Radiative Transfer Modeling of Magnetic Fluxtubes

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Abstract. Exploration of the detailed structure of the small-scale magnetic elements called flux tubes that make up a large fraction of the surface magnetic field on the solar surface is still hampered by a lack of spatial and polarimetric resolution. Although this shortcoming may be alleviated in the near future with new facilities like the Advanced Technology Solar Telescope (ATST), real progress can only be made with an accompanying effort in theoretical ab-initio modeling. As examples I discuss the evaluation of hydrogen ionization and electron density in the context of a two-dimensional magneto-static fluxtube model, and the formation of G-band intensity through a snapshot from a three-dimensional magneto-hydrodynamic simulation of the solar convection.

1. Introduction

A large fraction of magnetic flux outside sunspots is concentrated in small-scale magnetic elements that have field strengths greater than approximately 1000 G at the formation height of most diagnostic lines. Because these flux elements (or fluxtubes) cannot be observed at their relevant spatial scales, which fall below the resolution limit of current solar instruments, their internal structure is largely unclear. Yet, precise knowledge of this structure, and of its evolution would provide much needed constraints on the fundamental mechanisms of solar magnetic activity. For example, the generation of these small-scale strong field concentrations, the amplification of magnetic field strength through interaction with convection, and the destruction of the field concentrations is poorly understood. It is not clear whether flux elements are the product of a local surface dynamo, or the result of diffusion off larger scale active regions that emerge from the bottom of the convection zone. Another example is the question of what causes variations in solar radiative output, which modulates terrestrial climate. Solar luminosity, especially in the UV, increases with solar magnetic activity. Since most of the radiative excess is contributed by small-scale elements it is of particular importance to understand what the fundamental physical processes are that are responsible for these elements. Finally, it is well established that chromospheric and coronal heating are intimately correlated with the presence of magnetic flux concentrations. However, our insight into the processes that are responsible for this heating are still incomplete.

To address such vital questions on the nature of solar magnetic activity an impressive array of new instrumentation has been proposed which includes
ground based facilities like the 4m Advance Technology Solar Telescope (ATST, see papers by Rimmer, and Keller in these proceedings) and the improved 1m Swedish Vacuum Tower and 1.5m GREGOR telescopes, as well as forthcoming space based missions like Solar-B. Not only do these new instrument concepts aim at improving spatial resolution through larger apertures and new technologies like Adaptive Optics (AO), they will also lead to better polarimetric capabilities by correspondingly increasing the photon flux at a given spatial resolution. To exploit the improved quality of observations to its fullest potential an accompanying development of diagnostic techniques should take place. Of particular importance is the development of forward modeling techniques that use detailed numerical simulations to investigate how physical information is encoded in the emergent radiation, and how accurately this information can be retrieved by analyzing the spectrum.

The goal of this paper is to elucidate the importance of forward modeling from first principles and to show two examples of such modeling in the context of small-scale magnetic fields. Section 2 first describes the advantages of models constructed from physical principles compared to the more traditional approach of semi-empirical modeling based on fitting free parameters to match the observed spectrum. In Sections 3 and 4 forward modeling of hydrogen ionization in a two-dimensional magnetostatic fluxtube, and spectral formation of the molecular G-band spectrum through a magneto-hydrodynamic simulation of the solar convection are described, respectively. Conclusions are presented in Section 5.

2. Semi-Empirical versus Ab-Initio Modeling

Empirical models of the solar atmosphere are derived directly from observations by varying a number of free parameters in the context of a simple physical description of the atmosphere to optimize the match between observed and synthetic spectrum. Sometimes the best match is achieved through trial-and-error, sometimes through a more formal method of minimization of the sum of the squared differences (χ²). An excellent example of the latter method is given by Keller et al. (1990), who attempt to determine the structure of solar magnetic fluxtubes from the inversion of observed Stokes I and V spectra. Since their method employs the Stokes V signal from different spectral lines, which only originate in the magnetic part of the atmosphere, the observations do not require the magnetic elements to be spatially resolved. However, this approach implies that all magnetic elements within the resolution element have similar internal structure. As a consequence this structure is only described in a statistical manner, and the method cannot track formation and evolution of individual elements.

Although the synthetic spectra are calculated through a physically consistent model, in the case of Keller et al. (1990) a magnetostatic thin fluxtube embedded in a non-magnetic atmosphere, the semi-empirical model does not contain any additional physical laws which explain the thermal structure of the fluxtube. In addition, as is usual in semi-empirical modeling, constraints are put on the smoothness of the solution. This limits the part of parameter space where solutions can be found. For these reasons the physical insight that can
be gained through semi-empirical modeling is limited, although of course good first estimates can be obtained for field strength and various thermodynamic quantities.

Models that are built on the basis of physical principles alone are generally much harder to build as they require the inclusion of all relevant physical processes consistently, requiring complicated numerical solutions. Because these models ideally do not have any free parameters, they allow us to better distinguish between important and less relevant physical processes the contribute to the observed structures. Since the whole physical structure and evolution is encoded in the computer it is much easier to investigate how a certain physical process contributes to the emergent spectrum, and how accurate the information is we retrieve from the spectrum with a given method of analysis. Prominent examples of numerical modeling of small-scale magnetic elements are two-dimensional calculations by Deinzer et al. (1984a, b), Knölker et al. (1988), Knölker and Schüssler (1988) and Steiner et al. (1998), and the recent three-dimensional magneto-hydrodynamic calculations presented by Bercik (2002).

3. Hydrogen Ionization in a Two-Dimensional Fluctube

In this section I give an example of the need for realistic forward modeling showing how the electron density in a two-dimensional fluctube depends on the detailed radiative transfer in the hydrogen Ly-α line.

3.1. A Two-Dimensional Magneto-Static Fluctube Model

The fluctube in this example is a two-dimensional axisymmetric magnetostatic fluctube developed in cooperation with van Ballegooijen (2001). It takes the traditionally used thin fluctube model one step further and allows thermodynamic quantities to vary over the cross section of the tube. Electron densities are solved consistently with the structure of the magnetic field by iteratively solving for the equations of magnetostatic equilibrium and full two-dimensional Non-LTE radiative transfer in hydrogen, including the effects of PRD in the Ly-α and Ly-β lines.

The procedure to create the model is set up as follows. A fluctube is placed in a cylindrical computational domain which is placed vertically in the solar atmosphere. At low heights the radius \( R(z) \) of the tube is small compared to the radius \( R_0 \) of the cylinder, and the tube is embedded in a field-free surrounding medium. There is a sharp boundary between the tube and its surroundings, and the tube radius increases with height \( z \). At the “canopy” height \( z = z_c \) the boundary of the fluctube reaches the cylinder wall and intersects the cylinder at a right angle. Above this intersection the magnetic field is vertical at the cylinder wall. This simulates the effect of neighboring fluctubes which prevent further expansion of the tube with height. At large heights the field inside the tube is nearly uniform and vertical.

The fluctube is assumed to be in magnetostatic equilibrium:

\[
- \nabla p + \rho g + (4\pi)^{-1}(\nabla \times B) \times B = 0,
\]

where \( B \) is the magnetic field, \( p \) is the gas pressure, \( \rho \) is the density, and \( g = -g\hat{z} \) is the acceleration due to gravity. This equation implies hydrostatic equilibrium.
along the field lines. Furthermore, pressure balance with the surroundings requires:

\[ p_{\text{int}}(z) + \frac{B_{\text{int}}(z)^2}{8\pi} = p_{\text{ext}}(z), \quad \text{for} \quad z \leq z_c, \]  

(2)

where \( B_{\text{int}} \) is the magnetic field strength just inside the flux tube boundary, and \( p_{\text{ext}} \) is the external gas pressure.

The employed pressure distributions are prescribed functions of height, and are take from the models of Fontenla et al. (1993), and Avrett (1995). Specifically the pressure stratification on the axis, inner tube wall, and outer tube wall are derived from models F, A, and XCO, respectively, where the latter is a model derived to fit disk center CO lines. It is a cool model that is used only below the canopy. The height scale of the pressure scale on the tube axis is shifted down in height so that the tube is in pressure equilibrium deep in the atmosphere.

It can be shown that the solution to Equations 1 and 2 corresponds to the minima of a Lagrangian which represents the magnetic and thermal energy of the flux tube plus a term that represents the work done against the external pressure when the fluxtube radius \( R(z) \) is increased. The minimization problem can be solved with a variational method which yields the magnetic field \( B(r, z) \), gas pressure \( p(r, z) \) and mass density \( \rho(r, z) \).

To determine the temperature \( T(r, z) \) and electron density \( N_e(r, z) \) for given \( p(r, z) \) and \( \rho(r, z) \), it is necessary to determine the ionization state of the plasma. The rates of ionization of hydrogen and other species depend in part on the radiation fields in various continua and spectral lines, i.e., there is nonlocal coupling between different parts of the flux tube and its surroundings. Therefore, the temperature and electron density are determined iteratively. For a given temperature and density distribution we solve the equation of radiative transfer and compute radiative ionization rates in the Lyman and Balmer continua of hydrogen and excitation rates in Ly-A. Then the hydrogen ionization balance is recomputed, and new values of \( T(r, z) \) and \( N_e(r, z) \) are derived. We find that this process converges in a few iterations.

3.2. Effect of PRD in Ly-\( \alpha \) on Hydrogen Ionization

When a photon is scattered by a line of which the excited state is unperturbed by elastic collisions the photon is emitted coherently with conservation of frequency with respect to line center. If part of the wings of such a line form in a region in the atmosphere where radiative excitation dominates over collisional excitation we have to account for the shape of the spectrum by which the upper level is populated, since this spectrum is generally not constant over the line. This is certainly the case for the strongest hydrogen Lyman lines in the solar spectrum, including Ly-\( \alpha \) and \( \beta \). The effects of coherent scattering are accounted for in these lines through the partial redistribution (PRD) formalism.

The emission profile \( \psi^{ij}(\nu) \) due to PRD in line \((i, j)\) depends on the radiation field \( J(\nu) \) in the following way:

\[ \psi_{ij}(\nu) = \phi^{ij}(\nu) \left\{ 1 + \frac{n_i B_{ij}}{n_j P_j} \int \left[ \frac{R_{ij}(\nu, \nu')}{\phi^{ij}(\nu)} - \phi^{ij}(\nu) \right] J(\nu')d\nu' \right\}. \]  

(3)
Figure 1. Sample curves of the quantity $R^{II}(x, x')/\phi(x)$ for different values of emission frequency $x$ (in Doppler units) as function of absorption frequency $x'$. It describes the probability that, if a photon is emitted at frequency $x$, it is absorbed at frequency $x'$. The area under each curve is normalized to unity. The dashed curve is the absorption profile for CRD, a Voigt function with the same damping parameter $A_{\text{damp}} = 10^{-2}$.

Here $n_{i,j}$ are the populations of the lower and upper level of the transition respectively, $\gamma$ is the fraction of photons that is scattered coherently (i.e., without perturbation of the upper state during scattering), $B_{ij}$ is the Einstein coefficient for absorption, and $P_j$ is the total transition rate (radiative plus collisional) out of the upper level $j$. The quantity $g_{II}(\nu, \nu') = R_{II}(\nu, \nu')/\phi(\nu)$ describes the probability distribution for absorbing a photon at frequency $\nu'$ prior to emission at frequency $\nu$ during a line-scattering event. It is plotted in Figure 1 for different values of the emission frequency. In case of CRD in the laboratory frame the function $g_{II}$ reduces to the line absorption profile, which is plotted in Figure 1 with the dashed curve. Major characteristics of the function $g_{II}$ are almost complete redistribution in the line core with a shape close to the line absorption profile for $x$ close to 0, and coherent scattering for $x > 4$. In the latter case the emitted frequency has a high probability of being close in frequency to the absorbed frequency.

The profound effects of PRD on the emergent profile of the Ly-$\alpha$ line are obvious in Figure 2 (note the logarithmic scale). Most notably, the wings are
Figure 2. Comparison of CRD (dashed curve) and PRD (solid curve) profiles of the hydrogen Ly-\(\alpha\) line through a standard one-dimensional model of the quiet-Sun.

much depressed under PRD compared to the CRD solution calculated with the same atmosphere. The wings of the CRD profile are so much higher because photons that are absorbed in the core of the line can be reemitted in the line wings, raising the wing source function to the same level as the core source function. When coherent scattering is prevalent on the other hand core photons are much less likely to be reemitted into the wing (only the small fraction that is scattered incoherently can be converted). The reduction of wing intensity in the Lyman-\(\alpha\) line in particular has a strong influence on the hydrogen ionization balance, which in turn determines the electron density in the chromosphere once hydrogen dominates other electron donors.

In the relatively low temperature photosphere and temperature minimum most electrons are donated by low ionization-potential metals. When temperatures rise towards the chromosphere the number of electrons donated by hydrogen ionization starts to dominate. Most of the ionization in hydrogen takes place in the Balmer continuum short-ward of 364.5 nm. Because there is relatively little excitation of the \(n = 2\) level of hydrogen (the lower level of the Balmer H-\(\alpha\) line and continuum) in the cool temperature minimum region the Balmer continuum is radiatively decoupled from local conditions there. The angle-averaged mean radiation field \(J_\nu\) at the Balmer edge is set in the photosphere, and is constant throughout the temperature minimum and chromosphere. As a consequence, the the ionization rate in the Balmer continuum is set by the number
of hydrogen atoms in the $n = 2$ level, which in turn is set by the Ly-α line of which this is the upper level. This is precisely why PRD in the Ly-α line has to be taken into account for the evaluation of the electron density.

![Graph showing departure coefficients of hydrogen levels and the continuum.](image)

**Figure 3.** Departure coefficients of the first two hydrogen levels and the continuum. The plot compares values calculated with either CRD (thin curves) or PRD (thick curves) in the Lyman-α line.

Although the core of the Lyman-α line is optically thick throughout most of the solar atmosphere, including the chromosphere, photons in the wings of this line can travel considerable distance through the atmosphere due to the high amount of scattering. As a result there is an appreciable amount of Ly-α line wing photons radiated back from the hot transition region into the lower lying chromosphere, and these photons produce additional population of the $n = 2$ level of hydrogen increasing ionization from this level.

A simplifying assumption of CRD for the Ly-α radiative transfer would overestimate this effect since it overestimates the intensity in this line’s wings considerably. The difference PRD makes for the first two hydrogen levels and the ionized state is shown in Figure 3, in which the population numbers $n_i$ of these levels are plotted normalized to their LTE values $n_i^{\text{LTE}}$. The bump in the departure coefficient of protons (the dot-dashed curve) between column masses $m$ of 10.0 and 0.1 kg m$^{-2}$ is the result of the supra-thermal radiation field at the Balmer edge through the temperature minimum, which leads to over ionization compared to the LTE solution. The smaller bump in the $n = 2$ departure coefficient (dashed curve) is the result of back radiation of Ly-α wing photons into the lower chromosphere. The effect of this back radiation is much
smaller when calculated with PRD in the Ly-α line (thick curves) than with CRD (thin curves). The corresponding reduction in wing intensity in this line results in less over-ionization in the lower chromosphere under PRD (see the difference between the thick and thin dot-dashed curves in Figure 3), and thus in a reduction of electron density there.

3.3. Non-LTE Electron Densities in the Fluxtube

In the converged axisymmetric flux tube model the effects of PRD on the back radiation in the Ly-alpha wings are fully accounted for since the model is based on full two-dimensional Non-LTE transfer in the relevant hydrogen transition. The effect is especially noticeable in the regions away from the flux tube axis, directly under the magnetic canopy. Due to the hot transition region that is now placed on top of the cool XCO atmosphere of the surrounding non-magnetic medium due to the expansion of the tube with height, the electron density just below the canopy is raised considerably compared to the original values of the XCO model. Figure 4 shows the calculated electron densities on the axis of

![Figure 4](image)

Figure 4. Stratification of electron density at the flux tube axis (thick solid curve) and away from the axis (thick dashed curve) compared with original values from models F (thin solid) and XCO (thin dashed curve).

the tube and away from the axis, compared with the stratification of electron densities of the initial models F and XCO for the tube axis, and the non-magnetic surrounding medium, respectively. The Wilson depression of the material in the
tube is evident in the downward shift of the thick solid curve. A strong increase in electron density of up to an order of magnitude is shown just below the canopy at $z = 1000$ km in the thick dashed curve compared to the thin dashed curve.

4. G-Band Intensity as Tracer of the Small-Scale Magnetic Field

Wide band imaging in the G-band has become the prime means of studying the structure and dynamics of small scale solar magnetic fields at the highest available spatial resolution. Several new techniques like Adaptive Optics, and speckle reconstruction, which are now very successfully applied to correct for image quality deterioration by the earth atmosphere, work better for such broadband imaging due to the relatively short exposure times involved than for spectroscopy, which requires much longer exposures at high spectral resolution. Many imaging observations notwithstanding, the reason why small-scale magnetic elements appear bright in the G-band is not clear at the moment.

![Graph showing comparison of spectrum calculated through a standard one-dimensional model of the quiet Sun (thick solid curve) with the observed spatially averaged solar spectrum (thin solid curve) in the wavelength region of the G band. Also plotted is a typical transmission curve of a wide band G-band filter. Vertical lines at the bottom of the graphs indicate positions of CH lines.](image)

Figure 5. Comparison of spectrum calculated through a standard one-dimensional model of the quiet Sun (thick solid curve) with the observed spatially averaged solar spectrum (thin solid curve) in the wavelength region of the G band. Also plotted is a typical transmission curve of a wide band G-band filter. Vertical lines at the bottom of the graphs indicate positions of CH lines.
Most of the line opacity in the G-band around 430 nm is due to electronic transitions of the CH molecule from the $X^2\Sigma$ ground state to the $A^2\Pi$ and $B^2\Sigma$ excited states. Figure 5 shows the spectrum in the wavelength range of the G-band along with the position of CH lines in the region, and the transmission function of a typical 1 nm wide G-band filter. Several atomic lines, most notably a group of lines due to Fe I and Ca I near 430.8 nm, also contribute to the opacity in this region. In this section I investigate the reason for the brightness in the G-band of small-scale magnetic elements by forward modeling of the G-band intensity through a snapshot from a three-dimensional magneto-hydrodynamic simulation of the solar convection.


A state-of-the-art numerical simulation by Bercik (2002, see also Stein & Nordlund 2000; Stein, these proceedings) was used to provide a realistic environment to calculate the G-band brightness of small-scale magnetic elements in the solar photosphere. These simulations account for the interaction between convection and the magnetic field through calculations of compressible magneto-convection. The equations of mass, momentum and energy conservations, and the induction equation are solved. The physical domain extends 12 Mm in both horizontal directions with a resolution of 96 km, extends 2.5 Mm below the surface and spans 3 Mm vertically. Periodic boundary conditions are used in the horizontal directions, and vertical boundary conditions are specified that are as transmitting as possible. Radiative contributions to the energy balance are calculated by solving radiative transfer in LTE using a four bin opacity distribution function.

![Continuum and G-band](image)

**Figure 6.** Calculated intensities in the 430 nm continuum (left panel) and the G-band proxy (right panel) for a single three-dimensional magneto-hydrodynamic snapshot.

Figure 6 shows the calculated emergent intensity in the 430 nm continuum (left panel) next to the integrated G-band intensity from one MHD simulation.
snapshot. The G-band intensity emergent from the snapshot is evaluated as follows. Given the instantaneous thermodynamic structure of the snapshot the non-linear set of chemical equilibrium equations for a set of molecules, including H$_2$, CH, CO, CN, OH, O$_2$, N$_2$, and H$_2$O, is solved with a Newton-Raphson solver. Thus it is assumed that chemical equilibrium sets in instantaneously, i.e., establishes itself in the dynamic atmosphere on a time scale that is much shorter than a typical dynamical time scale. With the CH concentration and assuming that the electronic states as well as the vibration-rotation sublevels of each state are populated according to Saha-Boltzmann statistics we can calculate the intensity at any given wavelength in the G-band region. To calculate intensities throughout the whole G-band region would require approximately 3000 wavelength points, which would constitute a prohibitively large computational task through a three-dimensional snapshot of dimension 120×120×64. Instead, the intensities for 100 points were calculated distributed over a representative section of the G-band that contained both lines and continuum. It was verified with one-dimensional calculations through different atmospheres that the contrast between this G-band proxy and intensity at a neighboring continuum wavelength is very similar to the contrast between continuum and the integrated G-band intensity folded with a typical G-band filter transmission function.

4.2. G-Band Brightness Through the MHD Snapshot

The left-hand image of Figure 6, the G-band proxy intensity, only has a few bright points (at $[x, y] = [13, 9]$, $[5, 7]$, and $[8, 11]$ arcsec for instance), despite the abundance of strong field in this particular simulation (the average vertical field was 400 Gauss). Moreover, the bright points are not brighter than the brightest granules, which is opposite to what is observed (cf. Berger & Title 2001). Clearly, the simulations and subsequent G-band modeling do not predict the correct appearance of G-band bright points.

To investigate why some of the magnetic elements appear as bright points, if not as clearly as observed, we take a close look at one of the most obvious ones located at $[x, y] = [13, 9]$ arcsec in Figure 6. Figure 7 shows the concentration

![Figure 7](image_url)  

Figure 7. Relative concentration of the CH molecule in a vertical cross section through the three-dimensional simulation snapshot. One of the stronger bright points is located at $x = 8.9$ Mm.
$n_{\text{CH}}$ normalized to the local hydrogen density $n_H$ as a function of height $z$ along a cross section at $y = 9$ arcsec through the three-dimensional snapshot. The granular upflows are clearly outlined at the bottom of this graph by the deficit in relative CH concentration due to their higher temperature. Just above the granules the CH concentration (and molecular concentration in general) is considerably higher because of the rapid expansion cooling that the upflowing material undergoes when it runs into the exponentially stratified stable overshoot layer. The bright point that appears at $[x, y] = [13, 9]$ arcsec in Figure 6 is associated with the dark plume at $x = 8.9$ Mm, which indicates that the relative CH concentration is considerably reduced at the location of what was verified to be a high magnetic field concentration.

Figure 8. Map of the (LTE) source function in the core of one of the CH lines in the G-band region in the same cross section as in Figure 7. The solid curve delineates the location of optical depth unity at this wavelength, and the dot–dashed curve shows the value of the source function along the optical depth curve. This value is a rough measure of the emergent intensity at this wavelength.

The reason for the reduction in molecular concentration is the partial evacuation of the gas in the magnetic flux tube required by pressure balance of the exterior gas with the interior gas plus magnetic pressure (see also Eq. 2). Apart from non-linearities due to the presence of a mixture of molecules, with constituents being able to combine into different molecules, the concentration of a diatomic molecule depends quadratically on density and is particularly sensitive to a reduction of the latter. In LTE the concentration of an atom that is appreciably ionized also depends quadratically on density. In the solar pho-
atmosphere, however, ionization is already largely controlled by radiation (see the discussion in Section 3.2. on hydrogen ionization for instance), in which case the dependence is closer to linear.

![Graph showing emergent intensities and continuum intensity](image)

**Figure 9.** Bottom panel shows emergent intensities in Gband (solid curve) and continuum (dashed curve). Top panel shows ratio of G-band over continuum intensity.

The evacuation of the flux tube and the resulting dissociation of the CH molecule allow radiation from deeper layers to escape at the location of the magnetic field concentration. This is evident in Figure 8, which shows a map of the source function at the wavelength of one of the CH lines in the G band. The location of optical depth unity at this wavelength for each position along the cross section is marked by the white curve. Particularly clear is the deep dip in line formation height in the flux tube at $x = 8.9$ Mm. The values of the source function along the optical depth unity curve provide a rough measure of the emergent intensity at this wavelength as function of position along the cross section. The source function values sampled by the optical depth curve are plotted with the dot–dashed curve. It shows a sharp peak in the fluxtube, which will appear bright in the CH line core, and contribute to the G-band integrated signal, because locally the optical depth unity curve samples hotter material deeper down, namely the walls of the flux tube. The G-band integrated
intensity (solid curve) for the cross section is shown in the lower panel of Figure 9 along with the continuum intensity (dashed curve). The upper panel shows the ratio of the two intensities, which accentuates the location of the bright points much better than the individual intensities.

5. Conclusions

Our best chance at understanding the interaction between plasma and magnetic fields in a more general astrophysical context lies in the construction of detailed numerical models that can be verified at their relevant scales on the solar surface. To achieve this goal requires the significant increase in spatial resolving power and polarimetric sensitivity that is promised by new facilities that are now on the drawing board. These instrumental developments should go hand in hand with progress in theoretical modeling that enables us to interpret the data of the future. This paper provides just two limited illustrations of such modeling.

In the discussion of hydrogen ionization presented in Section 3 it is found that back radiation in the wings of the Lyman-α line contributes significantly to hydrogen ionization and electron density in the low chromosphere. This effect is somewhat reduced when coherent scattering in the Ly-α line is accounted for because of the considerably lowered intensity in the wings of that line under PRD compared to CRD. In a flux tube with increasing radius with height the Ly-α wing back radiation is capable of increasing the electron density directly under the magnetic canopy by almost an order of magnitude in the example studied here.

Small-scale magnetic elements appear to be bright in the integrated light of a G-band filter primarily because of the evacuation of the flux elements, which causes a shift in the chemical equilibrium of molecules in general, and CH in particular, towards dissociation. The resulting decrease of CH line opacity in the flux tube causes the source function of these lines to sample deeper, hotter layers and raises the emergent intensity in the line core by a larger fraction than the continuum intensity. The bright points in our sample snapshot seem to occur less frequent than observed, given the abundance of strong fields, and appear to be less bright. It may be that the limited resolution of the simulation (horizontal grid size of 96 km) prevents the flux tube from having sharp boundaries. If the interface between flux tube and exterior would be sharper the optical depth unity curve (see Figure 8) would come closer to the hot material of the surrounding photosphere and sample even hotter material raising the emergent intensity closer to observed values in the G band.

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References

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