HIGH-DEGREE P-MODES AND THE SUN'S EVOLVING SURFACE

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ABSTRACT

Two of the most glaring problems in contemporary helioseismology are the limited availability of high-degree p-mode frequencies for use in inversions of solar internal structure and the lack of high-degree frequency splitting coefficients for use in inversions of solar internal dynamics. A third major problem is the lack of a consensus regarding the mechanism underlying the temporal shifts of the p-mode frequencies. The lack of high-degree frequencies and the lack of similar high-degree frequency splittings have occurred because of the inherent difficulties in measuring such frequencies and frequency splittings in high-degree power spectra without the inclusion of numerous systematic errors. We will first point out the importance of high-degree p-modes to helioseismic inversions. Next, we will describe recent progress we have made in estimating high-degree frequencies, including the use of both asymmetric and symmetric profiles in our fits. We will then demonstrate that the inclusion of corrections for the eigenfunction distortion due to latitudinal differential rotation removes long-standing discontinuities in the high-degree frequency splittings. We will go on to describe our recent efforts of increasing the sensitivity of the p-mode frequencies to changing levels of solar activity through the use of observing runs which are as short as three days in duration. Finally, we will also describe planned efforts at verifying recent discoveries in solar internal dynamics through the reduction and analysis of full-disk Dopplergrams obtained during Solar Cycle 22 prior to the beginning of the GONG and MDI projects. Taken together, the recent improvements in the estimation of high-degree frequencies, frequency splittings, and the availability of useful data from Solar Cycle 22 indicate that a renaissance in global helioseismology is now at hand.

Key words: solar oscillations; high-degree frequencies; temporal frequency shifts.

1. INTRODUCTION

Within the past five years the field of helioseismology has matured to the point where inversions of solar internal structure and dynamics (the so-called “global” methods) have been joined by several different techniques (among them ring diagram analysis, time-distance analysis, acoustic holography, and acoustic imagery) which attempt to infer the properties of the solar interior beneath individual active regions (the so-called “local” methods). While many exciting new results have indeed been obtained with the local methods, there is still much important solar physics to be studied with global methods. The importance of continued research in both areas of helioseismology is emphasized elsewhere in these Proceedings in the closing lecture (Thompson, 2003).

2. IMPORTANCE OF HIGH-DEGREE P-MODES TO HELIOSEISMIC INVERSIONS

2.1. Distinctions Between Structural, Abundance, and Rotational Inversions

Some of the most exciting results that helioseismology has provided have come from numerical inversions of the solar p-mode oscillations. Specifically, these results have come from inversions of datasets of either the oscillation frequencies themselves or of the so-called frequency splittings of the tesser(al and sectoral oscillation modes. The inversions of the various frequency datasets have yielded inferences of the thermodynamic structure of the solar interior, most notably of the sound speed there, while the inversions of the frequency-splitting datasets have provided inferences of the two-dimensional angular velocity of the solar interior.

The inversions of the different frequency datasets have been referred to as “structural” inversions (see e.g. Gough et al., 1996; Basu et al., 1996; Kosovichev et al., 1997), and “abundance” inversions (see e.g. Basu and Antia, 1995; Kosovichev, 1996; Antia and Chitre, 1998), while those of the frequency-splitting datasets have become known as “rotational” inversions (see e.g. Thompson et al., 1996; Schou et al., 1998). With the exception of a small number of structural inversions which have included limited sub-sets of the high-degree ($\ell \geq 150$) p-mode frequencies, such as the pioneering sound speed inversion of...
Christensen-Dalsgaard et al. (1985), the vast majority of the structural and abundance inversions have only included frequencies of the low- ($\ell \leq 4$) and the intermediate-degree ($5 \leq \ell \leq 150$) oscillations.

As we will illustrate below, this limitation has prevented helioseismologists from accurately probing the thermodynamic structure of the outer solar convection zone. In particular, the persistent absence of high-degree frequencies from past inversions has prevented us from inferring the sound speed, density, and adiabatic gradient in the outermost three to four percent (by radius) of the solar interior. In fact, it is these very shallow sub-photospheric layers which are the most sensitive to the details of the formulation of the so-called "equation of state" (EOS) of solar material. The structure of these layers is also affected by the amount of helium that is present in them and hence any inversion for the helium abundance should include them. In spite of the exciting progress which has been provided by the recent helioseismic structural inversions, the nearly-complete exclusion of the high-degree frequencies sorely limits us from taking full advantage of the unique plasma physics laboratory which the outermost layers of the solar interior represent.

### 2.2. Potential Improvements in Determination of Solar Internal Sound Speed

The importance of extending the frequency datasets which serve as the inputs to solar structural inversions to include large numbers of high-degree frequencies was emphasized by Christensen-Dalsgaard (1998), who argued that there is an urgent need for a better understanding of the potential sources of error in the calculation of solar models and he also pointed out that most of the $\ell$-$\nu$ diagnostic plane has not yet been utilized in probing solar models through structural inversions. More recently, Christensen-Dalsgaard and some of his colleagues have presented detailed calculations which show the improvements which can be made in the mathematical functions known as kernel functions which are utilized in the structural inversions to provide the spatial sampling of the solar structural variables (Rabello-Soares et al., 2000). These kernel functions are important in structural inversions because the inversion results represent localized estimates of the sound speed or other variables which can be interpreted as convolutions of the true solar quantities with the localized averaging kernels. Clearly, a set of narrow averaging kernels is desirable in order to maximize the radial resolution of the inverted sound speed profile.

Rabello-Soares and her co-authors computed these kernel functions for both a set of MDI frequencies which did not include any high-degree frequencies whatsoever (from Schou, 1998; which they referred to as their Intermediate-Degree, or ID, set) and they also re-computed these same kernel functions using a dataset in which they replaced these intermediate-degree frequencies with a sub-set of the 7480 intermediate- and high-degree MDI frequencies which our team presented at the SOHO-6 Workshop (from Rhodes et al., 1998; which they referred to as their High-Degree, or HD, set). These two different frequency datasets are shown together in Figure 1a, which is taken from Rabello-Soares et al. (2000). In

![Figure 1a](image)

**Figure 1a.** (top) Comparison of $\ell$-$\nu$ coverages of ID mode set in gray and the additional points which were combined with the ID mode set to comprise HD mode set in black. Except for the $f$-ridge all of the ID frequencies corresponded to $\ell < 200$. (bottom) Four pairs of radial averaging kernels computed using the HD mode set in black and using the ID mode set as the dashed curve.

In Figure 1a the points which are common to both the ID and to the HD sets are shown in gray, while the additional points in the more extensive HD set are shown in black. Some of the results of these calculations are shown in Figure 1b. This Figure, which is also taken from Rabello-Soares et al. (2000), shows the manner in which the inclusion of just these few additional high-degree frequencies causes the kernel functions to become sharper and higher than when only the intermediate-degree modes are employed. The kernels computed from the ID mode set are shown as the dashed curves in Figure 1b, while the corresponding kernels which were derived from the complete HD dataset are shown in black. Not only are the black kernels sharper and taller than are the dashed kernels, but the kernels for the two outermost target radii (e.g., $r/R_\odot = 0.99$ and 0.9975) are positive everywhere rather than having negative contributions as in the case of the dashed curves.

### 2.3. Importance of High-Degree p-Modes to Studies of the Equation of State

While the solar convection zone is adiabatically stratified, the solar material is not adiabatic at the very top of that zone. Rather, the gas is undergoing the transition from being largely-neutral to largely-ionized in the hydrogen and helium ionization zones which are located in the outermost few percent (by radius) of the solar interior. It turns out that helioseismology can probe important effects of the equa-
tion of state through the ionization-induced decrease of the adiabatic gradient $\Gamma_1$ that occurs in these ionization zones. For such a study, the most promising region is the second ionization zone of helium, which is sufficiently deep to be largely unaffected by the remaining near-surface uncertainties. A number of early papers (Berthomieu et al., 1980; Lubow, Rhodes and Ulrich, 1980; Ulrich, 1982; Ulrich and Rhodes, 1983; Shibahashi et al., 1983, 1984; Noels et al., 1984) suggested that improvements in the solar equation of state could reduce the discrepancies which then existed between the theoretical and observed solar oscillation frequencies. Later, Christensen-Dalsgaard, Däppen, and Lebreton (1988) showed that the MHD equation of state reduced these discrepancies for a large range of oscillation modes. Still later, in a comparison between solar models based on the “EFF” (Eggleton et al., 1973) and the Mihalas-Hummer-Däppen (MHD) (Hummer and Mihalas, 1988; Mihalas, Däppen, and Hummer, 1988; Däppen et al., 1988) equation of states, helioseismology clearly showed the effect of the leading-order Coulomb corrections (Christensen-Dalsgaard and Däppen, 1992; Däppen et al., 1993; Christensen-Dalsgaard et al., 1996).

An alternative approach to the equation of state was independently developed for the OPAL opacity project at the Lawrence Livermore National Laboratory (Rogers, 1986; Rogers et al., 1996). In the initial set of helioseismological tests of the MHD and OPAL equations of state it initially appeared that the OPAL EOS was providing a closer fit to the oscillation frequencies than was the MHD EOS (Christensen-Dalsgaard et al., 1996). A dramatic demonstration of the improvement which the inclusion of the high-degree frequencies can make in such EOS testing is shown here in Figure 2a, which is from Rabello-Soares et al. (2000). This Figure shows the radial profile of the difference in the squared sound speed between two different solar models (in the sense MHD-OPAL) as the solid line and two different inversions which were carried out to infer this profile as the two sets of points. The inversion which employed only the 1D mode set is shown as the set of gray triangles, while the inversion which incorporated the entire HD mode set is shown as the set of black circles. It is clear that the inversion which included the high-degree frequencies can measure the sound speed difference between these two solar models much more closely to the surface than can the other inversion. Furthermore, from the agreement between the solid line and the black circles, it would appear that the inclusion of the high-degree frequencies allows the inversion to do an excellent job of measuring this important difference. The addition of a complete set of high-degree frequencies should improve the accuracy of this comparison even more.

Another Figure which also shows the value of the high-degree frequencies in discriminating between the MDI and OPAL equations of state is Figure 2b. This Figure shows the differences in the intrinsic $\Gamma_1$ instead of the sound speed differences for the two models that were used to generate Figure 2a. Here the solid line is the radial profile of the differences in the intrinsic $\Gamma_1$ of the two models, while the gray set of triangles is the result of the inversion which incorporated only the intermediate-degree frequencies, and the black set of circles is again the result of the inversion which incorporated the high-degree frequencies as well. The solid line clearly shows the locations of the two different helium ionization zones and the black set of circles clearly does a better job of matching the solid line at the solar surface. Clearly, high-degree frequencies with even smaller errors will be helpful in better resolving the two ionization zones in future inversions.

2.4. Importance of High-Degree Frequency Splittings for Internal Dynamics Inversions

An area of great difficulty in the studies of the solar internal rotation just below the photosphere is the estimation of accurate rotational frequency-splitting.
coefficients for \( \ell \geq 120 \). For example, Korzennik (1990) demonstrated that both the odd- and even-ordered frequency-splitting coefficients computed for such degrees exhibited sizeable discontinuities as functions of degree. Later, Rhodes et al. (1998) demonstrated the existence of similar jumps in the high-degree splitting coefficients computed from 1996 MDI power spectra. The lack of understanding of the true cause of these discontinuities has led to a rejection of ad hoc correction schemes and that rejection has led in turn to the near-absence of high-degree internal dynamic inversions. Instead, our knowledge of the dynamics of the shallow sub-photospheric layers has primarily come from the inversion of \( f \)-mode splittings for \( \ell < 300 \) (Kosovichev and Schou, 1997; Schou, 1999), from time-distance analyses such as those described by Kosovichev et al. (2001), and from the inversion of \( p \)-mode frequency splittings determined from ring-diagram analyses (Haber et al., 2002). Since inversions of accurate high-degree rotational splitting coefficients will be necessary for the confirmation of the results of these local helioseismic techniques, our new-found ability of computing rotational splitting coefficients which do not have large discontinuities in them, which we will demonstrate in Section 3.3, will be essential for such comparisons.

3. RECENT IMPROVEMENTS IN THE FITTING OF SOLAR POWER SPECTRA

3.1. Development of Our First \( p \)-Mode Fitting Method

In attempts to generate multiple sets of \( p \)-mode parameters which span widely different solar conditions which are also highly accurate, we have developed a total of three different methods for fitting the peaks in solar oscillation power spectra. We began our efforts by developing the first of our three generations of power spectral fitting techniques in order to fit the power spectra which were computed as part of the MDI experiment’s Medium-\( \ell \) Program. The first-generation technique which we developed has come to be known as the “single-peak, averaged spectrum” method (which we will refer to from now on as Method 1). In this method we employed a single, symmetric Lorentzian profile to fit each peak in each set of \( m \)-averaged spectra. However, the use of only a single Lorentzian profile to represent a set of closely-spaced oscillation peaks and sidelobes introduces errors into all of the \( p \)-mode parameters as is shown here in Figure 3a. In this panel we show that the use of a single Lorentzian profile to represent the \( \ell = 50, n = 10 \) oscillation mode does not adequately fit the lower portions of this peak due to the proximity of the spatial sidelobes to the central peak of interest. The scaled residuals of this fit are also shown in Figure 3b.

Isolated \( p \)-mode peaks can be measured only for degrees less than about 220, with the exception of the fundamental mode, which has been detected as a set of isolated peaks up to \( \ell = 296 \). In addition, even for degrees for which some individual modal peaks can be identified at low frequencies, these peaks become blurred together as the frequency is increased. This overlap of peaks and sidelobes produces a so-called “ridge” of observed power which must be fit. The problems inherent in the fitting of broad ridges rather than sharp, isolated peaks of observed power were first addressed by Libbrecht and Kaufmann (1988), who showed how these problems resulted in a so-called “frequency pulling” of the measured ridge-fit frequencies away from the “true” solar frequencies for the high-degree modes. Due to the severity of such frequency-pulling effects, we attempted to develop a frequency-correction procedure in which we fit the low- and intermediate-degree power spectral peaks with both a set of wide fitting ranges and with a set of narrow fitting ranges for our Lorentzian profiles. An example of the profile which resulted when the wide fitting range was employed to fit the same \( \ell = 50, n = 10 \) mode is shown here as Figure 3c. Even at the scale of this Figure it should be evident that the vertical dashed line which represents the frequency that Method 1 determined when the wide fitting range was employed does not agree with the location of the central peak in the observed spectrum. The scaled residuals between this “wide fit” and the observed profile are shown in Figure 3d. These residuals show the obvious disagreement between the fitted profile.
and the individual sidelobes. We stress that while this disagreement is obvious in this case of a set of resolved peaks, the same problems are occurring in the cases of the broad ridges of power at the higher frequencies and higher degrees even though the observed peaks do not show evidence of the individual sidelobes. We tried to correct our “wide fit” frequencies for the effects of the mode-blending using the differences between the narrow- and wide-fit frequencies, but by the fall of 1998 we became convinced that such an approach would never be successful because it relied on an extrapolation of such frequency differences to much higher degrees than those modes for which both fitting methods could be carried out.

3.2. Development of Second Fitting Method: The WMLTP Method

The second-generation fitting method is our so-called “Windowed, Multiple-Peak, Averaged-Spectrum” Method, which we will refer to herein as either Method 2 or our WMLTP method. As its name implies, the WMLTP method is applied to m-averaged power spectra, just as was the case with Method 1. Initially, our WMLTP method incorporated a sum of five symmetric Lorentzian profiles and it included the m-averaged leakage matrix for the instrument which generated the power spectra being fit; however, due to the relatively high duty cycle of the MDI spectra which we developed this method to fit, we did not convolve the theoretical profile with the power spectrum of the so-called temporal window function of each observing run. An example of a fit using this early version of Method 2 is illustrated here in Figures 3e and 3f. These panels indicate that the absence of the temporal convolution caused the fitted profile to miss about one-half of the peaks in our observed MWO power spectrum. Subsequently, we modified the code to increase the number of profiles which were included to as many as nine and we also convolved the theoretical profile with the power spectrum of the actual temporal window function of each observing run. The dramatic improvement in the fitted profile which resulted when we computed the actual window function for our 1996 MWO time series is illustrated here in Figures 3g and 3h. The fitted profile now can be seen to pass through all of the peaks in the observed spectrum and the scaled residuals are relatively small and do not depend upon frequency. We also note that this version of Method 2 provided more accurate amplitudes and widths than did Method 1 or the earlier versions of Method 2.

In our most recent improvement to this WMLTP fitting method we have replaced the symmetric profiles with the asymmetric profile of Ngiam and Kosovichev (1998). This change has removed one of the previous limitations to widespread use of the WMLTP method. The details of how the Ngiam and Kosovichev profile is implemented in the WMLTP fitting method are given by Reiter et al. (2002a). An example of how well the asymmetric WMLTP Method fits observed power spectral peaks when the individual modal peaks can be separately resolved is given in Figure 1 of Reiter et al. (2002a). Additional comparisons of an observed power spectral ridge with fitted profiles which were computed using both the symmetrical Lorentzian and the asymmetrical profiles are shown here in Figure 4. In Figure 4a we show the observed ridge for $\ell = 750$, $n = 2$ with the fit using the symmetric profile superimposed as the solid curve. In Figure 4b we show the residuals between the observed ridge and the symmetric profile. These residuals show obvious, systematic, frequency-dependent trends. In Figure 4c we show the same observational ridge as in Figure 4a, but with the asymmetric profile employed instead of the symmetric Lorentzian profile. In Figure 4d we show the residuals between the observations and the asymmetric profile. In contrast to the residuals of Figure 4b these residuals are much smaller in general and they do not show such large frequency-dependent trends.

In Figure 4a we show the $\ell$-$\nu$ diagram which resulted when we fit the m-averaged power spectra computed from the 1996 MDI Dynamics Run time series using the WMLTP method and the asymmetric profile of Ngiam and Kosovichev (1998). In Figure 5b we present the frequency dependence of the Full-Width-at-Half-Maxima, FWHM, of the fits using the same spectra and the same profile. In Figure 5c we present the frequency dependence of the logarithm of the fitted power densities computed from the same spectra and the same profile. In Figure 5a we show the frequency dependence of the asymmetry parameter, $B$, as defined in equation (2) of Reiter et al. (2002a).

In Figure 6b we show the frequency dependence of the differences between the frequencies computed using the symmetric Lorentzian profiles and the asymmetric profiles. Inspection of both panels of Figure 6 shows that the frequency differences have a frequency dependence which is very similar to that of the asymmetry parameter, with a strong oscillation in both quantities occurring for frequencies above 5000 $\mu$Hz. We have also compared the FWHM values and the power densities computed using the symmetric and asymmetric profiles and we have found that the FWHM values show the largest relative differences at the same high frequencies. However, the power densities have large relative differences both at low frequencies where the fitted power densities are low and at frequencies above 5000 $\mu$Hz, where the power densities are also low.
3.3. Development of the “Maximum-Likelihood, Multi-Peak, Tesseral-Spectrum” Method

We have already discussed the large discontinuities which have been seen in the odd-ordered, rotational frequency splitting coefficients for degrees running from roughly \( \ell = 140 \) to \( \ell = 220 \). In an effort to compute splitting coefficients which did not suffer from such discontinuities, we developed a third-generation method of frequency estimation which employs the zonal, sectoral, and tesseral power spectra rather than using \( m \)-averaged spectra. This method, which we refer to as our Method 3 allows to simultaneously obtain both splitting coefficients and frequencies. Some details of Method 3 are given in Reiter et al. (2002b). As was the case for Method 2, either the symmetric Lorentzian profile or else the asymmetric profile of Nigam and Kosovichev (1998) can be invoked in Method 3.

Recently, a preprint written by Korzennik, Rabello-Soares, and Schou (2002) called attention to the much-earlier paper by Woodard (1989) in which he described the distortion which solar latitudinal differential rotation introduces into the spherical harmonics which govern the spatial behavior of the solar \( f \)- and \( p \)-modes. Specifically, Woodard (1989) showed that the distortion of high-degree \( p \)-mode eigenfunctions by a slow, axisymmetric differential rotation can be expressed as a superposition of the unperturbed eigenfunctions of the same radial order \( n \) if Coriolis forces are neglected. Upon seeing this preprint, we immediately modified our Method 3 to correct the spatial leakage matrices which it employs to include the corrections for this distortion.

When we employed this modified version of Method 3 to the fitting of our reference set of 1996 MDI Dynamics Run zonal, sectoral, and tesseral power spectra, we discovered immediately that the discontinuities had disappeared from the odd-ordered splitting coefficients. Comparisons of our original, uncorrected odd splitting coefficients with our new, distortion-corrected splitting coefficients for the \( n = 1 \) \( p \)-mode ridge are shown here in Figure 7a, which is taken from Reiter et al. (2003). Even a cursory inspection of Figure 7a shows clearly that the corrections which Woodard (1989) described have removed the jumps in the odd coefficients. We are elated to have finally solved one of the most long-standing problems in the use of high-degree splittings in solar dynamics inversions. In addition to removing the jumps in the odd splitting coefficients, the inclusion of the differential rotation correction also improves the multiplet-averaged frequencies which result from Method 3. This is shown here in Figure 7b, which is also taken from Reiter et al. (2003), where we have compared the average multiplet frequencies with the theoretical frequencies of Model S calculated by Christensen-Dalsgaard et al. (1996). As can be seen the combination of radius corrections and optical distortion corrections caused by MDI instrumental problems with the perturbation of the resulting leakage matrix by solar differential rotation smooths the fitted frequencies significantly.

4. STUDIES OF SHORT-DURATION P-MODE FREQUENCY SHIFTS

While it is now a well-established fact that the frequencies of the low- and intermediate-degree solar \( p \)-mode oscillations do indeed change with time in...
response to changing levels of solar activity, there is currently no consensus as to the solar origin of these changes. This situation was summarized clearly by Kuhn (2001), who described the various changes which have been seen in solar $f$- and $p$-mode frequencies, frequency splittings, horizontal flow velocities, and solar diameter measurements as diagnostics of what he called the "acoustic solar cycle". Consequently, we have been working to improve this situation by employing short-duration time series in the computation and fitting of $p$-mode frequencies and widths. We have adopted this strategy because the use of lengthy time series as in most past studies of the temporal frequency shifts of these modes has averaged out the daily variations in the corresponding activity indices during those time series. Such extensive temporal averaging has likely reduced the sensitivity of the resulting $p$-mode frequencies to the changing levels of activity. It has been our hope that the use of short-duration observing time series, during which solar activity is more constant, may allow us to increase our sensitivity of the frequency changes to the mechanism which causes them.

The $p$-mode frequency datasets which we employed in this study were computed with our WMLTP fitting method. In this study this fitting method was applied to a total of five sets of $m$-averaged MDI power spectra which were computed from observing runs which were either 4320 or 4800 minutes in length. This fitting method was also applied to a total of six MWO sets of $m$-averaged power spectra which were all computed from observing runs which were slightly less than 4320 minutes long due to the time of sunset at the end of the third observing day of each run.

To extend previous studies of $p$-mode frequency shifts so that both the high-frequency and high-degree modes would be included in the comparisons, we compared all of our various MDI datasets and we compared the four of our six MWO datasets which had the highest duty cycles; however due to systematic shifts between the MDI and MWO frequency datasets at frequencies above 5000 $\mu$Hz, we chose not to cross-compare any of the MDI datasets with any of the MWO datasets. These inter-comparisons yielded a total of ten tables of MDI frequency shifts and a total of six tables of MWO frequency shifts. In order to study the dependence of these frequency shifts upon changing levels of solar activity, we also computed the average 10.7-cm radio flux and the average magnetic plage strength index for each observing interval. In the top panel of Figure 8 we show the average 10.7-cm radio flux values as a function of time for all 11 of our different frequency datasets and for other, longer-duration observing runs available to us from MWO, MDI, and GONG+ in order to place the levels of solar activity in the context of the most recent 12 years. This Figure indicates that we have been able to obtain frequency datasets covering more than one entire solar cycle. In the middle panel of Figure 8 we show the daily 10.7-cm flux values for each of the five MDI time observing runs. In the bottom panel of Figure 8 we show the daily 10.7-cm flux values for the six MWO observing runs along with the 3-day average values, which are connected by a dashed line. The four MWO frequency sets which we compared were from the first four of these six runs.

In Figure 9a we show the frequency dependence of the unbinned frequency shifts between two datasets
which correspond to minimum and maximum levels of solar activity in order to illustrate the frequency dependence of frequency differences corresponding to very large differences in levels of activity. Past studies of the temporal behavior of the p-mode frequencies have shown that such frequencies increased along with the rising level of activity between the two runs. The "dip" in the frequency shifts is very similar to a similar dip in a comparison of unpublished 1990 and 1987 South Pole frequencies which was included in a review talk at the SOHO-4 Workshop by Harvey (1995). It is also similar to a dip in a comparison of 1991 Mees Obs. and 1987 South Pole data which was reported by Ronan, Cador, and LaBonte (1995). We note that this "dip" in the frequency shifts is roughly centered at the location of the acoustic cut-off frequency as shown by Steffens et al. (1995). On the other hand, the dip is somewhat lower in frequency than the calculated frequency shifts published by Johnston, Roberts, and Wright (1995), and by Jain (1995) all of who found a strong dip in the shifts at a frequency of about 6200 μHz. In Figure 9b we show a similar comparison between two of our 3-day MDI frequency datasets. Once again, solar activity was increasing, but in this case the difference in activity was much smaller since both runs in Figure 9b were obtained near solar minimum; hence, the frequency differences in this panel are considerably smaller than are the comparable differences in Figure 9a.

To better illustrate the anti-correlation of the shifts in the lower and higher frequency regimes, we have plotted the binned frequency shifts as functions of frequency for all 10 of the MDI frequency-difference datasets in Figure 10. In Figure 10a we show the binned differences for the four intervals during which solar activity increased between the two runs by the largest amount, while in Figure 10b we show three of the sets of binned differences for time intervals when the activity was relatively unchanged, and in Figure 10c we show three sets of the differences which corresponded to decreases in the level of activity from the first run to the second run of each pair. The curves in Figure 10a show that in each of these sets of frequency differences the lower frequency differences did increase as expected, but they also show that the higher frequencies ranging between 5000 and 6000 μHz dropped as shown in Figure 9. On the other hand, the curves in Figure 10c show clearly that the low-frequency shifts were in fact negative as would be expected from the earlier studies. Figure 10c also shows that above 5200 μHz the frequency shifts changed sign and the "dip" mentioned above was replaced by a "peak" in the frequency differences.

In order to study the response of our MDI and MWO frequency shifts to changing levels of solar activity in more detail, we first selected three different points along the curves of Figure 10. Specifically, we selected the frequencies of 3625, 4875, and 5625 μHz as being representative of the low- and high-frequency regimes of our curves. Next we subtracted the mean 10.7-cm flux values of the different observing runs from one another and we generated a table of differences in the radio flux. Next, we performed linear regression analyses of the different sets of frequency shifts upon the differences in 10.7-cm flux. In addition we also subtracted the frequency shifts located at the minima of the curves in the top panel of Figure 10 (i.e. at 5625 μHz) from the shifts located at the peaks of the curves at 4875 μHz. These differences illustrated the difference in behavior of the two different frequency regimes along the curves shown in Figure 10. The results of regressing the 16 binned shifts at 3625 μHz upon the 16 differences in the 10.7-cm flux are shown in upper-left panel of Figure 11, while the results of regressing the 16 shifts at 4875 μHz on the same 16 10.7-cm flux differences are shown in the upper-right panel of Figure 11. Furthermore, the results of regressing the 16 frequency shifts in the "dip" at 5625 μHz upon the 16 10.7-cm flux differences are shown in the lower-left panel of Figure 11 and the results of regressing the 16 differences of the
frequency shifts at 4875 and 5625 μHz are shown in the lower-right panel. As with most past studies of the low- and intermediate-degree shifts, the shifts at 5625 and at 4875 μHz show positive slopes of 35.8 and 61.5 nHz/SFU, respectively, where SFU = 1 solar flux unit. These slopes are roughly ten to 20 times larger than the slopes found in most of the long-term studies, which were generally less than 4 nHz/SFU. In the two lower panels of Figure 11 the slopes were -137.7 and 199.2 nHz/SFU, respectively. In both of these cases the absolute values of the slopes were substantially larger than the slopes in the upper two panels, suggesting that the higher frequencies have greater sensitivity to differences in solar activity. We also note that the fourth slope is simply equal to the difference in the second and third slopes. In all four panels of Figure 11 the ten mean MDI frequency shifts are shown as the xs, while the six mean MWO frequency shifts are shown as the triangles.

The regression analyses summarized in Figure 11 are merely examples of the relationship between the frequency shifts and changes in solar activity. In Figure 12 we show the frequency dependence of the regression slopes, intercepts, and correlation coefficients which resulted from our correlations of only the ten sets of MDI binned frequency shifts illustrated in Figure 10 with their associated changes in 10.7-cm flux. The upper two panels of Figure 12 show how the linear regression line rotates rapidly with increasing frequency above 5000 μHz, while the lower panel shows that the higher-frequency shifts were definitely anti-correlated with the lower-frequency shifts.

6. CONCLUSIONS

We have demonstrated the importance of including the high-degree frequencies in structural inversions and we have demonstrated that we can now compute high-degree frequency splittings without the large discontinuities which were present in all previous studies of the high-degree splittings. We have also addressed the current controversy over the mechanism of the p-mode frequency shifts by our use of shorter time series than have generally been used in such studies. Specifically, we have shown that our different sets of computed frequencies of both the

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intermediate- and high-degree modes have such high signal-to-noise ratios that we have been able to compare time series as short as three days in duration. We have also shown that many of the frequencies above 5000 µHz are anti-correlated with changes in solar activity and that they are also more sensitive to such changes in activity than are the frequencies below 4000 µHz. Hence, we have demonstrated the response of the p-modes to short-term changes in solar activity. Both of these results indicate that short-duration, high-frequency shifts are more sensitive to changes in solar activity than are the shifts measured by long-duration studies which use only the lower frequencies. Lastly, we have described plans to study the dynamics of the interior during Solar Cycle 22.

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