The Crab Glitches: Incidence and Cumulative Effect

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Abstract. The fourteen glitches observed during 33 years do not show the simple pattern expected from a relaxation oscillator. They may however be regarded as three major events separated by about 12 years, the third being a group of smaller glitches. There is a step increase in slowdown rate at each glitch, whose cumulative effect makes a significant contribution to the second differential \( \dot{\nu} \). The braking index \( n \) has previously been evaluated only between glitches: the effect of the glitches is to reduce \( n \) from 2.51 to 2.45. This extra effect due to the glitches would be explained by an increase in dipole field at the fractional rate of \( 1.5 \times 10^{-5} \) per annum.

1. Introduction

I start by reminding you that we do not fully understand the rotational slowdown of pulsars.

The rotational slowdown of neutron star losing angular momentum by pure magnetic dipolar radiation should follow a power law

\[
\dot{\nu} \propto -\nu^n
\]

where the braking index \( n = 3 \). This can be tested when a value of \( \dot{\nu} \) can be measured, by finding

\[
n = \frac{\nu \ddot{\nu}}{\dot{\nu}^2}
\]

Where measurements are possible, no pulsar has been found with a braking index of 3. The discrepancy is usually attributed to complications in the magnetosphere, and particularly to angular momentum loss by particle outflow in the polar regions.

The slowdown is punctuated by glitches. Measurements of \( \dot{\nu} \) require long runs of data, which for most pulsars include glitches; we do not know how these affect the long-term behaviour. Taking the view that glitches are temporary steps followed by a complete recovery, we can smooth over several glitches to find \( \dot{\nu} \); for the Vela pulsar this gives \( n = 1.4 \pm 0.2 \). For the Crab pulsar the glitches are small and there are sufficiently long stretches of data between them to allow measurement of \( \dot{\nu} \), giving remarkably consistent values of braking index around \( n = 2.51 \). But we know that for the Crab and possibly other pulsars there is a persistent step in slowdown rate \( \dot{\nu} \) at glitches. What is the effect of these steps on the overall slowdown law?
Figure 1. The sequence of glitches in the Crab pulsar 1969-2002.

We now have available an almost complete record of the rotational slowdown of the Crab pulsar over 33 years (Lyne et al 1993, Wong et al 2001), including 14 glitches above a well-defined size limit. We expect to publish details of the most recent glitches shortly (Jordan, Smith and Lyne in preparation). These Crab pulsar glitches are comparatively small and infrequent, offering the possibility of comparing their effect on the overall long-term slowdown with the smooth behaviour between glitches.

2. Incidence of the Glitches

The Julian dates of occurrence of the glitches are shown in Fig 1. Can we characterise this sequence in terms of a simple stick-slip behaviour of a two-component system, comprising the solid crust and part of the superfluid? We would expect to see a relaxation oscillator, with a more regular sequence. Must we consider a multicomponent system which would produce some sort of random or chaotic sequence of occurrence and size of glitches? If we consider only the intervals between glitches, without regard to their sequence, we do not see anything like a relaxation oscillator; in contrast the intervals between glitches are spread in a reasonable approximation to a Poisson distribution. But the actual sequence does not seem to be random.

Fig 2 shows the size of each glitch as measured by the initial step $\Delta \nu$. A possible description is now in terms of three events, the first two being the large glitches in 1975 and 1989, and the third being a closer spaced group of smaller glitches. These major events were spaced by about 12 years, so it will take some time before we can confirm this interpretation.

Is it reasonable to consider the Crab glitches as characteristic of a simple two-component system? The more random behaviour of the Crab pulsar might be indicating a many-component system, typified by earthquakes or the classical sandpile. In such systems the frequency of events $N$ is related to event size $S$ by a power law $N \propto S^{-\tau}$, where $\tau$ is commonly in the range 1 to 2 (Per Bak 1996). Figure 3 shows the statistics of glitch size on a log plot, with a line corresponding to $\tau = 1$. There is an obvious lack of both small and large events for this type of behaviour. We conclude that the system is basically a two-element system, with some variation in the triggering which led to the group of smaller glitches.

3. The Cumulative Effect on Slowdown Rate

We turn now to the steps $\Delta \dot{\nu}$ in slowdown rate at the glitches. Fig 4 shows the cumulative effect of the steps in slowdown rate at the glitches. This provides a significant contribution $\dot{\nu}_g$ to the overall second differential $\ddot{\nu}$; we identify this
Figure 2. The size of each glitch measured by the height of the initial step.

Figure 3. The logarithmic size/number relation for Crab pulsar glitches.
glitch contribution as the overall slope of Fig 4. The line arbitrarily drawn has a slope corresponding to \( \dot{\nu}_g = -6.2 \times 10^{-23} \text{ Hz s}^{-2} \).

The derivative of the slowdown rate has previously been evaluated in the smooth runs between widely separated glitches, giving \( \dot{\nu}_g = 1.185 \times 10^{-20} \text{ Hz s}^{-2} \); this led to the widely quoted value \( n = 2.51 \) for the braking index. The contribution \( \dot{\nu}_g \) is a 3\% reduction in \( \dot{\nu} \), giving a new value \( n = 2.45 \) for the long-term slowdown.

This new value can be compared with the braking index of the Vela pulsar, which can only be evaluated by integrating the effect of many glitches (Fig 5, Lyne et al. (1996) to produce a value of \( \dot{\nu} \). This is the origin of the measured value of the index \( n = 1.4 \pm 0.2 \).

4. The Dynamics of Slowdown

We return to the question: why is the braking index different from 3? Following Allen and Horvath (1997), there are evidently two separate contributions, which we call interior and exterior, due respectively to glitches and to the magnetosphere. The magnetospheric contribution is due to the overall configuration of the magnetic field, including the effect of angular momentum loss by particle outflow. The glitches provide the interior contribution, which we expect to be a relaxation oscillation with no average effect. But in fact they produce an accumulating effect on slowdown rate; how does this happen? Are any of the parameters in the dipole slowdown law Equation 3 changing at each glitch?

\[
\dot{\nu} = -M^2 \sin^2 \alpha I^{-1} \nu^3
\]
where $M$ is the dipole moment, $\alpha$ is the inclination of the dipole to the rotation axis and $I$ is the moment of inertia.

Is $I$ changing? For the Crab the glitches increase the slowdown rate by a fraction $0.34 \times 10^{-4}$ per year. This is too large to be due to a reduction in ellipticity, since the equilibrium value of ellipticity is $e = 10^{-4}$.

An apparent reduction in $I$ might be due to a continuous accumulation of pinned vortices, in ‘capacitors’ (Alpar et al. 1996), eventually locking up a large fraction of the superfluid. This would have to persist and accumulate through a succession of catastrophic glitches, which seems unlikely. We therefore look at a change in $M_\perp$, either in the dipole moment or in the misalignment angle $\alpha$.

Is $\alpha$ changing? The slowdown rate is proportional to $\sin^2 \alpha$, so that the required fractional change in $\alpha$ is greater by $\frac{1}{2} \tan \alpha$ than $\dot{\nu}_e/\dot{\nu}$. Romani & Yadigaroglu (1995) show that $\alpha \approx 70^\circ$, giving a required increase of about $10^{-4}$ radians per year. According to their model the spacing between the two main pulse components is sensitive to such a change; putting $\alpha = 70^\circ$ we find the spacing would increase by about 2 microseconds per year. This probably allows a check to be made, but in any case the distribution of $\alpha$ amongst older pulsars does not support the idea of evolution towards orthogonality.

So we arrive again at the proposal for an increasing dipole field, which has already been suggested for the Vela pulsar (Lyne, Shemar & Smith 2000). For the Crab pulsar the fractional increase would be $1.5 \times 10^{-5}$ per annum. Considering that we know very little about the generation of the dipole field, there seems to be no argument against this suggestion in a young pulsar like the Crab.

Accounting for the step changes in slowdown at glitches in this way does not, of course explain the major part of the deviation of the braking index from

Figure 5. The slowdown rate of the Vela pulsar over 25 years (Lyne et al. 1996), showing the slow change in the second differential $\dot{\nu}$. 
the theoretical value \( n = 3 \). Glitches are a phenomenon of the neutron star; the main anomaly in the Crab pulsar is evidently concerned with the configuration of the magnetosphere. We do not know if this true also of the Vela and other pulsars.

References