ON THE PROPERTIES OF SOLAR-LIKE OSCILLATIONS: APPLICATION TO PROCYON

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ABSTRACT

In order to interpret the ground-based observations of solar-like stellar oscillations and prepare the scientific exploitation of future spatial asteroseismic observations, we have studied the theoretical properties of the frequencies of possible models for a given stellar target, taking into account its observational constraints in the Herzprung-Russell diagram. We have computed a set of "calibrated" stellar models satisfying these constraints for a star of solar metallicity. We present some results on the sensitivity of the oscillation frequencies to the description of the convective transport and to physical processes entering in the stellar model, like core overshoot. We compare the seismic properties of our models with the observations of Procyon by Martić et al. (2001).

1. Models

We have computed models and oscillations in range of luminosity and effective temperature in H-R diagram, corresponding to Procyon:

\[ \log L/L_\odot \leq 0.89, \]

\[ T_{\text{eff}} = (6530 \pm 90) \text{K} \quad (3.809 \leq \log T_{\text{eff}} \leq 3.821). \]

This work has been motivated by recent observations of solar-like oscillations in Procyon (Barban et al. 1999, Martić et al. 1999, Martić et al. 2001). Procyon is also one of the solar-like targets for the MONS telescope (MONS 2000).

About 20 models satisfying the above prescriptions have been computed with the CESAM code (Morel 1997), with a given physics: nuclear data from Caughlan & Fowler, EFF equation of state, Eddington law for the atmosphere description. The convection is described either with classical mixing-length theory (the Böhm-Vitense prescription – BV) or with the Canuto-Mazzitelli one (CM), with mixing-length parameter respectively \( \lambda = 1.7 H_p \) (BV) and \( \lambda = 1.0 H_p \) (CM). The microscopic diffusion is neglected. The metallicity is close to the solar one, as it is for Procyon (van’t Veer & Mégevand 1996).

Previous models have been computed by Barban et al. (1999) and Chaboyer et al. (1999). Here, in order to study the effect of possible overshoot of the core into the radiative zone on a distance \( d = \min(\zeta H_p, r_{\text{core}}) \), we have computed models with overshoot parameters \( \zeta \) ranging from 0 to 0.3.

The range of mass \( M \) we obtained is from 1.47 to 1.53 \( M_\odot \), well in agreement with the recent redetermination of the Procyon mass by Girard et al. (2000). The positions of the models in the H-R diagram are plotted in Fig. 1. All our models are in the central hydrogen burning phase.

2. Oscillations

2.1. Frequency separations

Several combinations of frequency of low degree \( p \) modes inform us on different parts of the stellar structure. The large separation \( \Delta \nu_{n,\ell} \) between the frequency of modes of given degree \( \ell \) and consecutive radial orders:

\[ \Delta \nu_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell}, \quad (\ell) \]

depends on the external layers and is mainly related to the stellar radius \( R \) or to the dynamical frequency \( \Omega_p = \sqrt{GM}/R^2 \). It is roughly approximated by its mean value \( \Delta \nu_{n,\ell=0} \) (see below Eq. 5).

The small separations between frequencies of modes with degree \( \ell, \ell + 2 \) or \( \ell, \ell + 1 \), which penetrate differently in the central layers, are mainly sensitive to the stellar core:

\[ \delta \nu_{n,\ell+2} = \nu_{n,\ell+2} - \nu_{n,\ell+1} \quad \text{for } \ell = 0, 1; \]

\[ \delta \nu_{n,\ell+1} = 2\nu_{n,\ell} - (\nu_{n,\ell+1} - \nu_{n-1,\ell+1}) \quad \text{for } \ell = 0. \]

The second order differences:

\[ \delta^2 \nu_{n,\ell} = \nu_{n,\ell} - 2\nu_{n+1,\ell} + \nu_{n+2,\ell} \]

depend on the external layers, specially the He ionization zone and the base of the convection zone (e.g. Gough 1991). All these frequency differences are given as function of the frequency in Fig. 5 for a model with \( M = 1.5 M_\odot \) and \( \zeta = 0.1 \).

In the high frequency range (i.e. low degree modes with \( n \gg 10 \)), the \( p \)-mode spectrum can be characterized by the mean of large and small frequency separations, usually estimated by analytical fits of the frequencies and of the small frequency separations. The numerical

frequencies are fitted by the following polynomial expression (Berthomieu et al. 1993):

$$\nu_{n,t} = \nu_{0,t} + \Delta \nu_{t}(n + \frac{\ell}{2} - n_0) + a_{\ell}(n + \frac{\ell}{2} - n_0)^2,$$

(5)

around a radial order $n_0 = 21$. The quantities $\Delta \nu_{t+2}$ and $\Delta \nu_{t+1}$ vary almost linearly with the frequency or the radial order. Mean small spacings $\bar{\Delta} \nu_{t,i}$ are estimated by using the following linear fit:

$$\bar{\Delta} \nu_{t,i} = \bar{\Delta} \nu_{t,0} + S_{t,i}(n - n_0) \quad (i = 1, 2).$$

(6)

The mean large spacing $\bar{\Delta} \nu_{t,0}$ and the mean small spacings $\bar{\Delta} \nu_{0,1}$, $\bar{\Delta} \nu_{0,2}$ and $\bar{\Delta} \nu_{1,3}$ have been estimated for all the models. Note that the mean large spacing $\Delta \nu_{t}$ does not depend much on the degree.

The results are given in Figs. 2 and 3. As expected, we find that the mean large frequency spacing $\Delta \nu_{t}$ is a linear function of $\Omega_p$. Thus it gives a measure of the mean stellar density. The radius of the convective core $r_{\text{core}}$, including the overshoot region, is a function of the central hydrogen content $X_c$. At the considered evolutionary stage, the radius of the convective core $r_{\text{core}}$ decreases with the age. Fig. 2 shows that these two linear functions do not depend on the amount of overshoot.

Upper line plots in Fig. 3 show that, for a given overshoot parameter $\zeta$, there is a well defined relation between the radius of the convective core $r_{\text{core}}$ and the mean small spacings $\bar{\Delta} \nu_{0,1}$, $\bar{\Delta} \nu_{0,2}$ and $\bar{\Delta} \nu_{1,3}$. The quantities $\bar{\Delta} \nu_{0,2}$ and $\bar{\Delta} \nu_{1,3}$ are increasing functions of $r_{\text{core}}$. In the range of considered stellar parameters these mean small spacings vary respectively within $1 \mu$Hz and $1.5 \mu$Hz. The mean small spacing $\bar{\Delta} \nu_{0,1}$ is a decreasing function of $r_{\text{core}}$. It is much less dependent on the overshoot parameter and it varies within $2 \mu$Hz. Thus it seems to be a better indicator of the size of the convective core. Lower line plots represent some relations between the different mean frequency spacings which are “observable” quantities. The plots of $\bar{\Delta} \nu_{1,3}$ and $\bar{\Delta} \nu_{0,1}$ as a function of $\Omega_p$ show that for a given overshoot parameter the points corresponding to our different models, which have a given chemical composition, are rather well aligned independently on the mass, age and mixing-length parameter.

2.2. Stellar structure rapid variations

Rapid variations in stellar structure located at any radius denoted by $r_{\text{env}}$ induce oscillatory behavior of the frequencies of p modes, with a “period” equal to the inverse of twice the travel time of the sound from the surface to the location of rapid variation (e.g. Gough 1991):

$$r_{\text{env}}^{-1} = 2 \int_{r_{\text{env}}}^{R} \frac{dr}{c}.$$

Such rapid variations can occur at the limit of the convective core, at the base of the convection zone and in the HeII ionization zone. This oscillatory behavior can be seen in some frequency separations. $\Delta \nu_{\text{core}}$ has a sinusoidal behavior with two different “periods” of order $150 \mu$Hz and $360 \mu$Hz, as illustrated in Fig. 5 (right panel). The lower period is due to a discontinuity in the derivative of the sound velocity at the base of the convection zone. The larger period has the largest amplitude and is due to the variation of the adiabatic index $\Gamma_1$ in the HeII ionization zone. This large period is also visible in the variation of $\Delta \nu_{\text{core}}$ with the frequency. The measure of these periods in the observations would provide an estimation of the position of the HeII ionization zone and of the convection zone (Fig. 4). The rapid variation of the sound speed at the frontier of the convective core gives rise to a much longer period (e.g. Audard & Provost 1994), i.e. from 2500 to $3300 \mu$Hz for the set of considered models. Thus it cannot be observed from frequency differences of a given model directly.

3. Preliminary comparisons with observations

Solar-like oscillations have been detected in Procyon by Martić et al. (1999). Using the CLEAN algorithm and the Comb response, they give a most probable spacing between the peaks in the p mode spectrum of order $55 \mu$Hz. Using longer runs, Martić et al 2000 estimated this spacing around $54 \mu$Hz. Here we compare observations and theory, using echelle diagrams. Fig. 6 present an echelle diagram from 1999 observing runs, constructed from the CLEANed spectra by adding the values over the threshold for the frequency modulo $\Delta \nu_0 = 54 \mu$Hz (from Martić et al 2001 in preparation). The frequencies from a standard model (Chaboyer et al 1999) taking into account the microscopic diffusion are indicated (see figure caption). Fig. 7 presents the theoretical echelle diagram for our model with $\bar{M}=1.5\bar{M}_\odot$ and $\zeta=0.1$. Note the different behavior at low frequency of the models with and without diffusion, probably due to different helium content in the HeII ionization zone. In order to better constrain the stellar physics from Procyon’s oscillations, more precise observations and models are needed.

References

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Figure 2. Global characteristics of the model and of the stellar acoustic frequency spectrum, for all our Procyon models. The symbols indicate the values of overshoot parameters ($\zeta = 0.3$ open circle; $\zeta = 0.2$ full triangle; $\zeta = 0.10$ open square; $\zeta = 0.05$ open star; $\zeta = 0$ full circle). Left panel: mean large frequency spacing $\Delta \nu_0$ as a function of $\Omega_0$. Right panel: radius of the convective core $r_{\text{core}}$ as a function of the central hydrogen content $X_c$.

Figure 3. Plots of global quantities characteristics of the model and of the stellar acoustic frequency spectrum, for Procyon models. Same symbols as in Fig. 2 for the values of overshoot parameter $\zeta$. Upper line plots: mean small spacings $\delta \nu_{0.1}$, $\delta \nu_{0.2}$ and $\delta \nu_{1.3}$ as a function of the radius of the convective core $r_{\text{core}}$. Lower line plots: left (respectively right) panels mean small frequency spacing $\delta \nu_{0.1}$ (respectively $\delta \nu_{1.3}$) as a function of $\delta \nu_{0.2}$; middle panel mean small frequency spacing $\delta \nu_{0.2}$ as a function of the mean large frequency spacing $\Delta \nu_0$.

Figure 4. Plots of global characteristics quantities for all the stellar models of Fig. 1. $\nu_{\text{ZC}}$ and $\nu_{\text{HeII}}$ are the frequencies of the oscillatory components of the frequency due to rapid variations of the stellar structure, respectively located at the base of the convection zone $r_{\text{ZC}}$ and at the HeII ionization zone $r_{\text{HeII}}$. © European Space Agency • Provided by the NASA Astrophysics Data System
Figure 5. Frequency separations for a model with $M=1.5 M_\odot$ and $\zeta = 0.1$. Left panel: large separations $\Delta \nu_{\nu, \ell} = \nu_{\nu, \ell} - \nu_{\nu, \ell+1}$ for $\ell = 0$ (full symbol) and 1 (open symbol); middle panel: small separations $\delta \nu_{\nu,3}$ (full circle), $3/\sqrt{5} \delta \nu_{\nu,3}$ (open circle), and $3/4 \delta \nu_{\nu,3}$ (full triangle); right panel: second differences $\delta^2 \nu_{\nu, \ell}$ for $\ell = 0, 1, 2, 3$.

Figure 1. Position of the models in H-R diagram. Evolutionary tracks for stellar models of $M=1.47M_\odot$ (dashed line), $1.50M_\odot$ (full line) and $1.55M_\odot$ (dot-dashed line), for different core overshoot parameters $\zeta$ from 0 to 0.3 ($\zeta = 0.3$ open circle; $\zeta = 0.2$ full triangle; $\zeta = 0.10$ open square; $\zeta = 0.05$ open star; $\zeta = 0$ full circle). The error box is the one of Procyon. The dotted lines represent lines of constant radius. For a star of given mass, they are also lines of constant dynamical frequency $\Omega_\nu$. From one line to the next towards the right, the variation of the radius induces a decrease of $\Omega_\nu$ by about 2%.

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Figure 6. Echelle diagram of Procyon 99 (nights 1–10) showing the frequency modulations $\Delta \nu_f = 54 \mu$Hz (from Martic et al. 2001 in preparation). The frequencies from a standard model taking into account the microscopic diffusion (Chaboyer et al. 1999) are indicated by asterisk, plus sign, square and triangle respectively for $l = 0, 1, 2, 3$. Dashed lines are at $l = 0, 1 \pm 1, 0, 1 \pm 1, 3$ corresponding to the day aliases.

Figure 7. Echelle diagram for the model presented in Fig. 5, with same symbols for the degrees as in Fig. 6.

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