EFFECT OF LINE ASYMMETRY ON DETERMINATION OF HIGH-DEGREE MODE FREQUENCIES

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ABSTRACT

Accurate measurements of frequencies of high-degree p-modes are important for diagnostics of the structure and dynamics of the upper convective boundary layer, and understanding the nature of the solar-cycle variations detected in low- and medium-degree mode frequencies. Neglecting line asymmetry in the peak-bagging approach may lead to systematic errors in the determination of the mode characteristics and, hence, may affect the results of inversions. Here we demonstrate how the p-mode frequencies are systematically changed in the range of \( \ell \leq 1000 \), \( \nu \leq 7 \text{ mHz} \) when line asymmetry is taken into account in the fitting of the spectral power peaks. The results reported are based upon spectra that were created from observations obtained from the MDI Full-Disk Program during the 1996 SOHO/MDI Dynamics Run.

Key words: Methods: data analysis, Sun: oscillations.

2. THE TECHNIQUE

The results presented in this paper are based upon spectra that were created from observations obtained from the MDI Full-Disk Program during the 1996 MDI Dynamics Run (Scherrer et al., 1995). Specifically, a 60.75-day time series of full-disk MDI Dopplergrams was converted into \( 2\ell + 1 \) time series of spherical harmonic coefficients using the software of the MDI Project Staff. In the initial spherical harmonic step the focal distance of the MDI optical path was assumed to be a constant and the solar radius was assumed to vary only because of the constantly-varying, but known, distance of SOHO from the Sun. However, as has been recently pointed out by Rabello-Soares, Korzennik, and Schou (2001), the effective focal length of the MDI instrument did not remain constant, at least through the middle of 2000. This variability of the MDI image scale means that the solar radii which were used as inputs to the spherical harmonic decomposition software were not correct throughout the 1996 Dynamics Run. In
turn, the temporally-varying ratio of the assumed solar radius to the true solar radius means that the power spectra which were computed from these sets of spherical harmonic coefficients were also slightly in error. Nevertheless, these time series of spherical harmonic coefficients were converted into a set of 2£ + 1 zonal, tesseral, and sectoral power spectra for 0 £ £ 1000 using standard FFT techniques.

These unaveraged power spectra were then converted into a set of 1,001 m-averaged power spectra in a four-step procedure. In the first step of this procedure we estimated the frequency shift which was introduced into each spectrum of azimuthal order, m, by solar internal rotation and asphericity by expanding the 2£ + 1 frequency shifts for a given £ in a polynomial expansion similar to that of Duvall, Harvey, and Pomerantz (1986):

\[ \nu_{ntm} = \nu_{nt} + L \sum_{k=1}^{s} a_k^{(nt)} P_k(m/L), \tag{1} \]

where \( \nu_{nt} \) is the mean frequency of the modal multiplet of degree £ and radial order n, where \( L^2 = \ell(\ell + 1) \), where the \( a_k^{(nt)} \) are the so-called frequency-splitting coefficients, and where the \( P_k \) are the Legendre polynomials of degree k. We employed the Legendre polynomials in this work since our goal in computing the splitting coefficients was only the computation of m-averaged spectra. An alternative expansion in terms of different orthogonal polynomials, \( P_k^{(L)} (m) \), was introduced by Ritzwoller and Laveley (1991). For £ \( \gg k \), \( P_k^{(L)} (m) \rightarrow LP_k(m/L) \).

In this limit, the \( a_k^{(nt)} \) coefficients are equivalent to the coefficients obtained from the Ritzwoller and Laveley expansion. Hence, for most of the degrees considered here, the m-averaged power spectra are effectively identical for both expansions. To generate the set of frequency shifts, \( \nu_{ntm} \), at each £ we used an iterative cross-correlation method very similar to that described in Korzennik (1990). Because we have generally employed a wide range of frequencies when applying this method in the past, we cross-correlated the individual spectra at each £ over 1800 \( \leq \nu \leq 4800 \) mHz in this case. The use of such a wide frequency range effectively averaged the expansion coefficients over the radial orders of the strongest p-mode peaks at each degree; hence, the resulting expansion coefficients can be thought of as n-averaged frequency-splitting coefficients.

In the second step of this procedure we adjusted these raw rotational splitting coefficients to remove jumps in some of the coefficients which appeared to be due to the effects of differential rotation on the p-mode eigenfunctions as first suggested by Woodard (1989) and we also removed other trends using low-order polynomial fits. In the third step we then shifted each tesseral and sectoral spectrum by the calculated frequency shift for that m value using the adjusted frequency splitting coefficients. Finally, in the fourth step of this procedure we averaged the shifted spectra together with the unshifted zonal (\( m = 0 \)) spectrum in an unweighted fashion to create the m-averaged spectrum for each degree.

To determine the mode characteristics from the rotationally corrected, m-averaged spectra, we employed a model profile which is based upon the asymmetric fitting formula of Nigam & Kosovichev (1998), viz.

\[ P_{\nu, \ell}(\nu) = \sum_{\ell'=\ell-1}^{\ell+M} A_{\ell, \ell'}^n \left( \frac{1 + B z_{\nu, \ell'}^n}{1 + z_{\nu, \ell'}^n} \right)^2 \Lambda(\ell, \ell') \otimes \Psi(\nu) + b_0 + b_1 \nu + b_2 \nu^2, \tag{2} \]

Here, \( z_{\nu, \ell'}^n = \frac{2(\nu - \nu_{\ell', \ell})}{w} \), \( P_{\nu, \ell}(\nu) \) is the fitting model profile of the target mode of radial order n and degree \( \ell, \nu \) is the frequency, \( 2M + 1 \) is the total number of fitted peaks, \( \Lambda(\ell, \ell') \) is the m-averaged leakage matrix which relates the relative power density of the target mode to that of the spatial sidelong of degree \( \ell' \), \( A_{\ell, \ell'}^n \) and \( \nu_{\ell', \ell} \) are, respectively, the amplitude and the frequency of the peak with radial order \( n \) and degree \( \ell' \), \( \Psi(\nu) \) is the power spectrum of the temporal window function of the actual observational time series from which the spectra were computed, \( b_0, b_1, \) and \( b_2 \) are parameters which describe the distribution of the background power in the fitting box, and \( \otimes \) denotes convolution. The asymmetry of the model profile is defined by the parameter \( B \) which is positive for positive asymmetry, i.e. excess of power in the low-frequency wing of the p-mode line, and which is negative for negative asymmetry. By setting \( B = 0 \) the traditional symmetrical Lorentzian profile is recovered. In Equation (2) we have assumed that the \( 2M + 1 \) p-mode lines have the same width, \( w \), and the same asymmetry parameter, \( B \). This may not be strictly true, but the variation in both width and asymmetry is not very large between neighboring peaks. Moreover, the leakage matrix \( \Lambda(\ell, \ell') \) is approximated by a Gaussian profile. Since we are dealing with m-averaged spectra we employ a least-squares approach for fitting the model profile (2) to the measured power spectra.

In this work we employed an m-averaged leakage matrix which was computed using three key assumptions: 1) the ratios of the horizontal and vertical components of the solar p-mode eigenfunctions were assumed to be equal to theoretically-predicted ratios as described by Rhodes et al. (2001); 2) the amount of shift in the position of each pixel of each MDI Dopplergram due to distortion in the MDI optical path was given by the amount of cubic distortion computed using an optical ray-tracing program, and 3) the ratio of the value of the solar radius which was input to the spherical harmonic decomposition program was constant over the 1996 Dynamics Run. By employing a different version of our fitting code in which the eigenfunction ratio can be determined as one of the fitted parameters and then comparing the resulting ratios with the predicted ratios on a mode-by-mode basis for \( \ell \leq 200 \), we concluded that the ratio of assumed and true solar radii during mid-1996 was approximately equal to 1.00023. Even though
we have already pointed out that the MDI image scale was varying during this time interval, we also note that Figure 6 of Rabello-Soares, Korzenieki, and Schou (2001) indicates that the ratio of assumed and true solar radii varied by less than 0.0002 and hence our assumption of a constant ratio for the assumed and true solar radii is not likely to affect the results shown here in any significant way.

Each of the 1,001 spherical harmonic time series used in this work covered a total of 87,480 minutes. The initial time series contained a few missing coefficients and had a common duty cycle of 95.2%. After we filled a few of the shortest gaps using standard gap-filling techniques, the final time series contained a total of 85,114 spherical harmonic coefficients for an effective duty cycle of 97.30%. We computed the temporal window function of these time series by first zeroing-out an 87,480-element array and then replacing 85,114 of those zeros with the number 1.0 whenever we had a valid coefficient in the gap-filled time series. We then computed the power spectrum of this window function, $\Psi(\nu)$, in the normal manner.

3. INITIAL RESULTS

In Figure 1 we show two sample cases in which we have included the observed power spectrum, the original fitted profile which was computed using a symmetric profile (i.e., with $B = 0$), and with the fit using the asymmetric profile. The improvement in the agreement between the observed and fitted profiles, particularly for the $\ell = 300$, $n = 1$ case, when asymmetry is included in the model is striking.

In Figure 2 we show the degree dependence of $B$ for the $n = 0$ (top) and $n = 3$ (bottom) ridges. For the $n = 0$ ridge $B$ is negative for all but 28 of the 909 cases shown in the Figure, while for the $n = 3$ ridge $B$ changes sign at roughly $\ell = 850$. This sign change in $B$ may be helpful in locating the source of $p$-mode excitation more precisely. Also, the small jump in $B$ that is evident for the $n = 0$ ridge suggests that we might not have found the optimum set of leakage matrices for use with these power spectra yet. However, the jump in $B$ for the $n = 3$ is much less noticeable than is the jump in $B$ for the $n = 0$ ridge.

In Figure 3 we plot the degree dependence of the differences in the fitted frequencies as computed using both the symmetric and asymmetric profiles. The frequency differences are plotted in the sense $\nu_{\text{sym}} - \nu_{\text{asym}}$. The changes in the fitted frequencies can be seen to be substantial at the higher degrees. Just as in Figure 2 for $B$ the small jump in the frequency differences for the $n = 0$ ridge that is visible near $\ell = 400$ suggests again that we still need to compute additional leakage matrices and employ them in future fits in place of the matrix which we used for these fits. Finally, in Figure 4 we show the frequency dependences of both the asymmetry parameter (top) and the frequency differences (bottom) for 30 of the
p-mode ridges ranging from \( n = 0 \) through \( n = 29 \). We also give attention to the significant oscillations in both \( B \) and in the frequency differences for frequencies above 4500 \( \mu \text{Hz} \). The presence of these oscillations makes the importance of the inclusion of the effects of asymmetry in the fitting of high-degree p-mode ridges evident.

![Graph](image1.png)

**Figure 3.** Degree dependence of the differences between the frequencies obtained with the symmetrical profile and the frequencies obtained with the asymmetrical profile in the sense “symmetric minus asymmetric” for radial orders \( n = 0 \) (top) and \( n = 3 \) (bottom). The dashed line in each panel is for a frequency difference equal to zero. Note the change in vertical axis scales between the top and bottom panels.

**Figure 4.** (top) Frequency dependence of the asymmetry parameter \( B \) for radial orders \( n = 0 \) through \( n = 29 \). The dashed line is for \( B = 0 \). (bottom) Frequency dependence of the differences between the frequencies obtained with the symmetrical profile and the frequencies obtained with the asymmetrical profile in the sense “symmetric minus asymmetric” for radial orders \( n = 0 \) through \( n = 29 \). The dashed line is for a frequency difference equal to zero.

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