X-RAY OBSERVATIONS WITH RHESSI AND COLLISIONAL THICK TARGET MODEL WITH NONUNIFORM TARGET IONISATION

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ABSTRACT

Past analysis of flare Hard X-Ray (HXR) spectra have largely ignored the effect of nonuniform ionisation along the electron paths in the thick target model, though it is very significant for well-resolved spectra. The fit to RHESSI data on four flares for a single power-law \( F_0(E) \) is much improved when ionisation structure is included. The expression involves the column depth \( N_\alpha \) of the transition region in the flare loop as one of the parameters.

Key words: Sun : flares, hard X-rays, spectroscopy.

1. INTRODUCTION

To utilise fully the high spectral resolution of RHESSI in diagnosing flare photons and particles, it is essential to correct the observed radiation spectra (already instrument corrected) for any effects at the sun which contaminate or distort them. In the case of thick target model interpretation (Brown, 1971; Brown, 1973a) of hard X-ray (HXR) spectra, one effect, which has been largely ignored in data modelling till now, is that of varying ionisation along the thick target beam electron paths. As first discussed by Brown (1973a) the fall of ionisation with depth in the atmosphere reduces long range collisional energy losses and so enhances the HXR bremsstrahlung efficiency there, elevating the high energy end of the HXR spectrum by factors of up to 2.8 above that for an ionised target. The net result is that a power-law electron spectrum of index \( \delta \) produces a photon spectrum of index \( \gamma = \delta - 1 \) at low and high energies but with \( \gamma < \delta - 1 \) in between. The upward knee occurs at fairly low energies, proably masked in data by the tail of the thermal component while the downward knee occurs in the few deka-keV range, depending on the column depth of the transition zone.

2. THICK TARGET SPECTRUM FOR NON-UNIFORM TARGET IONISATION

We will denote the electron injection spectrum by \( F_0(E_0) \) (electrons s\(^{-1}\) per unit injection energy \( E_0 \)) and the emitted bremsstrahlung HXR spectrum at the Sun by \( I(\varepsilon) \) (photons s\(^{-1}\) per unit photon energy \( \varepsilon \)). Following Brown et al. (1998) we treat the electron propagation as 1-D and ignore trapping, relativistic, and directivity effects. The bremsstrahlung cross-sectional differential in \( \varepsilon \)

\[
Q(\varepsilon, E) = Q_0 \frac{q(\varepsilon, E)}{\varepsilon E},
\]

yields for the Kramers approximation \( q = 1 \)

\[
J(\varepsilon) = \frac{Q_0}{K' \varepsilon} \int_0^\infty \frac{F_0(E_0)}{\lambda + \varepsilon ((E_0^2 - \varepsilon^2)/2K')},
\]

where \( K' = 2\pi e^4 \lambda_\Lambda \), \( \Lambda = \lambda_{ee} - \lambda_{ee} \) and \( \lambda = \lambda_{ee}/\Lambda \) (numerically \( \lambda_{ee} = 20, \lambda_{ee} = 7.1 \) so \( \lambda = 12.9 \) and \( \lambda = \lambda_{ee}/\Lambda \approx 0.55 \)). \( x(M) \) is the atmospheric ionisation at an ionisation weighted target column density \( M \) (cm\(^{-2}\)) defined in Sec.2 of Brown et al. (1998).

Though \( x \) changes from 1 to 0 over a substantial spatial range on the Sun, the change is rather abrupt in column density range and, following Brown et al. (1998), we approximate \( x(M) \) as a step function at depth \( M_\ast \)

\[
x(M) = \begin{cases} 
1, & M < M_\ast \\
0, & M > M_\ast 
\end{cases}
\]

for which Eq. (2) and writing

\[
F_0(E_0) = (\delta - 1) \frac{F_1}{E_1} (E_0/E_1)^{-\delta - 2}
\]

where \( F_1 \) is the total injection rate (s\(^{-1}\)) above \( E_0 = E_1 \) results, after one integration, in

\[
J(\varepsilon) = \frac{Q_0 F_1}{(\lambda + 1) K' \varepsilon} \left[ \frac{1}{\delta - 2} \left( \frac{\varepsilon}{E_1} \right)^{\delta - 2} \right]
\]
\[ + \frac{1}{2\lambda} \left( \frac{E_x}{E_1} \right)^{(\delta+2)} B \left( \frac{1}{1 + (\epsilon/E_x)^{2/\delta}}, \frac{\delta}{2} - 1, \frac{1}{2} \right) \] (5)

where \( K = 2\pi e^6 A_1 e, E_c = (2K'M_e)^{1/2} \) and \( B \) is the Incomplete Beta function

\[ B \left( y, \frac{\delta}{2} - 1, \frac{1}{2} \right) \equiv \int_0^y x^{\frac{\delta}{2} - 2} (1 - x)^{-\frac{1}{2}} dx, \] (6)

and is well tabulated.

3. APPLICATIONS TO RHESSI DATA

We have chosen four examples of RHESSI flare spectral data sets (Kontar, Brown and McArthur, 2002). These are the events of February 20 at 11:06 UT, March 17 at 19:26 UT, May 31 at 22:52 UT, and June 1 at 00:06 UT. The corresponding light curves are presented in Fig. 1 while the the locations and classes of the flares are given in Table 1. Note that the event of May 31 is very near the limb and if this involves any footpoint occultation our analysis based on the whole thick target source volume may be inappropriate.

<table>
<thead>
<tr>
<th>Date</th>
<th>Class</th>
<th>Location</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 Feb 2002</td>
<td>C7.5</td>
<td>N15 W77</td>
<td>9825</td>
</tr>
<tr>
<td>17 Mar 2002</td>
<td>M4.0</td>
<td>S21 E18</td>
<td>9871</td>
</tr>
<tr>
<td>31 May 2002</td>
<td>M2.4</td>
<td>S30 E90</td>
<td>9973</td>
</tr>
<tr>
<td>1 Jun 2002</td>
<td>M1.5</td>
<td>S19 E29</td>
<td>9973</td>
</tr>
</tbody>
</table>

3.1. Fully ionised target with single powerlaw \( F_0(E_0) \)

After allowance for all instrumental effects (as in Aschwanden et al., 2002) the spatially integrated RHESSI spectra in these events can be modelled well as the sum of an isothermal component and a nonthermal component. The energy range below around 10-20 keV (depending on the flare) can be best fit with an exponential function while the range well above this is approximately a power law, though with some steepening at higher energies. The best fit parameter (\( \delta = \gamma + 1, kT \)) values for a single powerlaw are presented in Table 2 while the spectral fits, made with the help of SPEX, are shown in Fig. 1.

The figures clearly show that the greatest deviation from a single powerlaw occurs in the range 30-60 keV (see fig. 1) where the spectra show a downward 'knee'. To improve the fit we need to take a more complex function than the simple powerlaw. This spectral 'knee' in the delakeV range of \( \epsilon \) is often attributed to some sharp feature in \( F_0(E_0) \) such as a broken powerlaw or spectral bump (e.g. Johns and

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>( \delta )</th>
<th>( kT(\text{keV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 Feb 2002</td>
<td>11:06:00-11:06:40</td>
<td>4.89</td>
<td>1.47</td>
</tr>
<tr>
<td>17 Mar 2002</td>
<td>19:27:30-19:29:10</td>
<td>4.84</td>
<td>1.27</td>
</tr>
<tr>
<td>31 May 2002</td>
<td>00:06:40-00:08:00</td>
<td>3.79</td>
<td>2.02</td>
</tr>
<tr>
<td>1 Jun 2002</td>
<td>03:53:10-03:54:30</td>
<td>4.26</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Lin, 1993). However these interpretations are based on the fully ionised target approximation. Here we consider how well the photon data are fit by reverting to a single powerlaw in \( F_0(E_0) \) but allowing for the ionisation structure which is certainly present - i.e by the form predicted in Eq. 5.

3.2. Nonuniformly ionised target with powerlaw \( F_0(E_0) \)

We therefore fitted the data with the following model prediction (Eq. 5)

\[ J(\epsilon) = \frac{I_0}{(\lambda + 1)\epsilon} \left[ \epsilon^{\gamma+2} + \frac{E_c^{-\gamma+2}}{2\lambda} \right] \times B \left( \frac{1}{1 + (\epsilon/E_c)^{2/\delta}}, \frac{\delta}{2} - 1, \frac{1}{2} \right) \] (7)

where \( \lambda = 0.55 \) and \( I_0, E_c \) and \( \delta \) are adjustable parameters. Note that \( I_0 \) is just a scale factor depending on \( F_0, E_1 \) - see Eq.(5).

<table>
<thead>
<tr>
<th>Date</th>
<th>( E_\gamma ) (keV)</th>
<th>( N_\gamma ) (cm(^2))</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 Feb 2002</td>
<td>5.29</td>
<td>37.4</td>
<td>2.7 \times 10^{20}</td>
</tr>
<tr>
<td>17 Mar 2002</td>
<td>4.99</td>
<td>24.4</td>
<td>1.1 \times 10^{20}</td>
</tr>
<tr>
<td>31 May 2002</td>
<td>4.15</td>
<td>56.2</td>
<td>6.1 \times 10^{20}</td>
</tr>
<tr>
<td>1 Jun 2002</td>
<td>4.46</td>
<td>21.0</td>
<td>8.4 \times 10^{18}</td>
</tr>
</tbody>
</table>

The best fit results for \( E_\gamma \) and \( \delta \) are presented in Table 3, along with the column depth \( N_\gamma \) equivalent to \( E_\gamma \) and \( \chi_{\text{nonuni}}^2/\chi_{\text{uni}}^2 \) is the ratio between total \( \chi^2 \) deviations of the model prediction from the data,

\[ \chi_{\text{model}}^2 = \sum_{\text{int}} \left( J_{\text{model}}(\epsilon) - J_{\text{RHESSI}}(\epsilon) \right)^2, \] (8)

for the two models - nonuniformly (nonuni) and uniformly (uni) ionised targets. In the four panels of Fig. 1 we show the best fit spectra superimposed on the data for each of the four events. The overall \( \chi^2 \) value summed over all data points Eq. (8) is much lower for the model allowing for the ionisation structure than for the fully ionised case (see Table 1) and the individual residuals are also much smaller.
Figure 1. The RHESSI light curves are shown for the low-energy channel 20 – 50 keV, binned in steps of 1.0 s (first row). The spectra and spectral fits with power law plus thermal component in the energy range 10-100keV (top panel). The fit parameters are given in Table 2. Normalised residuals are presented in the bottom panel showing the deviation of real data from the power law. The time intervals taken for the spectral fits and fit parameters are given in the Tables 1-3.
(note the log scale) in the range 20-60 keV. Above 60 keV the data are progressively noise dominated and therefore both models give similar residuals. Thus inclusion of the ionisation effect allows a much better fit to these RHESSI spectra with a single powerlaw \( F_0(E_0) \) than does the usual ionised approximation and removes the need to invoke features in the acceleration process producing \( F_0(E_0) \). The inferred best fit \( \delta \) values are slightly higher in the more realistic ionisation model (compare Tables 2 and 3).

3.3. Location of the transition region

By assuming that the main spectral feature observed in the HXR spectra is caused by the increased bremsstrahlung efficiency of the chromosphere and not an original feature in the electron spectra, then the HXR spectra can be expressed analytically as the combination of a power law and an incomplete beta function (7). The HXR spectra can be fitted with (7) to determine the three variables of this model i.e. photon flux constant \( I_0 \), electron spectral index \( \delta \) and the transition zone electron stopping energy \( E_\delta \). For these flares, the best fit values of the transition region energy parameter \( E_\delta \) lie in the 20-60 keV range. The stopping energy can be readily converted into a column density of the total material the electron passed through to reach the chromosphere, this fitting procedure allows estimation of the coronal column density, \( N_\delta = (2k_\text{B}M_e)_0^{1/2} \). This corresponds to \( M_e = (1.2-11) \times 10^{20} \text{ cm}^{-2} \) with the equivalent \( N_\delta \) in the range \((0.8-7) \times 10^{20} \text{ cm}^{-2} \). Thus the spectral power of RHESSI enables us to derive the transition region column depth (McArthur, 2000). By extending the work of Aschwanden et al. (2002) using RHESSI spectral images to derive hard X-ray source heights for limb events and combining this with the spectral estimate of \( N_\delta \), it should be possible to determine the geometric height of the transition region inside the thick target flare loop. This is the subject of a future paper where we will also attempt to follow the evaporative evolution of \( N_e \) as the flare progresses, using our spectral fitting technique.

4. DISCUSSION AND CONCLUSIONS

Starting from the basic principles of collisional propagation of a thick target beam in the nonuniformly ionised solar plasma we derived a simple model for its bremsstrahlung spectrum for a general electron injection spectrum \( F(E_0) \). For a pure powerlaw \( F(E_0) \), with two adjustable parameters \( F_1, \delta \) plus the stopping energy \( E_\delta \) we can physically explain the observed deviation from a power law photon spectrum, agreeing much better with RHESSI data than the pure powerlaw obtained in the usual uniformly ionised target analysis. The method is also transparent and easy to use for general \( F(E_0) \). Finally we note that there are known additional effects which should be included in refined spectral modelling which might lead to deviations from a single photon power law. In particular there is the significant but smaller effect of solar HXR albedo (Alexander and Brown 2002). In summary, the results show that

- Detailed RHESSI spectral data show a deviation from a simple power law in the range 20-100 kev often attributed to a feature in \( F_0(E_0) \), i.e. in the acceleration process.
- Inclusion of the effect of target ionisation change across the transition region removes the need for any such feature in \( F_0(E_0) \) which can then be a pure powerlaw.
- Using the technique presented in the paper one can determine the energy \( E_\delta \) corresponding to the transition zone depth and hence the value of that column depth.

ACKNOWLEDGMENTS

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REFERENCES