A NEW METHOD FOR MEASURING FREQUENCIES AND SPLITTINGS OF HIGH-DEGREE MODES

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ABSTRACT

A novel peak-bagging method is presented that operates by fitting a theoretical profile (symmetric or asymmetric) to the separate peaks of each multiplet within each unaveraged power spectrum. This new approach allows a separate frequency, width, and amplitude to be obtained for each m value at each value of ℓ, n. Hence, the frequency splittings due to solar rotation for each multiplet can be measured directly. We present some of our initial results obtained with this new method in the range 45 ≤ ℓ ≤ 300, ν ≤ 7 mHz when applied to data from the Michelson Doppler Imager (MDI) onboard the Solar and Heliospheric Observatory (SOHO). Also discussed are some instrumental and methodological problems of high-degree mode measurements.

Key words: Methods: Data analysis, Sun: oscillations.

1. INTRODUCTION

For the helioseismic probing of the near-surface layers of the Sun high-degree oscillations are of great importance. So far frequencies of high-degree oscillations can only be obtained by fitting m-averaged power spectra while rotational splitting coefficients of high-degree oscillations are obtained by employing the traditional cross-correlation method of Brown (1985) and Tomczyk (1988). While the fitting of m-averaged spectra is rather straightforward (cf. Korzennik, 1990) the following problems are encountered in the generation of m-averaged spectra. First, the issue is raised if unweighted or weighted averages of the spectra should be used. Second, in the computation of both unaveraged and n-averaged splitting coefficients using a cross-correlation approach it is found that these coefficients undergo dramatic jumps or discontinuities for degrees ranging anywhere from ℓ = 150 to ℓ = 270 or thereabouts. Hence, the issue is raised whether or not the splitting coefficients should be used with these discontinuities present in them or whether some attempt should be made to adjust or correct the splitting coefficients for these discontinuities.

2. THE “MULTI-PEAK, TESSERAL-SPECTRUM” METHOD

In order to avoid the above-mentioned problems in the computation of frequencies and rotational frequency splittings a novel peak-bagging method has been developed by one of us (JR). This new method operates by fitting an accurate theoretical model profile to the separate peaks of each multiplet within each unaveraged spectrum. Hence, it is called the “multi-peak, tesseral-spectrum” method. For both brevity and historical reasons this new method will be referred to simply as Method 3 in the remainder of this paper.

Since the errors in the tesseral, sectoral, and zonal power spectra are not distributed normally as they are in the case of the m-averaged spectra (at least for ℓ ≫ 1), a maximum-likelihood fitting approach is employed in Method 3. Also, in the model profile which is fit to the peaks in each spectrum, both the ℓ- and the m-leaks are included. However, the n-leaks from adjacent ridges have not yet been incorporated in the model profile. Hence, degrees ℓ ≤ 45 cannot be fitted accurately with the current version of the Method 3 code. Also, problems are encountered when fitting low-ℓ, low-n modes if their widths are on the order of, or even smaller than, the spectral resolution.

There are two options implemented into Method 3. One option allows to use either a symmetrical Lorentzian or an asymmetrical line profile (cf. Nigam & Kosovichev, 1998). The other option allows to expand the rotational frequency splittings in either the orthogonal polynomials \( P_\ell^m \) first suggested by Ritzwoller and Lavelly (1991) or in Legendre polynomials \( P_\ell \). Unless otherwise stated the symmetrical Lorentzian line profile and the Ritzwoller and Lavelly
polynomials were used in the calculations presented in this paper. More details of Method 3 will be given elsewhere.

3. INITIAL RESULTS

The results presented in this paper are based upon spectra that were created from observations obtained from the MDI Full-Disk Program during the 1996 MDI Dynamics Run (Scherrer et al., 1995). In Figure 1 the coverage of the $\ell$-$\nu$ plane which we have generated using Method 3 is shown. Only degrees in the range $45 \leq \ell \leq 300$, $0 \leq n \leq 29$, $\nu \leq 7000$ $\mu$Hz have been fit with this method due to the large amount of time consumed by the Method 3 code. For example, on the JPL Origin 2000 supercomputer the fitting of the $p$-modes in the above-given range of $\ell$, $n$, and $\nu$ values takes 1180.6 hours of CPU time. The corresponding wallclock time strongly depends on both the load of the machine and the number of processing units invoked. Speedup factors of 9 or greater can easily be achieved.

![Figure 1. Coverage of the $\ell$-$\nu$ plane obtained with Method 3. The fitted frequencies are shown as the diamonds. Only degrees in the range $45 \leq \ell \leq 300$ have been fit with this method due to the large amount of time consumed by the Method 3 code.](image1)

In the above-given range of $\ell$, $\nu$, and $n$ values we attempted to fit 1,591,543 modes in the unaveraged spectra using Method 3. It turned out that in 7,244 cases the Method 3 code failed to converge giving rise to a total of 5,007 multiplet frequencies and splittings. About two third of the failed cases are for the $n = 0$ ridge. This ridge is hard to fit mainly because of the small widths of its modes particularly for $\ell \leq 150$. From the set of 5,007 multiplets 198 had to be flagged as being outliers for various reasons. As a result, the final table of frequencies, widths, amplitudes, and rotational frequency splittings comprises 4809 multiplets. The frequencies of these multiplets are shown in Figure 1.

From the above-given numbers it follows that the Method 3 code takes on average just 2.67 seconds to fit one mode. This amount of time is comparable to the time which is required to fit one mode in an $m$-averaged spectrum. Hence, the large amount of

![Figure 2. Runs of frequency splittings (top), logarithm of width (middle), and logarithm of amplitude (bottom) versus $m/L$, where $m$ is the azimuthal order and $L^2 = \ell(\ell+1)$, for the $\ell = 300$, $n = 1$ multiplet. The measured values are shown by diamonds while outliers are indicated by triangles. The full line in both the top and bottom panel represents a fit to the measured data. However, in the top panel the full line is not to be seen at the scale of the figure. In the middle panel the full line represents the average value of $\ln(w)$ of the multiplet.](image2)

CPU time required by the Method 3 code is mainly a consequence of the huge amount of modes to be fitted rather than due to any inefficiency of the code.

Method 3 allows us to obtain a separate frequency, width, and amplitude for each $m$ value at each value of $\ell$, $n$. Hence, the frequency splittings due to solar rotation for each multiplet can be measured directly. Samples of the frequency shifts measured by Method 3 for $\ell = 300$, $n = 1$ are shown in Figure 2. For this multiplet the target peak and the spatial sidelobes are blended to form ridges of power. Also shown in Figure 2 are plots of both the logarithm of fitted width and fitted amplitude of the modes as a
function of \( m/L \), where \( m \) is the azimuthal order and \( L^2 = \ell (\ell + 1) \). While the run of the frequency splittings with \( m/L \) looks smooth, there is a rather large scatter in both amplitude and width. Moreover, a periodic variation of the width with \( m/L \) shows up. Such periodic variations are not to be seen for those multiplets for which the target peak and the spatial sidelobes are well separated. For each multiplet the width is averaged to give a mean value of the width for that multiplet, while the logarithm of the amplitude is fitted by a parabolic-shaped curve which is assumed to be an even function of \( m/L \).

Typical fits to tesseral spectra employing both the symmetrical and asymmetrical model profile are depicted in Figure 3. Please note both the ragged appearance of the tesseral spectra and the shift of the fitted frequencies towards higher frequencies when the asymmetrical model profile is used. In Figure 4 the frequency differences between frequencies calculated with the asymmetrical profile and frequencies obtained with the symmetrical profile are shown for the \( \ell = 300, n = 1 \) multiplet. It is interesting to note that \( \nu_{\text{asym}} - \nu_{\text{sym}} > 0 \) for the majority of the modes of that multiplet.

![Figure 3](image)

*Figure 3.* Fit to \( \ell = 185, n = 1, m = -150 \) (top) and \( \ell = 300, n = 3, m = -290 \) (bottom) tesseral spectrum. The thin line is for the observed spectrum. The thick line is for the fitted model using the symmetrical line profile while the dashed line is for the fitted model using the asymmetrical line profile. The fitted frequencies are indicated by tick marks at both the top and bottom axis in each plot. The short tick marks correspond to the symmetric case while the longer tick marks are for the asymmetric case. Note the shift in the fitted frequencies for both oscillation modes if the asymmetrical line profile is used.

In Figure 5 the parameter \( B \), which defines the asymmetry of the model profile (cf. Nigam & Kosovichev, 1998), is shown as a function of \( m/L \) for the \( \ell = 150, n = 1 \) multiplet. A negative value of \( B \) means that there is excess of power in the low-frequency wing of a \( p \)-mode line while a positive value of \( B \) indicates an excess of power in the high-frequency wing. For \( B = 0 \) the model profile consists of symmetric Lorentzian peaks.

![Figure 5](image)

*Figure 4.* Frequency differences for the \( \ell = 300, n = 1 \) multiplet between frequencies calculated with the asymmetrical model profile and frequencies obtained with the symmetrical model profile as a function of \( m/L \), in the sense \( \nu_{\text{asym}} - \nu_{\text{sym}} \). The dashed line is for a frequency difference equal to zero.

In Figure 5 the parameter \( B \), which defines the asymmetry of the model profile (cf. Nigam & Kosovichev, 1998), is shown as a function of \( m/L \) for the \( \ell = 150, n = 1 \) multiplet. The asymmetry parameter is marked with diamonds. The full line represents the average value of \( B \) of the multiplet.

4. DISCONTINUITIES IN THE ROTATIONAL SPLITTING COEFFICIENTS

For both low- and high-\( \ell \) values the rotational frequency splittings are a smooth function of the azimuthal order \( m \) (cf. Figure 2). Computationally, this is for low-\( \ell \) values because the target peak and the spatial sidelobes are well separated in the spectra while it is for high-\( \ell \) values because the spectra consist of well defined ridges of power. In the transition region of \( \ell \) values, however, where individual modal peaks begin to blend together to form ridges
of power this is no longer the case. For the \( n = 1 \) ridge, say, this transition region ranges roughly from \( \ell = 160 \) to \( \ell = 210 \). As \( n \) increases this region moves towards lower \( \ell \) values. One result of this blending of peaks into ridges is that outliers in the fitted frequencies of a multiplet \( \ell, n \) do occur in a highly systematic manner. This is demonstrated here in Figure 6 where the run of the frequency splittings versus \( m/L \) is shown for the \( \ell = 170, n = 1 \) multiplet. The full line is for the expansion of the frequency splittings in the Ritzwoller & Lavelly (1991) orthogonal polynomials. Since a rather sophisticated technique has been implemented into Method 3 to flag outliers in the measured rotational frequency splittings in a very efficient manner the curve fitted to the measured frequency splittings remains unaffected by the outliers to a great extent. However, if too many outliers do occur in a range of \( m \) values even this sophisticated method breaks down. For the \( n = 1 \) ridge this occurs at about \( \ell = 190 \). As a result, the fit to the frequency splittings is heavily distorted.

![Figure 6. Run of frequency splittings versus m/L for the \( \ell = 170, n = 1 \) multiplet. The measured values are shown by diamonds. Outliers are indicated by triangles. The full line represents a smooth fit to the measured frequency splittings. Note that the outliers predominantly occur for negative values of m.](image)

Since the rotational splitting coefficients for a multiplet \( \ell, n \) are derived from a fit to the measured frequency splittings of that multiplet such distortion in the measured frequency splittings results in jumps (discontinuities) in the splitting coefficients. This is demonstrated here in Figure 7 for the \( a_1 \) splitting coefficient for the \( n = 1 \) ridge. In this Figure it is also demonstrated that the discontinuity is also present in the \( a_1 \)-coefficient if the asymmetrical model profile is employed. Similar discontinuities do occur for the higher-order \( a \)-coefficients as well as for other ridges.

As we already mentioned discontinuities do also occur in the \( a \)-coefficients if a cross-correlation technique is employed in the estimation of the high-degee rotational frequency splittings. However, in the two sets of splitting coefficients computed with Method 3 and the cross-correlation technique systematic differences are to be noted. For example, the discontinuities do not necessarily occur in the same range of \( \ell \) values. It is of most concern that similar jumps in the rotational splitting coefficients are introduced even if different methods are employed for estimating them. One effect that may cause these jumps is the coupling of modes due to the differential rotation of the Sun as described by Woodard (1989). However, also instrumental effects such as image distortion and plate scale errors must be considered as possible causes, as has been shown by Rhodes et al. (2001). These issues will be addressed in an upcoming study.

![Figure 7. Degree dependence of two sets of \( a_1 \) splitting coefficients for the \( n = 1 \) ridge. The set marked with diamonds and error bars is generated employing the symmetrical model profile while the set marked with triangles is based upon calculations using the asymmetrical model profile. The error bars correspond to 1 standard deviation of the measurements.](image)

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