ACCURATE MEASUREMENTS OF SOH/MDI HIGH-DEGREE FREQUENCIES AND FREQUENCY SPLITTINGS

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ABSTRACT

We present accurate measurements of high-degree p-mode frequencies and frequency splittings obtained from the Full-Disk Program of the Michelson Doppler Imager (MDI) experiment onboard the Solar and Heliospheric Observatory (SOHO). The frequencies and frequency splittings are computed from unaveraged zonal, tesseral, and sectoral power spectra using a new fitting method of Reiter et al. (2002) based upon a maximum-likelihood fitting approach. In this method, both the spectral power distribution and contributions of the various observational and instrumental effects to the spatial leakage matrices are modelled accurately. We demonstrate that one of the most long-standing problems in high-degree helioseismology, viz. the jumps in the frequency splitting coefficients, can be solved by taking into account the distortion of the leakage matrix by the solar differential rotation. The results of inversion of the initial frequency set determined using this new method in the range of angular degree $\ell = 45 - 300$ show a substantially better resolution of the subsurface layers compared to the previous studies with $\ell$ below 220.

Key words: Methods: Data analysis, Sun: oscillations.

1. INTRODUCTION

Since the acoustic modes of high angular degree are trapped near the solar surface they represent an important diagnostic tool for probing the structure and dynamics of the upper convective boundary layer. This region is of great interest for many reasons. For example, in this region the ionization zones of hydrogen and helium are located, and they can provide rather sensitive tests of the solar equation of state. Also, solar cycle variations and complex near-surface rotational dynamics detected in low- and intermediate-degree modes can only be fully understood if high-degree modes are included in the analysis. In addition, the high-degree modes have the potential to significantly improve the resolution of the solar structure in the near-surface region, provide detailed tests of the equation of state and constrain the envelope helium abundance (e.g. Rabello-Soares et al., 2000; Di Mauro et al., 2002).

However, the high-degree mode frequencies are difficult to measure reliably. This is because high-degree mode parameters cannot be determined in a straightforward manner using simple peak-finding algorithms. A power spectrum computed for a specific target mode with degree $\ell$ and azimuthal order $m$ contains contributions of power from modes with neighbouring $\ell$ and $m$ as well. These contributions are measured by the so-called leakage matrix, which is essentially defined as the overlap integrals between the target mode and modes with neighbouring $(\ell, m)$. With increasing degree the separation of the spatial leaks decreases. Moreover, the widths of the lines increase with frequency and degree. As a consequence, individual modes blend into ridges of power. Since the amplitudes of the spatial leaks are asymmetric the central frequency of a ridge is significantly offset from the frequency of the targeted individual mode frequency, resulting in unphysical jumps in the fitted parameters (e.g. Korzennik, 1990). The p-modes typically begin to blend into ridges for degrees ranging anywhere from $\ell = 140$ to $\ell = 210$ or thereabouts depending on the overtone number $n$. We show that to resolve the offset problem the leakage matrix calculations at high degrees have to take into account the distortion of the eigenfunctions by the solar differential rotation first noted by Woodard (1989).

2. DATA AND FITTING TECHNIQUE

The results presented in this paper are based upon spectra that were created from observations obtained from the MDI Full-Disk Program during the 1996 MDI Dynamics Run (Scherrer et al., 1995). Specifically, a 60.75-day time series of full-disk MDI Dopplergrams taken at a cadence of one minute was converted into $2\ell + 1$ time series of spherical harmonic coefficients. The initial time series had a common duty cycle of 95.2%. After filling a few of the shortest gaps using standard gap-filling techniques, the duty cycle of the final time series was increased to 97.3%. These time series of spherical harmonic coefficients were used to calculate a set of $2\ell + 1$ zonal, tesseral, and sectoral power spectra for $0 \leq \ell \leq 1000$ by a stan-
standard FFT method. The initial results presented here are obtained for modes in the range $45 \leq \ell \leq 300$.

The zonal, tesseral, and sectoral power spectra generated in the above-described manner were fitted employing the so-called “Maximum-Likelihood, Multi-Peak, Tesseralf-Spectrum” method (Reiter et al., 2002). Unless otherwise stated a theoretical model profile based upon the asymmetrical line profile of Nigam & Kosovichev (1998), an expansion of the rotational frequency splittings in orthogonal polynomials first suggested by Ritzwoller & Lavelle (1991), the theoretical value of $c_1$, the ratio of the horizontal and vertical components of the solar $p$-mode eigenfunctions, and the so-called reference set of leakage matrices (cf. Section 4) were used in the calculations presented in this paper.

3. EFFECT OF PERTURBATION OF THE LEAKAGE MATRIX BY SOLAR DIFFERENTIAL ROTATION

One of the long-standing problems in high-degree helioseismology is the fact that the rotational splitting coefficients and the mean multiplet frequencies appear to undergo jumps or offsets for degrees ranging anywhere from $\ell = 140$ to $\ell = 250$. The exact location of the range of $\ell$ values where the jumps occur depends on the radial order $n$. These discontinuities were noticed by Korzennik (1990) more than a decade ago, and more recently by Rhodes et al. (1999) in attempts to generate rotational splitting coefficients employing the cross-correlation method of Brown (1985) and Tomczyk (1988). Most often mode-blending effects were blamed for causing these jumps. Reiter et al. (2002) speculated that one effect that may cause these discontinuities is the coupling of modes due to the differential rotation of the Sun (see also Korzennik et al., 2002).

Woodard (1989) has shown that the distortion of high-degree $p$-mode eigenfunctions by a slow, axisymmetric differential rotation can be expressed as a superposition of the unperturbed eigenfunctions of the same radial order $n$ if Coriolis forces are neglected. Applying Woodard’s theory, we have found that taking into account this effect in the leakage matrix has a very significant impact upon the resulting frequency-splitting coefficients. This is demonstrated in Figure 1. While for the lower degrees the splitting coefficients remain almost unchanged the notorious jumps can be seen to disappear if the mode coupling due to the differential rotation of the Sun is taken into account.

There is also an effect to be noted upon the fitted mean multiplet frequencies. This is shown in Figure 2 where we have compared the average multiplet frequencies with the theoretical frequencies of Model S calculated by Christensen-Dalsgaard et al. (1996). As can be seen the inclusion of the perturbation of the leakage matrix by the solar differential rotation smoothes the distribution of the fitted frequencies significantly.

Figure 1. Degree dependence of two sets of $a_1$, $a_3$, and $a_5$ splitting coefficients for the $n = 1$ ridge. The splitting coefficients marked with the diamonds and the error bars were generated using a leakage matrix which took into account only the corrections for the distortion due to latitudinal differential rotation. The triangles were computed without the inclusion of these corrections. The error bars are $\pm 1 \sigma$.

4. EFFECT OF INSTRUMENTAL IMAGE DISTORTION

In addition to taking into account the distortion of the eigenfunctions by the solar differential rotation, several instrumental effects and processing problems, which have been disregarded in the initial generation of the spectra, have to be taken into account. However, rather than re-doing the spherical harmonic transform (SHT) which is computationally expensive we instead chose to correct the leakage matrix by passing distorted spherical harmonic images through the SHT code. Among the problems encountered are the plate scale error, the fact
that the MDI optics introduces a cubic distortion, the fact that the CCD is tilted and the width and spatial nonuniformity of the instrumental point spread function (PSF).

In the following we compare the results of using the so-called reference set of leaks, i.e. leaks which have been generated without taking into account any of these effects, with those using our best estimate of the cubic distortion and a previous estimate of the best plate scale (radius) correction for the MDI 1996 Dynamics Run. Given the problems of estimating the CCD tilt and the point spread function we have not yet corrected for them.

The inclusion of both the plate scale error and the image distortion as well as the solar differential rotation effect in the leakage matrix employed has a rather strong impact upon the fitted frequencies. This is shown in Figure 3. As can be seen by comparing Figures 2 and 3 the frequencies become much smoother if both the plate scale error and the image distortion are taken into account in the leakage matrix. The impact of these instrumental effects upon the $a_1$ splitting coefficient is shown for the $n = 1$ ridge in Figure 4. As can be seen the $a_1$ splitting coefficient is strongly affected for $t \gtrsim 200$. Similar effects are to be noted on the higher-order splitting coefficients as well as on the splitting coefficients for other ridges.

Figure 2. Degree dependence of differences between average multiplet frequencies and the theoretical frequencies calculated using Model S of Christensen-Dalsgaard et al. (1996). The top panel is for the $n = 1$ ridge, while the bottom panel is for the $n = 2$ ridge. Note the different vertical scales in the plots of the two panels. The frequency differences marked with the diamonds and the error bars were generated with the inclusion of the corrections to the leakage matrices due to differential rotation. The triangles were computed without the inclusion of these corrections. The error bars are $\pm 1 \sigma$.

Figure 3. Same as Figure 2 aside from the fact that the frequency differences marked with diamonds and error bars were generated by taking into account both the solar differential rotation and the instrumental effects in the leakage matrix employed. The leakage matrix used in computing the triangles included neither the corrections for the effects of differential rotation nor any corrections for the instrumental problems. The triangles shown here are identical to those shown in Figure 2.

Figure 4. Degree dependence of the difference of $a_1$ splitting coefficients obtained by employing two different sets of leakage matrices, for the $n = 1$ ridge. The splitting coefficients denoted by $a_1(\text{lkref})$ were generated using the so-called reference set of leakage matrices, which did not include any corrections for the instrumental problems, while the coefficients denoted by $a_1(\text{lkfull})$ were generating using leakage matrices which did include such instrumental corrections. The corrections for the solar latitudinal differential rotation were included in both cases. The error bars are $\pm (\sigma_1^2 + \sigma_2^2)^{1/2}$ where $\sigma_1$ and $\sigma_2$ are the standard deviations of the measurements of the two sets of $a_1$ splitting coefficients.
While the inversion shown in Rhodes et al. (1997) stops at 0.96 \( R_\odot \) the new inversion goes out to 0.99 \( R_\odot \) and, hence, includes the turn-around in the difference of \( \delta v^2 \) at 0.98 \( R_\odot \). Resolving the structure of this feature in detail is important for studying the equation of state and the chemical composition of the Sun.

Finally, this method provides also accurate estimates of rotational frequency splittings which are important for understanding the angular momentum transport and dynamics of the near-surface shear layer. The range of our measurements can be extended up to at least \( \ell = 1000 \) for the MDI Dynamics data.

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