THEORETICAL HYDROGEN POPULATION RELATIONS FOR HORIZONTAL CLOUD-LIKE SOLAR STRUCTURES

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ABSTRACT

A large set of parameterized NLTE models has been computed for a 1D horizontal cloud-like structure standing above the solar surface. The used input model parameters are temperature, velocity, microturbulence, electronic density and geometrical thickness of the structure. From the output of our computations which is the hydrogen level populations as a function of the line center optical depth, we calculate several relations between the hydrogen population levels and the considered electronic density. These relations are important for the study of prominence-like structures from observations.

The result of these calculations is the level populations as a function of the line center optical depth $\tau_c$. For further details of the calculations we refer the reader to Molowny-Horas et al. [5] and Tziotziou et al. [7].

We construct a grid of models that depends on several parameters which are presented in Table 1. We have fixed our microturbulent velocity to a value of 5 km/s which is extensively used in literature for such structures. We also consider only static structures ($V = 0$ km/s). For all figures presented in this paper we use the following symbols for temperature: 6000 K (plus signs), 7000 K (asterisks), 8000 K (diamonds), 9000 K (triangles), 10000 K (squares) and 11000 K (circles).

Table 1: Parameters used for the calculation of the grid. $N_e$ is the electronic density, T the temperature, Z the geometrical thickness of the structure, $\zeta_m$ the microturbulent velocity and V the bulk velocity. The total number of computed MALI models is $9 \times 11 \times 5 = 495$. h is the height of the bottom of the 1D slab above the solar surface.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Range</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_e$</td>
<td>cm$^{-3}$</td>
<td>2 x 10$^{10}$ - 10$^{11}$</td>
<td>10$^{10}$</td>
</tr>
<tr>
<td>T</td>
<td>K</td>
<td>6000-11000</td>
<td>500</td>
</tr>
<tr>
<td>Z</td>
<td>km</td>
<td>1000-5000</td>
<td>1000</td>
</tr>
<tr>
<td>V</td>
<td>km/sec</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$\zeta_m$</td>
<td>km/sec</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>h</td>
<td>km</td>
<td>20000</td>
<td>-</td>
</tr>
</tbody>
</table>

3. RESULTS

We introduce the average level populations over all depths $<N_i>$, $<N_{II}>$ and $<N_{III}>$ as well as the average total hydrogen population $<N_{HI}>$ given by

$$<N_i>=\frac{1}{Z} \int_0^Z N_i \, dz \quad i=1,2,3,4$$

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Table 2: Coefficient D and exponent $\alpha$ of Eq. 4, coefficient E and exponent $\beta$ of Eq. 7, coefficient F and exponent $\gamma$ of Eq. 10, and coefficient G and exponent $\delta$ of Eq. 11 as a function of temperature.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>D/10^5</th>
<th>$\alpha$</th>
<th>E/10^7</th>
<th>$\beta$</th>
<th>F</th>
<th>$\gamma$</th>
<th>G</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>1.542</td>
<td>0.521</td>
<td>4.086</td>
<td>0.404</td>
<td>2.17 $10^{16}$</td>
<td>2.644</td>
<td>5.38 $10^{16}$</td>
<td>2.608</td>
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<tr>
<td>6500</td>
<td>1.371</td>
<td>0.522</td>
<td>4.170</td>
<td>0.404</td>
<td>1.16 $10^{18}$</td>
<td>1.876</td>
<td>3.16 $10^{18}$</td>
<td>1.837</td>
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<tr>
<td>7000</td>
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<td>0.522</td>
<td>4.247</td>
<td>0.404</td>
<td>0.002</td>
<td>1.344</td>
<td>0.005</td>
<td>1.315</td>
</tr>
<tr>
<td>7500</td>
<td>1.617</td>
<td>0.523</td>
<td>4.317</td>
<td>0.403</td>
<td>1.9</td>
<td>1.048</td>
<td>2.56</td>
<td>1.042</td>
</tr>
<tr>
<td>8000</td>
<td>1.636</td>
<td>0.524</td>
<td>4.381</td>
<td>0.403</td>
<td>50.98</td>
<td>0.892</td>
<td>33.67</td>
<td>0.919</td>
</tr>
<tr>
<td>8500</td>
<td>1.648</td>
<td>0.525</td>
<td>4.438</td>
<td>0.403</td>
<td>269.06</td>
<td>0.804</td>
<td>77.04</td>
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</tr>
<tr>
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<td>4.487</td>
<td>0.403</td>
<td>633.04</td>
<td>0.751</td>
<td>74.57</td>
<td>0.860</td>
</tr>
<tr>
<td>9500</td>
<td>1.664</td>
<td>0.528</td>
<td>4.527</td>
<td>0.403</td>
<td>959.37</td>
<td>0.717</td>
<td>47.96</td>
<td>0.869</td>
</tr>
<tr>
<td>10000</td>
<td>1.663</td>
<td>0.529</td>
<td>4.556</td>
<td>0.404</td>
<td>1122.63</td>
<td>0.695</td>
<td>26.50</td>
<td>0.887</td>
</tr>
<tr>
<td>10500</td>
<td>1.665</td>
<td>0.531</td>
<td>4.573</td>
<td>0.405</td>
<td>1130.79</td>
<td>0.680</td>
<td>14.57</td>
<td>0.906</td>
</tr>
<tr>
<td>11000</td>
<td>1.637</td>
<td>0.533</td>
<td>4.575</td>
<td>0.406</td>
<td>1058.61</td>
<td>0.669</td>
<td>8.59</td>
<td>0.924</td>
</tr>
</tbody>
</table>

As known, the line center optical thickness for Hα is given by the relationship

$$d\tau(H\alpha) = \alpha_0(H\alpha) N_2 dz$$ (2)

where $\alpha_0(H\alpha)$ is the absorption cross section at Hα line center, $N_2$ the second level population and $Z$ the geometrical thickness of the structure. However, our computations indicate that the major contribution for $\tau(H\alpha)$ comes from depths which almost have a constant $N_2$ and hence Eq. 2 can be rewritten as

$$<N_i> = \sum_{i=1,2,3,H} \int_0^{\tau_0} N_i d\tau$$ (3)

where $\tau_0$ is the maximum line center optical depth.

3.1 $N_e$ vs. $<N_2>$

Assuming population and depopulation of the second hydrogen level through the Balmer continuum (by ionization and radiative recombination), the relationship between $<N_2>$ and $N_e$ is

$$N_e = D(T) <N_2>^\alpha \quad \alpha \approx 0.5$$ (4)

where $D(T)$ is a coefficient which depends on temperature, both kinetic temperature $T$ and radiation temperature $T_e$ of the Balmer continuum. In Fig. 1 we show a comparison between $N_e$ and $<N_2>$ for all our calculated models.

By taking into account all models for all temperatures we find that

$$N_e = 1.78 \times 10^8 <N_2>^{0.52}$$ (5)

In Table 2 we show the calculated values of coefficient $D(T)$ and exponent $\alpha$ for different temperatures. Using a polynomial fit we find an approximate expression for $D$ as a function of temperature – for temperatures between 6000 and 11000 K - given by

$$D(T) \approx 7.95 \times 10^7 + 18056 \ T - 0.94 \ T^2$$ (6)

![Fig. 1: $N_e$ vs. $<N_2>$ for all MALI models computed with the parameters presented in Table 1. The solid line corresponds to the relation given by Eq. 5.](image)

3.2 $N_e$ vs. $<N_i>$

In Fig. 2 we show the relationship between $<N_i>$ and $N_e$. Assuming a linear relationship

$$N_e = E(T) <N_i>^\beta$$ (7)
we find for all calculated MALI models that

\[ N_e = 4.39 \times 10^9 \langle N_i \rangle^{0.4} \]  

(8)

In Table 2 we present the calculated values of \( E(T) \) and exponent \( \beta \) for different temperatures. Using again a polynomial fit we find the approximate expression for \( E \) as a function of temperature

\[ E(T) \approx 2.34 \times 10^9 + 392932 \ T - 17.21 \ T^2 \]  

(9)

![Graph](image1.png)

Fig. 2: \( N_e \) vs. \( \langle N_i \rangle \) for all MALI models computed with the parameters presented in Table 1. The solid line corresponds to the relation given by Eq. 8.

### 3.3 \( N_e \) vs. \( \langle N_i \rangle \)

The relationship between \( N_e \) and \( \langle N_i \rangle \) which corresponds to the number density of neutral hydrogen is shown in Fig. 3. If we assume a linear relationship of

\[ N_e = F(T) \langle N_i \rangle^\gamma \]  

(10)

we find for coefficient \( F \) and the exponent \( \gamma \) the values presented in Table 2. We should point out that the spread of points for a certain temperature is due to different values of the geometrical thickness of the structure.

![Graph](image2.png)

Fig. 3: \( N_e \) vs. \( \langle N_i \rangle \) for all MALI models computed for the parameters presented in Table 1.

### 3.4 \( N_e \) vs. \( \langle N_{hi} \rangle \)

In Fig. 4 we present the relationship between \( N_e \) and \( \langle N_{hi} \rangle \) which represents the number density of total hydrogen. \( \langle N_{hi} \rangle \) is practically equal to \( \langle N_i \rangle + \langle N_e \rangle \). If we assume a relationship of

\[ N_e = G(T) \langle N_{hi} \rangle^\delta \]  

(11)

we find for coefficient \( G \) and the exponent \( \delta \) the values shown in Table 2.

![Graph](image3.png)

Fig. 4: \( N_e \) vs. \( \langle N_{hi} \rangle \) for all MALI models computed for the parameters presented in Table 1.

### 4. DISCUSSION

The average value over all temperatures for coefficient \( D \) of \( 1.78 \times 10^8 \) is almost half that of the value of \( 3.2 \times 10^8 \) found by Giovanelli [1], Yakovkin & Zel'dina [8] and Poland et al. [6] for vertical structures. However it is almost the same as the value found by Heinzel et al. [3]
for vertical structures studied with a 20-level atom. Our calculations indicate that it is not the considered geometry (horizontal or vertical) which is more important but the radiation temperature $T_r$ of the Balmer continuum. The value of coefficient $D$ reported in our work is computed for a radiation temperature of 5480 K. If we consider a radiation temperature of 5940 K, which was instead used in the aforementioned early vertical structure calculations, the value of $D$ becomes 2.6 $10^6$.

The reason why we didn't recompute the whole grid for an atom model with more levels is that transfer effects may play a role and this should be better investigated in detail in a future work.

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REFERENCES


