Determination of Fundamental Parameters

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Abstract. The location of a star in the physical ($\log T_{\text{eff}}, M_{\text{bol}}$) diagram needs first an accurate determination of the abscissa and of the ordinate, already not a trivial task. The interpretation of this location in terms of internal structure theory, is connected also to the initial uniform chemical composition of the object, to its mass, and to its age. We discuss here the determination of $T_{\text{eff}}$ and $M_{\text{bol}}$ and more briefly of some other parameters relevant to the HR diagram.

1. Introduction

At its birth a star has an initial mass, an almost homogeneous chemical composition, and an initial angular momentum. These initial parameters control the subsequent evolution of the object, age being an obvious fourth parameter. We shall not consider here the effects of angular momentum, which has a small incidence, if the initial momentum is not too large. At each phase of its evolution the star has two major fundamental parameters: its total radiative power output $L$ and its radius $R$. The traditional HR diagram uses $\log(L)$ as ordinate, with often the bolometric luminosity of the Sun $L_{\odot} = 3.846 \times 10^{26}$ W, taken as unit, and a combination of $L$ and $R$ as abscissa, called the effective temperature $T_{\text{eff}}$ of the star, defined in section 2. Note that the ordinate is also often expressed as a magnitude, a linear combination of $\log(L)$:

$$M_{\text{bol}} = -2.5 \log(L/L_0)$$

where $L_0$ is an arbitrary power $L_0 = 3.055 \times 10^{28}$ W (Andersen 1996). This choice, recommended by Commissions 25 and 36 of the IAU, makes the absolute bolometric magnitude of the Sun equals to 4.750. In this talk, I shall mostly concentrate on the parameters of the HR diagram, which are chemical composition, effective temperature and bolometric luminosity. The determination of stellar masses is a subject which would deserve a special talk. It is still a big worry that so few stellar masses are known with a decent accuracy, but the development of interferometry with new instruments opens hope for the future. The next section deals with the determination of the effective temperature and of the bolometric luminosity. The third section considers the determination of the chemical composition, which is multi-parametric, but is often reduced, for the HR diagram, to the concept of helium content and “metallicity”, in first approximation. The last section gives the conclusions of this review.
2. Effective Temperature and Bolometric Luminosity

Rightly, effective temperature $T_{\text{eff}}$, the abscissa of the theoretical HR diagram, is considered as the most critical fundamental parameter. It is the dominant parameter in shaping the spectrum of a star, controlling colour and the line spectrum. The spectral classification from O, B, ..., to M stars, is a decreasing effective temperature sequence, from 40 000 K to about 3000 K. By definition the effective temperature $T_{\text{eff}}$, is defined by equation (1):

$$F = \sigma T_{\text{eff}}^4$$

in which $\sigma$ is the Stefan-Boltzmann constant, and $F$ is the bolometric flux in the atmosphere of the star. Chemical composition is also a very fundamental parameter, but, historically its role has been recognized 50 years later, when Chamberlain & Aller (1951) discovered that stars can have a chemical composition quite different from that of the Sun. Bolometric luminosity $L$, the ordinate in the theoretical HR diagram, characterizes the total power radiated by a star. Traditionally, both effective temperature and bolometric luminosity are plotted on a logarithmic scale in the HR diagram, indeed quite necessary for the bolometric luminosity, which varies by 8 orders of magnitudes, or 20 astronomical magnitudes, from white dwarfs to supergiants.

2.1. Effective Temperature and Bolometric Luminosity by the Direct Method

The principle of the direct method is quite simple. If $L$ is the bolometric luminosity of a star, $R$ the radius and $d$ the distance of the star, $F$ the bolometric flux in the atmosphere of the star and $f$ the bolometric flux observed on earth from the star, $L$ can be expressed as (with no interstellar absorption):

$$L = 4\pi R^2 F = 4\pi d^2 f$$

From these two expressions one gets:

$$F = \left(\frac{d}{R}\right)^2 f = \left(\frac{2}{\phi}\right)^2 f$$

where $\phi$ is the angular diameter of the star, seen from Earth. Relation (1) allows to express $F$ as the effective temperature of the star. If $f$ is measured by photometry, and $\phi$ is measured by interferometry, the effective temperature is given by (1) and (3), and the bolometric luminosity by (2), provided the distance of the star is known. So the star is completely located in the HR diagram. Actually, the application is not as clean, as it looks. First, angular diameter are not measured directly, but are derived from "fringe visibility" curves. The transformation involves the knowledge of the limb-darkening of the stellar disk. Most often, this limb-darkening is borrowed from stellar atmosphere computations, so the method is somewhat dependent on stellar atmosphere theory. Only in exceptional cases (Arcturus, Quirrenbach et al. 1996) the limb darkening can be also derived from the interferometric measurements. The second limitation is the still small number of stars for which the angular diameter can be measured with an accuracy better than 2 per cent, inducing an error bar less than
1 per cent on the effective temperature. Recent lists count about 25 angular
diameters obtained with this accuracy, with no dwarf cooler than the Sun. The
calibration of secondary methods versus the direct method rely on a very small
sample dominated by population I giants or subgiants (see Di Benedetto 1998).

![Graph showing calibration of $T_{eff}$ versus $V-K$ for dwarfs according to Alonso (1996b). Metallicity as parameter. The curves are for $[\text{Fe/H}]$ 0., -0.65, -1.5, -2.5 respectively, from the full line to the dash-dot line.]

**2.2. Effective Temperatures from the Infrared Flux Method (IRFM)**

Because of the meager number of effective temperatures obtained by the di-
rect method, other methods have been widely used. Among them, the so-called
IRFM, proposed by Blackwell & Shallis in 1977, but redescribed in Blackwell &
Linas-Gray (1994, 1998), is often considered as the second best method. Equa-
tion (2) is true not only for the bolometric flux, but for any flux, monochromatic
or in any photometric band. Let us write it for a photometric band $i$:

$$F_i = \left( \frac{d}{R} \right)^2 f_i$$  \hspace{1cm} (4)

From (3) and (4) one gets:

$$\frac{F_i}{F} = \frac{f_i}{f}$$  \hspace{1cm} (5)

The right side is observed and the left side is a peculiar colour index, which has
no name, one colour is at will, and the other is the widest band: the bolometric
flux. If stellar model atmospheres were perfectly reliable, for a given gravity and chemical composition, only one effective temperature would give a value equal to the observable of the right side, and this unique effective temperature would be the effective temperature of the star. In order to achieve a good accuracy, it is possible to play with the free choice of the band $i$. The choice of the IRFM is to have $f_i$ in the infrared, where the flux has a slow dependence upon $T_{\text{eff}}$. It is well known that in the Rayleigh-Jeans tail, $f_i$ is just proportional to $T_{\text{eff}}^4$, whereas the denominator varies in $T_{\text{eff}}^4$. So any relative error, observational or in model atmosphere prediction is divided by $3$. The weak point is the dependence not so much upon temperature but upon gravity and chemical composition, which must be known too, to get the ratio $F_i/F$. In spite of that, the IRFM can be tested when the direct method can be applied (see Van’t Veer & Mégessier 1996), and performs fairly well. Another consistency test is to use not one, but several IR bands (for example J, H, K), and check if the same effective temperature is obtained (Alonso 1996a). Of particular importance is the extensive work of Alonso & al. (1996b, 1999), which supplies the only available calibration of many photometric indices, for a wide range of metallicities, based on measurements of several hundreds of stars. Fig. 1 shows the calibration of the index $V - K$ for dwarfs, with metallicity as parameter.

![Figure 2](image.png)

**Figure 2.** Left: Comparison of Alonso et al. 1996b, Blackwell & Linas-Gray 1994, 1998 and Bessell & al. 1998 (BCP98) calibrations, for dwarfs of solar metallicity. Right: comparison of Alonso (IRFM) and Di Benedetto (ESB) results for dwarfs of solar metallicity.

Other authors have published IRFM, calibrations, in particular Blackwell & Linas-Gray 1994, 1998. The 1994 calibration is in agreement with Alonso et al., whereas the 1998 calibration is in disagreement with it, and with most other calibrations (Fig.2). Alonso et al. calibrations allow to study the effect of gravity as well as metallicity on the $T_{\text{eff}}, V - K$ relation. Fig. 3 shows that there is little gravity effect at $[\text{Fe/H}]=0$, but more effect at $[\text{Fe/H}]=-2$. 

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2.3. **Empirical Surface Brightness method (ESB)**

This method is somewhat similar to the IRFM, but not identical. It tries to be less dependent on model atmosphere theory. It relies on the principle that the surface brightness at a given wavelength is a function (i) of the effective temperature (ii) of the chemical composition which control the opacity (iii) the gravity which acts for example on the Balmer jump and on the blanketing. Instead of *computing* the surface brightness as a function of these parameters, one calibrates its value, using the direct method, as the surface brightness $F_V$ for example, can be derived from the visual apparent flux $f_V$ by relation (5) if the angular diameter is known. Di Benedetto (1998) has applied the method in deriving effective temperatures as a function of the index $V - K$, from stars having more or less a solar chemical composition, for giants (easy) and for dwarfs (more difficult by the lack of dwarfs with well measured angular diameter). Fig. 3 shows a comparison of the $T_{\text{eff}}$ versus $V - K$ from the IRFM method (Alonso et al.) and from the ESB (Di Benedetto). The agreement is satisfactory for stars of solar metallicity: there is no comparison for metal-poor stars, because there is no calibration of the ESB by the direct method for those. Fig. 4 compares Di Benedetto calibration with Gratton et al. (1996) calibration, which is partly based on detailed analysis work (see subsection 2.7).

2.4. **A Generalisation of the Photometric Methods**

A useful method consists in measuring a larger set of fluxes, from the visible or the UV up to the IR. Model atmospheres predict the fluxes $F_i$ in the atmosphere, and observations can supply the fluxes $f_i$ at the Earth. The ratios between the fluxes are conserved, when interstellar reddening is negligible. These ratios primarily depend upon effective temperature, so a model which reproduce all the ratios, with the proper gravity and chemical composition, is likely to have the right effective temperature. The IRFM is a restriction of the general method to the use of only two fluxes: the bolometric flux and a flux in an IR band. But more bands introduces a useful redundancy.

2.5. **The $T_{\text{eff}} V - K$ Calibration from Theoretical Model Atmosphere**

It is interesting to study also these dependences from a theoretical point of view, using grids of model atmospheres. We have done that using a grid of Kurucz model atmospheres computed by F. Castelli, with no overshoot. The theoretical results support the empirical results of Alonso et al. For example the left panel of Fig. 5 shows that the predicted metallicity effects are very similar to the empirical results of Fig. 1. The right panel shows the same diagram for giants instead of dwarfs. Fig. 6 presents the effect of gravity at two different metallicities, to be compared with Fig. 3.

2.6. **Effective Temperature from Balmer Lines**

We noted in the preceding sections than photometric methods apply directly only to stars with a negligible interstellar reddening. This quantity is difficult to obtain accurately for distant stars, especially near the galactic plane. Therefore it is often useful to have a method independent of interstellar reddening. It is the case when lines can be used, as the reddening is negligible over the small
Figure 3. Left: Comparison dwarf-giants at solar metallicity: full line Alonso-dwarfs, dotted line Alonso giants. Right: Same, but at metallicity $[\text{Fe/H}]=-2.0$

Figure 4. Comparison Gratton et al. 1996 with Di Benedetto 1998. The second reference is pure ESB; the first one involves detailed analyses of stars in addition to IRFM.
Figure 5. Theoretical side-effect of metallicity on the $T_{\text{eff}} - V - K$ calibration. The left panel shows the influence of metallicity for dwarfs. Note the similarity with Fig. 1. The right panel shows the same influence but for giants ($\log(g) = 1.0$).

Figure 6. Theoretical side-effect of gravity on the $T_{\text{eff}} V - K$ calibration. Left panel is at solar metallicity; right panel is at metallicity [Fe/H] = -1.5, typical of pop. II. The effects are to be compared to the empirical results of fig.3. The effect of gravity is less at [Fe/H] = 0 than in metal-poor stars in both cases, but there is a slight offset theory/empirical at [Fe/H] = -1.5.
portion of the spectrum occupied by an absorption line and its nearby continuum or pseudocontinuum. Of particular interest are the Balmer lines because they are strong and have a fast variation with effective temperature for late F, G and early K stars. The method has been discussed by Fuhrmann, Axer & Gehren (1994), by Van't Veer-Menneret & Mégevand (1996), and by Van't Veer-Menneret & Katz (1999). In short, the method is efficient, but needs some caution, because the structure of the wings of the Balmer lines are sensitive to the structure of the convective zone. Hα is more robust than the other Balmer lines thanks to its self-resonance broadening. It is therefore the recommended line to use. For Hβ it is necessary to adapt the value of the convection parameter l/H (mixing length over pressure scale height) in Kurucz models, to recover the true effective temperature. A value of l/H of 0.5 instead of the standard value of 1.25, is needed. Fuhrmann has shown that this choice holds for metal-poor stars as well.

2.7. Excitation Temperature from metallic lines

Particularly in abundance analysis, it is required that the selected effective temperature leads to an abundance independent of the excitation potential of the line used. If it is not the case the choice of the effective temperature is reconsidered. However, if departures from local thermodynamical equilibrium exist, there is no guarantee that the departure coefficient bₐ are the same for all levels. The concept of excitation temperature, as criterion of effective temperature, becomes fragile in that case.

3. Determination of the initial chemical composition

It is currently assumed that the chemical composition of a stellar atmosphere is still the original one. However, gravitational settling phenomena are predicted to occur (Michaud & Proffitt 1992), and it is now generally accepted that helium has settled in Sun from a mass fraction of 0.28 to 0.25 after 4.5 Gyr (see Richard et al. 1996). These effects may be more important in the pop. If stars, for which more time is available (Lebreton et al. 1999). We leave this point here, but it must not be forgotten. Chemical composition fixes the location of a star in the HR diagram through the opacity of the stellar matter and the energy generation rates. It is no more sufficient to describe this composition with two independent parameters: Y helium fraction and Z "metallicity". The ratio of the so-called α-elements to the iron group metals is different in the Sun and in pop. If by at least a factor 2. Grids of internal structure models have already included this fact, but Nissen & Schuster (1997) have shown that this ratio is a supplementary parameter, not a mere function of Z.

3.1. Atmospheric Gravity

Abundance analyses require the knowledge of gravity, and in internal structure gravity is a signature of the degree of evolution of a star. Unevolved stars have a remarkably stable value of their atmospheric gravity, from log(g) = 4.0 (g in CGS) for B stars, to 4.6 for M dwarfs. By contrast, log(g) can be as low as 0. in a cool supergiant, or 1.0 in a yellow giant. At very low gravity the sphericity of the atmosphere must be taken into account, and the new OSMARCS grid,
does it. There are several usable gravity criteria. One is the state of ionisation of elements present by two stages of ionisation in the spectrum, often Fe I and Fe II. The abundance ratio of an ionised level to a neutral level is controlled by the value of the gravity and of the temperature. If the temperature has been already determined, the gravity is the remaining unknown. In the Sun there is empirical evidence that the method works well. But theoretical work (Thévenin & Idiart 1999) predicts that the LTE ionisation ratio is incorrect for metal-poor stars. This is in line with the empirical evidence that gravities derived from Hipparcos data for metal-poor stars tend to be larger than those derived from ionisation equilibria (Nissen et al. 1997). See Ivans et al. 2001 for giants, as well.

3.2. Methods for Chemical Composition Determinations

The classical approach consists in assuming LTE, and using the true effective temperature derived from the direct method or the IRFM as a first approximation only, and iterating, by imposing that the abundance of iron, for example, be the same from lines of all excitation potentials (revising so the value of \( T_{\text{eff}} \)), from lines of all equiv. widths (fixing so the microturbulent velocity parameter), and requesting that abundances derived from neutral and ionised lines be the same (fixing the value of the gravity). All of that being performed with 1-D radiative-convective constant flux models. We already noted that taking into account departures from LTE is a first possible improvement. But the major improvement, demonstrated so far mainly on the solar case, is to go to 3-D radiative hydrodynamical codes (Stein & Nordlund 1998 ; Ludwig, Freytag & Steffen 1999) which bypass the mixing length algorithm, and supply a full description of the atmospheric velocity field, removing the crude classical treatment using the dichotomy micro-macro “turbulence”, taken most of the time constant with depth. Limitations still existing in 1-D models are hopefully going to vanish when grids of 3-D models will become available. Three ad hoc parameters will then disappear: mixing length, macroturbulence and microturbulence.

4. Conclusions

The strong points that the author would like to be remembered are as follows:

- For placing a star in the HR diagram, it is highly recommended to determine \( T_{\text{eff}} \) by a method linked as directly as possible to the definition itself of \( T_{\text{eff}} \). In order of decreasing preference, come (i) the direct method based on the measurement of the angular diameter and of the bolometric flux (ii) the IRFM or even better the IRFM complemented by the measurement of the flux at several wavelengths covering a wide spectral range (iii) calibrations of the index \( V - K \), that of Alonso & al. being the only one covering the full range of metallicities present in the Galaxy, for dwarfd and giants (iv) calibration of other indices

- There is a dramatic shortage of \( T_{\text{eff}} \) determined by the direct method. It is hoped that a VLTI Large Programme will be dedicated to the determination of many more angular diameters, including cool dwarfs and metal-poor stars, completely absent so far in the direct method.
• Accurate stellar masses from binary orbits remain much too rare. They are vital data to check the correctness of internal structure computations. A major effort is required in this field, and is possible with the development of interferometric techniques.

• Chemical composition: helium content by mass fraction $Y$ and “metallicity” are no more sufficient to describe the chemical composition of the stellar matter. At least a separate abundance for the so-called $\alpha$-elements is requested. Abundances crucially depend upon the validity of the model atmospheres used. A major step is the fast progress of radiative hydrodynamic 3-D codes, removing several \emph{ad hoc} approximations, as the mixing length algorithm, and the reduction of the atmospheric velocity field to macro and micro turbulence.

References


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