DAMPING OF CORONAL LOOP OSCILLATIONS: RESONANT ABSORPTION AT WORK

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ABSTRACT

Motivated by recent TRACE observations of damped oscillations in coronal loops, we consider analytically the motion of an inhomogeneous coronal magnetic tube of radius \(a\) in a zero \(\beta\) plasma. An initially perturbed tube may vibrate in its kink mode of oscillation with the frequency \(\omega_k\) but those vibrations are damped. The damping is due to resonant absorption, acting in the inhomogeneous regions of the tube, which leads to a transfer of energy from the kink mode to azimuthal oscillations within the inhomogeneous layer. We determine explicitly the decay decrement \(\gamma\) for a coronal flux tube whose plasma density varies in a thin layer of thickness \(\ell \ll a\) on the tube boundary. We apply our results to the observations, suggesting that loop oscillations decay principally because of inhomogeneities in the loop. It follows from our theory that only those loops with density inhomogeneities on a small scale (confined to within a thin layer of order \(\alpha \ell / \omega_k\) in thickness) are able to support coherent oscillations for any length of time, and so be observable. Loops with a more gradual density variation, on a scale comparable with the tube radius \(a\), do not exhibit pronounced oscillations.

Key words: Solar corona; magnetic loops; oscillations; damping.

1. INTRODUCTION

This paper has been motivated by the recent observations of standing oscillations of solar coronal loops (Aschwanden et al. 1999, 2002; Nakariakov et al. 1999; Schrijver & Brown 2000; Schrijver et al. 2002), detected by the Transition Region and Coronal Explorer (TRACE) spacecraft. An understanding of coronal oscillations is especially important because such oscillations may shed light on the puzzle of coronal heating and furthermore they may provide seismic information about the coronal plasma (see, for example, Roberts 2000). Aschwanden et al. (1999) and Nakariakov et al. (1999) interpreted the observed oscillations, which followed on from an earlier flare in a nearby location, in terms of the kink mode of oscillation of a coronal loop. The theory of coronal loop oscillations has been developed in some detail for the special case of a straight cylinder of magnetic field embedded in magnetized plasma surroundings (see, for example, Edwin & Roberts 1983). Nakariakov et al. (1999) noted that the loop oscillations were strongly damped, decaying in about 14.5 minutes (compared with an oscillation period of 256 s). If such a rapid decay is due to viscous (or ohmic) damping, then the coefficient of the shear viscosity must be several orders of magnitude larger than that given by the classical Braginskii (1965) value (Nakariakov et al. 1999). On the other hand, many other effects may also bring about decay and they require careful assessment (Roberts 2000). An understanding of what brings about the decay is an important step in understanding coronal oscillation phenomena in general, with coronal seismology (Roberts et al. 1984) a natural goal (Nakariakov & Ofman 2001).

Here we consider the manner in which energy in the global mode of oscillation of a coronal loop is transferred into motions (predominantly azimuthal) in a thin layer at the boundary of the loop where the plasma density falls from high values to match its lower density surroundings. The process is an example of resonant absorption. Indeed, it may be that the observed decay of loop oscillations provides a first explicit solar illustration of resonant absorption. By solving an initial value problem, we determine the decay rate of this process. Our treatment is similar in spirit to the initial value discussion of an incompressible transitional layer by Goedbloed (1979), Rae & Roberts (1981) and Lee & Roberts (1986), drawing on the analysis by Sedlacek (1971) of electrostatic oscillations and by Isonson (1978) of coronal oscillations. For a weakly dissipative plasma, the decay rate is independent of dissipative coefficients; instead, the decay rate provides a direct measure of how strongly inhomogeneous is a coronal loop.

In the next section we formulate the problem and in Section 3 we outline its solution. In Section 4 we compare our theoretical results with the observations of damped oscillations of coronal loops, presenting our conclusions in Section 5.
2. FORMULATION

We consider oscillations of a magnetic tube in a cold viscous plasma. We aim to apply our results to the oscillations of solar coronal loops. In accordance with the classical Braginskii’s expression for the viscosity tensor in a magnetized plasma (Braginskii 1965), under typical coronal conditions the coefficient of the shear viscosity is at least ten orders of magnitude smaller than that of the compressional viscosity. However, in the problem of oscillations of coronal loops dissipation is only important in an Alfvénic dissipative layer embracing an ideal resonant magnetic surface. Numerical studies by Ofman et al. (1994) and Erdélyi & Goossens (1995) have shown that in Alfvénic dissipative layers only the shear viscosity is significant, all other terms in Braginskii’s tensorial expression being neglected. This fact enables us to write the viscous force in the momentum equation in the simplified form \( \rho v \nabla^2 v \), where \( v \) is the velocity, \( \rho \) the plasma density and \( v \) the kinematic viscosity.

![Figure 1. A sketch of an oscillating loop, showing a magnetic flux tube with plasma density \( \rho_1 \) embedded in a plasma with density \( \rho_e \). The equilibrium magnetic field is everywhere of strength \( B \). The equilibrium density varies in the annulus region \( a - \ell < r < a \) from \( \rho_1 \) to \( \rho_e \). The dashed lines show the perturbed magnetic tube in its kink mode of oscillation, line-tied at its ends.](image-url)

Inside the flux tube the plasma density is \( \rho_1 \) and outside it is \( \rho_e \). The two regions are connected by a thin layer, \( a - \ell < r < a \) with \( \ell \ll a \), where the plasma density varies monotonically from \( \rho_1 \) to \( \rho_e \), with \( \rho_1 > \rho_e \). The equilibrium magnetic field \( B \) is everywhere uniform and in the \( z \)-direction, \( B = Bz \) (see Fig. 1). The nonuniformity in plasma density \( \rho(r) \) produces a nonuniform Alfvén speed, allowing resonant wave effects to occur. It is in such nonuniform layers that viscous effects are likely to be most important.

In what follows we adopt the cylindrical coordinates \( r, \varphi, z \) with the \( z \)-axis aligned with the equilibrium magnetic field. In the dissipative layer, there are large gradients in the radial direction only. This observation enables us to use the approximation \( \nu \nabla^2 v \approx \nu \partial^2 v / \partial r^2 \). Then the linearized MHD equations for a cold (zero \( \beta \)) plasma take the form

\[
\begin{align*}
\frac{\partial u}{\partial t} & = -\frac{1}{\rho} \left( B \frac{\partial b_z}{\partial z} - \frac{1}{r} \frac{\partial P}{\partial r} \right) + \nu \frac{\partial^2 u}{\partial r^2}, \\
\frac{1}{\rho} \left( B \frac{\partial b_z}{\partial z} - \frac{1}{r} \frac{\partial P}{\partial r} \right) & = \nu \frac{\partial^2 v}{\partial r^2}, \\
\frac{\partial b_r}{\partial t} & = B \frac{\partial u}{\partial z}, \\
\frac{\partial b_z}{\partial t} & = B \frac{\partial v}{\partial z}, \\
\frac{\partial P}{\partial t} & = -\frac{\rho V^2}{A} \left( \frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial \varphi} \right). 
\end{align*}
\]

Here \( v = (u, v, 0) \) is the velocity and \( b = (b_r, b_z, b_z) \) is the perturbed magnetic field, with \( P = Bb_z / \mu \) being the perturbation in magnetic pressure. The Alfvén speed is \( V_A(r) = B(\mu \rho(r))^{-1/2} \), with \( \mu \) the magnetic permeability.

We assume that the magnetic tube is bounded at \( z = 0 \) and \( L \) by dense, ideal, infinitely conducting plasmas with the magnetic field frozen in these plasmas. This together with the continuity of \( b_z \) across the boundaries implies the boundary conditions

\[
\begin{align*}
u & = v = 0 \quad \text{at} \quad z = 0, L. 
\end{align*}
\]

Equations (4) and (5) imply that \( P = 0 \) at \( z = 0, L \).

We consider the kink mode where perturbations are proportional to \( e^{ib_\varphi} \) (with the coefficients of proportionality real for \( u, b_r \) and \( P \), and purely imaginary for \( v \) and \( b_z \)). This choice corresponds to a linearly polarized kink oscillation of the tube. In addition, we restrict our analysis to the fundamental mode. Then it follows from the boundary conditions (5) that perturbations of \( u, v, P \) are proportional to \( \sin(\pi z / L) \). To complete our formulation, we assume that all perturbations vanish as \( r \to \infty \), and we specify initial conditions at \( t = 0 \). The initial conditions are taken to be arbitrary except that it is assumed that the characteristic scale of their variation with respect to \( r \) is of order or larger than \( a \).

3. SOLUTION

In this section we outline briefly the solution of the initial value problem. Details can be found in Ruderman & Roberts (2002). To solve the problem we eliminate \( b_r \) and \( b_z \) from the system of Equations (1)–(4), take \( u, v \) and \( P \) proportional to \( e^{ib_\varphi} \sin(\pi z / L) \), and apply the Laplace transform with respect to time. As a result we obtain a system of three ordinary differential equations determining the radial dependence of \( u, v \) and \( P \). We solve this system separately in the external \((r > a)\), internal \((a - \ell < r < a)\), and intermediate \((a - \ell < r < a)\) regions.

The assumption \( \ell \ll a \) enables us to neglect the variation of \( P \) in the intermediate region, which strongly simplifies the solution (see also Hollweg 1987; Hollweg & Yang 1988). We match the solutions at the
boundaries \( r = a \) and \( r = a - \ell \) using continuity of \( P \) and \( u \). As a result we obtain expressions for \( u, v \) and \( P \) in terms of Bromwich integrals. Our main goal is to study the asymptotic state of oscillation for long times. Using an appropriate closure of the Bromwich integration contour, we calculate the asymptotics of the solution for \( t \gg \omega_k^{-1} \), where \( \omega_k \) is the frequency of kink oscillations given by (e.g. Edwin & Roberts 1983)

\[
\omega_k^2 = 2\rho V_A^2 k^2 \rho_e + \rho_i, \quad k = \frac{\pi}{L}
\]

(6)

Note that \( \rho V_A^2 = B^2/\mu \) is independent of \( r \) in a zero \( \beta \) plasma. This asymptotics show that, for \( t \gg \omega_k^{-1} \), the magnetic tube oscillates harmonically with the frequency \( \omega_k \), and the amplitude of this oscillation decreases in time with the decrement \( \gamma \) given by

\[
\gamma = \frac{\pi \rho_A |w_k| \rho_i (\rho_i - \rho_e)^2}{2a|\Delta| (\rho_i + \rho_e)^3} = \frac{k^2 dV_A^2}{dr} \mid_{r=r_A}.
\]

(7)

Here the resonant position \( r_A \) is determined by the condition \( kV_A(r_A) = \omega_k \), and \( \rho_A = \rho_A(r = r_A) \). We see that \( \gamma \) is determined by the quantity \( \Delta \), which is proportional to the rate of variation of the Alfvén speed \( V_A \) at the resonant position \( r_A \). This fact has a deep physical meaning: the damping of the global kink mode occurs due to resonant interactions between the kink mode and local Alfvénic motions in a thin dissipative layer embracing the ideal resonant position \( r = r_A \), where the frequency of the global mode matches the Alfvén frequency. This resonant interaction results in an energy transfer from the global mode to local azimuthal motions and thus in a global mode damping. The mechanism of global mode damping is called resonant absorption. In weakly dissipative plasmas it is independent of dissipation and can be described in the framework of ideal MHD.

We have studied the motion in the dissipative layer for large times (\( t \gg \omega_k^{-1} \)). Our main results can be described as follows. After a time of order \( \omega_k^{-1} \) the amplitude of the motion reaches its maximum value, which is of order \( \omega_k/\gamma \) times the maximum amplitude of the global mode oscillation. The further evolution of the motion in the dissipative layer depends strongly on the value of the Reynolds number \( R = \frac{AV_A d}{\nu} \). When \( R^{1/3} \ll \omega_k/\gamma \sim \alpha/\ell \), this motion is a quasi-stationary Alfvénic oscillation (the velocity is predominantly azimuthal) exponentially decaying on the time-scale \( \gamma^{-1} \). On the other hand, when \( R^{1/3} \gg \omega_k/\gamma \) (valid for typical coronal conditions, at least if we use the classical expressions for the viscosity coefficients in collisionless plasmas), then the motion amplitude remains approximately constant (or even slightly increases) for \( t \leq t_m \), where \( t_m \gg \gamma^{-1} \) (\( t_m \approx 15\gamma^{-1} \) for typical coronal conditions). Simultaneously the characteristic radial spatial scale of motions decreases as \( 1/t \). Thereafter, the motion amplitude exponentially decreases on the time-scale \( \omega_k^{-1} R^{1/3} \gg \gamma^{-1} \).

4. APPLICATION TO CORONAL LOOP OSCILLATIONS

Formula (7) gives the calculated decay rate of oscillations in the kink mode. It has been obtained also using a different approach applied to a Cartesian geometry by Hollweg & Yang (1988) and for a cylinder by Goossens et al. (1992). Its use may be conveniently illustrated by taking the density profile in the annulus in the form

\[
\rho(r) = \frac{\rho_i + \rho_e}{2} - \frac{\rho_i - \rho_e}{2} \sin \left( \frac{2\pi r + \ell - 2a}{2\ell} \right),
\]

\[
\rho = \rho(r) - \rho_e - \rho_e < \rho_i < \rho_e.
\]

Using Equation (6), we obtain \( r_A = a - \ell/2 \) and \( \rho_A = (\rho_i + \rho_e)/2 \). Then it follows from Equations (7) that

\[
\gamma = \frac{\omega_k (\rho_i - \rho_e)}{4a (\rho_i + \rho_e)}. \quad \gamma = \frac{\omega_k (\rho_i - \rho_e)}{4a (\rho_i + \rho_e)}. \quad (9)
\]

In terms of the period \( \tau = 2L/c_k \) of the fundamental kink mode with wavenumber \( k = \pi/L \) and kink speed \( c_k = \omega_k/k \); see Roberts et al. (1984), we obtain an oscillation decay rate \( \tau_{\text{decay}} = (\gamma^{-1}) \) of

\[
\tau_{\text{decay}} = \frac{2}{\pi} \left( \frac{a}{\ell} \right) \left( \frac{\rho_i + \rho_e}{\rho_e - \rho_i} \right) \tau.
\]

(10)

We consider this result in relation to the observational data recorded by Nakariakov et al. (1999), who reported a coronal loop oscillation with frequency \( \omega_k \approx 0.024 \text{ s}^{-1} \) (\( \tau = 256 \text{ s} \)) and decrement \( \gamma \approx 0.0011 \text{ s}^{-1} \) (\( \tau_{\text{decay}} = 870 \text{ s} \)). Taking \( \rho_i = 10\rho_e \), we obtain from equation (9) that \( \ell/a \approx 0.23 \).

It follows from Equation (9) (and more generally from Equation (7)) that the condition \( \gamma \ll \omega_k \) is equivalent to \( \ell \ll a \). It can be shown that, in the general case where the density varies through the whole tube cross-section, the condition \( \gamma \ll \omega_k \) is equivalent to \( |\Delta| \gg \omega_k^2/a \). This inequality means that the characteristic scale of the density variation near the resonant position is much smaller than the tube radius. When this condition is not satisfied, eigenmodes corresponding to the kink oscillations have imaginary parts of the same order as real parts. As a result all tube perturbations damp aperiodically (or almost aperiodically) and an external perturbation fails to produce pronounced tube oscillations.

5. DISCUSSION AND CONCLUSIONS

We have studied the plane-polarized kink oscillations of a straight cylindrical magnetic tube with its footpoints embedded in a dense immovable plasma. We have assumed that the equilibrium plasma density varies only in a thin annulus at the tube boundary, the density falling smoothly from \( \rho_i \) to \( \rho_e \) (< \( \rho_i \)). We have used the cold plasma approximation in viscous MHD, and restricted the analysis to the fundamental mode.
We have determined the time evolution of the magnetic pressure in the annulus in terms of a Bromwich integral, thus obtaining the global mode of the tube oscillation (see also Ruderman and Roberts 2002). We have shown that this global mode describes the asymptotic state of tube motions for times much larger than $\omega_k^{-1}$, the inverse eigenmode frequency $\omega_k$. Our theoretical results have been compared with observations of coronal loop oscillations.

The main conclusions of our analysis are as follows:

(i) The asymptotic state of motion of an arbitrarily perturbed thin magnetic tube is a damped harmonic oscillation with the frequency of the global kink mode $\omega_k$.

(ii) The observed decay of coronal loop oscillations may be explained as a consequence of resonant absorption (which is the conversion of the energy of the global mode into the energy of local motions in the dissipative layer embracing the ideal Alfvén resonant position) if we assume that the thickness of the inhomogeneous region is some 23% of a loop radius.

(iii) The decrement $\gamma$ of the tube oscillation is independent of the Reynolds number $R$ when $R \gg 1$; $\gamma/\omega_k$ is of order of $\ell/a$, so the condition $\gamma \ll \omega_k$ is equivalent to $\ell \ll a$.

(iv) After the characteristic damping time $\gamma^{-1}$ of the tube oscillation, the energy of the tube oscillation is converted into the energy of Alfvén oscillations in the dissipative layer; at this time the amplitude of oscillations in the dissipative layer is of the order $a/\ell$ times the maximum amplitude of the tube oscillation.

(v) Under the assumption $\gamma/\omega_k \gg R^{-1/3}$, as in applications to the solar corona, the amplitude of oscillations in the dissipative layer continues to grow until a time of order $t_m$; thereafter, it exponentially decreases on the time-scale $\omega_k^{-1} R^{1/3}$. For typical coronal conditions, $t_m \approx 15 \gamma^{-1}$.

(vi) The Reynolds number $R$ affects only the behaviour of wave motions in the vicinity of the ideal resonant position, so that we cannot here draw any conclusions about the value of the viscosity in the corona on the basis of observations of damping of the coronal loop oscillations. In the case where $\gamma/\omega_k \gg R^{-1/3}$ motions in this region are characterized by strong spatial oscillations in the radial direction. However, if due to certain physical processes (e.g., turbulence) the viscosity becomes anomalously large, so that $\gamma/\omega_k \lesssim R^{-1/3}$, then motions in the vicinity of the ideal resonant position are characterized by a monotonic spatial behaviour in the radial direction. If future observations are able to resolve spatial scales down to a few percent of a loop radius, then it may prove possible to make qualitative estimates of the value of $R$ on the basis of this difference in the character of wave motions in the vicinity of the resonant position.

The conclusion that the condition $\gamma \ll \omega_k$ is equivalent to $\ell \ll a$ is of particular importance. It implies that, in the case where the density varies through the whole tube cross-section with a characteristic scale $a$, $\gamma \sim \omega_k$ and an external perturbation does not cause pronounced tube oscillations. This fact may explain why coronal loop oscillations are so rarely observed.

There are indications that loop structure in general is multi-threaded on a scale below the loop radius, so that $\ell \ll a$ (Aschwanden et al. 1999). It is thus of interest to consider such small-scale loop structures; we may expect that resonant absorption will then occur not in one but in $n$ resonant layers, and the damping rate will accordingly be scaled by a factor of order $n^2/\ell a$. A detailed calculation of the damping rate for such a model, and its comparison with observations, is a natural extension of the present study.

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