Constraints derived from binary systems calibration: some examples

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Abstract. Based on the reasonable assumption of a common origin for both components, i.e. same initial composition and age, the calibration of a binary system consists in determining a consistent evolutionary history for a double star, given the positions of the two components in a HR diagram, the stellar masses and the present day surface chemical abundances. Some examples, α Centauri and ζ Herculis, are considered to see how deeply these calibrations may test stellar physics. A seismological analysis of the calibrated star is considered to estimate the contribution of asteroseismology to this test.

1. Introduction

Along the last decades, the constraints on stellar modeling became more stringent due to significant improvements on our knowledge of fundamental stellar data. However, with exception of the Sun, modeling of a single star is not a closed problem because the number of indeterminate parameters is larger than the observed ones. Five quantities are needed to construct an evolutionary stellar model: the mass $M$, the age $t$, the initial mass fraction of helium $Y$, the heavy elements $Z$ (or metallicity $\text{[Fe/H]}$) and the mixing length parameter $\Lambda$ which describes the transport of convective energy, while generally, three observable quantities only, effective temperature $T_{\text{eff}}$, luminosity $L$, and surface metallicity $\text{[Fe/H]}$ at present day are available.

The situation is better if one considers binary systems or clusters because of the additional constraints due to the common history of the stars. The first part will present the way binary systems are calibrated. In the second section, we recall the global constraints brought by asteroseismology. Sections 3 and 4 give in more details the calibrations of α Centauri (Morel et al. 2000) and ζ Herculis (Morel et al 2001).

2. Calibration of a binary system

High quality measurements of the parallax $\varpi$, with the great contribution of Hipparcos, and photometry provide accurate values of the luminosity and masses. If the orbit is known, the knowledge of the period $P$, the semi major axis $a$, the parallax, gives the sum of masses $S_M = (M_A + M_B) = a^3/\varpi^3P^2$. The mass fraction $B_M = M_B/(M_A + M_B)$ may be derived from astrometry or precise radial velocity data. Spectroscopic observations allow for the determination of the
metallicity and effective temperature. However, the more interesting systems concern stars with different masses, with not too large orbital periods, which are close systems and therefore make the observations harder.

With the reasonable hypothesis of a common origin for both components, i.e. same initial composition and age, the number of constraints is equal or larger than the unknowns. Thus it is possible to confront observations and evolved stellar models and to determine the age and the initial helium abundance which are important to understand the galactic chemical evolution. It is also possible to constrain the mixing length parameter $\Lambda$ and try to determine if the known solar value applies for stars with different masses, ages and chemical compositions. Different methods can be used.

The differential method initiated by Noels et al. (1991) for the binary system $\alpha$ Centauri is a generalization of the solar calibration technique. Let $\varphi = \{t, M_A, M_B, Y, [\text{Fe/H}]_i, \Lambda_A, \Lambda_B\}$ be the set of parameters to be determined and $\nabla_{\text{obs}} = \{T_{\text{eff}},(A,B), \log (L/L_\odot)(A,B), [\text{Fe/H}]_s(A,B), S_M, B_M\}$ be the set of observable quantities to be reached by the calibration. A couple of evolved models (corresponding to $\varphi_{\text{mod}}$) characterized by the vector of the observable quantities $\nabla_{\text{mod}}$ and the numerical derivatives of these observable quantities relatively to the initial parameters $\varphi$ are computed. An iterative Newton method is then used to estimate the correction of the parameters and compute a new solution: $\nabla_{\text{obs}} = \nabla_{\text{mod}} + (\partial \nabla_{\text{mod}}/\partial \varphi)(\varphi - \varphi_{\text{mod}})$.

The $\chi^2$ functional minimization method consists in minimizing the sum of the square of the differences between theoretical vector of observable quantities and their observed values weighted by the observational errors $\sigma$ taking into account the mass constraints (see Lastennet et al. 1999).

$$\chi^2 = \sum_{i=A,B} \left( \frac{T_{\text{eff}}(i)_{\text{mod}} - T_{\text{eff}}(i)}{\sigma(T_{\text{eff}}(i))} \right)^2 + \left( \frac{\log (L/L_\odot)(i)_{\text{mod}} - \log (L/L_\odot)(i)}{\sigma(\log (L/L_\odot)(i))} \right)^2$$

$$+ \left( \frac{[\text{Fe/H]}(i)_{\text{mod}} - [\text{Fe/H}](i)}{\sigma([\text{Fe/H}](i))} \right)^2$$

$$M_A + M_B = S_M \pm \Delta S_M ; \quad M_B/M_A + M_B = B_M \pm \Delta B_M$$

3. Asteroseismic constraints

Asteroseismology is a powerful tool to study the internal properties of the stars. The solar-like stars are good candidates because, as in the Sun, oscillations of such stars may be stochastically excited by the convection. Indeed ground observations show some indication of the presence of such oscillations and different space missions (COROT, MONS, Eddington) are planned to look for these oscillations. For binary systems where one star is a solar-like star, global quantities extracted from the acoustic spectrum will bring additional constraints to the calibration.

The oscillations are acoustic waves ($p$-modes) in spherical geometry characterized, when there is no rotation, by their degree $\ell$ inversely proportional to horizontal wavelength, radial order $n$ and frequency $\nu_{n,\ell}$. In the high frequency
range, the frequencies are regularly distributed. The mean large separation \( \Delta \nu \), proportional to the dynamical frequency of the star \( \Omega_g = \sqrt{GM_*/R_0^3} \) is given by:

\[
\nu_{n, \ell} \sim \nu_{0 \ell} + \Delta \nu (n + \frac{\ell}{2} - n_0)
\]

(2)

The knowledge of this mean large separation strongly constrains the radius or the mass of the star independently of the chemical composition.

More difficult to extract from observations, the small mean difference, \( \overline{\Delta \nu} \), defined by \( \nu_{n+1, \ell} - \nu_{n, \ell+2} \sim \overline{\Delta \nu}_{\ell+2} + S_\ell (n - n_0) \), depends on the properties of the stellar core and is an indicator of the age at a given chemical composition.

4. Calibration of \( \alpha \) Centauri binary system

As the Sun’s nearest stellar neighbors, the two members of the binary \( \alpha \) Centauri A & B provide the most accurate potentiality of testing stellar physics in slightly different conditions from solar ones and then deserve undivided attention for the modeling and oscillation frequencies calculations. A recent determination of the orbit by Pourbaix et al. (1999) leads to an increase of the mass of 5.5% relatively to those used previously (see Guenther & Demarque 2000). This increase is mainly due to the 1.5% disparity between the revised Hipparcos parallax of Söderhjelm (2000) and the orbital parallaxes determination of Pourbaix.

Seismic observations, which were up to now controversial, seemed to indicate the presence of acoustic oscillations in the power spectrum. Kjeldsen et al. (1999) found tentative evidence for \( p \)-mode oscillations in \( \alpha \) Cen. A. Very recently Bouchy & Carrier (2001) have detected such oscillations.

Morel et al. (2000) computed detailed evolutionary models of the binary \( \alpha \) Centauri, using the recently determined masses and including pre main-sequence evolution, updated physics with microscopic diffusion and either Böhm-Vitense's mixing length theory, (MLT\(_{BV}\)) or Canuto & Mazzetti (1992) theory (MLT\(_{CM}\)) to describe convection. The effective temperatures, surface gravities and metallicities, have been revisited using published detailed spectroscopic data. The \( \chi^2 \) minimization has selected the best fits taking into account the accuracy of the observational constraints.

4.1. Results of the calibration

A calibration with the constraints of Guenther & Demarque (2000) is computed for comparison (\( A_{GP} \)). Model \( A_{PV} \) includes core overshoot. The most striking fact in the results given in Table 1 are the small values obtained for the ages which are notably smaller than previous estimates except those of Pourbaix et al. The difference with the recent models of Guenther & Demarque is mainly due to the mass discrepancies resulting from the small differences in distances.

The determination of the age appears to be more sensitive to the mass differences than to the basic observational atmospheric constraints namely, effective temperature, gravity – or luminosity – and metallicity. Thus, none really satisfactory criterion allows to discriminate among sets of different modeling parameters which all generate models which fit the observational constraints based on H-R diagram analysis and metallicities. The Böhm-Vitense’s mixing length
parameters decreases with decreasing age. For models computed with MLT<sub>CM</sub> convection theory, the mixing length parameters are both close to unity and to the convection parameter of the solar model calibrated with the same physics. This indicates that the Canuto & Mazitelli’s theory may support the assumption of a universal convection parameter.

As a conclusion, even for α Cen., the best known binary system, the models are not strongly enough constrained by the available present day observations unless additional information concerning the internal structure can be used. Asteroseismology is now in position to bring such information.

4.2. Seismic analysis of α Cen. A

The large and small mean separations of α Cen. A oscillations are presented in Table 1. As expected they decrease with the mass. At given mass, the variation of the large mean separation is explained by the radius difference, while the dispersion of the values of small separations are mainly due to differences in central hydrogen, hence in age.

<table>
<thead>
<tr>
<th>t (Myr)</th>
<th>Y&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Z/X&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Λ (μHz)</th>
<th>ν&lt;sub&gt;00&lt;/sub&gt; (μHz)</th>
<th>Δν&lt;sub&gt;0&lt;/sub&gt; (μHz)</th>
<th>Διν&lt;sub&gt;0.2&lt;/sub&gt; (μHz)</th>
<th>Διν&lt;sub&gt;1.3&lt;/sub&gt; (μHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&lt;sub&gt;BV&lt;/sub&gt;</td>
<td>2710</td>
<td>0.284</td>
<td>0.044</td>
<td>1.53</td>
<td>2391.0</td>
<td>108.1</td>
<td>8.9</td>
</tr>
<tr>
<td>A&lt;sub&gt;ov&lt;/sub&gt;</td>
<td>3530</td>
<td>0.279</td>
<td>0.045</td>
<td>1.64</td>
<td>2365.9</td>
<td>106.8</td>
<td>9.1</td>
</tr>
<tr>
<td>A&lt;sub&gt;CM&lt;/sub&gt;</td>
<td>4086</td>
<td>0.271</td>
<td>0.045</td>
<td>0.96</td>
<td>2397.3</td>
<td>108.1</td>
<td>7.5</td>
</tr>
<tr>
<td>A&lt;sub&gt;GD&lt;/sub&gt;</td>
<td>5640</td>
<td>0.3</td>
<td>0.048</td>
<td>1.86</td>
<td>2256.9</td>
<td>101.7</td>
<td>4.4</td>
</tr>
</tbody>
</table>

The different estimations for the large and small spacings Δν<sub>0</sub> and Διν<sub>0.2</sub> by Kjeldsen et al. are not able to discriminate between all the models. The very recent results of Bouchy & Carrier Δν<sub>0</sub> ~ 103.8 μHz are intermediate values between those of models with old and new masses.

5. Calibration of ζ Herculis binary system

ζ Herculis is a well known bright visual and single lined spectroscopic binary system of naked-eye brightness. Lebreton et al. (1993) tried to determine both age and chemical composition of the system, modeling the two components simultaneously, but they did not succeed to model consistently the secondary. Based on precise spectroscopic analysis, Chmielewski et al. (1995) modeled both components and derived an age of 4.0 ± 0.4 Gyr and masses of the components respectively equal to 1.3<sub>M</sub> and 0.9<sub>M</sub>. Since 1995, the Hipparcos’s parallax of ζ Her have been recently improved by Söderhjelm.

Morel et al (2001) have reestimated the effective temperature, surface gravity and metallicity of ζ Her A, using published spectroscopic data and new photometric data of the TYCHO catalogue. The luminosity of ζ Her A and the effective temperature and luminosity of ζ Her B are calculated according to TYCHO and Hipparcos photometric data and Bessel calibrations. The sum of masses are estimated using updated astrometric relative orbit and parallax
determinations. The mass fraction is derived from spectroscopic and astrometric orbits. The constraint on surface metallicity of ζ Her B is not available.

5.1. Results

Evolutionary models of ζ Herculis which do not include microscopic diffusion have been computed (Table 2). The calibration leads to an age of ~ 3400 Myr. As a sub-giant, in the HR diagram ζ Her A is located beyond the main-sequence in the “Hertzsprung gap”, where the star loci move rapidly. So, all other modeling parameters fixed, the locus of ζ Her A spends only a couple of ten Myr to cross over the error box. This explains why the error bars for the age are so artificially small. The calibration is discussed in Morel et al (2001). The mixing length parameters of ζ Her A & B, are of the order of unity as expected with the Canuto & Mazzitelli, convection theory.

<table>
<thead>
<tr>
<th></th>
<th>(m/M_\odot)</th>
<th>(Y_1)</th>
<th>(\Lambda)</th>
<th>(T_{\text{eff}}) (K)</th>
<th>(L/L_\odot)</th>
<th>(X_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζHerA</td>
<td>1.45 ± 0.01</td>
<td>0.243 ± 0.002</td>
<td>0.92 ± 0.05</td>
<td>5813</td>
<td>6.70</td>
<td>0.737</td>
</tr>
<tr>
<td>ζHerB</td>
<td>0.98 ± 0.02</td>
<td>0.243 ± 0.002</td>
<td>0.90 ± 0.10</td>
<td>5413</td>
<td>0.65</td>
<td>0.737</td>
</tr>
</tbody>
</table>

Fixing the masses, \(m_A = 1.50\,M_\odot\) and \(m_B = 1.00\,M_\odot\), it was not possible to obtain simultaneous satisfactory adjustments, within the error boxes, for both components. Realistic solutions are found with \(m_A = 1.45\,M_\odot\). That may indicate that the suspected duplicity of ζ Her A is perhaps real. In that case, the mass of the unseen component, ζ Her Aa, would be of the order of 0.05\,M_\odot, a value significantly smaller than previously expected corresponding to a brown dwarf even a giant planet.

5.2. Seismic analysis of ζ Her A

Seismic observations from Martić et al. (2000), made with the ELODIE echelle spectrograph on the 1.93m telescope at OHP, show a narrow excess of power in the power spectrum around maximum peak at 0.675 \(\mu\)Hz. An average large separation is estimated from echelle diagram of the order of 43.0 \(\mu\)Hz.

The sub-giant star ζ Her A is an evolved star which has burnt all the hydrogen core and presents a radiative core and a convective envelop which may stochastically excite oscillations. The convective core which is present during the main sequence evolution has disappeared but a zone of varying chemical gradient remains between the outer edge of the initial convective core and the center. This gives a rapid variation of sound speed and a large value of the maximum of Brunt-Väisälä frequency which greatly complicates the oscillation spectrum.

The large frequency differences characterizing the \(p\)-mode oscillations according to Equation 2 are given in \(\mu\)Hz by:
\[
\begin{align*}
l = 0 & \quad \Delta \nu = 40.7; \\
l = 1 & \quad \Delta \nu_1 = 38.8; \\
l = 2 & \quad \Delta \nu_2 = 40.8; \\
l = 3 & \quad \Delta \nu_3 = 40.7
\end{align*}
\]

These values are to be compared to the value around 43\(\mu\)Hz given by the observations of Martić et al. (2000).
6. Conclusion

Binary systems are good candidates to test stellar evolution to derive the galactic chemical evolution. However models are still not enough constrained by present day observations and asteroseismology appears to be a new tool to constrain the calibration. Small variations of mixing length parameter are obtained and it is reasonable to adopt the solar value for modeling solar-type stars. For models computed with MLT$_{CM}$ convection theory, the mixing length parameters are both close to unity and to the convection parameter of the solar model calibrated with the same physics, which indicates that the convection theory of Canuto & Mazitelli’s may support the assumption of a universal convection parameter.

For the future, an improvement of the accuracy of observational data is needed. GAIA will provide measurements of global stellar quantities, covering a large variety of independent parameters. On the other hand, the development of asteroseismic observations (COROT, MONS, Eddington,..) will allow to put stronger constraints on the models. Finally improvements in stellar physics theory particularly diffusion, convection, turbulence are also needed.

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