AXISYMMETRIC MAGNETIZED WINDS AND STELLAR SPIN-DOWN

B. van der Holst\textsuperscript{1}, D. Banerjee\textsuperscript{1}, R.Keppens\textsuperscript{2}, and S. Poedts\textsuperscript{1}

\textsuperscript{1}Centre for Plasma Astrophysics, Katholieke Universiteit Leuven, Celestijnenlaan 200B, B-3001 Leuven, Belgium
\textsuperscript{2}FOM-Institute for Plasma-Physics Rijnhuizen, P.O.Box 1207, 3430 BE Nieuwegein, The Netherlands

ABSTRACT

We present 2.5D stationary solar/stellar wind numerical simulation results obtained within the magnetohydrodynamic (MHD) model. This is an extension of earlier work by Keppens & Goedbloed (1999, 2000), where spherically symmetric, isothermal, unmagnetized, non-rotating Parker winds were generalized to axisymmetric, polytropic, magnetized, rotating models containing both a 'wind' and a 'dead' zone. We study the influence of stellar rotation and coronal magnetic field strength on the wind acceleration. Since dynamos in cool stars are thought to operate more efficiently and to produce a stronger coronal magnetic field with increasing stellar rotation rate, we assume this increase is linear. We quantify the stellar angular momentum loss via the magnetized wind with an equatorial dead zone. The obtained spin-down rates are much smaller than values obtained from Weber–Davis wind estimates. The need to invoke a dynamo with magnetic field saturation to lower the spin-down rates for fast rotators is re-evaluated in view of these results.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Observed line of sight rotational velocity distributions for three clusters of known age, taken from Keppens et al. (1995). Only stars with mass $0.8-1M_\odot$ are shown. The cluster ages are indicated.}
\end{figure}

1. INTRODUCTION

From observations of stars in open clusters of various ages the rotational history of stars can be investigated. In Fig. 1, the observed line of sight rotational velocity distributions of stars of mass $0.8 - 1M_\odot$ are shown for several clusters of known age. From these data one can deduce:

1. Early on the main-sequence there is a broad range of rotation rates, with an excess of slow rotators.

2. Rapid spin-down of the fastest rotators at a timescale of $\sim 20$ Myr.

3. At 600 Myr all stars have low rotational velocities.

The surface rotation rate of a solar-type star changes via different mechanisms, namely: (1) Angular momentum loss from the convection zone via a magnetized stellar wind; (2) Outward angular momentum transport from the radiative core to the convection layer; and (3) Spin-up due to contraction and internal structural changes in the pre-main sequence phase.

In this paper we will focus on the spin-down effect due to a magnetized wind. We assume a phenomenological dynamo that prescribes the base coronal magnetic field linearly with the rotational velocity of the star. Such a dependence is expected when the stellar magnetic field is sustained by the interplay of differential rotation and convection ($\alpha\nu$-dynamo). Instead of using the usual 1D Weber & Davis (1967) wind solution, where a kind of dynamo saturation

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is needed to fit the observations, we exploit a more realistic 2.5D stellar wind to quantify the spin-down rate. We address the question: Do we still need the magnetic field saturation to explain the 20 Myr spin-down timescale? A similar question was posed by Solanki et al. (1997), still using a purely parametric prescription of spin-down by magnetized winds.

2. SOLAR CASE

The gross macroscopic behavior of the solar wind can be qualitatively described by the MHD equations. The stationary wind solution is modelled by choosing at the reference position $r_* = 1.25 R_\odot$; temperature $T_0 \approx 1.5 \times 10^6$ K, number density $N_0 \approx 10^{30}$ cm$^{-3}$, angular velocity $\Omega \approx 3 \times 10^{-6}$ s$^{-1}$, and radial magnetic field $B_r \approx 2 G$. Far away from the sun the plasma outflow is superfast and continuous. Throughout this paper we assume a polytropic state, $p/\rho_0 = \rho/\rho_0$, where $p$ is the pressure, $\rho$ is the density, and $\alpha$ is the polytropic index taken to be 1.13.

To compare the angular momentum loss of the 2D solar wind with that of a 1D case, we first start with the 1D Weber–Davis wind solution. This is a steady-state ($\partial / \partial t = 0$) axisymmetric ($\partial / \partial \phi = 0$) wind for the equatorial plane (where the polar angle $\theta = \pi/2$, further assuming $v_\theta = 0$, and $B_\phi = 0$). This magnetized polytropic solar wind solution, shown in Fig. 2, passes through two critical points of the radial momentum equation, namely the slow $r_s = 9.2 R_\odot$ and fast $r_f = 39.1 R_\odot$ magnetosonic critical point. The Alfvén point where the radial velocity equals the radial Alfvén speed is at $r_A = 36.5 R_\odot$.

\begin{equation}
\frac{dM}{dt} = -\frac{4\pi r^2 \rho v_r}{\rho v_r} dM/dt,
\end{equation}

where $-dM/dt = -4\pi r^2 \rho v_r$ is the total rate of change of mass of the sun. The second equality shows that the rate of angular momentum change $dJ_z/dt$ is directly related to the Alfvén radius. The factor $2/3$ comes from moments of inertia calculation. For the 1D solar solution in Fig. 2 this results in a total rate of change of angular momentum of $dJ_z/dt = -2.39 \times 10^{31}$ (dyne cm) and rate of change of mass of the sun is $dM/dt = -2.94 \times 10^{-14} M_\odot$ yr$^{-1}$.

The 1D Weber–Davis solution is unrealistic in the sense that the actual solar wind has open field lines along the poles and closed field lines about the equator. In the following we will use the more realistic 2.5D axisymmetric polytropic solar wind as described by Keppens & Goedbloed (1999). There they used an initial condition with a wind zone along the polar open field lines (monopolar), where the radial magnetic field strength $B_r$ at the coronal base $r_* = 1.25 R_\odot$ is the same as the Weber–Davis wind, and a dead zone about the equator with a 60° opening angle, dipolar field, and initial zero flow field. The resulting wind pattern is shown in Fig. 3 and has a rate of change of solar mass $dM/dt = -4\pi \int_0^{\pi/2} r^2 \sin \theta \rho v_r \sin \theta d\theta = 1.99 \times 10^{-14} M_\odot$ yr$^{-1}$. As already pointed out by Keppens & Goedbloed (1999), in the polar regions the Alfvén and fast critical curves almost coincide whereas near the equator the Alfvén and slow critical curves coincide. Note also that the Alfvén radius changes with solar rotation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The 1D Weber–Davis solar wind for the equatorial plane. Shown is the radial dependence of the poloidal Alfvén Mach number, $M_A = v_r / v_A$, and the poloidal slow and fast Mach numbers, determining the Alfvén point $r_A$ and the critical slow and fast points.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The 2.5D axisymmetric MHD solar wind, containing a wind and dead zone. The poloidal magnetic field lines (solid lines) and poloidal flow vectors are shown. The grey-scale contours indicate log(density). The slow (dotted lines), Alfvén (solid lines), and fast (dashed lines) critical transitions of the solar wind are also drawn. The length scales are normalized to the base height 1.25$R_\odot$.}
\end{figure}

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Figure 4. The radial variation of the solar angular momentum flux of the 2.5D wind. The advective and magnetic tension part of the flux are also shown. The contribution of the advective part at the coronal base is only 1.4% of the total, so the magnetic tension completely dominates the loss rate there.

The total rate of change of angular momentum of the sun due to the wind is equal to minus the angular momentum flux at any given radius. For a 2.5D axisymmetric wind this can be written as

$$\frac{dL_z}{dt} = -2\pi \int_0^\infty r^3 \sin^2 \theta (\rho v_r v_\phi - B_r B_\phi) d\theta.$$  (2)

The first term between brackets represents the angular momentum flux due to advection, the second term is due to the magnetic tension. Whereas the angular momentum loss of the 1D Weber–Davis wind is directly related to the Alfvén radius, for the 2.5D case this is less obvious. In Fig. 4 the radial variation of the angular momentum flux is shown. The total flux is $1.19 \times 10^{36} \text{dyn cm} \text{s}^{-1}$ and is one order of magnitude smaller than the 1D Weber–Davis solution. This conclusion was also obtained by Priest & Pneuman (1974) and Keppens & Goedbloed (2000). Near the solar surface the flux is almost entirely due to the magnetic tension, so that the angular momentum loss of the sun via the solar wind is basically magnetic.

3. STAR WITH $3\Omega_\odot$ AND $3B_\odot$

From Fig. 1 it was clear that stars rotate faster when they are early on the main-sequence than e.g. at the time of 600 Myr or later. We now investigate a solar type star with mass $1M_\odot$ and radius $1R_\odot$ modelled with at the coronal base $r_* = 1.25R_\odot$: temperature $T_0 \approx 1.5 \times 10^6 K$ and number density $N_0 \approx 10^6 \text{cm}^{-3}$ both assumed to be reasonable estimates for the star. However, we take for the angular velocity three times the solar rotation velocity, i.e. $\Omega = 3\Omega_\odot \approx 9 \times 10^{-6} \text{s}^{-1}$, and we assume a linear dynamo law so that the radial magnetic field is $B_r \approx 6G$.

The polytropic 1D Weber–Davis solution for this star is shown in Fig. 5. The radius of the Alfvén $r_A = 78.25R_\odot$ and fast $r_f = 278.9R_\odot$ point increases drastically at higher rotation rate as compared to the solar case, mainly due to the larger assumed magnetic field strength, and they also become more separated. The slow $r_s = 8.04R_\odot$ point on the other hand has moved inward. The mass loss rate is $3.29 \times 10^{-14}M_\odot \text{yr}^{-1}$ and the rate of change of angular momentum is $dJ_z/dt = -1.47 \times 10^{32} \text{dyne cm}$. This more than 15 times increase with respect to the solar case is not only due to the 3 times higher rotation rate and the only slightly higher mass loss rate, but for a large part also due to the more than 2 times larger Alfvén radius.

Figure 5. As in Fig. 2: The 1D Weber–Davis star wind for the equatorial plane with $\Omega = 3\Omega_\odot$ and $B_r = 3B_\odot$ at $r_* = 1.25R_\odot$.

Figure 6. As in Fig. 3: The 2.5D axisymmetric MHD star wind solution with $\Omega = 3\Omega_\odot$ at $r_* = 1.25R_\odot$ and at the base of the poles $B_r = 3B_\odot$.

The 2.5D axisymmetric polytropic star wind is shown in Fig. 6. The dead zone about the equator has like the solar wind an opening of $60^\circ$. The rate of change of mass of the star is larger than the solar case, namely $dM/dt = -4.13 \times 10^{-14}M_\odot \text{yr}^{-1}$. Whereas the slow critical curve is more flattened in the $Z$-direction as compared to the solar wind, the Alfvén and fast critical curves are more elongated in this direction. The equatorial inward shift of the slow
and Alfvén curves is less pronounced. The fast and Alfvén critical curves are now more detached than was the case with the solar wind, but they connect again at the poles.

![Graph](image1)

**Figure 7.** The radial variation of the angular momentum flux of the star rotating at three times the solar rate. The contribution of the advective part at the solar base is only 1.23% of the total, so the magnetic tension completely dominates the loss rate.

The radial dependence of the angular momentum flux for the 2.5D star wind is shown in Fig. 7. The total rate of change of angular momentum of the star is equal to the outgoing flux, \( \frac{dJ}{dt} = -1.05 \times 10^{31} \text{ (dyn-cm)} \). This is almost a factor 9 larger than the solar case and again almost entirely due to the magnetic tension. Also the angular momentum loss rate is again one order of magnitude smaller than the \( \Omega = 3\Omega_\odot \) Weber–Davis star wind solution.

### 4. SPIN-DOWN

In Keppens et al. (1995) the evolution of the distribution of rotational velocities for solar-type stars was investigated, before and during their main sequence phase. To quantify the angular momentum loss via a stellar wind, they used the 1D Weber–Davis model. One then assumes that the coronal magnetic field strength entering the model scales linearly with the stellar rotation rate. However, this linear dynamo relation was not able to reproduce the observed changes in the rotational velocity distribution. A dynamo saturation was needed to reduce the angular momentum loss via stellar winds for strongly magnetized, fast rotators, and thereby reproduce the large spread at the ages of α Persei and Pleiades. On the other hand, the rotation rate \( \Omega_{\text{sat}} \) above which the magnetic field does not increase anymore with increasing \( \Omega \) must be sufficiently high to explain the low rotation rates at the age of the Hyades. The 1D Weber–Davis angular momentum loss rates, assuming both a linear and a saturated dynamo relation, are shown in Fig. 8 together with the newly created values from actual 2.5D stellar winds containing a dead zone. The spin-down rates for the 2.5D case are an order of magnitude smaller than those from the 1D calculations.

![Graph](image2)

**Figure 8.** The angular momentum loss-rate (in dyne cm) as calculated from a Weber-Davis and a 2.5D MHD stellar wind model, for a 1M_\odot and 1R_\odot star, as a function of the surface rotation \( \Omega \). The curves for Weber-Davis solutions with field strength saturating above \( \Omega = \Omega_{\text{sat}} \) are also shown.

### 5. CONCLUSIONS

We are presently constructing a table of angular momentum loss rates from 2.5D MHD stellar winds with dead zones. The loss rates obtained so far are an order of magnitude lower than those from corresponding 1D Weber-Davis solutions of similar rotation rate and coronal field strengths. Hence, we may not need a field strength saturation to lower the spin-down rates at high \( \Omega \), usually invoked to explain the observed spin-down in rotational velocity distributions.

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### REFERENCES


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