Challenges in the Solution of the Transfer Equation in Multi-D Hydrodynamical Model Atmospheres for Cool Stars

Hans-Günter Ludwig

Lund Observatory, Box 43, S-22100 Lund, Sweden

Abstract. This paper gives a brief account of existing methods and future challenges in the solution of the radiative transfer equation (RTE) in the context of hydrodynamical model atmospheres. It focuses on the representation of the complex geometry of gas flow and radiation field, as well as the treatment of the frequency dependence of the radiation field.

1. Introduction

During the last two decades we witnessed large progress in the hydrodynamical modeling of the atmospheres of late-type stars. Multi-dimensional, time-dependent models have been constructed accounting for the detailed interplay of convective and radiative energy transport, avoiding approximations usually made in standard model atmospheres, like the assumptions of hydrostatic and radiative-convective equilibrium. Today, hydrodynamical models cover so different objects like white dwarfs and red supergiants, as well as A-type to M-type objects in vicinity of the main sequence. While having reached a level of realism allowing a direct confrontation with observations (Asplund et al. 2000), in particular the representation of the radiation field has not yet reached the same accuracy commonly achieved in standard model atmospheres. The rapidly increasing computing power will allow to overcome existing limitations in the near future. In the following, I will give a brief account of the presently adopted ways of describing the radiation field in hydrodynamical model atmospheres, and discuss possible directions of developments. The vantage point of cool stars makes NLTE issues less important here, they will not be a topic of this contribution.

2. Challenge: Geometry

Figure 1 visualizes the temperature field typically encountered in a hydrodynamical model for a late-type star. The computational domain is a Cartesian box representing a small volume at the stellar surface. A granulation pattern is apparent in layers around optical depth unity. Concentrated, plume-like downdrafts dominate the sub-photospheric flow, the optically thin layers are harboring a mixture of overshooting and wave motions with associated temperature fluctuations. We are confronted with a complex geometry devoid of symmetries which would alleviate the solution of the RTE. However, certain simplifying features can be identified: (i) Radiation flows primarily in the vertical direction, which
Figure 1. Temperature field in a 3D hydrodynamical model of the Sun. Lighter shades of gray correspond to higher temperature.

helps when optimizing the discretization of the solid angle. (ii) The energy density in the radiation field is low, and the flow evolution is slow in comparison to the light travel time. It is sufficient to solve for the time-independent RTE, and one can neglect aberration effects. (iii) Often, LTE is a good approximation, and scattering can be neglected. This leads to a decoupling of the RTE for different directions. (iv) The granulation pattern contains rarely very concentrated, point-like radiation sources. This alleviates demands on the space angle discretization. (v) The decoupling of most of the radiation field from the stellar matter occurs in a well defined, rather fixed layer — the “surface”.

In the present approach, the formal solution of the RTE is computed employing long characteristics (“rays”) and Feautrier’s method. Typically, $10^4$ to $10^5$ rays are used to model the geometry of the radiation field. Since most of the rays do not coincide with the grid on which the hydrodynamical equations are discretized, one is confronted with a complex interpolation problem between the rays and the hydrodynamical mesh. The interpolation has to fulfill an important constraint — the conservation of energy. While energy conserving interpolations schemes have been developed, the question arises whether short characteristics would allow a more direct inclusion of this property. Furthermore, it is unclear whether long or short characteristics might ultimately prove more efficient, in particular in the view of parallel computer architectures. At first glance short characteristics appear more localized, i.e more attractive for parallel architectures where performance is often limited by the necessary communication among processing units. However, it might turn out that the communication overhead even for long characteristics is not crucial, since the hydrodynamical simulations today are rather CPU time than memory limited.
3. Challenge: Spatial Resolution

Entangled with the issue of how to best represent the geometry of the radiation field, is the issue of the limited spatial resolution of hydrodynamical models. Presently, of the order of $10^2$ mesh points per spatial dimension can be afforded in a hydrodynamical model atmosphere. For an explicit time stepping scheme computing time scales as the forth power of this number (3 powers for the spatial dimensions, 1 accounting for the reduction of the time step to preserve stability). Hence, only a rather modest improvement of the spatial resolution will occur in the foreseeable future. Moreover, hydrodynamical flows have the tendency to develop flow discontinuities (shocks) which always introduce large changes of the flow structure over a few mesh points irrespective of resolution. Whatever method one chooses for the solution of the RTE, it must be robust enough to cope with this situation. In particular, the method should ensure reasonable physical behavior. E.g. a hot region embedded in a cool background should cool and heat the surroundings. Non-monotonic interpolation schemes are prone to violate such basic demand.

4. Challenge: Frequency Resolution

A great number of atomic and molecular lines is typically present in the spectra of late-type stars. For an exact reproduction of such spectra the RTE would have to be solved for an even larger number of frequencies which would be computationally very demanding. However, for determining the thermal structure of an atmosphere only the frequency integral of the radiative energy exchange matters. The large number of spectral lines becomes an advantage if one resorts to statistical techniques to compute this integral. Opacity sampling and opacity distribution functions (ODFs) are today's standard techniques for an economic evaluation of the radiative energy exchange. While reducing the effective number of frequencies to the order of $10^4$ this is still prohibitively expensive in the context of hydrodynamical model atmospheres, since the frequency-dependent RTE has to be solved along all characteristics (taking care of the geometry), and this at every time step (of the order $10^4$ for a model run). For hydrodynamical model atmospheres a multi-group technique (hereafter dubbed opacity binning method, OBM) has been introduced (Nordlund 1982; Ludwig 1992) which allows a reduction of the number of effective frequency points (or frequency bins) to only a few while retaining a reasonable accuracy.

The left panel of Fig. 2 (perhaps challenging your eyesight) shows an example of how the OBM works in the case of 4 frequency bins. Somewhat related to the ODF approach frequencies are grouped according similar properties. The depth in which a particular frequency reaches optical depth unity serves as criterion into which bin it is grouped. In the figure this $\tau_\nu = 1$ level is depicted. The atmosphere is usually divided into equally spaced intervals in logarithmic Rosseland optical depth $\tau_R$ (horizontal lines) giving the demarcations among the groups. Effectively, this leads to a classification of frequencies into groups representing the continuum, weak, and increasingly stronger lines. Once the grouping is done the group integrated source function and group averaged opacities are calculated and tabulated as a function of thermodynamic variables. The RTE
Figure 2. **Left panel:** diagrammatic representation of the opacity binning method (OBM), for details see text. **Right panel:** comparison of 1D solar radiative-convective equilibrium models based on high frequency resolution (ODFs with 1220 frequencies, solid line) and the OBM with 5 bins (dash-dotted line). The maximum temperature difference between the models amounts to 70 K.

has to be solved only once per group. Note, that the groups do not form contiguous bands; furthermore, — similar to ODFs — a successive refinement of the grouping does not guarantee the convergence to the exact solution.

The right panel of Fig. 2 shows a comparison among 1D solar model atmospheres based on ODFs with high frequency resolution and the OBM, here employing 5 bins. The correspondence is quite satisfactory, the most difficult region for the OBM turns out to be the transition optically thick to thin layers, in which most of the radiation detaches from the stellar plasma. The increase in computing power will allow to move from a few frequency groups to about ten times the number in the near future. One important aspect, that needs to be addressed, is, whether Doppler shifts along the line of sight might systematically alter the radiative energy balance, in particular in higher atmospheric layers. A course opacity sampling would offer a direct way to include such effects. On the other hand, it may well be that a statistical treatment of effects of Doppler shifts in line ensembles might prove more efficient and accurate, as long as one is still constrained to a modest number of frequencies. Such a treatment would follow more closely the methodology of the OBM.

References