WAVES AND OSCILLATIONS IN THE CORONA: THEORY

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ABSTRACT

Recent observations of oscillatory phenomena in the corona have provided considerable impetus to the theoretical development of magnetohydrodynamic wave theory in magnetic structures. This review aims to lay out some of the basic ideas and unifying principles that underpin many of the theoretical models of waves and oscillations in magnetic structures. The combination of such theories and the observations leads towards the development of a coronal seismology and moreover promises to provide a greater understanding of coronal heating.

Key words: Solar corona; magnetic loops; oscillations; damping.

1. INTRODUCTION

Theoretical aspects of magnetohydrodynamic (MHD) waves in the solar coronal plasma have been investigated for decades, but it is only very recently, with the unambiguous detection of such oscillations, that those theories take on a new vigour. Despite a number of early reports of coronal oscillations, in radio wavelengths (e.g., Rosenberg 1970; Trottet, Pick and Heyvaerts 1979; see the review by Aschwanden 1997), in eclipse observations (e.g., Pasachoff and Ladd 1987), in large magnetic arches (Svestka et al. 1982; Koutchmy, Zhugzhda and Locans 1983, McKenzie and Mullan 1997), and indeed in x-ray active red dwarf stars (Mullan, Herr and Bhattacharya 1992), the spatial detection of oscillations by the Transition Region and Coronal Explorer (TRACE) spacecraft (Schrijver et al. 1999; Aschwanden et al. 1999; Nakariakov et al. 1999; Schrijver and Brown 2000) has demonstrated unequivocally the presence of oscillations in coronal loops. The oscillations rapidly decay (Nakariakov et al. 1999). It has now proved possible to systematically study coronal loop oscillations and their decay (Schrijver, Aschwanden and Title 2002; Aschwanden et al. 2002). Oscillations in hot ($>10^6$ K) loops have also been very recently detected by SUMER, the Solar Ultraviolet Measurements of

Emitted Radiation spectrometer on the Solar and Heliospheric Observatory (SOHO) (Kliem et al. 2002; Wang et al. 2002a-c).

The theory of coronal loop oscillations has recently been reviewed by Bray et al. (1991), Roberts (1991b, 2000, 2001), Nakariakov (2001), Goossens, De Groof and Andries (2002), and Roberts and Nakariakov (2002). However, it is evident that the subject is developing apace, led by the recent observational discoveries which have prompted a re-examination of theoretical aspects.

Loops may also carry upwardly propagating waves, detected with SOHO’s Extreme ultraviolet Imaging Telescope (EIT) (Berghmans and Clette 1999; Robbrecht et al. 2001) and TRACE (Berghmans et al. 1999; De Moortel, Ireland and Walsh 2000; De Moortel et al. 2002a-c). Indeed, De Moortel et al. (2002b) conclude that longitudinal intensity oscillations are a commonly occurring phenomena in coronal loops, and that loops connecting to sunspots seem to carry 3 minute oscillations whereas loop legs that appear not to connect to spots carry modes with periods in the 5 minute range. Sunspots themselves have long been known to carry a variety of oscillations, including both 3 minute and 5 minute modes as well as running penumbral waves (see the overview of sunspots by Thomas and Weiss (1992), and specifically articles by Lites (1992) and Roberts (1992)).

Loops are not the only objects seen to oscillate in the corona: prominence oscillations have long been studied (see reviews by Oliver (1999, 2001), Engvold (2001), and Oliver and Ballester (2002); see also Roberts (1991a) and Joarder and Roberts (1992, 1993)), and coronal plumes are seen to support waves (Olman et al. 1997; DeForest and Gurman 1998).

All of these oscillatory events refer to local coronal structures, but large-scale events also arise. These are the EIT waves (Thompson et al. 1999; Thompson 2000; Wills-Davey and Thompson 1999), first observed with the Extreme ultraviolet Imaging Telescope on SOHO. These large-scale waves are generally associated with flares or coronal mass ejections and propagate with speeds of 200-500 km s$^{-1}$. EIT waves may be associated with Moreton waves which are sometimes observed propagating in the photospheric layers of the Sun.
Both slow and fast magnetoacoustic waves have been proposed to explain these various observations. Slow waves have been implicated in sunspots (see reviews in Thomas and Weiss (1992)), polar plumes (Ofman, Nakariakov and DeForest 1999), the legs of coronal loops (De Moortel, Hood and Ireland 2002; De Moortel et al. 2002a-c), and in hot loops (Ofman, Wang and Solanki 2002). The fast kink wave has been invoked in the interpretation of transverse loop oscillations in TRACE (Aschwanden et al. 1999; Nakariakov et al. 1999), and impulsively excited fast waves are thought to be involved in an interpretation of rapid pulsations detected in an eclipse (Williams et al. 2001; Pasachoff et al. 2002). Fast waves are also invoked to explain EIT waves (e.g., Wang 2000; Murawski, Nakariakov and Peli novsky 2001; Ofman and Thompson 2002) and Moreton waves (e.g., Uchida 1974).

There are two important aspects of coronal oscillations we should mention. Firstly, their detection allows a greater scrutiny of the idea that waves may play a role in coronal heating. So far direct evidence for significant wave heating seems slight, the observed amplitudes being too small. Nonetheless, the very presence of waves is significant and adds to their potential role as a means of heating. Secondly, the detection and analysis of waves makes possible the development of coronal seismology (Roberts, Edwin and Benz 1983, 1984; Roberts 1988; Edwin and Roberts 1986; De Moortel, Hood and Ireland 2002), and with it the potential determination of magnetic field strengths in local coronal structures (Nakariakov and Ofman 2001).

How are we to understand in theoretical terms the various oscillatory phenomena so far detected? In this review we point out some general aspects of wave propagation in a magnetically structured atmosphere, and how these aspects may be applied to coronal oscillations. In the main we confine ourselves to loop oscillations, though much of what is discussed has implications for oscillations more widely. We emphasise a basic model for magnetoacoustic waves in a magnetic flux tube, developed for coronal oscillations by Edwin and Roberts (1989) and Roberts, Edwin and Benz (1984).

2. WAVE EQUATIONS

The starting point for our discussion is the linear magnetohydrodynamic wave equations in a magnetically structured atmosphere. In this discussion we ignore a number of complications such as nonlinearity, non-adiabaticity, loop curvature, parametric wave coupling, gravitational stratification, and flows. All these effects are likely to prove important in various circumstances, but a full assessment of their significance in coronal oscillations is not yet available, though a number of aspects have been examined. For example, flows and nonlinearity have recently been discussed in a general form by Ballai and Erdelyi (1998, 1999) and Ballai, Erdelyi and Goossens (2000), and parametric mode coupling has been discussed in Zaqarashvili (2001) and Zaqarashvili and Roberts (2002). But here our aim is to bring out in its simplest form the general features of wave propagation in a structured medium as they apply directly to coronal oscillations, and this is most conveniently done in terms of the linear equations of wave motion.

Accordingly, we consider the linear equations of motion \(\mathbf{v} = (v_x, v_y, v_z)\), written in the form (see, for example, Roberts 1991b, 2000)

\[
\rho_s \left( \frac{\partial^2}{\partial t^2} - c_A^2 \frac{\partial^2}{\partial z^2} \right) v_L = -\nabla \cdot (\rho \frac{\partial \mathbf{B}}{\partial t})
\]

(1)

\[
\rho_s \left( \frac{\partial^2}{\partial t^2} - c_A^2 \frac{\partial^2}{\partial z^2} \right) v_z = - \left( \frac{c_A^2}{c_L^2 + c_A^2} \right) \frac{\partial}{\partial z} \left( \rho \frac{\partial p_T}{\partial t} \right)
\]

(2)

where the total pressure variations \(p_T\), given by

\[
p_T = p + \frac{1}{\mu} B_0 \cdot \mathbf{B},
\]

(3)

are related to the flow \(v\) through

\[
\frac{\partial p_T}{\partial t} = \rho_s c_A^2 \frac{\partial v_z}{\partial z} - \rho_s (c_A^2 + c_L^2) \text{div} \mathbf{v}.
\]

(4)

Here we have taken a non-uniform equilibrium magnetic field \(\mathbf{B}_0 = B_0 \hat{z}\) which is aligned with the \(z\)-axis of our coordinate system. The field strength \(B_0\) is such as to balance plasma pressure \(p_s\), through

\[
\text{grad}(p_s + \frac{B_0^2}{2\mu}) = 0.
\]

(5)

The flow is \(\mathbf{v} = v_x \hat{x} + v_L \hat{z}\), for component \(v_x\) along the applied magnetic field and flow \(v_L\) in the plane perpendicular to the \(z\)-axis. The perturbed magnetic field is \(\mathbf{B}\), and \(\nabla_L\) refers to the gradient operator perpendicular to the applied magnetic field. Written in this way, Equations (1)-(5) may be applied both to flux tubes, described by a cylindrical coordinate system \((r, \theta, z)\), and magnetic slabs, described by Cartesian coordinates \((x, y, z)\).

Three basic speeds immediately arise in Equations (1)-(4): the sound speed \(c_s = (\gamma p_s/p_s)^{1/2}\), the Alfvén speed \(c_A = (B_0^2/\mu p_s)^{1/2}\), and the slow magnetoacoustic speed \(c_t\) defined through

\[
c_t = \frac{c_s c_A}{(c_A^2 + c_s^2)^{1/2}}.
\]

(6)

Each of these speeds is in general a function of the coordinate perpendicular to the applied magnetic field, be it the radius \(r\) in cylindrical coordinates or \(x\) in Cartesian coordinates. Equations (1)-(4) are the basic equations describing MHD wave propagation in a magnetically structured medium, with radial structuring in cylindrical geometry and structuring in \(x\) in a slab geometry (cf. Roberts 1981).

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There is a rich structure in Equations (1)-(4), and this structure is important for coronal oscillations. The corona is generally considered to be a low β-plasma, so that \( c_s \ll c_A \); consequently, the ordering of speeds is typically \( c_i < c_s < c_A \). When applied to a discrete flux tube with Alfvén speed \( c_A \) inside the tube and an environment with Alfvén speed \( c_{Ae} \), a further speed becomes important; it is the mean Alfvén, or kink speed, \( c_k \), defined through

\[
c_k = \left( \frac{\rho_o c_A^2 + \rho_e c_{Ae}^2}{\rho_o + \rho_e} \right)^{1/2}
\]

where \( \rho_o \) denotes the plasma density within the tube and \( \rho_e \) is the density in the tube's magnetic environment. The speed \( c_k \) is intermediate between \( c_A \) and \( c_{Ae} \). In the extreme of a magnetic field \( B_0 \) being everywhere uniform with the tube defined simply by virtue of a strong density enhancement (\( \rho_o \gg \rho_e \)), then \( c_k = \sqrt{2} c_A \).

The speeds \( c_i \) and \( c_k \) have been recognised as important in a number of early studies in connection with photospheric flux tubes (Defouw 1976; Ryutov and Ryutova 1976; Roberts and Webb 1978; Parker 1979; Spruit and Roberts 1983; Ryutova 1990) and coronal tubes (Wilson 1980; Spruit 1981, 1982; Edwin and Roberts 1983). The speed \( c_k \) also arises in the description of waves and instabilities on magnetic interfaces (e.g. Kruskal and Schwarzschild 1954; Chandrasekhar 1961; Miles and Roberts 1989).

3. PROPERTIES

3.1. Phase mixing

The simplest solution of (1)-(4) is provided by Alfvén waves, with \( p_f = 0, u_v = 0, u_\theta = 0 \) and \( \nabla \cdot \mathbf{v} = 0 \). Then torsional oscillations \( \mathbf{v}_\theta \) satisfy

\[
\frac{\partial^2 v_\theta}{\partial t^2} = c_A^2 (r) \frac{\partial^2 v_\theta}{\partial z^2}.
\]

This equation exhibits phase mixing (Heyvaerts and Priest 1983), whereby the gradient \( \partial v_\theta / \partial r \) across the field grows rapidly in time even though \( v_\theta \) itself remains finite. Viscous (or other damping effects) would then soon act to dissipate the motion.

To illustrate the effect, we consider the Cartesian form of Equation (8) adding also the effect of a kinematic viscosity \( \nu \); motions \( v_\theta \) then satisfy (Heyvaerts and Priest 1983)

\[
\frac{\partial^2 v_\theta}{\partial t^2} = c_A^2 (x) \frac{\partial^2 v_\theta}{\partial z^2} + \nu \left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) \frac{\partial v_\theta}{\partial t}.
\]

Noting a mathematical analogy between the processes of phase mixing and flux expulsion of magnetic field from within a convective cell, Roberts (1988) wrote down an illustrative approximate solution of (9) which allows a simple inspection of the two timescales that are likely to operate in the dissipation of the wave. An approximate solution is

\[
u_\theta = u(t) \sin(k_z z) \cos(k_A(x) t),
\]

where the amplitude \( u(t) \) is given by

\[
u(t) = u_0 \exp\left(-\frac{1}{2} k_z^2 \nu (t + \frac{1}{3} c_A^2 \nu^2) \right),
\]

the prime (') denoting the derivative of the Alfvén speed \( c_A (x) \).

For very short times (\( t^2 c_A^2 \ll 1 \) or in a uniform medium (\( c_A' = 0 \)), damping proceeds slowly. For a standing wave in a loop of length \( L \) we may set \( k_z = \pi / L \) (for the principal mode) and then the decay proceeds on a timescale \( \tau_d \) where

\[
\tau_d = \frac{2}{\nu k_z^2} = \frac{2 L^2}{\nu \pi^2}.
\]

For example, with \( L = 10^8 \) m and \( \nu = 4 \times 10^9 \) m² s⁻¹, this produces a decay time of \( \tau_d = 5 \times 10^8 \) s.

However, spatial variations in the Alfvén speed \( c_A \) quickly render this process obsolete. With \( c_A' \approx c_A / L \), we expect that \( t^2 c_A^2 \ll 1 \), viz. \( t^2 c_A^2 \gg \eta^2 \), for all but the very shortest of times or most uniform of equilibrium states. For example, with an Alfvén speed \( c_A = 10^3 \) km s⁻¹ and a spatial variation scale \( l \) of 10³ km, phase mixing dominates after only a few seconds. Decay is then determined by the \( t^2 \)-term in Equation (11), giving

\[
\tau_d = \left[ \frac{6 L^2}{\nu \pi^2 c_A^2} \right]^{1/3}.
\]

This result has recently been applied to loop oscillations by Ofman and Aschwanden (2002). Write \( \tau = \sqrt{2} L / c_A \), the period of a kink wave in a high density (\( \rho_o \gg \rho_e \)) loop (see discussion below in Sect. 3.2), and suppose that the transverse inhomogeneity scale \( l \) is a fraction \( \alpha \) of the length of a loop, so \( l = \alpha L \) (with \( \alpha < 1 \)). Then

\[
\tau_d = \left( \frac{3 \alpha^2}{\nu \pi^2} \right)^{1/3} (L \tau)^{2/3}.
\]

If, statistically, variations in \( \alpha \) from loop to loop are insignificant, then phase mixing gives a decay time \( \tau_d \) that is proportional to the \( 2/3 \)-power of the product \( \eta L \). We return to this discussion in Section 4.

3.2. Guided modes

Returning to equations (1)-(5) we consider the possibility of magnetosonic waves guided by inhomogeneities, such as coronal loops. Standard studies of MHD waves in a uniform medium (e.g., Roberts 1985) show that the Alfvén wave is field guided: the group velocity of the Alfvén wave is along the applied magnetic field. Similarly, the slow wave is field.
guided in that its group velocity lies in a cone close to the direction of the applied field. By contrast, the fast magnetoacoustic wave spreads almost isotropically in a uniform medium, its group velocity diagram being similar to the sphere of a pure sound wave. However, in an inhomogeneous medium even the fast wave may become wave guided through refraction in regions of low Alfvén speed. Coronal loops are usually regions of enhanced plasma density, corresponding to regions of low Alfvén speed, so we may expect fast waves to be wave guided by coronal loops (Uchida 1974; Habbal, Leer and Holzer 1979; Edwin and Roberts 1983; Roberts, Edwin and Benz 1984).

To see this specifically, consider a magnetic flux tube of radius $a$ and field strength $B_0$, embedded in a magnetic environment of strength $B_0$. The plasma density inside the tube is $\rho_0$ and that in the environment is $\rho_e$, so that

$$B_0(r) = \begin{cases} B_0, & r < a, \\ B_e, & r > a, \end{cases} \quad \rho_0(r) = \begin{cases} \rho_0, & r < a, \\ \rho_e, & r > a. \end{cases}$$

(15)

The Alfvén, sound and tube speeds within the tube are $c_A$, $c_s$, and $c_t$, and their values in the external medium are $c_{Ae}$, $c_{se}$ and $c_{te}$.

The dispersion relation for this configuration follows by solving Equations (1)-(4) inside and outside the tube (both uniform media) and matching those solutions across the tube boundary. The resulting dispersion relation is (e.g., McKenzie 1970; Spruit 1982; Edwin and Roberts 1983; Cally 1986; Evans and Roberts 1990)

$$\frac{n_0}{\rho_0(k_2^2c_A^2 - \omega^2)} \frac{J_\nu(n_0 a)}{J_\nu(n_0 a)} = \frac{m_e}{\rho_e(k_2^2c_{Ae}^2 - \omega^2)} \frac{K'_\nu(m_0 a)}{K_\nu(m_0 a)}$$

(16)

where

$$n_0^2 = \frac{(\omega^2 - k_2^2c_A^2)(\omega^2 - k_2^2c_s^2)}{(c_s^2 + c_A^2)(\omega^2 - k_2^2c_t^2)}$$

and

$$m_e^2 = \frac{(k_2^2c_{Ae}^2 - \omega^2)(k_2^2c_{se}^2 - \omega^2)}{(c_{se}^2 + c_{Ae}^2)(k_2^2c_{te}^2 - \omega^2)}.$$

In deriving (16) we have written

$$p_T(r, \theta, z, t) = p_T(r) \exp i(\omega t + n\theta - k_z z)$$

(17)

for frequency $\omega$, azimuthal number $n = 0, 1, 2, ..., \ldots$, and longitudinal wavenumber $k_z$.

Dispersion relation (16) describes the magnetoacoustic modes ($p_T \neq 0$) of a magnetic flux tube. Our interest here is in coronal tubes, with $c_t \ll c_A < c_{Ae}$, corresponding to a dense tube ($\rho_0 \gg \rho_e$) in a strong magnetic field. Equation (16) requires that $m_e > 0$, corresponding to waves being radially trapped within the tube. In the extreme of a zero sound speed (cold plasma), this corresponds to $c_{Ae} > c_A$, and regions of high plasma density are wave guides for fast magnetoacoustic waves (Roberts, Edwin and Benz 1984). If the condition $m_e > 0$ is not met, then wave leakage may occur: radially propagating waves are excited in the tube's environment and propagate away from the tube. The result is a decay in the oscillations within the loop. However, considerations of this effect (e.g., Meerson, Sasarov and Stepanov 1978; Spruit 1982; Cally 1986; Tsap and Kopylova 2001) suggest that for long waves it is too weak to account for the observed decay in the oscillations (Roberts 2000).

Solutions of dispersion relation (16), subject to $m_e > 0$, show that the fast waves are strongly dispersive (with phase speed $c = k_z^2c_t^2$ varying with wavenumber $k_z$). There is a global mode of oscillation of the tube — the kink ($n = 1$) mode — which has $\omega \approx k_z^2c_t$. For a standing wave with no disturbance at the loop ends $z = 0$ and $z = L$, we obtain the period $\tau$ of the principal kink mode standing in a loop of length $L$ as (Roberts, Edwin and Benz 1984)

$$\tau = \frac{2L}{c_t}. \quad (18)$$

It may be noted that in the special case of a uniform magnetic field ($B_0 = B_e$), with the loop's identity being due principally to enhanced plasma density (or temperature inhomogeneity), the kink propagation speed is simply

$$c_k = \sqrt{2c_A(1 + \frac{\rho_e}{\rho_0})^{1/2}}, \quad (19)$$

Such standing kink modes have been observed by TRACE (Aschwanden et al. 1999, 2002; Nakariakov et al. 1999). Moreover, these relations have been exploited to deduce the magnetic field strength in a coronal loop (Nakariakov and Ofman 2001). This is the beginnings of coronal seismology (Roberts, Edwin and Benz 1984; Roberts 1986; Edwin and Roberts 1986). To see this, note that (18) and (19) can be rearranged to give the coronal field strength:

$$B_0 = (2\mu)^{1/2}(\rho_0 + \rho_e)^{1/2} \frac{L}{\tau}. \quad (20)$$

Equation (20) gives the field strength $B_0$ (in mks units, tesla) in terms of the loop length $L$, oscillation period $\tau$, and plasma densities $\rho_0$ and $\rho_e$; in cgs units, the field strength is given in gauss (G) with the factor $\sqrt{2\mu}$ replaced by $\sqrt{8\pi}$.

Note that the presence of a square root in the density term narrows the uncertainties in determining the field strength. Nakariakov and Ofman (2001) used (20) — but note their different definition of $L$ (taken in their formula as the loop height) — to deduced a loop field strength in the range $4 \sim 30$ G, uncertainties in the plasma density contributing to this range; considerations of plasma density refined their determination to a field strength of $13 \pm 9$ G.

The slow modes are also guided by the loop; they are only weakly dispersive, with $\omega \approx k_zc_t$. Thus, a standing slow mode has a period (Roberts et al. 1984)

$$\tau = \frac{2L}{c_t} \approx \frac{2L}{c_e}. \quad (21)$$
This mode may have been detected by SUMER. Wang et al. (2002a) point out that for a loop of length \( L = 140 \) Mm with temperature \( T = 6.3 \) MK (giving a sound speed of \( c_s = 295 \) km s\(^{-1}\)) the slow mode produces a period \( \tau = 15.8 \) minutes, consistent with their observed periods of 14 – 18 minutes. Wang et al. (2002a) also consider an interpretation in terms of the kink mode relation (18) but since this requires a plasma \( \beta \) of order unity and also a rather large loop displacement it seems more likely that the slow mode is implicated.

All the waves of a flux tube may be excited impulsively; indeed, the fact that some of these waves are associated with flares suggests precisely that. When impulsively excited, the dispersion in a wave is important for it determines the detailed nature of the resulting disturbance. The slow modes are only weakly dispersive, so little results from this effect, but the fast waves are strongly dispersive.

Roberts, Edwin and Benz (1983, 1984) have shown how dispersion results in a distinctive wave packet behaviour for the fast sausage mode. Linear and nonlinear numerical simulations by Murawski and Roberts (1993a-d, 1994) offer further illustration of the effect. Also, there is observational evidence from TRACE (see Aschwanden et al. 2002) that impulsively generated waves are involved in a number of events. When impulsively excited, the fast sausage mode produces a wave packet, the nature of which is defined by the internal and external Alfvén speeds of the flux tube together with the detailed form of the group velocity for the mode (which depends specifically upon the plasma density profile that is modelled; see Nakariakov and Roberts 1995). All the features of an impulsively generated wave can be determined theoretically: in particular, the wave packet exhibits rapid oscillations. The time scale \( \tau_{\text{pulse}} \) for such oscillations is of the order of the Alfvén travel time across a loop diameter; specifically,

\[
\tau_{\text{pulse}} = \frac{2\pi a}{j_0 c_s A \left(1 - \frac{\rho_a}{\rho_0}\right)^{1/2}}
\]  

(22)

where \( j_0 \approx 2.40 \) denotes the first zero of the Bessel function \( j_0 \). For example, in a tube of radius \( a = 10^3 \) km with internal Alfvén speed \( c_A = 10^3 \) km s\(^{-1}\), with the plasma density inside the tube being much larger than that in the environment (so \( \rho_0 \gg \rho_a \)), this produces a timescale of 2.6 s.

The coupling of coronal motions with the chromosphere and photosphere necessarily leads to some footpoint leakage. Early considerations (e.g. Hollweg 1984; Berghmans and De Bruyne 1995), predating the observations, suggested that generally the effect was small. De Pontieu, Martens and Hudson (2001), however, argued that the effect may be significant and comparable with observed damping times for transverse loop oscillations. A recent assessment by Ofman (2002) concludes, however, that De Pontieu et al. over-estimated the effect and that chromospheric footpoint leakage cannot explain the observed rapid damping. These are important considerations that justify further study to improve the modelling of loop footpoints and possibly to allow for the dynamical nature of the chromosphere. Indeed, in may be that dynamical aspects – so far not studied – are most important, modifying the effective density scale height that chromospheric layers provide (De Pontieu 2002).

We have discussed phase mixing in Section 3.1, showing that it produces a decay rate \( \tau_2 \) given by Equations (13) and (14). Ofman and Aschwanden (2002) applied this result to a subset of TRACE observations for which the period and decay is reasonably well determined. If statistical variations from loop to loop in the fraction \( \alpha \) are weak, then \( \tau_2 \) is proportional to \( \left(L_r\right)^{2/3} \), and so the decay time varies as the \( 2/3 \)-power of \( L_r \). Although there is some observational support for this trend, that support is inconclusive; a greater number of events will need to be studied to explore this relationship. Moreover, there seems no compelling physical reason why the transverse scale of inhomogeneity should not arbitrarily vary from loop to loop, independently of the length of a loop, and then (13) rather than (14) is more appropriate. Also, it should be borne in mind that (13) and (14) apply to an Alfvén wave, whereas the observations relate to the kink mode. Moreover, as Ofman and Aschwanden (2002) rightly point out, loop cross-sections may well roll-off, and so have inhomogeneities on a small scale. This effect certainly needs to be included in the various models.

The third effect we discuss is resonant absorption. The inhomogenous nature of a loop, whereby its internal density blends in with its environment, leads to wave coupling of the global mode and Alfvénic motions, and this produces a decay in the global mode. Such an effect has been extensively studied for MHD waves (e.g., Uberoi 1972; Goedbloed 1971, 1983; Tataronis and Grossman 1973; Ionson 1978; Rae and Roberts 1981; Lee and Roberts 1988), drawing on the seminal treatment by Sedlacek (1971) of...
the mathematically similar electrostatic oscillations; its importance for coronal physics was first stressed by Ionson (1978), mainly as a means of coronal heating. For recent reviews, see Goossens (1991) and Poedts (2002). Here we consider the initial value problem for the kink mode of a coronal flux tube with zero plasma $\beta$, following the treatment by Ruderman and Roberts (2002a, b). Ruderman and Roberts (2002a) show that the global kink oscillations of a tube decay on a timescale

$$\tau_d = \frac{2}{\pi} \left( \frac{\rho_0 + \rho_e}{\rho_0 - \rho_e} \right)^2 \frac{a}{\rho_A} \left| \frac{\partial}{\partial r} \right| \frac{\Delta}{\omega_k^2}. \quad (23)$$

Here $\omega_k = k_z c_A$ is the frequency of the kink mode and $\Delta = -d(\omega_k^2)/dr$, where $\omega_A = k_z c_A(r)$ is the Alfvén frequency; both $\omega_A$ and its slope $\Delta$ are calculated at $r = r_A$, where $\omega_A = \omega_k$ and $\rho_A = \rho(r = r_A)$.

In terms of the transverse scale height $H = \rho_0(r)/\rho_0(r)$ of density variations, calculated at $r = r_A$, we may rewrite (23) in the form

$$\tau_d = \frac{4}{\pi} \left( \frac{\rho_0 + \rho_e}{\rho_0 - \rho_e} \right)^2 \frac{a}{k_z c_A \rho A} \left| \frac{\partial}{\partial r} \right| \frac{H}{H}. \quad (24)$$

Ruderman and Roberts (2002a) apply (23) to a tube with density profile $\rho_0(r)$ which changes smoothly from a uniform density $\rho_0$ in $0 \leq r < a - l$ to a uniform value $\rho_e$ in $r \geq a$, the change being monotonic across a layer of width $l \ll a$ on the tube boundary; the resulting decay time of the standing kink mode is

$$\tau_d = \frac{2}{\pi} \left( \frac{\rho_0 + \rho_e}{\rho_0 - \rho_e} \right) \tau, \quad (25)$$

where $\tau$ is the period of the kink oscillation (given by (18)). Much the same formula arises if instead we consider a linear density ramp connecting $\rho_0$ to $\rho_e$ across a layer of width $l$ on the tube boundary; equation (23) then gives

$$\tau_d = \frac{4}{\pi^2} \left( \frac{a}{l} \right) \frac{\rho_0 + \rho_e}{\rho_0 - \rho_e} \tau. \quad (26)$$

In fact, both these results, (25) and (26), may be deduced from earlier studies. Hollweg and Yang (1988) looked at a single interface, and derived (26) results if their Cartesian geometry result is applied to a thin cylinder by writing the wavenumber as $1/a$. They obtained their result by exploitation of the resonant oscillation approach first pointed out by Hollweg (1987). Importantly, in the context of current coronal observations, Hollweg and Yang (1988) commented that “coronal loop oscillations would decay in only two periods”. The result (23) also arose in the work of Goossens, Hollweg and Sakurai (1992), who applied connection formulae to derive the decay rate of the kink mode (and indeed all modes with wavenumber $n \neq 0$). Finally, we note that in her study of the kink mode in an isolated photospheric flux tube, Ryutova (1977) also obtained the decay rate of the oscillation due to resonant absorption.

Applying (25) to the loop oscillation investigated by Nakariakov et al. (1999), Ruderman and Roberts (2002a) showed that it matches the observations if $l = 0.23a$. Thus, through resonant absorption, a density inhomogeneity on a scale of some 23% of the loop radius $a$ is sufficient to provide the observed damping. A sharper profile gives a smaller $l/a$ and with it a longer damping time, no damping occurring in the limit of a discrete (step function) density profile; a less distinct profile, with $l$ approaching $a$, leads to motions that are so rapidly damped that no coherent oscillations occur (and so are likely to be seen). This fact possibly explains the puzzling observation (Schrijver et al. 2002; Aschwanden et al. 2002) that certain loops seem to be particularly receptive to oscillations while neighbouring loops are apparently undisturbed.

Goossens, Andries and Aschwanden (2002) have explored (25) further, examining eleven cases selected in Ofman and Aschwanden (2003), and concluded that resonant absorption is able to reproduce the observed decay for a range of $l$ extending from 16% to 46% of the loop radius. Consequently, the idea seems to apply across a wide spectrum of observations.

The observed decay in SUMER loops (Wang et al. 2002a-c) has been considered by Ofman, Wang and Solanki (2002), who argue that the effect can be understood as a consequence of thermal conduction acting on the oscillations. Since the coefficient of thermal conduction along a magnetic field line depends upon the $5/2$-power of the temperature, thermal conduction is particularly efficient in the high temperatures (6 MK) recorded in SUMER loops. Thus the effect is plausible. Whether resonance effects are also significant, acting on the slow mode continuum, has yet to be investigated.

5. CONCLUSIONS

The observational discovery of loop oscillations has given great impetus to the study of MHD waves. The combination of observations with theory offers hope that waves will provide a probe of the local corona, giving a coronal seismology. Moreover, it should now prove easier to assess the role of MHD waves in the heating of active regions and coronal holes.

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