Magnetic Doppler Imaging of Solar-type Stars

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Abstract. We have developed and tested a Magnetic Doppler Imaging (MDI) code capable of reconstructing the distribution of field vectors over a stellar surface without additional (often artificial) assumptions. The new code was successfully applied to magnetic CP stars. Here we present the results of numerical experiments for active late-type stars in order to formulate the requirements for the observations and assess the properties of solutions. The simulations show that our code can be used to recover the general magnetic field distribution over the surfaces of active late-type stars provided that good quality observations in four Stokes parameters are available. For the models used in the simulations cyclic variations of the polarization profiles look more systematic and easier to observe than rotational modulation which may indicate a new way of studying stellar activity cycles.

1. Introduction

Following the development of theory for the formation and evolution of the lower main sequence stars one gets a feeling that whenever a model encounters difficulties magnetic fields are called to the rescue. Such "rescues" are hard to verify experimentally as direct observation of magnetic fields on solar-type stars other than the sun is a next-to-impossible task. Nevertheless, it is clear that magnetic fields play a major role in forming low-mass stars, in supporting their activity on the main sequence and in controlling mass loss during the last stages of their lives. One of the main obstacles in development of the theory is the gap between what is usually derived from observations (disk integrated temperature, gravity and abundances, rotation rate, short- and long-term modulation of these parameters) and what is predicted/required by the models (differential rotation, temperature and magnetic field distributions). Recent progress in observational techniques have allowed us to obtain very accurate time series of high-resolution spectra and spectropolarimetry and our goal is to create a tool capable of deriving from these data the physical parameters needed for the models. In order to achieve this goal we have designed a Magnetic Doppler Imaging code INVERS10 capable of recovering the distribution of magnetic field vectors over a stellar surface. In the next sections we briefly describe the code, the numerical experiments and the application to magnetic CP stars, followed by numerical experiments for active solar-type stars. We derive the requirements.
for the observations and discuss the information content which can be extracted from the field maps of these stars.

2. MDI procedure

MDI reconstructs the distribution of magnetic field vectors from a time series of spectropolarimetric data. The data include observations in four Stokes parameters with high spectral resolution covering the rotational period as evenly as possible. We assume that the surface distribution does not change during the observations and that we can synthesize the polarization spectrum from each surface resolution element with a single model atmosphere and a single magnetic field vector.
Figure 2. Comparison of the rotational and cyclic modulation of the circular and linear polarization profiles for the inclination angle of 60° and $v \sin i = 20 \text{ km s}^{-1}$.

The surface resolution is determined by the spectral resolution and the phase coverage with the ultimate limit set by the ratio of the projected equatorial rotational velocity $v \sin i$ to the Doppler width of the spectral lines.

The field map is constructed by solving the inverse problem similar to conventional Doppler Imaging (Piskunov & Rice 1993). However, extensive tests and numerical experiments with INVERS10 demonstrate significant differences of MDI. The most important difference of MDI is the ability to recover a unique solution without relying on regularization. This is not totally surprising as Piskunov & Kochukhov (2002) demonstrated that mathematically a continuous set of four Stokes parameters defines a unique surface distribution of magnetic fields. The second most important difference is the ability of MDI to recover magnetic maps of very slow rotators. This feature is explained by the fact that Stokes profiles are rotationally modulated not only due to the Doppler effect but primarily due to the change of orientation of magnetic fields with respect to the polarization analyzer. We also note that inhomogeneities often associated with magnetic fields do affect the shape of all Stokes profiles and for this reason we made INVERS10 capable of recovering one scalar parameter (e.g. abundance the in case of a magnetic CP star) simultaneously with magnetic fields.

A detailed description of the code is presented in Piskunov & Kochukhov (2002) where we also presented a comparison of the formal magnetic radiative transfer solvers and the implementation scheme on parallel computers.
Kochukhov & Piskunov (2002) presented the results of numerical experiments carried out in order to study the stability and the convergence of the code for realistic data sets. The code was successfully applied to a few magnetic CP stars ($\alpha^2$CVn, CS Vir and 53 Cam, Kochukhov et al. 2002) and proved to be capable of recovering even rather complex field geometries.

3. MDI of cool stars

The selection of magnetic CP stars for the first application of our MDI procedure was not a random choice. These objects are expected to have magnetic fields coherent on rather large scales which can be resolved with Doppler Imaging. They also show no evolution of the surface structures on time scales of typical observations and thus represent perfect targets for the MDI code. Late-type stars have an unknown but presumably very complex field geometry (supported by the solar example and by extremely low polarization signals even for the most active stars) with surface structures evolving on time scales of a few weeks. These circumstances make uncertain even the concept of applying MDI to late-type stars. In order to test the concept we decided to start with numerical experiments consisting of the selection of a field model, calculation of the synthetic "observations" in four Stokes profiles, application of MDI and comparison of the original and recovered field distributions.
3.1. Model star

Recent work by Schrijver & Title (2001) suggests that fields on more active stars could be more organized globally than on the Sun and we may hope to detect net polarization from such objects. Although this model is based on extrapolation from the solar case and may not have a deep physical basis it can be used as a first approximation for the field geometry on active stars. Moreover, Schrijver and Title carried out calculations through the whole solar activity cycle which allows us to test MDI for stars with different activity levels.

The models of magnetic fields produced by Schrijver and Title consist of magnetic field sources (dipoles) emerging 30 times more often than in the Sun. The surface distribution is taken from solar observations and the final field geometry is derived by computing the merging/annihilation of magnetic active regions. The models are computed throughout an 11 year activity cycle and thus can be directly compared with the observations of the Sun. For the line profile calculations we assume a radial field at the location of every source. The typical number of sources is 17000 to 50000 for the Sun and 35000 to 125000 for an active star model. Figure 1 shows an example of the distribution of the field vectors used in calculations. The stellar surface was subdivided into approximately 10000 elements of roughly equal areas and the total magnetic flux within each element was divided by the surface area in order to get the field strength.

3.2. Simulation of spectropolarimetry

The synthetic spectropolarimetry was computed using the forward mode of the INVERS10 code (2002) for 10 equispaced rotational phases. The selected resolving power is 100000. The calculations were performed for Fe I 5250.2 Å line using a
Kurucz (1993) solar model atmosphere ($T_{\text{eff}}=5770$ K and log $g=4.44$). Magnetic radiative transfer was solved with the quadratic DELO algorithm (Trujillo Bueno 2003). The Stokes profiles were computed for two inclination angles 30° and 60° and for four rotational velocities 5, 10, 20 and 30 km s$^{-1}$ for each of the six activity cycle phases available. We did not include differential rotation in the model. Examples of the resulting profiles are shown in Figures 2 and 3.

The amplitude of the rotational modulation of circular and linear polarization is very small compared to the amplitude of the polarization signal in particular for low inclination angles (Fig. 3). For larger inclinations we see an increase in the modulation of circular polarization while the linear polarization profile remains rather constant. On the other hand, cyclic modulation of the polarization profiles is significant implying that we should consider averaging throughout a single rotation in order to increase the signal-to-noise ratio of (linear) polarization. Such behaviour of the polarization signal reflects the properties of the magnetic field distribution and can be used to verify the Schrijver and Title models observationally.

3.3. MDI maps

The synthetic data described above was used as input for the MDI procedure. The inverse problem was solved on a surface grid consisting of 1800 elements with roughly constant element area (the number of elements per latitude decreases towards the poles). The solution used all four Stokes profiles while regularization was only imposed to maintain numerical stability of the optimization scheme early during first iterations. The Marquardt-Levenberg optimization algorithm is used to find the solution. The radiative transfer equation is solved "on the fly" for each element, rotational phase and wavelength. High perfor-
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Figure 6. The line profiles in four Stokes parameters computed from the resulting maps for rotational phase 0.2 (upper curves) and differences from the original synthetic data shifted and scaled by factor of 50 for $I$ and 20 for the three polarization profiles. Dash-dotted lines indicate the vertical shifts for the differences.

Performance is achieved by distributing individual surface elements between different processors on a parallel machine using the MPI library. The reconstructions described here were performed on a 12 CPU V-2200 server from Hewlett Packard. The comparisons of the original and resulting field maps derived for $i = 60^\circ$ and $v\sin i = 30$ km s$^{-1}$ are shown in Figures 4 and 5. Figure 6 compares the Stokes profiles for one phase.

4. Results

The excellent reproduction of the line profiles shown in Figure 6 is not surprising as the phase coverage was clearly insufficient to come anywhere near the spatial resolution of the original distribution and therefore even a lower resolution grid must be able to reproduce the observations. These values of the polarization and its rotational modulation are very small: 0.015/0.005 for $V$ and 0.002/0.0005 for $Q$ and $U$. This values set the precision requirements for observations to $10^{-4}$. Lower spectral resolution dramatically reduces the amplitude of the polarization signal. The maps shown in Figures 4 and 5 show a general resemblance e.g. in the latitude distribution of the field strength. Considering the meridional and azimuthal field components presented in Figure 4 to our great satisfaction we see very little crosstalk with the radial field (less than 100 G). It is much harder to study the two maps in detail and we find it more important to compare statistical properties of the two maps. Figure 7 shows the distribution of surface elements as a function of radial field, total field strength as well as the average field strength plotted against the latitude. For the histograms we have scaled down the number of surface elements in the original map to 1800. We see very good reproduction of the radial field distribution while for the field strength we lack the very weak fields. This is the result of the zero initial guess used for the MDI and it also gives an estimate for the uncertainty at a level of 100 G.
Figure 7. Statistical properties of the original and reconstructed maps.
The latitude field distribution is a slightly smoothed version of the original map consistent with the expected reduction in spatial resolution due to the limited phase coverage used in the experiment.

From these preliminary results we conclude that MDI is capable of recovering the general distribution of magnetic fields on an active late-type star. We should aim at targets with $\theta > 60^\circ$ and moderate rotational velocities and try to achieve $S/N$ of $10^4$ in all four Stokes parameters with high spectral resolution and dense phase coverage. We may also expect a significant modulation of polarization related to field evolution throughout the activity cycle. The detection of this modulation is within the reach of modern instrumentation and may prove to be our best source of information about stellar activity cycles outside the solar system. The final conclusion is that models similar to the one by Schrijver and Title can be directly compared with the results of MDI and thus we can finally bridge the gap between theoretical dynamo simulations and the observations.

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References

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Discussion

J.-F. DONATI: You mentioned that rotational modulation in Stokes $V$ is small. This is not what we observe on cool stars, we actually see that rotational modulation in Stokes $V$ is as big as $V$ itself.

N. PISKUNOV: The amplitude of rotational modulation was studied here for specific models which are directly extrapolated from the solar activity pattern. In reality, the distribution of magnetic active regions may be different. In these simulations we also ignored temperature variations over the surface which will cause rotational modulation of the polarization profiles. This is well known in the case of magnetic CP stars where ignoring inhomogeneous surface chemical composition hinders accurate reproduction of Stokes profiles.

J.-F. DONATI: You mentioned that the solution of the inverse problem is unique when all 4 Stokes parameters are available. This is not true. The uniqueness is only ensured by the regularization function (maximum entropy, Tikhonov
etc). This is particularly crucial for low multipolar field expansion for which e.g.
maximum entropy regularization fails reproducing the initial field structure even
when the 4 Stokes parameters are available (see Donati 2001, Astrotomography
workshop).

N. PISKUNOV: Unlike the conventional Doppler Imaging MDI mathematically
is not an ill-posed problem, that is any finite difference between two maps will
result in finite difference in the rotational modulation of the Stokes profiles (see
Piskunov & Kochukhov, 2002, A&A 381, 736). In practice, a discrete phase cov-
erage, limited spectral resolution and limited signal-to-noise reduce the spatial
resolution of MDI but if one uses the appropriate surface grid there is no need
for regularization in order to achieve a unique solution. We use regularization
during the starting iterations to prevent the optimization algorithm from going
astray, but the influence of the regularization function decreases as we approach
the solution.

G. WADE: The ability to recover the field down to $\sim 5 \text{ km s}^{-1}$ is a characteristic
of the mapping technique. One must still detect Zeeman signatures from such
stars, and it is in exactly such slow rotators that we have the most observational
challenge (due to cancellation of regions of opposite polarity, and due to the
inherent weakness of the dynamo in such stars). So although it is possible in
principle, it is highly unlikely in reality.

N. PISKUNOV: For typical field strengths on late-type stars it takes only a few
$\text{ km s}^{-1}$ to shift local line profiles by the width of its Doppler core and Zeeman
splitting. There are also active fast rotating stars visible at low inclination
angles.