2D Radiative Transfer in Magnetically Confined Structures

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Abstract. Magnetically confined structures in the solar atmosphere exhibit a large complexity in their shapes and physical conditions. As an example, we show the case of so-called magnetic dips in prominences which are in magnetohydrostatic equilibrium. For such models we solve the 2D non-LTE multilevel problem for hydrogen with partial redistribution in resonance lines. Here we describe an adequate iteration method for cases when the column-mass scale is used in one of the two dimensions while the other dimension has the geometrical scaling. Furthermore, we demonstrate how the proper selection of grids affects the spatial resolution of steep 2D gradients of the temperature and density.

1. Introduction

Solar prominences were initially modeled as 1D slabs in magnetohydrostatic (MHS) equilibrium for which the multilevel non-LTE transfer problem was solved using the complete linearization method (Heasley & Mihalas 1976) or MALI with a long-characteristics (LC) formal solver (Anzer & Heinzel 1999). In order to simulate the prominence fine structures, Heinzel & Anzer (2001) have generalized 1D MHS models to 2D models of local magnetic dips. As in 1D models, the pressure equilibrium along the magnetic field lines is expressed using the column-mass scale (x-direction). Perpendicularly to the field lines (y-direction), one can use the geometrical scale. 2D variations of the temperature, gas pressure and density are displayed in Heinzel & Anzer (2001) for one particular model. Note the very steep temperature gradient across the field lines and also the substantial decrease of gas density from the center of the dip towards its boundaries.

2. 2D Radiative Transfer

For structures representing 2D magnetic dips, Heinzel & Anzer (2001) solved the 2D transfer problem, using the approach of Auer & Paletou (1994). The iterative technique used is based on the Multilevel Accelerated Lambda Iteration (MALI) approach (Rybicki & Hummer 1991), with a diagonal Approximate Lambda Op-
erator (ALO) and Short-Characteristics (SC) formal solver as in Auer & Paletou (1994). To compute the hydrogen ionization balance, the preconditioned MALI equations are linearized with respect to atomic level populations and electron density and solved iteratively using the Newton-Raphson scheme (Heinzel 1994, Paletou 1995). Computational details are described in Heinzel & Anzer (2001), where the resulting synthetic line profiles of Lo, Lβ and Hα lines are shown for one particular model.

3. Adaptive MHS Grids

The requirement for 2D vertical MHS equilibrium in the dip leads to the pressure balance equation (Heinzel & Anzer 2001)

\[ p(m, y) = 4p_c(y) \frac{m}{M(y)} \left[ 1 - \frac{m}{M(y)} \right] + p_0, \tag{1} \]

where \( p \) is the total pressure, \( p_c(y) = B_{z1}^2(y)/8\pi \) and \( M(y) = B_x(y)B_{z1}(y)/2\pi g \). \( m \) is the column mass scale in \( x \)-direction, \( M(y) \) the column mass integrated along \( x \) and varying along \( y \), \( B_x \) is the horizontal field, \( B_{z1} \) the vertical field at the dip boundary, \( p_0 \) is the boundary coronal pressure and \( g \) is the gravitational acceleration at the solar surface.

As in 1D (Heasley & Mihalas 1976; Anzer & Heinzel 1999), the reason for introducing the column mass scale is that the pressure balance equation is then independent of the assumed temperature variations. However, in 2D the situation is much more complicated because we solve the transfer problem on a Cartesian grid \( (x, y) \) (using the SC technique) and for this we have to transform all quantities from \( m \)-scale to \( x \)-scale. In Heinzel & Anzer (2001) we have used an initial estimate of the ionization degree and did this conversion by proper interpolations. Then the Newton-Raphson iterations were performed on such a fixed Cartesian grid. However, for more realistic models one has to take into account that the computed ionization degree finally differs from the initial estimate and thus also the grid is either expanded or compressed along the \( x \)-direction, differently along each field line (i.e. for each value of \( y \)). Then one has to interpolate all quantities on the new grid.

In this paper we have performed iterative computations where the Cartesian grid was adapted several times, always after the inner Newton-Raphson loop converged and we obtained the new ionization degree. The convergence properties of such a procedure seem to be very favorable as we can infer from Figure 1a. After 5 iterations of the ionization degree the final MHS adaptive grid was fixed and 2D transfer solution finalized. The model used is the same as described in Heinzel & Anzer (2001). The redistribution matrices were recomputed consistently after each grid adjustment.

The initial estimate of the ionization degree used to construct an initial 2D Cartesian grid is (Heinzel & Anzer 2001)

\[ i = 1 - (1 - i_c) \left( \frac{T_{tr} - T}{T_{tr} - T_c} \right)^2, \tag{2} \]

where \( i_c \) is the ionization degree in the center of the magnetic dip (we took \( i_c = 0.3 \)), \( T \) is the temperature, \( T_c \) the central temperature and \( T_{tr} \) is the
transition-region temperature. The final ionization degree after 5 iterations is displayed in Figure 1b, where the central value changed from 0.3 to 0.23. At the boundaries, the hydrogen is almost fully ionized ($T=T_{\text{crit}}=50000$ K), large variations of $i$ are mainly due to steep temperature gradients.

4. 2D Grids – Discussion

Here we will mention various strategies in defining the grid lines for 2D transfer which uses Cartesian coordinates. The basic requirement is that all boundaries must be accurately resolved in the transfer equation. This practically means that the most opaque transition must become optically thin at the first boundary cells (L$\alpha$ in the case of the hydrogen atom). Paletou (1995) considered simple 2D isothermal and isobaric models of prominences where the standard logarithmic gridding can be used for both dimensions. Since the gas pressure is uniform, there is no need for adapting the grid as described in the previous section. On the other hand, physically complex structures like our 2D magnetic dips require a more careful setup of grids. We demonstrate this aspect in Figure 2. Figure 2a shows a steep decrease of the gas density (same for pressure) overlaid with the grid we used for such a model. The grid lines in $x$-direction are separated equidistantly, which is justified by a substantial decrease of the density and a steep increase of the ionization degree (Figure 1b). For a comparison, we show the logarithmic gridding for the same structure (Figure 2b). It clearly shows that a coarse grid in the central parts leads to very poor resolution, while at boundaries the grid is unnecessary fine. Along the $y$-direction, on the other hand, we must use the logarithmic grid because of the extremely steep temperature gradient at the boundaries. In Figure 2, note the somewhat different $x$-thickness - this is due to the change of MHS adaptive grid. Figure 2a corresponds to the final grid while Figure 2b to an initial grid.
Figure 2. a: Final density interpolated on lin-log 2D grid, b: initial density structure interpolated on log-log 2D grid (the iterative computations have been, however, performed using the lin-log grid as Figure 2a shows). Note an artificial broadening of the structure due to coarse-grid interpolations.

In general, one has to experiment with various combinations of equidistant (linear) and logarithmic grids, possibly combining them also locally in selected critical 2D regions. There is also a possibility to divide the whole 2D structure to several 'sub-atmospheres' and solve the coupled 2D transfer for them. Finally, a promising way is to use the Adaptive Mesh Refinement (AMR) techniques, which produce local grid refinements automatically with respect to spatial gradients of critical quantities (see e.g. Le Veque 1998).

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References