m}(\cos \theta) \sin \theta d\theta = \delta_{l+l'} \frac{2}{2l+1} \sqrt{(l + m)! (l - m)!} .$$

Therefore the spatial average is

$$\tilde{C}(\tau) = \pi \sum_{lm} \int d\omega a_{lm}(\omega)^{2} P_{l}(\cos \Theta) e^{-i\omega \tau}$$

$$= \frac{1}{2} \sum_{lm} \int d\omega a_{lm}(\omega)^{2} P_{l}(\cos \Theta) e^{-i\omega \tau} .$$

By defining

$$|a_{l}(\omega)|^{2} = \frac{1}{2l + 1} \sum_{m} |a_{lm}(\omega)|^{2} ,$$

we have the final form

$$\tilde{C}(\tau) = \sum_{l} (l + 1/2) \int d\omega |a_{l}(\omega)|^{2} P_{l}(\cos \Theta) e^{-i\omega \tau} ,$$

which is identical to $\langle C(\theta, \phi, \Theta, \Phi, \tau) \rangle$ if we identify the observed power to its ensemble average.

4. SUMMARY AND DISCUSSION

Time-distance autocorrelation function is often expressed as multi-dimensional Fourier-type transform of observed power spectrum, and this form permits faster computation. The expression is often derived through a statistical argument, in which the observed power spectrum is identified to its ensemble average, augmented by the assumption of statistical translational invariance. We, in the above, demonstrated that the same result can be obtained by spatially averaging the autocorrelation.
Where the equivalence comes from is obvious: it arises from the assumption of the statistical translational invariance, as a result of which a local quantity is statistically equal to its spatial average.

The result presented here can be obtained for fully discrete (and finite domain) cases as well, with a normal implicit assumption of periodicity in the wavefield, both in temporal and spatial domains.

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A METHOD FOR INVERTING HIGH-DEGREE MODES

Brooke Simmons and Sarbani Basu
Astronomy Department, Yale University, P. O. Box 208101, New Haven CT 06520-8101, U. S. A.
email: simmons@astro.yale.edu

ABSTRACT

In order to invert solar oscillation frequencies, the oscillation equation is linearized around a solar model. However, this linearization does not account for frequency differences due to near-surface uncertainties in the model. The usual method to deal with this is to add a slowly varying function of frequency to the inversion equation. This approach does not work for high-degree modes, because the surface-related uncertainty in the frequencies is no longer just a function of frequency. We present here a modified inversion equation that can be used to invert high-degree modes. This work is motivated by the desire to invert the high-degree mode frequencies obtained by ring diagram analyses.

Key words: Sun: oscillations; Sun: rotation; Sun: interior.

1. INTRODUCTION

Inversion for solar structure involves linearization of the oscillation equation around a solar model. The usual equation relates the relative differences between the frequencies of the Sun and the model, \( \delta \omega_i / \omega_i \), to the differences (\( \delta c^2/c^2 \), \( \delta \rho/\rho \)) in sound speed \( c \) and density \( \rho \), through the following equation (Antia 1995, Di Mauro et al. 2002, Rabello-Soares et al. 1999):

\[
\frac{\delta \omega_i}{\omega_i} = \int_0^1 K^{c^2}_{\lambda} (x) \frac{\delta c^2}{c^2} (x) \, dx + \int_0^1 K^{\rho}_{\lambda} (x) \frac{\delta \rho}{\rho} (x) \, dx + \frac{F_{\text{surf}}}{Q_i} + \epsilon_i,
\]

where \( x = r/R_\odot \) is the dimensionless radial distance from the Sun’s center, \( R_\odot \) is the solar photospheric radius, \( Q_i \) is the normalized mode inertia, and \( \epsilon_i \) are the observational uncertainties in the data. Each \( i \) corresponds to an \( (n,l) \) pair. The kernels \( K^{c^2}_{\lambda} \) and \( K^{\rho}_{\lambda} \) are calculated using the oscillation eigenfunctions of the reference model.

The surface term, \( F_{\text{surf}} \), is usually written as a function of only frequency, e.g.

\[
F_{\text{surf}} = \sum_{\lambda=0}^{\Lambda} b_\lambda P_\lambda (\omega),
\]

where \( P_\lambda \) are Legendre polynomials of degree \( \lambda \). However, the approximation that \( F_{\text{surf}} \) is independent of degree only holds for low- and moderate-degree modes. When high-degree modes are present, the approximation breaks down, and \( F_{\text{surf}} = f (\omega, l) \).

2. THE SOLA METHOD

The sound-speed inversion is carried out using the Subtractive Optimally Localized Averages (SOLA) method described by Pijpers & Thompson (1994). The application of this method to structure inversions can be found in Rabello-Soares et al. (1999). The technique consists of finding a set of coefficients \( c_i (r_0) \) such that the averaging kernel,

\[
K (x_0, x) = \sum_{i \in M} c_i (x_0) K^{c^2}_{i,\lambda} (x),
\]

peaks around \( x = x_0 \) and is small everywhere else; and that the cross-term kernel,

\[
C (x_0, x) = \sum_{i \in M} c_i (x_0) K^{\rho}_{i,\lambda} (x),
\]

a quantity which measures the contribution of \( \delta \rho/\rho \) to the \( \delta c^2/c^2 \) solution, is as small as possible. In the above equations, \( M \) is the set of observed solar oscillation modes being used, so that \( i \) again corresponds to pairs of \( (n,l) \) values.

The averaging kernel is used in all OLA methods, and the SOLA method specifically seeks to make the
averaging kernel resemble as closely as possible a target kernel \( T(x_0, x) \), which usually has the form of a Gaussian. The SOLA method minimizes the difference between the two kernels while also moderating the effect of errors in the data, by minimizing

\[
\Psi(x_0, x) = \int_0^1 [K(x_0, x) - T(x_0, x)]^2 \, dx + \beta \int_0^1 C^2(x_0, x) f(x) \, dx + \mu \sum_{ij} E_{ij} c_i c_j,
\]

(5)

where \( E_{ij} \) is the error covariance matrix of the observed frequencies, \( f(x) \) is designed to suppress the cross-term at the surface (We have used \( f(x) = (1 + x)^4 \)), and \( \beta \) and \( \mu \) are trade-off parameters that determine the relative importance of minimizing the first, second, and third terms in expression (5). The minimization is also subject to the conditions

\[
\int_0^1 K(x_0, x) \, dx = 1
\]

(6)

\[
\sum_i c_i P_\lambda(\omega_i) = 0
\]

for \( \lambda = 0, \ldots, \Lambda \).

Once these conditions are satisfied, then

\[
\left\langle \frac{\delta^2}{c^2} \right\rangle_{r_0} = \sum_i c_i \left( r_0 \right) \frac{\delta \omega_i}{\omega_i},
\]

(7)

where \( \left\langle \delta^2/c^2 \right\rangle_{r_0} \) denotes the inversion result.

The surface layers of the Sun appear to be where most of the solar cycle-related changes take place. Therefore it is useful to invert for the structure of the Sun’s surface. In order to do this, one needs high-degree modes (\( l \leq 1000 \)) in order to resolve structure so close to \( R/R_\odot = 1 \). In the presence of high-degree modes, the surface term becomes a function of the degree \( l \), and the traditional approximation is no longer representative of the true surface term. Thus if one wishes to use helioseismic inversion to probe the Sun’s surface structure, a new formulation for the surface term is needed.

3. PREVIOUS MODIFICATION OF THE SURFACE TERM

Antia (1995) compares the surface term computed from an adiabatic approximation and that computed from a non-adiabatic approximation. The difference between these surface terms is expressed as:

\[
\Delta F_{\text{surf}} \simeq F(\omega) + \frac{1}{(l+1)^2} F_l(\omega) + \frac{l}{(l+1)^2} F(\omega),
\]

(8)

where the unknown functions \( F(\omega) \) are expanded in terms of cubic basis functions. With this new surface term included, the scaled frequency differences between adiabatic and non-adiabatic frequencies is reduced considerably, although there are still some larger differences which are not fully accounted for by the new surface term representation.

Di Mauro et al. (2002) used an expansion for the surface term based on asymptotic theory:

\[
F_{\text{surf}} \simeq \sum_{\lambda=0}^{\Lambda} b_\lambda \mathcal{P}_\lambda(\omega) + \sum_{\lambda=0}^{\Lambda} \frac{c_\lambda \mathcal{P}_\lambda(\omega)}{w^2} + \sum_{\lambda=0}^{\Lambda} \frac{d_\lambda \mathcal{P}_\lambda(\omega)}{w^4},
\]

(9)

where \( \mathcal{P}_\lambda \) are scaled Legendre Polynomials of degree \( \lambda \). The maximum degree of the polynomial, \( \Lambda \), is assumed to be the same for all terms of the surface term. \( w \) is defined as \( w = \omega / (l+1/2) \) and contains the degree-dependence missing from traditional approximations to the surface term. The degree dependence was subtracted out of the surface term in pre-processing before the actual SOLA inversion, a method which makes the inversions computationally easier and gives good results. However, Di Mauro et al. failed to note that this process correlates the data errors, thus making the covariance matrix non-diagonal.

To keep the covariance matrix diagonal, we use the fully degree-dependent surface term explicitly in the inversion, without any pre-processing.

4. OUR SURFACE TERM MODIFICATION

The surface term used in our SOLA inversions has the same form as that of Di Mauro et al. (2002),

\[
F_{\text{surf}} \simeq \sum_{\lambda=0}^{\Lambda} b_\lambda \mathcal{P}_\lambda(\omega) + \sum_{\lambda=0}^{\Lambda} \frac{c_\lambda \mathcal{P}_\lambda(\omega)}{w^2} + \sum_{\lambda=0}^{\Lambda} \frac{d_\lambda \mathcal{P}_\lambda(\omega)}{w^4},
\]

(10)

except that in this case the above terms were inserted into the inversion matrix explicitly rather than determined with pre-processing and then subtracted out. In this way we preserve the diagonal form of the covariance matrix and assure that the errors remain uncorrelated.
Also, we constrain each term by requiring that

\[
\sum_{i \in \mathcal{M}} \frac{\alpha_i(x_0)}{Q_i} P_\lambda(\omega_i) = 0 \\
\sum_{i \in \mathcal{M}} \frac{\alpha_i(x_0)}{Q_i} \frac{P_\lambda'(\omega_i)}{\omega_i^2} = 0 \\
\sum_{i \in \mathcal{M}} \frac{\alpha_i(x_0)}{Q_i} \frac{P_\lambda(\omega_i)}{\omega_i^2} = 0, \quad \lambda = 0, 1, \ldots \Lambda.
\]

(11)

We assume that all three terms can be represented with the same \( \Lambda \) value.

5. SAMPLE RESULTS

Figure 1. Surface inversions using (a) the unmodified SOLA method, and (b) the SOLA method with our modified surface term. The red curve shows the actual sound speed difference between the reference and test models. The horizontal error bars show the spatial resolution of the inversion, and the vertical error bars show the inversion errors based on the MDI mode-set errors.

To test this method, we invert one model with respect to another. The models have a known value of \( c \) at each point, so we can explicitly calculate \( \delta c^2 / c^2 \) and compare it to the inversion results. In addition, we use an MDI mode-set to choose which frequencies to invert from the model, and we use errors from MDI data to estimate what we can expect the inversion errors to be like when we invert realistic data.

The reference model used in figure 1 above is a Bahcall-Pinsonneault pre-main sequence model, and

the test model is a Bahcall-Pinsonneault rotational mixing model (Basu, Pinsonneault, & Bahcall 2000). The mode-set is sampled from MDI data (Rhodes et al. 1998). We have sampled the mode-set instead of using the entire set to reduce processing time. We use 1741 total modes whose \( (n, l) \) values were chosen from the MDI data set, with a degree range of \( 0 \leq l \leq 1000 \) and a frequency range between 1.0 and 4.0 mHz.

Figure 1 shows a comparison between (a) a surface inversion using the unmodified SOLA method, and (b) a surface inversion using our modified degree-dependent surface term. In each figure, the red curve shows the actual sound speed differences between the two models. The vertical error bars are the inversion errors based on errors from the MDI mode-set. The horizontal error bars show the spatial resolution of the inversion. Although in both cases the inversion was run from 0.95\( R_\odot \) to 1\( R_\odot \), the unmodified technique was unable to invert closer to the surface than 0.975\( R_\odot \).

Figure 2 shows typical averaging kernels for the unmodified (dashed curves) and modified (solid curves) inversion. The kernels shown are those with target radii very close to \( r/R_\odot = 1 \). One can see that the unmodified inversion is unable to invert at radii larger than \( r \approx 0.975 R_\odot \).
6. CONCLUSIONS

Previous helioseismic inversions have not been able to invert reliably for the structure of the Sun at radii larger than about \( R = 0.96R_\odot \). This was partially due to the fact that high-degree mode data for the Sun was not available. However, even using high-degree (\( l_{\text{max}} = 1000 \)) modes, an unmodified SOLA inversion cannot invert for radii higher than 0.975\( R_\odot \). Our modified surface term, which is an expansion based on asymptotic theory, is able to invert reliably much closer to the surface than previous methods.

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THE SEISMIC RADIUS OF THE SUN, AND STRUCTURE INVERSIONS

M. Takata$^1$ and D. O. Gough$^2$

$^1$Department of Astronomy, School of Science, University of Tokyo, Bunkyo-ku, Tokyo, 113–0033, Japan
$^2$Institute of Astronomy and Department of Applied Mathematics and Theoretical Physics, Madingley Road, Cambridge CB3 0HA, UK

ABSTRACT

It is known (Schou et al., 1997; Antia, 1998) that the effective radius of the Sun determined by f-mode frequencies is different by a few hundredths per cent from the photospheric radius determined by direct photometric measurement (Brown and Christensen-Dalsgaard 1998). It is fair to say that we still do not fully comprehend the implications of the difference, save that the two radii are rather different quantities: the radius inferred from f-mode frequencies is determined by the location of the maximum in the f-mode energy (Gough, 1993), whereas the photospheric radius is determined by extrapolation to some prescribed optical depth from a fiducial point in the the limb-darkening function using a theoretical solar model. Both depend in particular on the structure of the upper superadiabatic convective boundary layer, the physics of which is not well understood. In this report we attempt to shed some light on the difference by determining a seismic radius from p-mode frequencies; the outcome depends predominantly on the variation of sound speed, and it is consistent with the f-mode value (Takata and Gough 2001). By considering the mathematical structure of an inversion process that does not explicitly distinguish f modes from p modes, we offer an interpretation of the seismic radius. This interpretation has led us to revise the method by which we carry out structure inversions.

Key words: solar radius; inversion; p modes.

1. INTRODUCTION

It has been pointed out that the f-mode frequencies suggest a value of the solar radius which is slightly different from the photospheric radius (Schou et al., 1997; Antia, 1998). Based on these analyses, Basu (1998) has demonstrated the sensitivity of the inferred seismic radius on some details of the inversions, and Antia et al. (2000) and Dziembowski et al. (2001) have sought to determine a variation with the solar cycle. In addition, we have shown that p-mode frequencies can also be used to determine a seismic radius (Takata & Gough 2001), although the difference between that and the f-mode radius is not entirely clear.

It is important to be explicit about the meanings of the solar radius determined by different methods, for, in particular, the result of the direct measurement of the photospheric radius recorded by Allen (1973), and its recent revision by Brown and Christensen-Dalsgaard (1998), is not consistent with the f-mode radius. It is not surprising at all that there is such an inconsistency, because the photosphere, which is defined in terms of optical depth, is not a special place for surface gravity waves, nor for acoustic waves, which are the constituents of the seismic oscillations. Schou et al. (1997) and Dziembowski et al. (2001) base their discussions on an interpretation by Gough (1993) that the f-mode radius is determined by the location of the peak of the kinetic energy density of the f modes. From this interpretation, it is evident that a naively defined f-mode radius can differ from one f mode to another. To relate them requires knowledge of the stratification of the outer layers of the Sun. This raises an issue which motivates us to discuss an interpretation of the p-mode radius, and its difference from the f-mode radius.

The rest of this paper consists of three sections. In section 2, we interpret the p-mode radius by examining formulae of the structure inversion that take uncertainities in the solar radius and the gravitational constant into account. We demonstrate the importance of the difference between the radius of the Sun and that of the reference solar model, in the light of which we revise, in section 3, our inversions for the sound speed.

2. SEISMIC RADIUS INFERRED BY P-MODE FREQUENCIES

We base our discussion of structure inversions on optimally localized averaging (OLA) (Backus and Gilbert, 1968), partly because it is one of the widely
used methods in the field, and partly because in some ways it is the most readily interpretable.

Takata & Gough (2001) have extended the standard formula for the differences $\delta \nu_i$ between the oscillation frequencies of the Sun and the eigenfrequencies of a reference theoretical model to include possible differences, ignored in conventional structure inversions, in radius $R$ and in the gravitational constant $G$. We present here an essentially equivalent (but more compact) formula:

$$\frac{\delta \nu_i}{\nu_i} = \int K_{c,\nu}^i \frac{\delta \xi(c/r)}{c/r} \, dx + \int K_{\rho,\nu}^i \frac{\delta \xi(G\rho)}{G\rho} \, dx.$$  

(1)

The meanings of the symbols in equation (1) are as followings: $\nu_i$ is the frequency of mode $i$, where the index $i$ refers to a set of the mode parameters $(n, l)$ ($n$ is the radial order and $l$ is the angular degree); $c$ is sound speed; $r$ is radius (the distance from the centre); $\rho$ is density; $x$ is the fractional radius $x/R$, where $R$ is a fiducial radius of the Sun whose meaning we discuss later; $\delta \xi f$ means the difference in some quantity $f$ between the Sun and the reference model at the same fractional radius $x$, and $K_{c,\nu}^i$ and $K_{\rho,\nu}^i$ are corresponding kernels. Although a surface term is usually added to the right-hand side of equation (1) to take account of various uncertainties in the subsurface region of the Sun, we omit it here because it is not important in the discussions in this section. However, it is actually included in the structure inversions presented in section 3. We should point out that the radius difference $\delta R$ is implicitly included in equation (1) through the definitions of the quantities $\delta \xi(c/r)$ and $\delta \xi(G\rho)$; in fact, these quantities are not well defined without specifying the radius difference. We note that Gough & Kosovichev (1993) have used a formula similar to equation (1) in the context of asteroseismology, where neither the radius nor the mass of the star is known.

Another basic equation used in structure inversions is the total mass constraint, which is usually included to ensure that the inversions are consistent with the observed value of the solar mass. Its extended version, which includes differences in the radius $R$ and the product $GM$ of the gravitational constant and the total mass, is given by

$$\frac{\delta (GM)}{GM} = \int \frac{4\pi R^3 x^2 \rho}{M} \frac{\delta \xi(G\rho)}{G\rho} \, dx + 3 \frac{\delta R}{R}.$$  

(2)

Note that $\delta R$ and $\delta (GM)$ appear explicitly here.

Since the form of equation (2) is similar to that of equation (1), in conventional inversions for the solar structure these equations are usually treated equivalently with no caution. It will become evident later that such treatment is justified only if the fractional radius difference $\delta R/R$ is negligible.

So far we have not specified what $R$ means physically in equations (1) and (2). In fact it has been introduced simply as a scaling factor. To assist in thinking about it, we first draw attention to the following annihilator relation:

$$\int K_{c,\rho}^i \frac{d \ln (c/r)}{d \ln r} \, dx + \int K_{\rho,\nu}^i \frac{d \ln \rho}{d \ln r} \, dx = 0,$$  

(3)

reflecting the property that adiabatic eigenfrequencies of stars are invariant under uniform stretching of the radial coordinate provided that the profiles of all dependent variables are appropriately scaled homologously. It implies the important fact that there is a series of (isospectral) structures that cannot be distinguished by their adiabatic eigenfrequencies alone.

In practice, however, one obtains results from OLA with no apparent ambiguity. Therefore the stretching factor is determined implicitly by the procedure. We need to know which specific value is chosen.

In summary, we have two questions to answer here: (i) How can we know the stretching factor, $1 + \delta R/R$, that is implicitly determined by the OLA method? (ii) What kind of principle is adopted in the process to pick up this specific value of the stretching factor? The total mass constraint (2), which is a physically different condition from the frequency equation (1), immediately gives us an answer to the first question. Once we have the density profile $\delta \xi(G\rho)/(\delta \xi(G\rho))$ without knowing what $R$, hence $x$, is, we can substitute this profile into equation (2) to get $\delta R/R$. Practically, we could perform another OLA inversion for $\delta R/R$ based on equations (1) and (2), which is what was actually done by Takata & Gough (2001). From a physical point of view, we can determine the size of the target star, which is otherwise ambiguous owing to the undetermined stretching of the radial coordinate, by constraining its total mass. Thus we propose here a procedure consisting of the following two steps: (step 1) carry out an inversion based on only equation (1), and (step 2) supplement this with an inversion for $\delta R$ based on both equation (1) and equation (2). This answers question (i); but we need to know more about the mathematics behind the OLA method before we can answer question (ii).

In the OLA method, we make inferences about quantities such as sound-speed differences (and/or density differences) in the vicinity of some specific point from integrals of the product of those quantities and appropriate averaging kernels, which are constructed as linear combinations of the kernels for each mode. Because relation (3) can be interpreted as the vanishing of the inner product of the kernel vector

$$\begin{pmatrix} K_{c,\rho}^i \\ K_{\rho,\nu}^i \end{pmatrix}$$  

(4)

and the annihilator vector

$$\begin{pmatrix} \frac{d \ln (c/r)}{d \ln r} \\ \frac{d \ln \rho}{d \ln r} \end{pmatrix}$$  

(5)

for all modes, we can say that the averaging kernels are orthogonal to the annihilator vector. This means that the inferences made by the OLA method without the total mass constraint (2) are never sensitive
to the annihilator component (5) of the target quantities \( \delta_x(c/r)/(c/r) \) and \( \text{or} \ \delta_x(G\rho)/(G\rho) \). We can understand this property by conforming attention to the projection of \( \delta_x(c/r)/(c/r) \) and \( \text{or} \ \delta_x(G\rho)/(G\rho) \) into the function space spanned by the kernels \( K_{c,r} \) and \( K_{p,c} \). Then the inferences of the structure differences by the OLA method are essentially averages of the following expansion:

\[
\begin{align*}
\begin{pmatrix} \delta_x(c/r) \\ \delta_x(G\rho)/(G\rho) \end{pmatrix} &= \sum_i A_i \begin{pmatrix} K_{c,r}^i \\ K_{p,c}^i \end{pmatrix},
\end{align*}
\]

in which the coefficients \( A_i \) are constant. Equation (6) is known as the spectral decomposition. Substituting into this equation the identity

\[
\begin{align*}
\begin{pmatrix} \delta_r c/c \\ \delta_r(G\rho)/(G\rho) \end{pmatrix} &= \begin{pmatrix} \delta_x(c/r) \\ \delta_x(G\rho)/(G\rho) \end{pmatrix} - \delta R \begin{pmatrix} \frac{d \ln(c/r)}{d \ln r} \\ \frac{d \ln(G\rho)/(G\rho)}{d \ln r} \end{pmatrix},
\end{align*}
\]

where \( \delta_r \) is the operator such that \( \delta_r f \) means the difference in quantity \( f \) between the target structure and the reference model at the same radius \( r \), we obtain the following relation:

\[
\begin{align*}
\begin{pmatrix} \delta_r c/c \\ \delta_r(G\rho)/(G\rho) \end{pmatrix} &= \sum_i A_i \begin{pmatrix} K_{c,r}^i \\ K_{p,c}^i \end{pmatrix} - \delta R \begin{pmatrix} \frac{d \ln(c/r)}{d \ln r} \\ \frac{d \ln(G\rho)/(G\rho)}{d \ln r} \end{pmatrix},
\end{align*}
\]

the left-hand side of which is independent of \( \delta R/R \). We can interpret the last term on the right-hand side of equation (8) as the annihilator term (5), which does not contribute to frequency differences at all. Taking the scalar product of both sides of equation (8) with the annihilator term (5), and integrating, yields, with the help of equation (3),

\[
\delta R/R = \lim_{x_0 \to x_s} \int_0^{x_0} \left( \frac{\delta_r c/c}{c} \frac{d \ln(c/r)}{d \ln r} + \frac{\delta_r(G\rho)/(G\rho)}{G\rho} \frac{d \ln r}{d \ln r} \right) dx,
\]

where \( x_s \) is the value of \( x \) at the ‘surface’ \( r = R \) of the star. If, as in a realistic stellar atmosphere, \( d \ln r/d\ln r \) almost diverges, as \( x \to x_s \) whereas \( d \ln c/d\ln r \) stays finite, equation (9) reduces to

\[
\delta R/R \approx \lim_{x_0 \to x_s} \int_0^{x_0} \frac{\delta_r(G\rho)/(G\rho)}{G\rho} d\ln r dx = \lim_{x \to x_s} \frac{\delta_r(G\rho)}{G\rho} \frac{d \ln r}{d \ln r},
\]

if \( \delta_r(G\rho)/(G\rho) \) is slowly varying. If, in addition, the differences in the structure are homologous, this expression tells us that \( 1 + \delta R/R \) is equal to the homology factor, and that the conventional OLA method (without the total mass constraint) should yield \( \delta_x(c/r)/(c/r) = \delta_x(G\rho)/(G\rho) = 0 \). Moreover, it is evident from equation (10) that the density profile near the surface is crucial in determining the p-mode radius, as indeed it is for the f-mode radius. Therefore the p-mode radius and the f-mode radius have something in common. However, we still have to think about what aspect of the surface density structure is most important. Because \( p \) modes are reflected at their upper turning points near the surface, we may naively think that the density profile in the vicinity of those turning points is the most important. If this interpretation is correct, then it must be that the p-mode radius is determined by the modes with the highest frequencies, because the upper turning point moves outward as frequency increases.

Now we consider the conventional method of structure inversion, in which the total mass constraint (2) is not distinguished from the frequency condition (1). By mixing those two kinds of constraint, we implicitly assume the following expansion of the seismically accessible component of the structure:

\[
\begin{align*}
\begin{pmatrix} \delta_r c/c \\ \delta_r(G\rho)/(G\rho) \end{pmatrix}_{\text{conv}} &= \sum_i B_i \begin{pmatrix} K_{c,r}^i \\ K_{p,c}^i \end{pmatrix} + B_\ast \begin{pmatrix} 0 \\ 4\pi R^3 x^2 \rho/M \end{pmatrix},
\end{align*}
\]

where the coefficients \( B_i \) and \( B_\ast \) are constants. Note that the last term on the right-hand side of this equation is relatively small at the surface, unlike the annihilator component (5), which is extremely large (possibly even divergent). Because the differences \( \delta_r c/c \) and \( \delta_r(G\rho)/(G\rho) \) generally contain an annihilator component, it is not a good idea to use equation (11) for representing such large differences at the surface. It will be seen in section 3 that so long as \( \delta R/R \) is comparable with the fractional sound-speed differences one is likely to generate large errors in localized averages constructed by the conventional method. We can still reinterpret the conventional inversions if they are performed without explicit use of the total mass constraint. In that case, we should replace the labels of the profiles \( \delta_r c/c \) and \( \delta_r(G\rho)/(G\rho) \) by \( \delta_x(c/r)/(c/r) \) and \( \delta_x(G\rho)/(G\rho) \), respectively. Note that we cannot then know the radius difference, which is required for the operator \( \delta_r \) to be well defined, without carrying out the additional inversion for \( \delta R \) using the total mass constraint as well.

3. REVISED SOUND-SPEED INVERSIONS

As described in the previous section, we can divide the inversion process into two steps to take proper account of the radius difference. Here we demonstrate that such treatment is actually needed in practice. We concentrate on inversions for sound speed.

We first demonstrate how different from conventional inversions the new inversions are by performing inversions of eigenfrequencies of a known test model. The test model is made by shrinking model S of Christensen-Dalsgaard et al. (1996) homologously.
by 0.0667%. Eigenfrequencies of this model were calculated for all modes included in the SOHO/MDI 360-day data set (Schou 1999), to provide artificial data for test inversions using the raw model S as the reference. The results are shown in the left panel of Figure 1. One can see that with only a small homologous difference there can be substantial error in the inversions. The conventional inversion with the total mass constraint is inaccurate especially in the outer regions of the star \((r/R_\odot > 0.7)\). On the other hand, without the total mass constraint the conventional inversion is in error by simply a constant, \(2\delta R/R\), as we anticipated at the end of section 2.

Next we invert real data, namely the SOHO/MDI 360-day data set (Schou 1999). The results are shown in the right panel of Figure 1. The constant difference between the revised inversions and the conventional inversions corresponds to \(2\delta R/R\), which has been estimated to be \(-8.6 \times 10^{-4}\) by Takata & Gough (2001). Note that we have plotted \(\delta_r c^2/c^2\) instead of \(\delta_r c^2/c^2\). The relation between these quantities is given by equation (7). This is why the conventional inversions with the total mass constraint look very different from others' results in the literature. In fact, the \(\delta_r c^2/c^2\) profile inferred from conventional inversions carried out with the total mass constraint is quite close to the \(\delta_r c^2/c^2\) profile of the revised inversion. We stress, however, that we have to distinguish the meanings of \(\delta_r c^2/c^2\) and \(\delta_r c^2/c^2\).

4. CONCLUSION

We have discussed the inference of the seismic radius determined by p-mode and f-mode frequencies of the Sun. The fractional radius difference determined by OLA-type inversions can be interpreted in terms of an homologous component of the difference in the density profiles of the Sun and the reference solar model particularly near the surface. Conventional inversions should be revised to take account of the radius difference appropriately.

ACKNOWLEDGEMENTS

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POWER SPECTRAL DENSITY OF ELECTRON FLUX FROM ULYSSES, 1999-2001

David J. Thomson 1, Carol G. Maclellan 2, and Louis J. Lanzerotti 3

1 Queen’s University, Kingston, Ontario
2 Bell Labs, Murray Hill, NJ
3 Bell Labs, Murray Hill, NJ and New Jersey Institute of Technology, Newark, NJ

ABSTRACT

Fluxes of low-energy (≈50 - 200 keV) electrons measured on the Ulysses HIGAILE instrument were of sufficient magnitude to compute reliable power spectra in the frequency range where low-order and degree g- and p- solar modes are expected. Fits to some well-determined \( l = 1 \) modes give an estimate of the asymptotic period spacing \( T_0 \approx 33.23 \) minutes.

Key words: Ulysses; g-modes; HISCALE.

1. INTRODUCTION

The nature of the interplanetary medium has long been of interest since the discussions of Biermann of the reasons for the orientation of Parker of the solar wind. This continued interest is driven by questions about basic physics of the tenuous ionized magnetized plasmas and the propagation behavior of higher energy charged particles in the medium. In the last several years we have called into question the commonly held view that the medium is totally stochastic. For example, Thomson et al. (2001b) assert that the energy content of the interplanetary magnetic field acquired by the Ulysses spacecraft may be as much as 25% to more than 50% discrete "modal" components.

The most likely source of such modal components is the sun. However, there are few optical observations of the Sun that would support the existence of the many numerous longer period oscillations that we have reported, and even if they existed, the detailed physics that would be needed for transporting these through the chromosphere into the corona and then into the interplanetary medium has not been examined. A recent review by Aschwanden (2002) of observations of waves in the solar corona contains only a few reports to date of coronal oscillations with periods of many to tens of minutes. Thus, the driving source(s) for the interplanetary modes that have been discussed in our research remain a subject for considerable further investigation.

In Thomson et al. (1995) we claimed that the spectrum of the interplanetary magnetic fields and particles is dominated by modes and not turbulence. Subsequent work, Thomson et al. (1997, 2000, 2001a) has reinforced our opinion. Because our hypothesis represents a radical break with what had been prevailing wisdom, it was, in the best traditions of science, vigorously attacked, e.g. Roberts et al. (1996); Riley and Sonett (1996); Hoogeveen and Riley (1998); Denison and Walden (1999). We have examined these arguments, and in Thomson et al. (1996, 2001b) shown the problems with them. The latter paper also shows specific examples where the frequencies identified in Thomson et al. (1995) are reproducible. Other agreements are shown in Thomson et al. (2000).

Here we give a preliminary set of \( l = 1 \) g-mode frequencies estimated from power spectra of interplanetary electron flux as measured by the HISCALE instrument on the Ulysses spacecraft for 1999 and 2000. At the beginning of this time period, Jan. 1999, Ulysses was at a heliographic latitude of 19° South and 5.19 AU from the sun. By the end of 2000, it had just completed its second pass of the south pole of the sun, and was at a heliographic latitude of −75.6° and was 2.05 AU from the sun. The data used in this paper were from the E1 and E2 channels of the LEPS-150 telescope, measuring electrons with energies of 42–65 and 64–112 keV, respectively. The raw data were cleaned, spin-averaged, at one hour increments. During the two years, there were only 78 missing hourly values, distributed over 24 different intervals. For reasons discussed in Lanzerotti et al. (1991), we used logarithms of the flux. This also transforms the statistical distribution of the data to something much closer to Gaussian, so that reliability of the power spectra is better than it is from the raw data. The missing values were interpolated as described in Thomson et al. (1995), and discussed in greater length in the Appendix of Thomson et al. (2001b). We also data at 1-minute resolution from the DE-2 detector, 53–103 keV, for days 70 to 170 of 2001. At one minute resolution, there are many outliers and a higher fraction of missing data than in the hourly cleaned data. Outliers and mis-
2. SPECTRUM ESTIMATES AND MODE IDENTIFICATION

To compute the spectrum, the log-data was passed through a prewhitening filter, and a multitaper power spectrum, Thomson (1982, 1990, 2001), was computed on three overlapping blocks each 8772 hours long offset by 4386 hours. Eight Slepian sequences were used on each section, with full adaptive weighting. The individual spectra were postwhitened with an AR–20 equivalent transfer function and averaged. This average has a chi-square distribution with about 96 degrees-of-freedom (df). Specifically, if the data are random, then it does not contain deterministic components within ±160 nHz. of the frequency under consideration, then the spectrum will have a central chi-square distribution with 96 degrees-of-freedom. The turbulent plasma hypothesis usually invoked to describe interplanetary magnetic fields and particles implies such a distribution.

Tests that these spectra have a central $\chi^2_{96}$ distribution, however, fail at high significance levels. This includes simple tests, such as the ratio of the 95% point to the 5% point of the distribution, and statistics such as Bartlett’s M-test for homogeneity (see §4.2 of Thomson (2001)) typically reject the central chi-square hypothesis at spectacular levels. Because the presence of deterministic or modal components at particular frequencies will cause the spectrum to have a non-central distribution at those frequencies so that, overall, the distribution will be a mixture of central and non-central $\chi^2$. In Thomson et al. (2001b) we show examples where the distribution of whitened spectra of interplanetary magnetic field data could not be reasonably fit with a central chi-square, but were very well fit with a mixture of central and non-central components.

In addition to these pre- and post-whitening operations, in the plots shown below, an additional levelling step of removing a linear trend from the log spectrum over the frequency range of the plot was made. Finally, using the 5% point of the distribution, the spectra were scaled to have an average value of 1.0 for a central chi-square distribution. This is done because the presence of modes affects the upper tail of the mixture distribution, but has relatively little effect on the lower tail, see Thomson (2000) or Thomson (2001).

In data from the current solar maximum, the electron data has been extremely rich and spectra from it, see Figure 1, are rich in modal structure. A spectrum of the DE-2 data shows the expected $p$-modes and, at lower frequencies, what appear to be low-order and degree $g$-modes. In this figure, one finds that there are 23 local maxima at levels above the 99% level. The frequency range 100 to 300 $\mu$Hz contains 17 modes for $l = 1$ and $l = 2$. The standardization was done using the 5% point. This results in an estimate with 63.3% of the estimates above the nominal 50% level. The ratio of average-to-median is 1.05, so results would not change much if either the average or median had been used for standardization. However 11.9% of the estimates are above the 99% level. Since the spectrum has a nominal $\chi^2_{96}$ distribution, the probability of this occurring by chance is tiny. Bartlett’s M-test, computed as mentioned above, has a probability of $2.3 \times 10^{-11}$, and the variance of $\ln \hat{S}$ is 1.81 times its expected value. Further work on the high-resolution data is in progress.

Given the presence of some obvious $l = 1$ modes, such as those shown in Figure 2 or 4, several of the others can be identified. To begin, the mode at 109.59$\mu$Hz falls in the range 105.49 to 112.8$\mu$Hz of predicted frequencies for the $n = -5$ mode (see Table 1 of Thomson et al. (2001b)) and close to 109.49$\mu$Hz in Provost et al. (2000). No other $l = 1$ mode falls in this frequency range, so we used it as a tentative starting point. We then scaled the “hare and hound” frequencies, see Appourchaux et al. (2000), in both period and splitting, to match and extended the scaling to adjacent modes. Matching was also done with $l = 2$ modes included (using the same scaling for both) and usually gave a good fit to other peaks in the frequency range with almost no change in the scaling parameters. Figure 5 shows an example with nearby $l = 2$ and $l = 3$ modes indicated. Typically, peaks in the spectrum that are above the 99% sig-
nificance level correspond to \( l = 2 \) and \( l = 3 \) modes. As in most such studies, not all modes are matched.

Figure 3 shows the periods of identified modes and a least squares fit. This fit has the form

\[ P(n) \approx 32.606 + 23.494n \]

giving an asymptotic period spacing of \( T_0 \approx 33.23 \) minutes. This is near the lower end of the 33 to 36 minutes predicted by standard solar models.

\[ \text{Figure 3. Plot of estimated periods for the } l = 1 \text{ modes as a function of radial order. The dashed line is a least-squares fit to the estimated periods.} \]

\[ \text{Table 1. Estimated mode frequencies for } l = 1 \]

<table>
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<tr>
<th>( n )</th>
<th>( m = -1 )</th>
<th>( m = 0 )</th>
<th>( m = +1 )</th>
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<tr>
<td>13</td>
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</table>

3. CONCLUSIONS

The power spectra of Ulysses E1' and E2' electron flux shows several \( l = 1 \) g-modes clearly. Fitting the estimated periods gives \( T_0 \approx 33.23 \) minutes.

Turning to higher frequencies, it appears that the discrepancies in the fit for \( 4 \leq n \leq 9 \) are probably
larger for low $n$. It also appears that either the very low radial order $g$ and $p$ modes are anomalously split into more than $2l + 1$ singlets or, alternatively, are so sensitive to solar activity that their frequencies change appreciably during the time required to estimate a spectrum, and so appear to be multiply split. We caution that, in fact, all the $g$-mode frequencies appear to vary systematically over a solar cycle.

REFERENCES


AN IMAGE MERGE FOR GONG+

C.G. Toner\textsuperscript{1}, D. Haber\textsuperscript{2}, T. Corbard\textsuperscript{1}, R. Bogart\textsuperscript{3}, and B. Hindman\textsuperscript{2}

\textsuperscript{1}NSO/GONG, Tucson, AZ, USA
\textsuperscript{2}JILA/Univ. Colorado, Boulder, CO, USA
\textsuperscript{3}CSSA-HEPL, Stanford Univ., Stanford, CA, USA

ABSTRACT

We are developing an algorithm for merging GONG+ velocity images. Here we describe the algorithm and present results of preliminary tests to investigate the utility of using the merged images for local-area helioseismology, focusing specifically on ring-diagram analysis.

Key words: Local Helioseismology; Ring Diagrams; Image Processing.

1. INTRODUCTION

The GONG Project recently completed an upgrade of its camera system from nominally 8x10 arc second pixels (GONG Classic) to 2.5x2.5 arc second pixels (GONG+). The major driving force behind the upgrade is the fairly recent development of various local-area helioseismology techniques, e.g., ring diagrams (Hill 1988, Haber et al. 2000, 2002), time-distance (Duvall 1997), and acoustic holography (Lindsey & Braun 2000), all of which require significantly higher spatial resolution than the original GONG Classic instrumentation provided.

The Project recognizes that there are several groups of researchers who could take advantage of the high resolution of GONG+ and the high duty cycle afforded by the GONG Network. To reduce the storage requirements and data handling complexity of dealing with multi-site observations the Project is developing a method for combining simultaneous velocity images recorded by the Network into a single, once per minute, set of registered merged images. These data will then be made available to the community.

2. METHOD

The process is divided into the following steps:

- All site-day images which contribute to a specified calendar day are staged to disk.
- A set of empty template files are generated, one file for each minute.
- To remove solar rotation and the affects of Earth Observer Motion a low-order polynomial is fitted to each input image and then subtracted.
- Each input image is then remapped to a circular shape using a user specified radius. During the remapping solar north is placed at the top and solar east at the left. The direction to solar north for each input image is determined by the method described in Toner & Harvey 1998 and Toner 2000.
- Each registered image is then summed into the appropriate template file.
- Once all of the input images have been processed the template files are normalized by the number of images summed into each one.
- Any template files which did not have observed data contributing to them are marked as “FILLED” and left simply as place holders.

3. PRELIMINARY TESTS

To test the utility of using the merged images for local-area helioseismology we have performed a “Dense-Pack” ring-diagram analysis (Haber et al. 2000, 2002) using 1664 minutes of merged data. The figures on the following page show the residual horizontal flows on Aug. 22, 2001 at a depth of 0.9 Mm (Figure 1) and 7.1 Mm (Figure 2). The velocity arrow diagram overlays a coeval magnetogram remapped to the same latitude-longitude grid.
4. DISCUSSION

This is a work in progress. We have demonstrated that we can merge GONG+ images and that ring-diagram analysis of the merged images produces not unreasonable horizontal flows. Comparison of the flows deduced from other experiments is currently under way. Preliminary results comparing the flows as measured using both MDI and merged GONG+ data are presented by Bogart et al. & Corbard et al. in these proceedings.

We still need to investigate the affect of the double interpolation which is inherent in this approach to analysing GONG+ imaged data. Does this introduce additional noise or artifacts? Does it influence the recovered flows? Will these images be useful for other local-area helioseismology techniques?

Also, there may be other avenues by which we can improve both the image merge and the ring-diagram analysis. Some possibilities are outlined briefly in the next section.

5. FUTURE ENHANCEMENTS

During the standard processing of GONG data an estimate of the Modulation Transfer Function (MTF) is determined for each image (see Toner & Jeffries 1993). Therefore, in principle, it should be possible to correct each image for the affects of the blurring introduced by the terrestrial atmosphere and the instrument optics. To this end, we have been experimenting with an image restoration method that can be applied to the GONG+ velocity images and which is quite fast (\(\sim 20\) seconds per image on a 450 MHz

Sun Ultra-80). Thus, it should be feasible to restore all of the site velocity images using existing Project hardware.

One would expect that once the images have been corrected for the MTF, simultaneous, restored data should match more closely than uncorrected images, and hence could improve the quality of the image merge.

Figure 3. On the left we compare a slice through ring spectra \((k_x - k_y)\) at 3606 \(\mu\)Hz for: unrestored data (top), unrestored data with multi-tapering (center), and restored data with multi-tapering (bottom). On the right side we compare a slice through the ring spectra at \(k_x = 0\) \((k_x - \nu)\) for: unrestored data (left), unrestored data with multi-tapering (center), and restored data with multi-tapering (right).

Also, it has been shown (Komm et al. 1998) that multi-tapering can help when fitting helioseismic
power spectra. Therefore, we have been investigating the use of multi-taper analysis applied to ring diagrams (Figure 3).

One can see from Figure 3 that multi-tapering produces much better defined rings and that image restoration combined with multi-tapering tends to equalize the power in all of the rings and improves the definition of the rings. Some preliminary tests have been performed using only multi-tapering (no restoration). Analysis of the multi-tapered spectra produces a larger number of good ring fits, leading to somewhat improved inversions and flow maps.

Finally, we note that this image merge routine also works well with GONG+ magnetograms (which, like the velocity data, are recorded every minute at each GONG site). Therefore, it is foreseeable that the project may also be able to provide merged magnetograms to the community.

ACKNOWLEDGMENTS

This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofisico de Canarias, and Cerro Tololo Interamerican Observatory.

REFERENCES


TRANSIENT OSCILLATIONS NEAR THE SOLAR TACHOCLINE

Juri Toomre¹, Jørgen Christensen-Dalsgaard², Frank Hill, Rachel Howe, Rudolf W. Komm³, Jesper Schou⁴, and Michael J. Thompson⁵

¹JILA and Dept. of Astrophysical & Planetary Sciences, University of Colorado, Boulder CO 80309-0440, USA; e-mail: jtoomre@solarz.colorado.edu; phone: 303-492-7854
²Theoretical Astrophysics Center & Aarhus University, DK-8000 Aarhus C, Denmark
³National Solar Observatory, P.O. Box 26732, Tucson AZ 85726-6732, USA
⁴Stanford University, CSSA-HEPL, Stanford, CA 94305-4085, USA
⁵Space & Atmospheric Physics, The Blackett Laboratory, Imperial College, London SW7 2BW, UK

ABSTRACT

We report on further developments in the 1.3-yr quasi-periodic oscillations reported by Howe et al. (2000). These are small (6 to 8 mHz peak-to-peak) oscillations in the inferred rotation rate near the bottom of the convection zone and in the outer part of the radiative interior. The oscillations are strongest and most coherent at about a fractional radius of 0.72 in the equatorial region. Further monitoring of the oscillations near the equator shows that they continued for a period after the end of the data analyzed by Howe et al., but appear to have now diminished in amplitude. This is reminiscent of the transient behavior of similar (1.3 to 1.4 yr) periodicities in solar-wind and geomagnetic datasets previously reported. We speculate that the near tachocline oscillation is associated with the rising phase of the solar cycle. We discuss tests performed to eliminate various possible explanations of the oscillations due to systematic errors in the data and in their analyses.

1. TACHOCLINE AND ITS VARIATIONS

The solar tachocline, which is a region of strong shear separating the differential rotation of the convection zone from the solid body rotation of the deeper radiative interior, has been one of the most significant discoveries of helioseismology. A tachocline affords a promising site for the operation of the solar global dynamo: the presence of prominent rotational shear combined with a stable density stratification makes it likely that strong toroidal magnetic fields can be created there from poloidal fields through distortion and stretching. The resulting toroidal fields can reside there for some time before magnetic buoyancy and other instabilities disrupt these flux concentrations and drag them upward, ultimately to emerge as loops at the surface. We have been very interested in seeking indications of possible temporal changes in the angular velocity Ω near the tachocline as the magnetic cycle advances. In the process of toroidal stretching, Lorentz forces will be generated that oppose the shear, causing it to weaken and thus alter the apparent local rotation rate. These forces are relaxed when a strand of the toroidal field breaks loose and the shear returns to its earlier state. Since toroidal flux is ejected irregularly but peaks within the 11-yr period of the sunspot cycle, we have little basis for expecting the time dependence to be periodic, except possibly with the time scale of the solar cycle. As first discussed in Howe et al. (2000), we have instead found a period of about 1.3 yr in angular velocity variations δΩ (Figs. 1A, D) that are in remarkable antiphase above and below the tachocline in the equatorial region. Since that paper, we have accumulated more MDI and GONG data intervals for analysis. Thus we have been able to revisit the quasi-periodic variations in the light of the extended sets of observations shown in Figure 1, and to assess possible systematic sources of error that might contribute to apparent temporal variations in the rotation rate.

Our procedure to detect temporal changes is based on analyzing the MDI global mode data in 72-day segments and the GONG data in overlapping 108-day intervals on centers separated by 36 days. The resulting mode frequency splittings for each interval are inverted both with OLA and RLS procedures to determine the angular velocity Ω profile with radius and latitude. The procedure is repeated independently for the MDI and GONG data for the full duration of both experiments. The time averages of Ω determined by RLS inversion are shown in Figure 2, both as contours with radius and latitude and in radial cuts at different latitudes. The mean Ω profiles deduced from MDI and GONG are in close agreement up to mid latitudes, or about 45°. At higher latitudes the MDI data fitting yields convoluted curves whereas the GONG profiles remain smooth. There are continuing studies to understand why there are small but systematic differences arising


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in the MDI and GONG peak fitting procedures to determine frequency splittings that sample the higher latitudes (Schou et al. 2002). However, at low and intermediate latitudes the deductions about Ω profiles are in good agreement, using any combination of data sets, fitting procedures and inversion methods.

Using these time-averaged Ω profiles as a reference, we have detected substantial variations δΩ in the rotation rates near the tachocline, with these being most pronounced near the equator and at high latitudes. Figures 1A, D display the time evolution of δΩ both above (at radius 0.72R) and below (0.65R) the tachocline near the equator, revealing oscillations with a period of about 1.3 yr (Howe et al. 2000). The temporal changes in δΩ/2π are of order 6 nHz and occur strikingly out of phase above and below the tachocline. These represent substantial variations compared to the 30 nHz drop in Ω/2π with radius across the tachocline at the equator (Fig. 2). The variations detected at higher latitudes (Figs. 1C, F) involve two-fold greater amplitudes for δΩ and have a period close to 1.0 yr.

The findings of Howe et al. (2000) have been questioned by Antia & Basu (2000) who reported that their analysis of GONG data yielded no evidence of periodic changes in the solar rotation rate near the base of the convection zone. Later using a different analysis technique, Basu & Antia (2001) found a signal in both the GONG and MDI data which looked essentially the same as the 1.3-yr quasi-periodic signal found by Howe et al. However, they concluded that their signal was not statistically significant, though the basis for this was unclear.

2. EFFECTS OF CHANGING MODE SETS

In addition to extending our analyses to the current epoch, we have also tested the effects of the changing mode sets that are available within each analysis interval. Such variations arise because stochasticity and mode beating can cause given modes to be detectable in some intervals and to be indistinguishable from noise in others. In order to assess the effect of changing mode sets on the inversion results, we prepared two sets of simulated rotation coefficients, each generated by forward calculation from a known profile. In one case, the underlying profile was simply the temporal mean of the RLS inversion results (Fig. 2, equator). In the other, the radial gradient in the rotation rate at the tachocline was artificially shap-
Figure 2. Mean rotation profiles $\Omega/2\pi$ deduced from RLS inversions separately of GONG and MDI frequency-splitting data over the full duration of the observations. Angular velocity is shown as contours (10 nHz intervals) with radius and latitude (on left), and as radial cuts (right) at indicated latitudes. Errors shown on radial cuts are multiplied by factor 3 for visibility.

ended to bring it closer to what is believed to be the real profile.

For each set of GONG and MDI observations, the artificial mode set was trimmed and given errors corresponding to the real data available. The resulting coefficient sets were then inverted using the same RLS and OLA codes and regularization parameters used for the real data. The results for such analyses of the effects of changing mode sets are shown in Figure 3, accompanied by the actual residuals $\Delta \Omega/2\pi$ in rotation rate for the four combinations of data and inversion approach. In all cases, the variations that may be attributable to changes in mode sets are very small compared to the deduced changes $\Delta \Omega$.

3. POSSIBLE IMPLICATIONS OF FINDINGS

Our finding of distinctive variations $\Delta \Omega$ in rotation rate near the tachocline shown in Figure 1 calls for continuing study as the solar cycle advances. It is striking that these variations with amplitudes of order 6 nHz are a substantial fraction of the average 30 nHz change in angular velocity across the tachocline at the equator, close to the presumed seat of the solar global dynamo. Clearly it is of interest to determine whether these are real periods or aperiodic wobbles, and whether they are present both in the rising and waning phases of solar activity. The presence of 1.3 yr periodic variations have been reported for some extended intervals in auroral data (Silverman & Shapiro 1983), and for variations seen in the higher velocity solar wind as observed from IMP-8 and Voyager 2 spacecraft (Richardson et al. 1994, Paularena et al. 1995), though it is unclear how such properties may be linked to variations within the deep solar interior.

The quasi-periodic variations in rotation rate that we have detected above and below the tachocline (with a period of about 1.3 yr near the equator, and nearly in antiphase across the tachocline) appear to be of solar origin. These variations cannot be accounted for by temporal changes in the node sets of frequency splittings in either data set. Variations detected in both GONG and MDI observations, and in two inversions, strengthen the case that the detected oscillations are a real phenomenon. The variations near the equator appear to be diminishing in amplitude late in this sequence when the solar cycle attained its maximum. Those at higher latitudes have persisted. The observations are continuing with the advancing cycle.

We have no explanation for what causes these tachocline oscillations, though we suspect that magnetic fields threading across the tachocline can allow Alfvén waves to communicate stresses, and these can lead to exchanges of angular momentum across the tachocline. The autocorrelation of $\Delta \Omega$ between the convection zone and the upper part of the radiative interior is suggestive of back and forth exchange of angular momentum while conserving overall angular momentum. A radial component of the magnetic field crossing the tachocline with a strength of 500 G could yield such 1.3 yr time scales (Gough 2000). We believe that these tachocline oscillations are very significant findings, for they are the first indications of detectable changes in rotation rate close to the likely site of operation of the global solar dynamo.

NSF and NASA have contributed to the support of both the GONG and SOI-MDI helioseismology experiments and to the various data analyses and interpretations. GONG with six contributing sites is operated by the National Solar Observatory through AURA on behalf of NSF. SOHO is a mission of international cooperation between ESA and NASA.

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Figure 3. Residuals in rotation rate with time obtained from GONG and MDI data with either RLS or OLA inversion (for 0.72 R at the equator, as in Fig. 1A), indicated in each panel with solid symbols and error bars. Shown also are estimates of the effects of changing mode sets in each if the tachocline were a relatively smooth function with radius at the equator (light curve) or a sharp one (dark curve), with those curves effectively overlapping.
Variation of VIRGO/ SOHO measurements during 1996-2001

Hari Om Vats
Astronomy Astrophysics Division
Physical Research Laboratory
Ahmedabad 380009, INDIA.
Fax: 00-91-79-630-1502
Email: vats@prlernet.in

Abstract:

Here, the correlation analysis of VIRGO/ SOHO measurements of total solar irradiance during 1996-2001 has been carried out. The correlograms of 1996 and 1998 show that there is clear and strong evidence of solar rotational modulation. The rotational modulation is more in 1998 than that in 1996, however, the data of 1998 shows a splitting in the modulation peak. The synodic rotation period is estimated to be about 27 days. The variation of rotational modulation is independent of solar activity and it is found to be drastically different than other solar emission e.g. solar Lyman-Alpha, radio emission etc. There exists a large yearly variation in the correlograms having periodic and aperiodic peaks.

Introduction:

The Sun – weather relationship is gaining more and more respect these days. It is true that our understanding of the Sun and solar processes has increased dramatically during recent years, however, it is realized that the Sun affects the Earth’s environment in a much more complicated manner than we have imagined. It is impossible to describe the effects of the Sun on Earth by just a few parameters. The most important solar parameter is the total power as irradiance received from the Sun at Earth. Until about 20 years ago this was believed to be constant, as the term “solar constant” indicates. Now it is possible to make high-precision measurements of the total solar irradiance outside the Earth’s atmosphere (Frohlich, 2000). This has been confirmed that there are indeed small but systematic variations in phase with the solar activity with an increase in irradiance from sunspot minimum to sunspot maximum of about 0.1 % corresponding to a global average of 0.3 W m². To derive this number is not an easy task as explained by Frohlich (2000). The VIRGO experiment on board ESA/NASA collaborative SOHO mission measures the total irradiance (TSI) with two absolute radiometers (DIARAD and PMO6). The solar irradiance is essentially the contribution from the small scale solar surface structures integrated over the solar disk. The data set for level 1.8 and 2 is available from the SOHO/VIRGO team site. The noise reduction in total solar irradiance has been carried out by Finsterle and Frohlich (1998). Here an attempt is made to investigate the short and long term variation of total solar irradiance during 1996-2001.

Results and discussions:

The data of the total solar irradiance (TSI) measurements during 1996-2001 was analyzed. This data is available from the cooperative ESA/NASA Mission SOHO site. Here we used the level 2 data (determined by comparison of level 1.8 data; of DIARAD and PMO6-V: non-exposure dependent changes). The formulation and scheme of analysis is same as those used by Vats et al 2001. The four correlograms for the period 1996-1999 are shown in Fig. 1.

Figure 1: Correlograms of total solar irradiance measured by VIRGO/ SOHO for the year 1996-1999.

... additional text and figures...

Figure 2: Correlograms of total solar irradiance measured by VIRGO/ SOHO for the year 2000-2001.
The correlograms of 1996 and 1998 are periodic whereas those of 1997 and 1999 are aperiodic. The correlograms of 2000 and 2001 are shown in the Fig. 2. These are also aperiodic in nature almost like 1997 and 1999. The correlograms of 1996 and 1998 show an obvious solar rotational modulation with a synodic period ~ 27 days. However, 1998 data shows some sort of splitting of one rotational peak into two at ~ 25 and 35 days.

To compare the correlograms of solar Lyman-Alpha with total solar irradiance, we carried out the similar analysis for solar Lyman-Alpha emissions. The results are shown in Fig. 3. The comparison of Fig. 3 with Fig. 1 obviously indicate that the evidence of solar rotation is quite different in both these solar emissions. There seems to be no systematic variation in them.

The rotational modulation was calculated for both the total solar irradiance as well as solar Lyman-Alpha. The variation of rotational modulation for total solar irradiance during 1996-2001 is shown in Fig. 4a. If the entire measurement is localized and periodic the modulation will be 2 whereas for a totally aperiodic signal it will be 0. The modulation for TSI is 0.3 in 1996 and decreases to almost 0 in 1997. The modulation is highest ~0.6 in 1998 wherein correlogram shows a splitting. For 1999 – 2001, it is again a very small value.

The Fig. 4b shows the variation of rotational modulation for solar Lyman-Alpha for the year 1996-1999. The rotational modulation in solar Lyman-Alpha is relatively stronger. The modulation in 1997 is highest ~ 1.3 which indicates the parameter is very much periodic. Here again the modulation shows a variation from year to year with value going from as low as 0.2 to 1.3. In this signature of rotational modulation is present in all the four years.

The comparison of Fig. 4a and b shows that the rotational modulation in both these seem to have opposite phase. The annual variation of sunspot numbers is shown in Fig. 4c which show that this period (1996-1999 or 2001 is the increase phase of the solar cycle), however, Fig. 4a and b clearly show that the solar rotational modulation does not have any systematic variation with the phase of the solar activity.

Conclusions:

The earlier observations of disk-integrated radio emissions (Vats et al 2001 and references there in) do show that radio emissions from the Sun have strong rotational modulation. Similarly at many other wavelengths continuum and the emissions (Altrock et al
2002 and references there in) do show strong evidence of solar rotational modulation suggesting rigid and non-rigid rotation of various regions of solar atmosphere. The total solar irradiance as such appears to have less evidence of solar rotational modulation; however in 1996 and 1998 it is present in a reasonable magnitude. Its comparison with chromospheric Lyman-Alpha is also presented here which is rather out of phase. What could be cause of very low rotational modulation in TSI and why is it out of phase with the chromospheric Lyman-Alpha ? are the questions that need further work. Similarly the rotational modulation for both total solar irradiance and Lyman-Alpha emissions seems to have no relationship with the phase solar activity, this is also a puzzle.

![Sunspots graph]

Figure 4c: Annual variation of Sunspots number during 1996 – 2001

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ON THE INFERENCE OF SUPERGRANULAR FLOWS BY TIME-DISTANCE HELIOSEISMOLOGY

Junwei Zhao and Alexander G. Kosovichev

W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA94305-4085

ABSTRACT

We have attempted to derive the internal flow fields of supergranules by inverting time-distance helioseismology measurements based on the ray theoretical approximation. Due to the “cross-talk” effect between the contributions of the vertical flows and horizontal divergent flows and also due to the measurement errors propagation in the inversion, we can derive reliably only the horizontal velocity distribution in the supergranules, but the vertical velocity which is much smaller than the horizontal velocity is rather uncertain. The preliminary results from our inversion show that the divergent flows extend to a few megameters below solar surface, and also present evidence of converging flows at the depth of ~10Mm. A simple estimation gives the average depth of supergranules is approximately 15Mm.

Another inversion technique Multi-Channel Deconvolution is also developed. The subsurface flow fields of a sunspot derived from this technique is compared with the results from algorithm LSQR, and a correlation of 95% and above is found.

Key words: helioseismology; inversions; supergranules.

1. INTRODUCTION

Time-distance helioseismology has provided us a powerful tool to investigate the three-dimensional interior structures and the subsurface flow fields of the Sun. Based on time-distance measurements and their inversion, both the large scale flows and local sound-speed structures and flow fields have been obtained. For the large scales, Giles (1999) obtained the meridional flows and solar rotation rates up to a depth of \( r = 0.80R_\odot \). For the small scales, the sound-speed perturbation structures and flow fields beneath the visible surface of the sunspots have been revealed by use of a ray approximation (Kosovichev et al., 2000; Zhao et al., 2001). The inversion based on Fresnel zone sensitivity kernels gave the similar structures inferred from the ray approximation (Jensen et al., 2001). Numerical modeling and forward problems of the time-distance helioseismology have also been addressed recently by Birch & Kosovichev (2000) and Gizon & Birch (2002).

It is also of great interest to investigate the interior flow fields of supergranules by the use of the time-distance inversion technique. It is well known that the vertical speed of supergranules is much smaller than its horizontal speed, therefore, a number of numerical tests should be made to test the validity of the inversion technique and its ability to recover small vertical speed. Based on the ray approximation, we have employed the algorithm LSQR (Paige & Saunders, 1982; Kosovichev, 1996) to test the artificial models and investigate the subsurface flows of supergranules.

In addition to the LSQR algorithm, we also employed a new inversion technique Multi-Channel Deconvolution (MCD) based on the algorithm proposed by Jacobsen et al. (1999). This new technique has been tested to derive the flow fields of a sunspot, and results are compared with those from algorithm LSQR.

In this paper, we test the accuracy of the LSQR inversion technique on various artificial models in Section 2. In Section 3, we discuss the flow fields of supergranules obtained from our inversions. In Section 4, we describe the MCD inversion technique and compare the inversion results with LSQR. A summary is given in Section 5.

2. INVERSION TESTS OF ARTIFICIAL MODELS

According to the measurement procedure of time-distance helioseismology, the travel time differences between outgoing and ingoing acoustic waves are

\[
\delta \tau_i = \delta \tau_i^+ - \delta \tau_i^- = -2 \int_{\Gamma_i} \frac{\mathbf{v}(x, t) \cdot \mathbf{n}}{c_0^2(x)} \, ds,
\]

where \( \delta \tau_i^+ \) and \( \delta \tau_i^- \) are outgoing and ingoing travel times, \( \mathbf{v}(x, t) \) is velocity of materials, \( ds \) is a length element along the ray path \( \Gamma _i \). This equation can then be discretized as

\[
\delta \tau_{\lambda , \mu , \nu , \alpha} = \sum_{i, j, k, \alpha} A_{ijk, \alpha}^{\lambda \mu \nu} \nu_{ijk, \alpha},
\]

where \( A_{ijk, \alpha}^{\lambda \mu \nu} \) can be obtained from solar models, indices \( \lambda \mu \nu \) label points in the solar surface, \( \nu \) denotes travel distances, and \( \alpha \) is the component of vector velocity. This
is an ill-posed linear system of equations, and regularization and constraints are usually needed to solve such kind of problems. Various numerical experiments are required to test the validity of solutions. We employ the LSQR algorithm (Paige & Saunders, 1982) to solve these equations.

We have made a variety of three dimensional velocity models to simulate the flow structures of the supergranules, and test the ability of the inversion code to reproduce the flow fields set in the model from a standard set of travel times. Figure 1 shows one example of the vertical cut of the calculations we have made.

It appears that strong divergent flows near the center of divergent flows can accelerate the outgoing waves and decelerate the ingoing waves in the way as the downward flows do, and the strong convergent flows can accelerate the ingoing waves and decelerate the outgoing waves the same way as upward flows do. In some cases, due to the strong divergence and weakness of vertical flows, the inversion results for the vertical flows can have incorrect sign due to the "cross-talk" effect between the divergence of horizontal flows and the vertical flows. Figure 1 shows that after 5 iterations of the LSQR code, the "cross-talk" effect results in incorrect vertical velocities near the surface, although some deeper flow structures can be accurately reproduced. But, after 100 iterations we find that the "cross-talk" effect near the surface is greatly reduced, or nearly totally removed.

As a summary after various artificial models experiments, we have found that based on different flow structures, "cross-talk" effects can be significant or not. But for sufficiently large number of iterations, the vertical flows can still be nearly or totally recovered for error-free data. However, for the real data it is usually not possible to make a large number of iterations because of magnification of errors.

3. INVERSION OF SUPERGRANULES

Inversions have been performed for a set of MDI high resolution data. Although the error-free numerical experiments have demonstrated that a large number of iterations can reduce the "cross-talk" effects, the error magnification of the real data may prohibit us from performing a large number of iterations. Therefore, the vertical flow speed may not be obtained accurately from our inversions, because these must be stopped after a few iterations. The inversion results of the MDI data for horizontal flows are shown in Figure 2, from which we find that the divergent flows extend from the surface to a few megameters beneath the surface, and convergent flows are found at the depth of 9 – 12 Mm.

Although the vertical flows have not been reliably obtained from our inversions, the convergent flows found at depth ~10 Mm may suggest that the supergranules are cellular convective structures. If this is true, we may estimate the depth of supergranules by the following means. In a quiet solar region of 200 Mm x 200 Mm, which includes about 30 supergranules, we calculate the divergence from the inverted horizontal velocities at different depths. Then we calculate the correlation coefficient of the divergence map at each depth with the divergence map of the first layer. Figure 3 shows the results of correlation coefficients relative to the depths, from which we may estimate the depth of supergranules are averagely around 15 megameters. Previously, Duvall (1998) estimated the depth of supergranules to be around 8 Mm by calculating the correlation coefficients between \(v_x\) and \(v_y\) components of velocities speed at different depths. It seems that the preliminary results from our inversions have significantly extended the estimated depth of supergranules.

4. MULTI-CHANNEL DECONVOLUTION

Jacobsen et al. (1999) have suggested an alternative to the LSQR algorithm for solving the inversion problems of time-distance helioseismology to obtain the sound speed
Figure 2. Horizontal flows of a supergranule from inversion results of the MDI data. The background image of each graph is the divergence of flows near the solar surface to indicate the location of the supergranule, with white as divergence and dark as convergence. The longest arrow in each graph represents 500 m/s, roughly.

Figure 3. The correlation coefficients between the divergence of each depth and the divergence in the top surface layer.

variations, which is called Multi-Channel Deconvolution (MCD). Here, we extend their approach to solve the inversion problem for three dimensional velocity field.

As shown in section 2, the travel time can be expressed as

\[ \delta_T^{\lambda \mu \nu, \alpha} = \sum_{ijk, \alpha} A_{ijk, \alpha}^\mu \nu \beta_{ijk, \alpha}^\nu \]

which can be converted into Fourier domain as

\[ d = G \cdot m, \]

where

\[ d = \{ \delta T^{\lambda \mu, \alpha}(k_\lambda, k_\mu) \}, \quad G = \{ \beta_{\mu, \alpha}(k_\lambda, k_\mu) \}, \quad m = \{ \upsilon_\alpha(k_\lambda, k_\mu) \}. \]

This is actually another linear inverse problem but with a much smaller coefficient matrix. Based on the proof given in Menke (1984), the equations can be solved as

\[ m = (G^H G + \varepsilon^2 V)^{-1} G^H \cdot d, \]

where \( G^H \) is the conjugate transpose of \( G \), and \( V \) is a unit matrix. The damping parameter \( \varepsilon \) is selected as the reciprocal of signal-to-noise ratio. After solving above linear problems in Fourier domain, the three dimensional velocities can be obtained by inverse Fourier transform of \( m \).

We applied this technique on the sunspot to derive the subsurface flow fields as in Zhao et al. (2001). Figure 4 shows the results obtained at the depth of 0 – 3Mm by MCD and by algorithm LSQR. Clearly, both horizontal velocities and vertical velocity agree very well with each other. The correlation coefficients between the three dimensional velocities obtained by these two different techniques are all above 95% at different depths above 12Mm.

5. SUMMARY

Based on the ray approximation, we have tested the accuracy of the LSQR inversion technique over various artificial models. We find that, for sufficiently large number of iterations, the inversion can eventually nearly or totally recover the flow structures set in the models including both the vertical and horizontal components. However, when applying the inversion to the real MDI data, the error propagation may prevent us from performing enough number of iterations. In this case, due to the “cross-talk” effect between the divergence of horizontal flows at the center of supergranules and vertical flows, we cannot derive the vertical flows of supergranules near the surface although horizontal flows can be well resolved.

From the inversion results obtained from a set of MDI high-resolution data, we mapped the horizontal flow fields of supergranules at different depth. We find that the divergent flows can extend to a few megameters in depth, and there is evidence of converging flows at the depth of ~10Mm. An estimation based on the inversion results for 30 supergranules gives an average depth of supergranules of about 15 Mm.

We also develop a new inversion technique for 3D veloc-
Figure 4. The comparison between the flow maps of an active region obtained by algorithm LSQR (left) and MCD technique (right). The map is obtained at the depth of 0 – 3 Mm. In each image, white represents downflow and dark represents upflow. Arrows in the graph represent horizontal flows, with longest arrow as 1 km/s approximately.

ities using Multi-Channel Deconvolution. The flow maps obtained from this inversion technique in a sunspot region agree very well with the results obtained from algorithm LSQR.

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List of Participants
Ashok Ambastha  
Udaipur Solar Observatory  
Physical Research Laboratory  
PO Box 198, Dewali, Badi Rd  
Udaipur 313001  
India  
ambastha@prl.ernet.in

Valentyna Abramanto  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City, CA 92314  
USA  
avi@bbso.njit.edu

Bo Andersen  
Norwegian Space Centre  
P.O. Box 113 Skøyen  
Oslo N-0212  
Norway  
bo@spacecentre.no

H.M. Antia  
Tata Institute of Fundamental Research  
Homi Bhabha Road  
Mumbai 400005  
India  
antia@tifr.res.in

Thierry Appourchaux  
Research and Science Support Department  
European Space Agency  
P.O.Box 299  
Noordwijk 2200AG  
The Netherlands  
Thierry.Appourchaux@rssd.esa.int

David Arnett  
National Solar Observatory  
950 N. Cherry Avenue  
Tucson AZ 85719  
USA  
darmet@noao.edu

James Armstrong  
Institute for Astronomy  
University of Hawaii  
2680 Woodlawn Drive  
Honolulu HI 96822  
USA  
armstrong@ifa.hawaii.edu

Enric Palle Bago  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City, CA 92314  
USA  
epb@bbso.njit.edu

Caroline Barban  
National Solar Observatory  
950 N. Cherry Avenue  
Tucson AZ 85719  
USA  
barban@noao.edu

Sarbani Basu  
Astronomy Department  
Yale University  
P.O. Box 208101  
New Haven CT 06520-8101  
USA  
basu@astro.yale.edu

John Beck  
Stanford University  
Stanford CA 94305-4085  
USA  
beck@moto.stanford.edu

Tim Bedding  
School of Physics, Room 309  
A28 Physics University of Sydney  
New South Wales  
20006  
Australia  
bedding@physics.usyd.edu.au
<table>
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<th>Name</th>
<th>Organization</th>
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<tr>
<td>Richard Clark</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
<td><a href="mailto:rclark@noao.edu">rclark@noao.edu</a></td>
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<td>Thierry Corbard</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
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<tr>
<td>Werner Däppen</td>
<td>Department of Physics &amp; Astronomy</td>
<td>Univerisity of Southern California</td>
<td><a href="mailto:dappen@usc.edu">dappen@usc.edu</a></td>
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<td>Leonid Didkovsky</td>
<td>Big Bear Solar Observatory</td>
<td>New Jersey Institute of Technonology</td>
<td><a href="mailto:leonid@bbso.njit.edu">leonid@bbso.njit.edu</a></td>
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<td>Oleksandr Dolgushyn</td>
<td>Big Bear Solar Observatory</td>
<td>New Jersey Institute of Technonology</td>
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<tr>
<td>Thomas Duvall</td>
<td>HEPL A209</td>
<td>Stanford University</td>
<td><a href="mailto:duvall@quake.stanford.edu">duvall@quake.stanford.edu</a></td>
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<td>Elena Dzičakova</td>
<td>Astronomical Institute</td>
<td>FMPhI Comenius University</td>
<td><a href="mailto:dzicakova@fmph.uniba.sk">dzicakova@fmph.uniba.sk</a></td>
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<tr>
<td>Patricia Eliason</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
<td><a href="mailto:peliason@noao.edu">peliason@noao.edu</a></td>
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<tr>
<td>Yvonne Elsworth</td>
<td>School of Physics &amp; Astronomy</td>
<td>University of Birmingham</td>
<td><a href="mailto:ype@bison.ph.bham.ac.uk">ype@bison.ph.bham.ac.uk</a></td>
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<tr>
<td>Bernhard Fleck</td>
<td>ESA</td>
<td>NASA/GSFC Mailcode 682.3</td>
<td><a href="mailto:bfleck@esa.nascom.nasa.gov">bfleck@esa.nascom.nasa.gov</a></td>
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<tr>
<td>Eric Fossat</td>
<td>Université de Nice</td>
<td>Sophia Antipolis, UMR 6525</td>
<td><a href="mailto:Eric.Fossat@unice.fr">Eric.Fossat@unice.fr</a></td>
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</tbody>
</table>
Claus Fröhlich  
Physikalisch-Meteorologisches Observatorium Davos  
World Radiation Centre  
Davos Dorf  CH-7265  
Switzerland  
cfrohlich@pmodwrc.ch

Alan Gabriel  
UMR 6529 du CNRS  
Observatoire de Nice  
BP 4229  
Nice CEDEX 4  06304  
France  
gabriel@ias.fr

Rafael A. Garcia  
CEA/DSM/DAPNIA/Sap  
CE de Saclay  
Gif-sur Yvette CEDEX  91191  
France  
rgarcia@cea.fr

Elena Gavryuseva  
Osservatorio Astronomico di Arcetri  
Università degli studi di Firenze  
Largo Enrico Fermi 5  
Firenze  50125  
Italy  
elena@arcetri.astro.it

Katya Georgieva  
Solar-Terrestrial Influences Lab.  
Bulgarian Academy of Sciences  
Bl.3, Acad G. Bonchev str.  
Sofia  1113  
Bulgaria  
kgeorg@bas.bg

Dali Georgobiani  
Department of Physics & Astronomy  
Michigan State University  
East Lansing  
MI 48824-1116  
USA  
dali@pa.msu.edu

Peter Gilman  
High Altitude Observatory  
3450 Mitchell Lane  
Boulder  CO 80301  
USA  
gilman@jabba.hao.ucar.edu

Laurent Gizon  
Annex A210  
W.W Hansen Experiemntal Physics Laboratory  
Stanford University  
Stanford  CA 94305-4085  
USA  /lgizon@solar.stanford.edu

Philip R. Goode  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City,  CA 92314  
USA  
pgoode@bbso.njit.edu

Gérard Grec  
UMR 6525  
Universite de Nice  
Sophia Antipolis  
Nice CEDEX 4  
France  
grec@obs-nice.fr

Sarah Gregory  
gregory@mdisas.nascom.nasa.gov

Katrina Gressett  
National Solar Observatory  
950 N. Cherry Avenue  
Tucson  AZ 85719  
USA  
kgressett@noao.edu
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Address</th>
<th>Email</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joseph Gurman</td>
<td>Goddard Space Flight Center</td>
<td>Solar Physics Branch Code 682.3</td>
<td><a href="mailto:gurman@gsfc.nasa.gov">gurman@gsfc.nasa.gov</a></td>
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<tr>
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<td>Greenbelt  MD 20771</td>
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<tr>
<td>Kerri Donaldson Hanna</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
<td><a href="mailto:khanna@noao.edu">khanna@noao.edu</a></td>
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<tr>
<td>Jack Harvey</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
<td><a href="mailto:jharvey@noao.edu">jharvey@noao.edu</a></td>
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<tr>
<td>Frank Hill</td>
<td>National Solar Observatory</td>
<td>950 N. Cherry Avenue</td>
<td><a href="mailto:fhill@noao.edu">fhill@noao.edu</a></td>
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<tr>
<td>Rachel Howe</td>
<td>School of Physics &amp; Astronomy</td>
<td>University of Birmingham</td>
<td><a href="mailto:rhowe@bison.ph.bham.ac.uk">rhowe@bison.ph.bham.ac.uk</a></td>
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<tr>
<td>Stuart Jefferies</td>
<td>Steward Observatory</td>
<td>University of Arizona</td>
<td><a href="mailto:stuartj@msrc.unm.edu">stuartj@msrc.unm.edu</a></td>
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<tr>
<td>Deborah Haber</td>
<td>JILA</td>
<td>Campus Box 440</td>
<td><a href="mailto:dhaber@solarz.colorado.edu">dhaber@solarz.colorado.edu</a></td>
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<td>Boulder  CO 80309-0440</td>
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<tr>
<td>Vasily Haneychuk</td>
<td>Crimean Astrophysical Observatory</td>
<td>P/O Nauchny 17-15</td>
<td><a href="mailto:han@crao.crimea.ua">han@crao.crimea.ua</a></td>
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<tr>
<td>David Hathaway</td>
<td>Marshall Space Flight Center</td>
<td>Mail Code SD50</td>
<td><a href="mailto:david.hathaway@msfc.nasa.gov">david.hathaway@msfc.nasa.gov</a></td>
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<tr>
<td>Bradley Hindman</td>
<td>National Center for Atmospheric Research</td>
<td>High Altitude Observatory</td>
<td><a href="mailto:hindman@solarz.colorado.edu">hindman@solarz.colorado.edu</a></td>
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</tr>
<tr>
<td>Stephen Hughes</td>
<td>Space &amp; Atmospheric Physics Group</td>
<td>The Blackett Laboratory</td>
<td><a href="mailto:stephen.hughes1@ic.ac.uk">stephen.hughes1@ic.ac.uk</a></td>
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<tr>
<td>Jesper Munk Jensen</td>
<td>Theoretical Astrophysics Center</td>
<td>Danish National Research Foundation</td>
<td><a href="mailto:jmunk@phys.au.dk">jmunk@phys.au.dk</a></td>
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<td>Aarhus C    DK-8000</td>
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<td>Denmark</td>
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</tr>
</tbody>
</table>
Alexander Ruzmaikin  
Jet Propulsion Laboratory  
4800 Oak Grove Drive  
Pasadena  CA 91109-8099  
USA  
Alexander.Ruzmaikin@jpl.nasa.gov

D. Salabert  
Departement d'Astrophysique  
UMR 6525  
Universite de Nice- Sophia Antipolis  
Nice Cedex 2  06108  
France

Metin Saltik  
Sakarya universitesi  
Mühendislik fakültesi  
Esentepe Kampüsü  
Sakarya  54187  
Turkey  
saltik@sakarya.edu.tr

Huguette Lacoste  
SER-CP  
European Space Agency  
P.O.Box 299  
Noordwijk  2200AG  
The Netherlands  
Huguette.Lacoste@esa.int

Philip Scherrer  
HEPL 4085  
Stanford University  
Stanford  CA 94305-4085  
USA  
pscherrer@solar.stanford.edu

Jesper Schou  
HEPL Annex A201  
Stanford University  
445 Via Palou  
Stanford  CA 94305-4085  
USA  
jschou@solar.stanford.edu

Takashi Sekii  
Solar Physics Division  
National Astronomical Observatory of Japan  
Mitaka  
Tokyo  181-8588  
Japan  
sekii@solar.mtk.nao.ac.jp

Hiromoto Shibahashi  
Department of Astronomy  
University of Tokyo  
Tokyo  113-0033  
Japan  
shibahashi@astron.s.u-tokyo.ac.jp

Sergiy Shumko  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City,  CA 92314  
USA  
shoumko@bbso.njit.edu

Brooke Simmons  
Astronomy Department  
Yale University  
PO Box 208101  
New Haven  CT 06520-8101  
USA  
simmons@astro.yale.edu

Herschel Snodgrass  
Department of Physics  
Lewis & Clark College  
0615 SW Palatine Hill Rd  
Portland  OR 97219  
USA  
hbs@lclark.edu

Jeneen Sommers  
Hausen Experimental Physics Laboratory  
Stanford University  
HEPL 4085  B124  
Stanford  CA 94305-4085  
jeneen@quiver.stanford.edu
John Varsik  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City, CA 92314  
USA  
varsik@bbso.njit.edu

Hari Om Vats  
Physical Research Laboratory  
Ahmedabad 380009  
India  
vats@prl.ernet.in

P. Venkatakrishnan  
Udaipur Solar Observatory  
Physical Research Observatory  
PO Box 198, Dewali, Badi Rd  
Udaipur 313 004  
India  
pvk@prl.ernet.in

Rajiv Vhatkar  
rajivvhatkar@yahoo.com

Sergei Vorontsov  
Institute of Physics of the Earth  
B. Gruzinskaya 10  
Moscow 123810  
Russia  
S.V.Vorontsov@qmw.ac.uk

Martin Woodard  
Northwest Research Associates  
CoRA Division  
3380 Mitchell Lane  
Boulder CO 80301  
USA  
mfw@co-ra.com

Vasyl Yurchyshyn  
Big Bear Solar Observatory  
New Jersey Institute of Technology  
40386 North Shore Lane  
Big Bear City, CA 92314  
USA  
vayur@bbso.njit.edu

Junwei Zhao  
Department of Physics  
Stanford University  
Stanford CA 94305-4085  
USA  
junwei@quake.stanford.edu