WAVE DAMPING IN HALL PLASMAS. APPLICATION TO THE SOLAR WIND

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ABSTRACT

The present study focuses on the damping of nonlinear magnetoacoustic waves in Hall viscous plasmas. Special attention is paid to solitary waves and their effect in the solar wind. In particular, the plasma acceleration by an obliquely propagating soliton is investigated. The results show that the solitons originating from the nonlinear steepening of slow waves can accelerate the plasma; the fast solitons obtained from fast waves can decelerate the plasma. For certain propagation angles, the solitons evolve into shock waves. The results are consistent with the observed non-thermal Doppler broadening of the ion emission line, observed by SoHO UVCS telescope in the acceleration region of the solar wind.

Key words: solitons, solar wind, wave damping, Hall MHD.

1. INTRODUCTION

The solar wind can be regarded as the extension of the solar corona into interplanetary space. We can distinguish a fast component which originates from coronal holes and is being accelerated and a slow component which emanates from equatorial regions. At large distances from the Sun, the fast solar wind streams, which always have a unique polarity, exhibit relatively steady speeds of 700 to 800 km s⁻¹, best demonstrated by Ulysses observations (see, e.g., Phillips et al. 1995, Woch et al. 1997). The slow component is denser than the fast one and at 1 AU its speed is roughly 400 km s⁻¹. The slow solar wind is much more complicated than the fast solar wind, since its properties vary on all time scales and it has a filamentary structure. Different source locations of the slow solar wind are under discussion: the edges of the polar coronal holes, the edges of streamer structures, or smaller loop structures in the equatorial region of the Sun. All these source locations need magnetic reconnection processes to open up magnetic field lines and release plasma into space.

The theoretical treatment of the solar wind acceleration has been initiated by Parker (1958), based on the concept of an expansion of the hot corona with temperatures of 10⁶ K. Later, it became clear that the heating of the solar corona and the generation of the solar wind are closely related and that a distributed energy source was required in the inner corona to account for the basic features of the solar wind. Despite of many attempts to explain the acceleration of the solar wind, the exact mechanism responsible for this effect remains unknown.

Solitons and solitary waves are known to occur in many laboratory and terrestrial circumstances and there are theoretical reasons to expect their occurrence in space plasmas. Solitons are finite-amplitude waves of permanent form which owe their existence to a balance between nonlinear wave steepening and dispersion. Nonlinearity is often a consequence of the large scales and is very likely to occur in astrophysical plasmas. Dispersion, on the other hand, can arise from an existing wave-guide (plasma structuring) often called geometrical dispersion, or from plasma effects (physical dispersion) introduced by, e.g., the generalized Ohm’s law. In general, these two effects give rise to different dispersive behaviours but they have the same general result: the creation of a length scale in addition to the natural length scale of the waves (their wavelength).

The study of compressional magnetohydrodynamic waves in Hall plasmas is a relatively new subject of the MHD wave theory in laboratory, solar plasmas (see, e.g., Ruderman 1987, Ghanash et al. 1996, Zhelyazkov et al. 1996, Zhelyazkov & Mann 1999). Hall MHD is relevant to plasma dynamics occurring on length scales comparable to the ion inertial length. Using realistic values, we can find that this length is a few cm in the photosphere, a few km in the solar wind, near to the Sun, and a few hundred km at 1 AU.

Observational evidence of compressional waves in the solar wind has been obtained recently. DeForest and Gurman (1998), using high-cadence EIT/SoHO observations, indicated some quasi periodic fluctuations with periods of 10 – 15 min. in solar plumes.
with a filamentary structure within the plume on a spatial scale of 3–5 arcsec. These fluctuations were identified as sound or slow magnetoacoustic waves propagating along the plumes at 75–150 km/s⁻¹. Ofman et al. (2000) and Banerjee et al. (2001) detected quasi periodic variations in the polarization brightness at 1.9 R☉, in both plume and inter-plume regions. Their Fourier power spectrum shows significant peaks around 1.6–2.5 mHz and additional smaller peaks at longer and shorter timescales. Their wavelet analysis of the polarization brightness time series shows that the coherence time of the fluctuations is about 30 min.

The paper is organized as follows: In Section 2 the basic equations are introduced and the main assumptions are discussed. The derivation of the KdV equation and its limiting cases are presented. Section 3 is devoted to a numerical study of the solution of the KdV-Burgers equation and to applying the solution of this equation to explain the observed Doppler-broadening of the ion emission line in term of solitary waves. Finally, in Section 5 we discuss our results.

2. NONLINEAR EVOLUTIONARY EQUATION

We model the solar wind by a viscous plasma penetrated by a uniform magnetic field. The magnetic field is unidirectional and situated in the xz-plane: \( \mathbf{B}_0 = B_0 \sin \alpha \mathbf{x} + B_0 \cos \alpha \mathbf{z} \), where \( B_0 \) is the field strength, \( \alpha \) is the angle between the magnetic field and the z-axis, and \( \mathbf{x}, \mathbf{z} \) are unit vectors.

We use the full nonlinear viscous MHD equations. The Hall term appears in the generalized Ohm’s law as an effect of a generated current (Hall current) perpendicular to the ambient magnetic field. This effect is produced by charged particles drifting across the magnetic field. The presence of this term introduces a new length-scale (the ion inertial length) which renders wave propagation as dispersive. Consequently, the induction equation modified by the presence of the Hall term is

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \frac{mi_e}{\epsilon \rho \mu} (\mathbf{B} \cdot \nabla) \nabla \times \mathbf{B},
\]

where \( \mathbf{v} \) and \( \mathbf{B} \) are the velocity and magnetic induction vectors, \( mi_e \), \( \epsilon \) and \( \rho \) are the ion mass, electron charge and density. The perturbations of the magnetic field and velocity are denoted by \( \mathbf{b} = (b_x, b_y, b_z) \) and \( \mathbf{v} = (v_x, v_y, v_z) \). All the other equations are similar to the usual MHD equations.

To derive the governing equation we use the standard method of multiple scaling. We suppose that the nonlinearity, the dispersive effects caused by the presence of the Hall term and the effect of viscous damping are all weak and are of the same order. We are then able to introduce the stretching variables \( \xi \) and \( \tau \) through

\[
\xi = \epsilon^{1/2}(z - Vt), \quad \tau = \epsilon^{3/2}t,
\]

where \( V \) is the phase velocity of slow or fast waves, \( \epsilon \) is a small parameter measuring the amplitude of the waves and \( \tau \) is the ‘slow’ time describing the wave evolution in a coordinate system moving together with the wave. Perturbations are expanded in series as

\[
f = \epsilon f_1 + \epsilon^2 f_2 + \ldots,
\]

for the sound variables \( \rho, p, v_x, v_z \) and \( b_x \), and

\[
g = \epsilon^{1/2} (g_1 + \epsilon^2 g_2 + \ldots),
\]

for the Alfvénic variables, \( v_y \) and \( b_y \).

Substituting the stretched variables (2) and the perturbation expansions into the system of MHD equations and equating coefficients of powers of \( \epsilon \), we obtain a sequence of equations for the variables \( f_1 \) and \( g_1 \). All variables (at the lowest order, \( O(\epsilon^2) \)) can be expressed in terms of the perturbed density, \( \rho_1 \). Having in mind the expression of the Hall term and its role in the process of wave propagation, we can say that the dispersion of the wave is generated by shear Alfvén waves through the Hall currents.

In the second order approximation we obtain a system of equation for variables with index ‘2’. These quantities are expressed in terms of the variables obtained in the first order of approximation. Eliminating variables, we obtain

\[
\frac{\partial \rho}{\partial \tau} + \alpha_1 \rho \frac{\partial \rho}{\partial \xi} - \alpha_2 \frac{\partial^3 \rho}{\partial \xi^3} - \alpha_3 \frac{\partial^2 \rho}{\partial \xi^2} = 0.
\]

Equation (5) is the Korteweg-deVries-Burgers equation which here describes the evolution of a density disturbance in a viscous plasma with the condition that \( \rho_1 \) vanishes at \( |\xi| \to \infty \). The coefficients \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are given by

\[
\alpha_1 = \frac{V[\gamma^2 - 1](V^2 - v_A^2 \cos^2 \alpha) + 3V^2(V^2 - c_A^2)]}{2\rho_0(2V^2 - v_A^2 \cos^2 \alpha)}
\]

\[
\alpha_2 = \frac{\nu^3 (V^2 - v_A^2 \cos^2 \alpha)(2V^2 - v_A^2 \cos^2 \alpha)}{2v_A^2 \nu^2 (V^2 - v_A^2 \cos^2 \alpha)}
\]

\[
\alpha_3 = \frac{(4V^4 - V^4 v_A^2 \cos^2 \alpha - 3v_A^2 \cos^2 \alpha)}{6(2V^2 - v_A^2 \cos^2 \alpha)}
\]

Equation (5) describes the nonlinear evolution of slow or fast magnetosonic waves in dissipative plasmas, but not their coupling. The coefficients \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) depend on the angle of propagation and the plasma beta, defined as \( \beta = 2c_A^2/(\gamma v_A^2) \) (the ratio of the kinetic and magnetic pressures). In the solar wind, near the Sun, \( \beta \) is small so we confine our attention to this case. The propagation angle varies between 0 (parallel propagation) and \( \pi/2 \) (perpendicular propagation).

In the case of fast magnetosonic waves, the nonlinear parameter \( \alpha_1 \) shows a strong dependence on \( \beta \). For a fixed value of \( \beta \) this parameter is practically independent of the propagation angle. In fact, the parameter shows a slight change with respect to \( \alpha \). The dispersive parameter \( \alpha_2 \) shows a very steep
variation only for small values of the propagation angle, i.e. for angles corresponding to almost parallel propagation. As the angle of propagation increases, the coefficient $\alpha_3$ saturates and tends to zero, i.e. for perpendicular propagation, waves are no longer dispersive. Here the wave evolution is described by the Burgers equation with solution in the form

$$\rho = \frac{\theta}{\alpha_1} \left\{ 1 - \tanh \left[ \frac{\theta}{2\alpha_3} (\xi - \theta \tau) \right] \right\}$$  \hspace{1cm} (7)

The solution given by (7) is a fast shock wave propagating perpendicular to the ambient magnetic field whose amplitude decays due to viscosity and the wavefront steepens due to nonlinearity. For propagation parallel to the field ($\alpha = 0$), the evolutionary equation shows a degeneracy (the dispersive parameter tends to infinity) and the evolution of a nonlinear dispersive fast magnetoacoustic wave is described by the derivative nonlinear Schrödinger-Burgers (DNLS-B) equation (Mjolhus and Hada 1997). The parameter of the dissipative term, $\alpha_3$, increases with propagation angle and is almost independent on the value of plasma-$\beta$. The damping parameter $\alpha_3$ is at a maximum perpendicular to the equilibrium magnetic field.

In the case of nonlinear waves which in the linear limit propagate as slow magnetoacoustic waves, the nonlinearity parameter $\alpha_1$ decreases with $\alpha$ and for large propagation angle the parameter is independent on plasma-$\beta$. Since there is no slow wave propagation across the field, $\alpha_1 = 0$ at $\alpha = \pi/2$. The coefficient of the dispersive term, $\alpha_2$, is negative for $0 < \alpha < \pi/2$ and for all $\beta$. Similar to the fast magnetoacoustic waves, for propagation parallel to the equilibrium magnetic field ($\alpha = 0$) waves are no longer dispersive and their nonlinear evolution is described by the Burgers equation, with a solution similar to the form given by (7) with the coefficients calculated for slow waves. The parameter of the dissipative term shows a slow decreasing tendency with respect to the propagation angle. Its value (similar to the case of fast waves) does not change significantly with respect to $\beta$. The damping parameter is at a maximum parallel to the equilibrium magnetic field.

Eq. (5) has an approximate soliton solution. In the frame of reference moving together with the soliton, this solution is

$$\rho = \tilde{\rho} \text{sech}^2 \left( \frac{\frac{\xi}{L} - \theta \tau}{\sqrt{12\alpha_2}} \right) \left( 1 + \frac{\tau}{t_d} \right)^{-1},$$  \hspace{1cm} (8)

where

$$L = \frac{12\alpha_2}{\tilde{\rho} \alpha_3}, \quad \theta = \frac{\tilde{\rho} \alpha_3}{3}, \quad t_d = \frac{15\alpha_2}{16\tilde{\rho} \alpha_1 \alpha_3},$$  \hspace{1cm} (9)

are the length of the soliton, the speed of the soliton in the moving frame of reference, and the decay time due to viscous dissipation. The constant $\tilde{\rho}$ is the amplitude of the soliton; the width of the soliton decreases with increasing amplitude, following a $\tilde{\rho}^{-1/2}$ law. As we go further from the Sun, the density decreases and so the width of this soliton increases. The effect of the viscosity is to induce a slow decay of the solitons. Comparing the damping times for solitons originating from the nonlinear steepening of fast and slow waves, we can conclude that the slow solitons damp much faster than the fast solitons for small plasma-$\beta$ and for all angles. The critical situations $\alpha = 0$ and $\alpha = \pi/2$ must be excluded since in these cases the solutions (and the damping times) are different.

3. DISCUSSION

Consider first solitary waves that arise from slow magnetoacoustic modes, for which $\alpha_1, \alpha_3 > 0$ and $\alpha_2 < 0$ (positive dispersion). Accordingly, the decaying soliton solution (8) gives a propagation speed (in the original coordinate system) of $V_s = \tilde{\rho} \alpha_3/3$, indicating an increase in speed (above the linear slow wave speed $V_s$) with increasing amplitude $\tilde{\rho}$ ($\tilde{\rho} > 0$). Thus, when the conditions of formation of a solitary wave are satisfied (the steepening due to nonlinearity is balanced by the broadening due to dispersion) a solitary wave appears which moves faster than the wave speed of the corresponding linear wave. If we identify the bulk motion of the plasma with the speed of the solitary wave, then we can conclude that the plasma is accelerated by the solitary wave.

Consider now solitary waves which in the linear limit propagate as fast magnetoacoustic waves, for which all three coefficients are positive. In this case, the propagation speed of the solitary wave (in the original coordinate system) is $V_s = \tilde{\rho} \alpha_3/3$, giving a speed which is smaller than the propagation speed of the corresponding linear fast magnetoacoustic wave and these waves have a negative dispersion.

We compare our theoretical results with the observational results found by the Ultra Violet Coronagraph Spectrometer (UVCS) on board the SoHO satellite. Kohl et al. (1996) and Noci et al. (1996) have reported an ion emission line that is Doppler-shifted by 300 km$^{-1}$ corresponding to unresolved motions at heliocentric distance 1.7 $R_\odot$. UVCS measurements suggest that the observed Doppler broadening is a result of thermal and non-thermal motions that have the same magnitude for H I and O VI ions and is about 300 km$^{-1}$. The fact that the Doppler-broadening is independent of the ion mass suggests the idea that the observed motions are due to the effect of unresolved wave motions integrated over several hours of observation.

As shown by Antonucci (1999), the proton temperature at 1.7 $R_\odot$ is about $2.5 \times 10^6$ K which corresponds to a thermal velocity of 207 km$^{-1}$. The distribution of a line intensity in terms of a velocity due to a thermal motion, $v_t$, is obtained by integrating the thermal emission shifted by the non-thermal wave velocity, $v_w(t)$ in the line of sight (in our case the
Figure 1. The Doppler broadening due to the unresolved wave motion. The intensity fluctuations are due to the velocity in the line of sight measured at the 'observation point' $z = 1.7 \, R_\odot$ and for $\beta = 5 \times 10^{-2}$.

$y$-component of the velocity), and is given by

$$\frac{I(v)}{I_0} = \int_{t_1}^{t_2} \exp \left( \frac{v - v_w(t)}{v_{th}} \right)^2 \rho^2(t) \, dt,$$

where $I_0$ is the maximal intensity and $t_2 - t_1$ is the observing time. The intensity of the line is weighted by $\rho^2$ to model the line emission dependence on the density. The integration is taken over times longer that the period of the waves and it is about 4 hours, which is of the order of the UVCS integration time. Based on the model proposed by Cranmer et al. (1999), we suppose that the plasma flow makes an angle with the vertical direction.

The result of the integration (10) for $\rho(t)$ given by (8) for the 'observation point' $z = 1.7 \, R_\odot$ and with $\alpha = 9^\circ$ is given in Figure 1; the shape of the model profile is very close to a Gaussian one. This model gives a profile width of 293 km s$^{-1}$ which corresponds to a kinetic temperature of $5 \times 10^{5}$ K, in agreement with the observations made by the UVCS spectrometer (Antonucci 1999 and Cranmer et. al. 1999, and references therein). This width decreases with decreasing $\alpha$ and increases very slightly for propagation angles larger than we have chosen.

4. CONCLUSIONS

In the theoretical model presented in this paper we have applied the theory of solitary waves in a viscous Hall plasma to explain the observed Doppler broadening in the emission line at $1.7 \, R_\odot$, measured in the solar wind. The presence of the Hall current perpendicular to the equilibrium magnetic field induces a dispersive effect which balances the nonlinear steepening of the wave amplitude. The nonlinear evolution of waves is described by a Korteweg-deVries-Burger equation and solutions have been found in the form of travelling solitary waves, the speed of which varies linearly with the wave amplitude and wavelengths vary inversely as the square root of wave amplitude. We supposed that the observed non-thermal Doppler broadening of the ion emission line is due to unresolved (linear and nonlinear) slow MHD waves which can accelerate the solar wind. The Doppler broadening of the model emission line, using the solutions in the form of KdV solitons is in a good agreement with the observed amplitude of Doppler-broadening in $H I$ and $O VI$ ions found by UVCS at $1.7 \, R_\odot$.

Using the solving method of the KdV equation we found that solitons which correspond to slow magnetoacoustic waves in the linear limit are able to accelerate the plasma, while those solitons which originate from fast magnetoacoustic waves decelerate the plasma.

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