OPTIMAL MASKS FOR G-MODE DETECTION IN MDI VELOCITY DATA

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ABSTRACT

We are applying spatial masks to MDI velocity data that are optimized for revealing g-modes in the frequency range 50 through 500 $\mu$Hz. These are masks that take into account the horizontal component of g-mode velocity eigenfunctions as well as the time dependent mode projection properties due to the changing solar B angle, and the varying noise level across the solar disk. The solar noise, which is likely to be caused by supergranulation, is assumed to be uniformly distributed over the solar surface, consisting of a dominant horizontal component and a small radial component. The resulting time series are examined for possible g-mode candidates and new upper limits for the surface amplitude of g-modes are obtained.

Key words: Sun: oscillations.

1. INTRODUCTION

The line of sight velocity on the sun's surface is determined from measurements of the Doppler shift of the Ni I absorption line at 6768 Å by the MDI instrument (Scherrer et al., 1995). Besides the usual spherical harmonic masks, other masks have been applied for different purposes (Toutain & Kosovichev, 2000). Up to now, no eigenmode below the g-mode limit of 470$\mu$Hz has been identified unambiguously. The g-modes exist below the peak of the Brundt-Väisälä frequency of around 470$\mu$Hz. Knowledge of the g-mode frequencies would be especially important for inferring the properties of the solar core. The increasing length of available time series of solar surface velocities and intensity variations without detection of any g-mode signature leads to lower and lower upper limits for the surface amplitude of these modes. Appourchaux et al. (2000) gave an upper limit of 10 mm/s at 200 $\mu$Hz in velocity and 0.5 ppm in intensity from whole disk observations. Our objective is to maximize the signal to noise ratio of possibly present low frequency modes taking advantage of the spatial resolution. Contrary to the p-mode range, the noise in the low frequency range is mostly horizontal, resulting in a low noise level at the disk center, where the movement is mostly perpendicular to the line of sight. On the other hand, within the frequency band where detection of g-modes is expected, a transition occurs for low l from mostly radial to mostly horizontal eigenmodes: higher frequency modes are radial, whereas lower frequency modes are horizontal. Therefore, the mask maximizing the signal to noise ratio depends strongly on the mode frequency. We decided not to use the optimal mask at theoretically predicted frequencies, but to cover a given frequency band by one mask, whenever neither the pattern of the eigenmode nor the noise changes substantially in this band.

2. SURFACE PATTERN OF THE EIGENMODE

Assuming a radially symmetric, time independent equilibrium state, the linearized adiabatic hydrodynamic equations of a star determine the dependence of the perturbations on the colatitude $\Theta$ and the longitude $\Phi$. The scalar quantities $\rho'$ (density perturbation), $p'$ (pressure perturbation), and $\Phi'$ (perturbation of the gravitational potential) form a pattern on the solar surface according to the real part of spherical harmonics, e.g. the pressure perturbation is given by:

$$\rho' (r, \Theta, \Phi, t) = \sqrt{4\pi R} \{ \tilde{\rho} (r) Y_l^m (\Theta, \Phi) \exp(-i\omega t) \}$$

(1)

($\tilde{\rho}$ denotes the amplitude), whereas the displacement vector is $\delta \mathbf{r}$ given by:

$$\delta \mathbf{r} = \sqrt{4\pi R} \left\{ \xi_r (r) Y_l^m \mathbf{e}_r + \xi_\theta (r) \left( \frac{\partial Y_l^m}{\partial \Theta} \mathbf{e}_\Theta + \frac{1}{\sin \Theta} \frac{\partial Y_l^m}{\partial \Phi} \mathbf{e}_\Phi \right) \right\} \exp(-i\omega t)$$

(2)
where $\mathbf{e}_r$, $\mathbf{e}_\Theta$, and $\mathbf{e}_\Phi$ are the unit vectors in radial, latitudinal and longitudinal direction, respectively, and $\xi_r$ and $\xi_\Theta$ are the respective amplitudes. The value of these amplitudes is unknown and depends strongly on assumptions on excitation mechanisms which are highly uncertain. However, the ratio $\xi_r/\xi_\Theta$ is a function of frequency and angular degree alone (Christensen-Dalsgaard, 1998). Given the boundary condition that the Lagrangian pressure perturbation vanishes at the solar surface, the ratio of the radial and horizontal root mean square (rms) displacement is given by:

$$\sqrt{\frac{\langle |\delta H|^2 \rangle}{\langle |\delta r|^2 \rangle}} = \sqrt{l(l+1)} \frac{g(R)}{R w^2} \equiv b(\omega),$$

where $g(R)$ denotes the gravitational acceleration at the sun’s surface, and $R$ is the solar radius. For $l$ greater than about 20 (Christensen-Dalsgaard, 1998), $b(\omega) = 1$ is the characteristic of the f mode ridge. For low $l$ however, we get well into the region of avoided crossings, where the f mode frequency is no longer given by $b(\omega) = 1$. Instead, the graph defined by $b(\omega) = 1$ in the $l-\omega$ diagram is within the area of the g-mode ridges.

3. OPTIMAL MASK

The velocity image we obtain by the MDI instrument is the displacement vector projected onto the line of sight. As our starting point we used medium-l data (Scherrer et al., 1995), already containing an apodization $A$. Thus, the pattern $\bar{Y}^m_l$ we expect to see from a mode of angular degree $l$ and tesseral order $m$ in the instrument’s image is:

$$\bar{Y}^m_l(\Theta, \Phi, t) = (\delta F \cdot \mathbf{e}_z) A,$$

where $\mathbf{e}_z$ is the unit vector in the line of sight direction. It’s important to note that $\mathbf{e}_z$ is not fixed as a function of the heliographic colatitude $\Theta$, but varies during the year with the solar B angle.

Additionally to the mode pattern, there is a noise contribution in the image. If we calculate a time series

$$y(t) = \int M(\Theta, \Phi) \left( \bar{Y}^m_l(\Theta, \Phi, t) + \epsilon(\Theta, \Phi, t) \right) d\Omega$$

made from an image consisting of a mode $\bar{Y}^m_l$ and a noise term $\epsilon$, the “optimal mask” $M(\Theta, \Phi)$ in terms of the best signal to noise ratio for a given mode $\bar{Y}^m_l$ is:

$$M(\Theta, \Phi, \omega) = \frac{\langle \bar{Y}^m_l \rangle^2}{\int \frac{|\bar{Y}^m_l|^2}{\sigma^2} d\Omega},$$

with

$$\sigma^2(\Theta, \Phi, \omega) = \langle \epsilon^2 \rangle.$$

The normalization is chosen such that the mode amplitude is reflected directly in the height of the peak in the power spectrum. Depending on the visibility of a mode, which can be very different for different modes, the resulting noise level varies accordingly.

4. MODEL FOR THE NOISE

We assume, that the noise is equally distributed over the surface of the sun. The varying noise level across the solar disk is due to the projection properties of horizontal and radial components of the noise.

The noise in the low frequency range is likely caused by supergranulation. Therefore, it consists mostly of horizontal flow, together with a smaller component of radial flow. Our model for the spatial variation of the noise is given by:

$$\sigma^2(\Theta, \Phi, \omega) = \sigma^2_R(\omega) \sin^2 \Theta \cos^2 \Phi + \sigma^2_\Theta(\omega) \cos^2 \Theta \cos^2 \Phi + \sigma^2_\Phi(\omega) \sin^2 \Phi.$$

Equation 8 implies that we assume the different vectorial components of the noise to be uncorrelated. This assumption has been made for the sake of simplicity and because the correlation coefficients turned out to be hard to estimate. As shown in Figure 1, the functional dependence of the noise on the surface coordinates $\Theta$ and $\Phi$ can be reproduced pretty well by expression 8.

As shown in Figure 2, the fitted coefficients show that the horizontal noise is indeed one order of magnitude higher in amplitude than the radial noise. The somewhat higher level of the horizontal noise after the SOHO gap may be caused by increased activity noise.

5. MASKS FOR $L = 1$ AND $L = 2$

Because both the eigenfunctions and the noise are frequency dependent, the masks are, too. To account for the rapidly changing eigenfunctions, we used masks with

$$\frac{\text{radial rms}}{\text{horizontal rms}} = \frac{1}{4^2} \frac{1}{2^1} 1, 2, 4, 8$$

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Figure 1. Estimator for the noise (solid line) at zero longitude as a function of colatitude and the fitted curve (dashed line) according to the model given by equation 8. Close to the limb we see the influence of the apodization. The values influenced by the apodization were not used in the fit.

Figure 2. Level of the horizontal noise (upper lines) and radial noise (lower lines) as a function of frequency. The horizontal noise after the SOHO gap (solid line) is a little bit higher than before the SOHO gap (dashed line). For the radial noise, the level before and after the gap is about the same.

Figure 3. Detection limits for a 90% probability of no mode detected in a 70μHz window from a spectrum made from a 765 day time series after the SOHO vacations. The limit has been calculated for all frequencies we have chosen for the construction of the masks. The points are connected by lines.

for modes of angular degree 1 and 2. Then we looked for peaks in the vicinity of the frequencies we calculated the masks for. If no mode peak can be identified, the detection limit can be lowered accordingly. Assuming a χ² distribution of the noise, we get a detection limit by estimating the level of a 90% probability of no noise peak to exceed this limit in a frequency band of 70 μHz. These values have been chosen to compare our results with the upper limit given by Appourchaux et al. (2000).

6. DETECTION LIMITS

Most of the peaks present can be explained by instrumental influence, where the most prominent peaks are harmonics of 52.125 μHz, a period which arises from beats between the spacecraft timing and the instrument sampling rate (Appourchaux et al., 2000). The few remaining peaks are far from theoretically predicted mode frequencies and their number corresponds to the expected number of statistical outliers. Therefore, from the noise level of the different spectra a new detection limit can be derived.

As we can see in Figure 3, the detection limit for the even modes is much lower than for the odd modes. The reason for this is that the radial component of the even modes have amplitude in the center of the disk, where the noise level is low. The l = 1, m = 0 mode is a somehow singular case. For equal amplitude of the horizontal and radial component at 119μHz it’s just a spatially uniform movement par-
allel to the sun's rotation axis, so it's hardly visible because the displacement is almost perpendicular to the line of sight.

To compare with Appourchaux et al. (2000), we denote explicitly the detection limits for the modes at 200\(\mu\)Hz:

<table>
<thead>
<tr>
<th>(m)</th>
<th>(l = 1)</th>
<th>(l = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>6.0</td>
<td>5.3</td>
</tr>
<tr>
<td>-1</td>
<td>6.0</td>
<td>11.7</td>
</tr>
<tr>
<td>0</td>
<td>21.3</td>
<td>7.5</td>
</tr>
<tr>
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<td>6.0</td>
<td>11.8</td>
</tr>
<tr>
<td>2</td>
<td>5.3</td>
<td></td>
</tr>
</tbody>
</table>

7. DISCUSSION

The upper limits given above lower the limit of g-mode visibility of \(l + m\) even modes given by Appourchaux et al. (2000) from MDI velocity time series of similar length (784 days). The even modes, having amplitude at the solar equator, profit more of the low noise level at the disk center, as illustrated in Figure 3. In contrast to the whole disk measurements, we can give an estimate of the upper limit for \(l + m\) odd modes, too, even though it's higher. The \(l = 1, m = 0\) mode has an untypical behavior, as explained above. The masks used here have a strong dependance on the ratio \(b(\omega)\) of radial and horizontal displacement. To increase the certainty about the physical relevance of our estimates, this ratio should probably be taken from numerical calculations in the future to account for deviations from equation 8.

The detection limit of 10 mm/s of Appourchaux et al. (2000), derived from whole disk measurements, can only be given for a whole multiplet, using the "collapsogram" technique, which assumes equal amplitude of all members of the multiplet. This is critical, because assumptions about excitation of g-modes are highly uncertain. The estimates given here are valid for each single member of the multiplet. Additionally, as shown in Table 1, the visibility of the low order modes is quite different, so the detection limit must be denoted for each single mode.

REFERENCES

Christensen-Dalsgaard, J. 1998, Lecture Notes on Stellar Oscillations