Stellar Dynamos: Scaling Laws and Coronal Connections

S. Saar

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

Abstract. I review recent empirical relationships between stellar magnetic dynamo characteristics, stellar coronae, rotation and other stellar properties, focusing in particular on relations between cycle frequency and rotational frequency in the context of dynamo theory.

1. Introduction

The ubiquitous presence of coronae among cool stars is one of the strongest pieces of evidence that they harbor magnetic fields, since coronae of any significance are difficult to sustain without fields to confine and heat them. Direct measurements of magnetic fields for a few stars seem to confirm this idea (e.g., Saar 1996, 2001; Johns-Krull & Valenti 2000). To create and sustain the fields, some sort of magnetic dynamo is thought to be necessary. But only one magnetic dynamo can be considered well studied – the Sun’s – and even it is still not fully understood. Study of magnetic cycles in stars may thus allow us to more fully investigate what stellar properties govern the dynamo process, and help weed out the best candidate models from the zoo of current possibilities.

With the advent of robotic telescopes and dedicated, longterm surveys, there has been a significant increase in the quality and quality of the cycle data available (e.g., Baliunas et al. 1995). Spurred by Hall (1991) and others, investigators have also looked into less “traditional” sources of cycle information, such as modulation of orbital periods in close binaries (Applegate 1992), or intervals between outbursts in CV systems. The welcome deluge of data from these efforts has led several groups (e.g., Baliunas et al. 1996; Ossendrijver 1998; Tobias 1998; Brandenburg et al. 1998; Saar & Brandenburg 1999 [=SB]; Lanza & Rodonò 1999) to revisit the connections between stellar magnetic cycles and other stellar properties. Expanding on initial work by Brandenburg et al. (1998), SB studied relationships between three non-dimensional quantities: the cycle-to-rotational frequency ratio \( \omega_{\text{cyc}}/\Omega \) (where \( \omega_{\text{cyc}} = 2\pi/P_{\text{cyc}} \) and \( \Omega = 2\pi/P_{\text{rot}} \), the normalized Ca II HK emission flux \( R'_{\text{HK}} = F'_{\text{HK}}/(\sigma T_{\text{eff}}^4) \), and the inverse Rossby number \( R_0^{-1} = 2\tau_c\Omega \) (\( \tau_c \) is the convective turnover time). They found that stars tended to cluster on three “branches” described by power laws between the non-dimensional parameters.

Here I expand on the analysis of SB. Ca II HK emission ceases to be an effective proxy for magnetic flux due to saturation at e.g., \( P_{\text{rot}} \sim 5 \) days in G dwarfs. It is thus not an effective diagnostic of dynamos for the many active dwarfs, RS CVns, Algols, and CV secondaries in the augmented SB sample.
I therefore concentrate on relationships between $\omega_{cyc}$ and rotation. I also include more cycles (collected in Saar & Brandenburg 2001), and explore how the expanded data set affects various parameterizations, both dimensional and non-dimensional, of the $\omega_{cyc}$ rotation relation.

2. How Common are Cycles?

Before launching into an analysis of cycle periods however, a reasonable first question is: how common are magnetic cycles? Answering this is a bit difficult, since none of the samples of cool stars studied for evidence of cycles are unbiased. There are three main ways that stellar cycles have been discovered: broadband photometry (e.g., Henry et al. 1995), narrowband photometry of Ca II HK emission (e.g., Baliunas et al. 1995), and cyclic modulation of the rotational period in close binaries (e.g., Lanza & Rodonó 1999), presumably through the Applegate mechanism (cycle-driven modulation of the stellar quadrupole moment via changes in mean magnetic pressure; see Lanza et al. 1998).

The Baliunas et al. (1985) sample is perhaps the best known cycle survey. Based largely on the original Wilson (1978) program stars, it is focused on optically bright dwarfs (and a few subgiants), and has $\sim 25$ years of data for most of its targets. Baliunas et al. (1985) note that 52 of their 112 stars (46%) show at least weak evidence for cycles. This figure, however, contains some stars with rather poorly defined cycles and 20 stars with shorter timeseries ($\leq 10$ years).

If we restrict ourselves only to stars with cycles ranked "fair" or better by the strength of their periodogram peaks and drop most of the short timeseries stars (with a few further additions and subtractions, e.g., HD 136202, from Donahue 1996), then 36 of 92 stars (39%) show relatively clear cycles. Most of these cycling stars (69%) are of low to moderate activity ($\log R'_{HK} < -4.6$). More active stars typically show aperiodic variability – at least in Ca II HK (but see below!). The fraction of stars with clear cycles rises dramatically with decreasing $T_{\text{eff}}$, from 19% in F stars, to 32% in G stars, to 79% in K stars.

Photometry is another primary tool for stellar cycle study, and is the "tool of choice" for uncovering cycles in more rapid rotators. Emission from plage (as seen in Ca II HK) saturates (by either reaching a maximum emission efficiency or maximum plage filling factor) at a much lower $\Omega$ than where the spot filling factor reaches a maximum (e.g., Messina et al. 2001). Thus, on rapid rotators, spots can show cycles at $\Omega$ well above the point where Ca II HK seems to merely record the flares and growth/decay of active regions on a star already nearly filled with plage. A good example is the very active near-pole-on BY Dra system V833 Tau, which shows a spot cycle (Hartmann et al. 1981), but no significant longterm activity modulation (Saar et al. 1990). Since spot coverage is tiny on the Sun and other older stars ($\sim 0.1\%$), photometrically determined cycles are biased towards the most active stars. To get a very rough idea of the frequency of cycles in active stars, I compare the number of spot cycles in BY Dra and RS CVn stars compiled in SB plus Olah et al. (2000) with the number of these stars catalogued by Strassmeier et al. (1993). If only half of the Strassmeier et al. stars have been studied thoroughly enough to reveal cycles, this yields a cycling fraction of 29% (30 out of 103), not too different from the less active stars with
clear plage (Ca II HK) cycles. If $P_{\text{orb}}$ modulation cycles are included (Lanza & Rodonó 1999), the fraction rises to 33%.

Spot cycles appear to be present in some ultra-fast rotating CV secondaries as well (Bianchini 1990). Most of these stars are late M dwarfs, and thus fully convective (or close to it) which immediately raises the question of how they can maintain a cycling dynamo (Küker & Rüdiger 1999). But the idea of cyclic dynamos in late M dwarfs cannot be summarily rejected; recent models of magnetic field transport by Dorch & Nordlund (2001) suggest an undershoot layer beneath the convection zone may not be needed for effective magnetic field storage and amplification. Turning to these CV secondary cycles, a comparison of number of photometric cycles compiled by SB (4) plus those from Ak et al. (2001; 21 additional) with the approximate number of non-nova CV systems with measured $\Omega$ in the Downes et al. (2001) CV catalog ($\approx 215$) yields a cycling fraction estimate of $\approx 12\%$. Thus, even in these unusual, fast rotating, largely fully convective (!) systems, there is a non-negligible fraction of stars with cycles.

An important caveat on all this is that it is not yet entirely clear that stellar cycles are truly analogous to the solar cycle in all respects. There is some evidence for migration of flux emergence latitudes over the course of the cycle, leading to something somewhat akin to the “butterfly” diagram (Donahue 1993, 1996; Donahue & Baliunas 1992). Noticeably lacking, though, is evidence for polarity reversals. Indeed, radio data for active stars shows no clear reversals (White 1996), even in some stars with photometric (spot) “cycles” (the strange case of UX Ari excepted; Massi et al. 1998).

This apparent contradiction is puzzling. Could the photometric variations not be due to spots after all? This seems unlikely, since a variety of investigations have shown that starspots are indeed cool, localized features akin to sunspots (or groups of them). Does this mean instead that stellar dynamos in active stars don’t reverse polarity? Do they only cycle in amplitude, suggesting something more akin to a $\alpha^2$-type dynamo, rather than the more expected $\alpha \Omega$ type? Not necessarily. Recent models by Schrijver & Title (2001) find that considerable flux from previous cycles can migrate towards the rotational poles in ring-like patterns, and remain near the poles for years while they slowly decay. These alternating polarity “rings”, the remnants of preceding cycles, could well mask any clear cycle-to-cycle reversal. Some results from Zeeman Doppler imaging (ZDI) may partly support this idea (though ZDI detects toroidal rather than radial flux “rings”; e.g., Donati et al. 1999). Further modeling and more precise ZDI may help resolve this interesting puzzle.

3. Analysis of Trends in Cycle Periods

To further explore cycle periods, I combine cycle and stellar data gathered by SB with more recent measurements collected in Saar & Brandenburg (2001), which includes new plage (e.g., Hatzes et al. 2000) and spot cycles (e.g., Oláh et al. 2000), plus a compilation of cyclic $P_{\text{orb}}$ changes in close binaries (Lanza & Rodonó 1999). Following SB, I use theoretical $\tau_c$ values (Gunn et al. 1998), and weight the $P_{\text{cyc}}$ by a “quality factor” $w$ ($0.5 \leq w \leq 4$) dependent on the strength of the periodogram signal or (more subjectively) the clarity of the cyclic variation. We set $w = 1$ for the $P_{\text{orb}}$-change cycles. Stars are assigned to branches.
(where appropriate) by eye to minimize the rms scatter of the fit. Evolved stars were not included in the fits due to less well determined $\tau_c$. Results for different classes of stars are shown in Figs. 1, 2, and 3, on the left using dimensionless $\omega_{\text{cyc}}/\Omega$ vs. $R_0^{-1}$ relations, and on the right using a dimensional formulation $\omega_{\text{cyc}}$ vs. $\Omega$.

**Figure 1.** Left panel: $\omega_{\text{cyc}}/\Omega$ vs. $R_0^{-1}$ for single dwarfs; symbols indicate the sun (○), F (△), G (○), and K (□) stars (filled if log $R'_{\text{HK}} \geq -4.75$; size $\propto \sqrt{w}$, the $P_{\text{cyc}}$ “reliability”). Dotted vertical lines connect two $P_{\text{cyc}}$ (a × marks $P_{\text{rot}}^{(2)}$), or $P_{\text{cyc}}$ with a long-term trend (i.e., a possible $P_{\text{rot}}^{(2)} > 25$ yr; arrow symbol). Weighted least square fits ($\omega_{\text{cyc}}/\Omega \propto R_0^\delta$) for the active (A) and inactive (I) branches are shown (solid); $\delta_I = -0.32$ and $\delta_A = -0.16$. Right panel: $\omega_{\text{cyc}}$ vs. $\Omega$ for single dwarfs. Fits ($\omega_{\text{cyc}} \propto \Omega^\delta$) for the A and I branches (solid) yield $\delta_I = 1.15$ and $\delta_A = 0.80$. Grey symbols not included in the fits.

Several main results arise from the various analyses of cycle/rotation relations:

1. Three branches – denoted I (inactive), A (active), and S (super-active) – appear in both the $R_0^{-1}$ and $\Omega$ parameterizations. For the $R_0^{-1}$ fit, the power law exponents are $\delta_I \approx -0.3$ (with a fit dispersion $\sigma_{\text{fit}} = 0.095$ dex), $\delta_A \approx -0.15$ ($\sigma_{\text{fit}} = 0.18$ dex), and $\delta_S \approx 0.4$ ($\sigma_{\text{fit}} = 0.26$ dex); for the $\Omega$ fit, $\delta_I \approx 1.15$ ($\sigma_{\text{fit}} = 0.093$ dex), $\delta_A \approx 0.8$ ($\sigma_{\text{fit}} = 0.17$ dex), and $\delta_S \approx 0.4$ ($\sigma_{\text{fit}} = 0.24$ dex). Thus the rms scatter is similar for the two parameterizations.

2. Secondary cycle periods ($P_{\text{rot}}^{(2)}$) seen in some stars often lie on one of the branches (though this is more rare in S branch stars). The solar Gleissberg “cycle” (∼ 100 years) appears to lie on the S branch. The preferred branch of the primary $P_{\text{cyc}}$ (with the strongest periodogram signal) may be mass and $\Omega$ dependent. Multiple $P_{\text{cyc}}$ may be due to multiple dynamo modes in an $a \Omega$ framework (Knobloch, Rosner & Weiss 1981), or may reflect different dynamos existing in separate latitude zones (note the two, separately evolving activity patterns in the double $P_{\text{cyc}}$ star β Comae; Donahue & Baliunas 1992). In Babcock-Leighton models, $P_{\text{rot}}^{(2)}$ may be excited by stochastic variations in the poloidal source term (Charbonneau & Dikpati 2000).
Figure 2. Left panel: same as Fig. 1 left, but now including binaries (BY Dra, CV secondaries; M stars = ⊙) and RS CVNs (+; not included in the fits). A "superactive" (S) branch appears, with $\delta_S = +0.43$ ($P_{\text{rot}} \leftrightarrow P_{\text{cyc}}$ lines shown only for new stars). A transitional regime between the A and S branches is indicated (dash-dot). Right panel: same as Fig. 1 right, but including binaries (like the left panel). The new S branch shows a power-law slope $\delta_S = 0.38$.

Figure 3. Left panel: same as Fig. 2 left, but including cycles based on $P_{\text{rot}}$ variation in RS CVNs (new +), CV secondaries (open ⊙), Algols (*), and contact binaries (gray ○). Right panel: same as Fig. 2 right, but including the additional $P_{\text{cyc}}$ in the left panel.

(3) A single power law can be fit to the data (e.g., $\omega_{\text{cyc}} \propto \Omega^{-0.09}$, SB; see also Baliunas et al. 1996) but only at the expense of a considerably higher dispersion about the fit ($\sigma_{\text{fit}} = 0.33$ dex), and lack of an explanation for the secondary cycle periods (since they no longer reside on another dynamo "branch").

(4) Evolved stars typically lie near branches, though show more scatter than the dwarfs. Since the increased scatter is seen in both parameterizations, it is unlikely to be due to less precise $\tau_c$ in evolved stars (indeed, arguably the scatter in evolved stars is reduced using $R_{\text{ot}}^{-1}$). The $P_{\text{cyc}}$ based on $P_{\text{rot}}$ variation (Lanza & Rodonò 1999) also follow the general trends. The branches are better separated using $R_{\text{ot}}^{-1}$. On the other hand, the $\Omega$ plot is simpler, lacking the "transitional" regime between the A and S branches seen in the $R_{\text{ot}}^{-1}$ diagrams. Contact binaries (gray ○; Fig. 3) are poorly fit in both schemes (worse if $R_{\text{ot}}^{-1}$ is used); their dynamos may be altered by turbulent energy transfer toward the
secondary (Hazlehurst 1985) which is independent of rotation. We will study evolved stars more thoroughly in a forthcoming paper.

(5) The branches may merge for small $R_\text{o}^{-1}$ or $\Omega$ (though at values which might not be achieved in typical stars). The ratio of the power law exponents for the $\Omega$ fits are $\delta_I : \delta_A : \delta_S \approx 3 : 2 : 1$; the significance of this (if any) is unclear.

(6) Since $\Omega$ and $R_\text{o}^{-1}$ decrease in time on the main-sequence, the relations between $\omega_{\text{cyc}}$ and $R_\text{o}^{-1}$ decrease in time on the main-sequence, the relations between $\omega_{\text{cyc}}$ and rotation trace evolution of dynamo properties with time. To graphically portray this, the left panels of the figures show an approximate age calibration along the top axes based on Donahue (1993). The overlapping branches and $P_{\text{rot}}^{(2)}$ suggest that $\omega_{\text{cyc}}$ evolves in time in a complex, sometimes multi-valued fashion.

4. Dynamo Models

There appears to be two main avenues for the interpretation of these results. First, one can opt for simplicity and discard the idea of branches being physically significant. Then, the single power law relation ((3) above) would apply. Standard mean-field $\alpha\Omega$ dynamo models where differential rotation (DR) and the $\alpha$ effect have only a weak dependence on $\Omega$ can be consistent with this result (Charbonneau & Saar 2001). The idea of a weak rotational dependence for DR (Kitchatinov & Rüdiger 1999) and $\alpha$ (Cattaneo 1999) do have some theoretical support. Fast rotators also seem to indicate a weakened DR dependence on $\Omega$ (e.g., Donati & Collier-Cameron 1997). Alternatively, a flux-transport dynamo model (i.e., one driven by meridional flow like the Babcock-Leighton type) predicts $\omega_{\text{cyc}} \propto u_\text{m}^{0.9}$ for solar-like dwarfs (where $u_\text{m}$ is the meridional flow velocity; Dikpati & Charbonneau 1999). If $u_\text{m}$ increases weakly with $\Omega$ (Kitchatinov & Rüdiger 1999), the weak $\omega_{\text{cyc}}$ dependence on $\Omega$ can be roughly matched. However, as noted above, this scenario yields a considerably larger scatter, has no explanation for the unusual "bunching" (branching?) of the data in the $\omega_{\text{cyc}}-\Omega$ plane, or for secondary cycle periods.

The other interpretation route accepts the "branches" as physically significant (SB). In this case, mean-field models with sufficiently strong $\Omega$ dependence for DR and the $\alpha$ effect can match the observed branches (SB; Charbonneau & Saar 2001). Observations of slow-to-moderate rotators support the idea of a significant dependence of DR on $\Omega$ (e.g., Donahue et al. 1996), and there are some ideas afloat that the $\alpha$ effect can be enhanced (rather than suppressed) by rotation in some regimes (e.g., Brandenburg & Schmitt 1998). Alternatively, a flux-transport dynamo model matches the I and A branches reasonably well if $u_\text{m}$ increases roughly linearly with $\Omega$ in slower rotators (e.g., Brummell et al. 1998). Changes in DR($\Omega$), $\alpha(\Omega)$, and $u_\text{m}(\Omega)$ between slow and fast rotators may be part of the reason for differing power-law slopes on the various branches.

Thus, it is not yet entirely clear which set of models best fit the data, or even which scheme of fitting the data to take! More and better cycle determinations, plus a better understanding (theoretical and observational) of the the dependence of DR, $u_\text{m}$, and the $\alpha$ effect on rotation would definitely be helpful. In the case of DR, for example, there is some disagreement and confusion between observations and theory. The argument (Kitchatinov & Rüdiger 1999; Collier-Cameron et al. 2001) that the strong observational DR($\Omega$) found by Don-
ahue et al. (1996), $\Delta \Omega \propto \Omega^{0.7}$, is due to a mix of spectral types in their analysis seems incorrect. Separate analysis of G and K stars from Donahue et al. (1996) yields $\Delta \Omega(G) \propto \Omega^{0.8}$ and $\Delta \Omega(K) \propto \Omega^{0.6}$, similar to each other, and quite unlike the $\Delta \Omega(G) \propto \Omega^{-0.56}$ and $\Delta \Omega(K) \propto \Omega^{-0.15}$ predicted for stars with $\Omega(\odot)$, or the constant $\Delta \Omega$ predicted for fast rotators (Kitchatinov & Rüdiger 1999). Indeed, a combined fit of the Donahue et al. (1996) results (ignoring close binaries and one outlier) with the Doppler imaging-derived DR from Collier-Cameron et al. (2001), yields $\Delta \Omega \propto \Omega^{0.28}$, similar to the value derived from spots ($\Delta \Omega \propto \Omega^{0.15}$, Hall 1991). The best observational fit is perhaps $\Delta \Omega \propto \Omega^{0.6}$ for $\Omega < 1$ rad day$^{-1}$, and $\Delta \Omega \approx$ constant for faster rotators, but clearly more work is needed!

We continue to study a variety of dynamo models to better understand the implications of the cycle-rotation relations seen here.

5. Coronal Cycles

To return more directly to the subject of the conference, is any evidence for stellar cycles in X-ray data? Until recently, the answer would have been “very little” (see Hempelmann et al. 1996). Coronal variability has clearly been clearly seen, but there are rarely enough observations of the less active stars where (by analogy with Ca II HK) cyclic variation should be apparent. Instead, X-ray telescopes concentrate on active stars, where the cycles are likely suppressed by activity “saturation” and/or masked by flares!

I note happily, though, that at this meeting there are some posters which begin to show new evidence for cycles in X-ray emission (Marino et al., Drake & Kashyap). And the future looks promising. With the advent of high sensitivity, high resolution X-ray spectrographs on Chandra and XMM-Newton, we can finally begin to explore coronal cycles in detail (e.g., how emission measure structure and flare rates vary with cycle phase).

Acknowledgments. This work was supported by NSF grant AST-9731652, HST grant GO-8143, and NASA Origins grant NAG5-10630. I am deeply indebted to A. Brandenburg, M. Dikpati and P. Charbonneau for enlightening discussions and stimulating collaborations, and to S. Baliunas and R. Donahue for their original advice and encouragement.

References

Ak, T., Ozkan, M.T., & Mattei, J.A. 2001, A&A 369, 882
Bianchini, A. 1990, AJ 99, 1941